

Rules of the Road  
Using adaptations of Nagel-Schreckenberg Cellular Automaton Traffic Simulations to Evaluate  
Passing Rules for Multi-Lane Freeways

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## Abstract

We present three conventions for regulating the behavior of automobiles on a freeway as potential alternatives to the “keep-right-except-to-pass” rule that is currently established in the United States and in many other nations. These conventions are tested against the established convention within a freeway traffic flow model we have designed based on Nagel-Schreckenberg models of traffic flow. We assume cars remain on the freeway and all drivers attempt to adhere to the rules of the road, but sometimes make mistakes that can lead to accidents. We also assume that an intelligent system controlling the actions of all cars on the roadway would not make those same mistakes.

The first of the three alternatives proposed presents drivers with no rules governing their lane-to-lane movement, allowing them to move to any lane that will let them go faster, and to stay there for exactly so long as that remains the case. This convention results in the highest flow rate of cars through the system, but also results in a higher accident rate for a typical three-lane freeway than does the “keep-right-except-to-pass” rule. However, when the number of lanes is increased, the accident rate drops significantly.

The second of the three alternatives forbids drivers from changing lanes at all, and results in the second highest flow rate seen, between the no rule and “keep-right-except-to-pass” systems, but results in a higher accident rate than either system for typical three-lane highways. The third model, mandating that drivers remain in the center lane of a three-lane or five-lane freeway, exhibited the lowest flow rate and highest accident rate of any of the four conventions studied.

Unfortunately, the model used was seen to be very sensitive to the rate governing accident probability and the number of lanes in a simulation. The way accidents were modeled meant that a single accident tended to promote the occurrence of many more. Additionally a software limitation in the function used to generate an initial matrix with unique car locations meant that it was impossible to create true gridlock within the model.

Based on the data from all of the models and the uncertainty in the accident data, we conclude that a no rules convention would result in greater flow rate through a typical three-lane freeway, but that no convention is safer than the “keep-right-except-to-pass” rule, supporting its status as the established rule in much of the world.

“不超车就得靠右行驶”是美国和其它许多国家的交通规则, 本文提出三种约束高速公路上汽车行为的规则, 作为“不超车就得靠右行驶”的潜在的替代规则. 本文基于 Nagel-Schreckenberg 交通流模型设计了一种高速公路交通流模型, 前用来测试了三种替代规则并与已有的规则 (不超车就得靠右行驶) 比较. 我们假设车辆待在高速公路上, 并且所有驾驶员都遵守道路规则, 但有时也会失误造成事故. 我们还假想了一种智能系统, 能够没有任何失误地控制道路上所有车辆的行为.

三个潜在的替代规则中的第一个规则是没有任何规则来控制驾驶员更换车道, 允许车辆移动到任何可以使其行驶得更快的车道上, 并且待在新的车道上直到不能再更快地行驶. 这个规则给出了车辆通过系统的车流量最大的结果, 但对于典型的三车道高速公路, 这个规则给出的事故率高于“不超车就得靠右行驶”的规则. 然而, 当车道数增加时, 事故率则会显著降低.

三个替代规则中的第二个规则禁止驾驶员换道, 该规则给出了第二大的交通流量的结果, 在无规则和“不超车就得靠右行驶”规则之间, 但对于三车道的高速公路, 其给出了相对其它规则最高的事故率. 第三个规则要求驾驶员待在三车道或五车道公路的中间车道上, 相对于所研究的其它四种规则, 其表示出最低的交通流量和最高的事故率.

可惜的是, 本文的模型看起来对模拟中控制事故概率参数和车道的数量非常灵敏. 本文中模拟事故的方式意味着: 单个事故倾向于造成更多的事故. 另外, 本文使用了软件中的函数来生成包涵一一对应的车辆位置的初始矩阵, 由于这个函数的限制, 使得模型不太可能模拟一个真实的完全阻塞的网格.

基于从所有模型中得到的数据和事故数据中的不确定性, 我们得到这样一个结论: 在典型的三车道的高速公路上, 无规则给出了最大的交通流量, 但是没有规则比“不超车就得靠右行驶”更安全, 这印证了“不超车就得靠右行驶”在世界的许多地方都是既定规则.

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## 1 Introduction / 引言

Due to the high speed at which freeway traffic travels, transport regulatory bodies impose rules on where in a multi-lane freeway drivers should be at any given time, with the purpose of maximizing the safety and efficiency of freeway travel. One such rule, which is commonly used in countries where automobiles drive on the right hand side of roads, requires drivers to stay in the right-most lane of the freeway unless they are attempting to pass another car. Cars that attempt to pass are required to safely move to the left, pass the car or cars they wish to pass, and then return to their original lane. An analogous rule can be used in countries such as the United Kingdom and Australia where automobiles are driven on the left hand side of the road.

In this paper we attempt to answer the question of whether or not the “stay right except to pass” rule is a safe and efficient method for controlling freeway traffic, and compare it to three other rules that could be used to regulate traffic. The three alternate rules have been called: “No Rules”, “No Lane Changes”, and “Middle Lane Rule”; and allow for free lane movement, no lane movement and randomized lanes, and free passing with mandatory return to the middle lane, respectively.

The three rules were all implemented into the same cellular automaton traffic control model, based on a model outlined by Nagel and Schreckenberg[1], under a variety of traffic conditions, with both heavy and light car density, fast and slow speed limits, and three different probabilities governing accident rates. For each rule and traffic condition, data was gathered on accident count and flow rate for 100 cell regions of 2 through 6 lane freeways across 250 time steps. The results of these simulations are presented in Section 4.2, followed by a discussion of the limitations and assumptions of the model in Sections 4.3 and 5.

## 2 Previous Work / 前人工作

### 2.1 Different Approaches to the Problem / 交通问题的不同方法

Modeling of traffic flow is by no means a new problem, and many models

由于高速公路上车量的行驶速度很高, 出于最大限度地提高安全性和通行效率的目的, 运输监管机构通过交通规则限制多车道高速公路上任何时候司机的行为. 比如有这样一条在汽车靠右行驶的国家普遍使用的交通规则: 除非司机正试图超越前方的一辆车, 否则必需在公路的最右车道上行驶. 想要超车的车辆必需安全的换到左边, 超过一个或多个要超过的车辆后, 再回到原来所在的车道上. 英国和澳大利亚等一些国家, 使用了一个类似的规则: 汽车靠路的左侧行驶.

本文尝试回答这样一个问题: “不超车就得靠右行驶”的规则是否是一个安全和有效的公路交通控制方法. 其它三种潜在的规则分别称为: “无规则”, “禁止换道”和“居中行驶”; 它们分别对应着允许车量在各车道间自由换道行驶, 随机分配车道并且禁止更换车道行驶, 和可自由超车, 但超完得回到中间车道行驶.

在不同的交通条件下, 包括较高和较低的车量密度, 较大和较小的限速, 以及三种不同的事故发生率, 以上三种规则都在相同的元胞自动机交通控制模型中实施, 元胞自动机交通控制模型是基于 Nagel 和 Schreckenberg[1] 提出的一种模型. 对于每种规则和交通条件, 事故数量和交通流量的数据是在 100 个单元区域, 2 到 6 条车道上经过 250 个时间步长的统计得到的. 这些模拟的结果见第4.2小节, 关于模型的局限性和假设合理性的讨论分别见随后第4.3和第5小节.

交通流建模不是一个新问题, 目前已经有很多模型被用



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have previously been used to describe and model the flow of traffic on various road types, from cities to freeways. The Wikipedia page on “Traffic Flow”[2] alone lists a huge number of model types and parameters that have been used in the past to describe traffic flow, with both discrete and continuous methods for approaching the problem. One model by Doboszczak and Forstall[3] used partial differential equations to model traffic flow for average densities of cars rather than looking at each individual car. These sorts of models use basic equations and assumptions to build up to more complex mathematical statements. Another model by a group at MIT examined Phantom Traffic Jams[4] and watched a group of cars and observed behaviors of traffic flow, then empirically developed algorithms to display the phenomena they observed.

## 2.2 The Nagel-Schreckenberg Model / Nagel-Schreckenberg 模型

The traffic model used in this paper was derived from the previous work by Kai Nagel and Michael Schreckenberg[1], originally published in 1992, in which cars are placed into a one-dimensional array wherein each cell may either be occupied or unoccupied, and the car in each occupied cell will have an associated speed between zero and the assigned maximum speed for the system. Multiple cars may not occupy the same cell, and the model is governed by four basic operations that are taken to occur simultaneously for each iteration. These four steps are:

### 1. Acceleration

All cars that are not at the maximum velocity,  $v_{\max}$  (speed limit for the road), and that have more than  $v + 1$  empty cells in front of them, accelerate by one unit.  $v \rightarrow v + 1$

### 2. Slowing Down for Safety

If a car has  $d$  empty cells ahead of it and its velocity after step one is greater than  $d$ , then it reduces its velocity to  $d$ .  $v \rightarrow \min\{d, v\}$

### 3. Randomization

For cars with a speed greater than 0, the velocity is reduced by one unit with

来描述和模拟不同类型道路 (从城市道路到高速公路) 的交通流. 维基百科上“交通流”[2] 的页面单独列出了大量已被用于描述交通流的模型和参数, 这些交通流的模型既有离散模型, 也有连续模型. 其中一种由 Doboszczak 和 Forstall[3] 提出的模型用偏微分方程组来模拟基于平均密度的交通流量, 而不是着眼于每个汽车. 这种类型的模型用基本的方程组和假设来取代更复杂的数学描述. 另一个模型由麻省理工学院的一个研究小组通过研究幽灵堵塞 [4] 现象建立, 他们通过观察一组汽车并观测交通流量的行为, 然后经验性的建立了规则来描述出他们观察到的现象.

本中所应用的模型是源于 Kai Nagel 和 Michael Schreckenberg [1] 在 1992 年发表的原创性工作, 在他们的模型中, 车辆被放置在一维的阵列中, 阵列中的每一个元胞要么被车辆占据 (非空), 要么为空, 每个非空元胞中的车辆都配有相应的速度, 速度被限定在零到系统的最大限速之间. 通常同一个元胞是不能同时被多个车量占据, 这个模型由每一个循环都同时执行四种基本的操作支配. 这四种基本操作分别为:

### 1. 加速

对于所有速度未达到最大值  $v_{\max}$  (道路的最大限速) 的车辆, 并且如果车辆前方有超过  $v + 1$  个空元胞, 则其速度增加一个单位.  $v \rightarrow v + 1$

### 2. 减速以防止碰撞

如果一辆车的前方有  $d$  个空元胞, 且该车在下一时间步的速度超  $d$ , 则该车必需减速至  $d$ .  $v \rightarrow \min\{d, v\}$

### 3. 随机化



probability  $p$ .  $v \rightarrow v - 1$

#### 4. Driving

After steps 1-3 the each car is assigned a new position  $x$  based on its current velocity  $v$ .  $x \rightarrow x + v$

Each car acts independently in the sense that the driver is concerned with reaching his or her own destination, but any given car's actions are also based in response to the cars around it. The model assumes that each driver wishes to drive as fast as allowable, but also that drivers wish to avoid accidents. It also introduces an element of imperfect to the control of the drivers by randomly reducing the speed of each car, though it should be noted that as cars slow down in response to the *position* of the car in front of them, rather than to its speed or expected position, this randomness will never result in the creation of accidents. It is also important to note that the model is designed for a single lane system, and so cannot be used to answer the problem addressed in this paper without modification to account for the probability of lane changes.

### 3 Model

#### 3.1 Simplifying Assumptions / 简化性的假设

- A simulation of a small number of cars in a small pre-defined region, can be taken to be representative of an entire freeway.
- No cars entered or exited the highway during each simulation, that is, the number of cars remained constant for each simulation, and therefore that the car density on the studied region remained constant for the entire simulation.
- All cars were identical, and they were separated by integer multiples of the cell length.
- Differing weather conditions can be accounted for by modifying the probability that governed accident rates.
- The reaction times of drivers to impending accidents are different for every

对于任何速度大于 0 的车辆, 其速度将以一定的概率  $p$  减小一个单位.  $v \rightarrow v - 1$

#### 4. 前进

对执行以上 1-3 步操作后, 根据当前速度  $v$  对所有车量指定一个新的位置  $x$ .  $x \rightarrow x + v$

驾驶员只关心他自己的目的地, 在这个层面上每一辆车的行为是独立的, 但是任何一辆给定的车辆的行为又取决于它对周围车辆的响应. 这个模型假定任何一位驾驶员都期望以最快的速度行驶, 但同时驾驶员又希望避免事故. 这个模型也引入了一个不完美的因素来控制驾驶员随机的降低他们所驾驶车辆的速度, 值得注意的是: 与车辆为避免与其前方车辆碰撞而强制减速类似, 这种随机的减速同样不会造成任何事故的发生. 同样需要注意的是这个模型是为单车道交通系统设计的, 因此如果不作改进来考虑变道的可能, 该模型并不能用来解决本文所需要解决的问题.

- 用少量车辆在一个预先定义好的较小的区域上的模拟, 可以被认为是一条完整高速公路的表示.
- 在每次模拟中, 没有任何车辆进入或离开高速公路, 也就是说, 对于任何一次模拟, 车辆的总数不变, 因此在整个模拟过程中, 所研究区域的车辆密度也保持不变.
- 所有车辆都是相同的, 并且它们在空间位置上相差车长的整数倍数.
- 可以通过修改事故发生的概率来考虑不同的天气状况 (对交通的影响).
- 驾驶员对即将发生事故的反应时间是因人而异的, 这里





driver and take random values.

- All accidents were taken to occur between two cars, and have the same effect on traffic in their lane. All cars return to the system after their accident is cleared.

The first assumption is necessary due to the fact that simulating large numbers of cars over a large distance is computationally expensive, and requires a complex system for measuring traffic flow, which may differ significantly along the length of the region. This assumption is one that has been previously made and discussed in a paper by Courage et.al.[5], wherein cars were taken to be members of small platoons that travelled together.

The second and third assumptions are inherent in the Nagel-Schreckenberg model on which ours is based, and is also necessary due to limited available computing power. Simulation of variable cell lengths and variable densities and car counts within each simulation would have been extremely resource intensive for the style of model used.

The fourth assumption was made due to the fact that the model does not explicitly consider how different weather conditions affect the accident rate on the freeway, despite the fact that weather conditions certainly do change driving conditions in regards to accidents a great deal. If asked to describe how each rule works in heavy rain or snowy conditions, we would address its performance under a condition that simulated high accident rates.

The fifth assumption was used to allow drivers in individual cars to prevent an accident from occurring. There was to be a fixed value for reaction time that was required in order to prevent a dangerous speed up, and each driver would randomly be assigned a reaction time to each incident. Benekohal and Treiterer present a justification for the use of random reaction times in their 1988 paper on their model, CARSIM[6]. While their model for assigning random reaction times is far more sophisticated and data based than our own, the reasons for using random reaction times remains the same.

The sixth and final assumption was another assumption that significantly reduced the amount of time required for each simulation. Though it would be possible for a car to collide with a car that had already collided with another, creating a three-car pileup, cars were assumed to never engage in a simultaneous three-car

我们取随机值.

- 任何事故都发生在两辆车之间, 并且任何一个事故对其所在车道上的交通流的影响是相同的. 发生事故的车辆在事故处理完后都将回到公路系统中 (继续前进).

第一个假设是必要的, 这是因为模拟大量的车辆在一个较长的距离上 (的行为) 在计算上的开销是非常昂贵 (主要是指费时间), 并且还需要一个复杂的, 并且还可能显著依赖区域长度的体系来衡量交通流. 这一假设在之前 Courage 等人 [5] 的论文中就被提出并被讨论过, 在他们的论文中, 车辆被看成是一小排区域上一起行驶的单元.

第二和第三个假设是作为本文基础的 Nagel-Schreckenberg 模型本身所固有的假设, 并且由于计算能力的限制, 这两个假设也是必要的. 每次模拟都对不同元胞长度, 不同车辆密度及车辆总数的进行计算, 对于这种模型, 这将会极度消耗资源.

做第四个假设是由于本文的模型并没有明确的考虑不同的天气条件对高速公路上的事故发生率的影响, 尽管天气条件的确会改变与大量事故相关的行车条件. 如果要求具体描述各种交通规则在大雨或暴雪天气中的表现, 我们将通过在某种较大的事故率条件下来测试其 (规则) 表现来解决这个问题.

第五个假设是用来允许车内驾驶员防止事故的发生. 必需存在一个固定的反应时间值来防止危险的加速, 因此将在任一步长内, 对每一个驾驶员随机的指定一个反应时间. 在 Benekohal 和 Treiterer 的 1988 年的论文里, 他们在 CARSIM 模型 [6] 中给出了一个使用随机反应时间的理由. 虽然他们对指定随机反应时间的模型和参数远比我们的复杂, 但 (我们和他们) 使用随机反应时间的原因是一样的.

第六个即最后一个假设也是另一个能有效地降低每次模拟所需的时间的假设. 虽然一辆车撞上已经和其它车辆发生碰撞的车是有可能的, 即搞一个三 P 连环碰. 但这里我们假设不存在三辆车同时发生碰撞, 并且假设车辆不会另一条车





collision, and cars were assumed not to collide with cars in other lanes. This concept specifically will be discussed in detail when discussing limitations of the model in Section 5.

### 3.2 The Model / 模型

The model we designed to compare the effectiveness of the different driving conventions used the concept of cells from the Nagel-Schreckenberg model and then added to the model to fit our needs. Our simulation of driving conditions was done by performing various operations on an  $n$  by 3 matrix, where  $n$  was the number of cars in the simulation. Each row in the matrix represented a car, where the first column's entry was the cell number of the car's location, the entry in the second column was the lane that car was located in and the third column's entry was the current speed of that car.

The largest change made from the Nagel-Schreckenberg system was the modeling of multi-lane systems, necessary for the comparison of lane switching strategies. It was made possible for multiple cars to occupy the same cell number, provided they had different lane numbers. The movement of the cars was studied on short, 100 cell regions, with a mechanism implemented to count the number of cars that left the 100 cell region and return them to the region in cell 1. A separate system counted the number of accidents that occurred in the timeframe of a simulation, and to lock down lanes behind accidents, forcing other cars to switch lane or stop and wait for the accident to clear.

本文这里的做法也很让我费解, 作者将超出第 100 个元胞区域的车辆返回到第 1 个元胞区域, 这无形在强制所有车辆都必需经过第 1 个元胞区域, 为何不使用周期性边界条件, 使 100 个元胞构成一个环道.

Accidents themselves occur due to a randomization step that was modified from the Nagel-Schreckenberg randomization step. In the original randomization step every car had fixed probability of randomly slowing down, which, as previously noted, would never cause accidents to be simulated in the system. In our model, the cars have a fixed probability of randomly speeding up, potentially causing them to rear end the car in front of them. Additionally, all the drivers will randomly respond to each potential accident, and will either succeed or fail in preventing the accident from occurring, as discussed in Section 3.1.

道上的车辆发生碰撞. 这个假设的细节将在第5中讨论模型的局限性时具体讨论.

本文的模型应用 Nagel-Schreckenberg 模型中元胞的概念并在此基础上作添加以满足我们的需求, 设计这个模型的目的是用来比较不同驾驶习惯的效力. 本文对不同行车条件的模拟, 是通过对于一个  $n \times 3$  的矩阵进行一系列的操作, 其中  $n$  是模拟中车辆数. 矩阵中的每一行表示一辆车, 其中第一列中的数字表示车所在位置的元胞标号, 第二列中的数字表示车辆所在的车道, 第三列中的数字表示车辆的当前速度.

在 Nagel-Schreckenberg 系统基础上做的最大的改进是多车道系统的建模, 这对于比较不同的换道策略是必要的. 这使得多个车辆可能具有相同的元胞标号, 如果它们的车道标号不一样. 车辆运动的研究是在只含 100 个元胞的短区域上进行的, 并包含一种机制来统计那些超出第 100 个元胞区域的车辆数, 并使这些车辆返回到第 1 个元胞区域. 一个独立的系统来统计在一次模拟中的某个时间步内事故发生的数量, 并且锁定车道上事故后方的车辆, 强制其它车辆换道或者停止以等待事故现场的清理.

事故的出现是由于随机化操作导致的, 这个随机化是由 Nagel-Schreckenberg 模型中的随机操作修改得到的. 在原始的随机化操作中每一辆车有一个固定的随机减速的概率, 前面已经提及, 这种方式永远不会在系统中模拟出交通事故. 在本文的模型中, 车辆有一个固定的随机加速的概率, 这导致它们可能会与它们前方的车辆追尾. 此外, 正如在3.1节中所述的那样, 所有驾驶员会对潜在的事故做出随机的反应, 当然这种反应可能会成功地防止事故的发生, 也有可能失败.



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Though the explicit step sequence differed for each rule within our model, the general sequence of steps in the cellular automaton traffic simulation, each of which modifies the  $n$  by 3 matrix discussed above, was as follows:

#### 1. Acceleration

All cars that are not at the maximum velocity,  $v_{\max}$  (speed limit for the road), and that have more than  $v + 1$  empty cells in front of them, accelerate by one unit.  $v \rightarrow v + 1$

#### 2. Lane Changing

If a car has  $d$  empty cells ahead of it and its velocity after step one is greater than  $d$ , and there is no adjacent car preventing lane changes as allowed by the rule system, the car will move over one lane and then repeat this step until no further productive lane movement is possible. In the Right Hand Rule system, cars can only move left in this step, while in the No Rules, and Middle Lane Rule systems the cars were free to move either left or right. In the No Lane Changes system this step is skipped.  $\text{Lane} \rightarrow \text{Lane} \pm 1$

#### 3. Slowing Down for Safety

If a car has  $d$  empty cells ahead of it and its velocity after step one is greater than  $d$ , then it reduces its velocity to  $d$ .  $v \rightarrow \min\{d, v\}$

#### 4. Randomization

For cars with a speed less than  $v_{\max}$ , the velocity is increased by one unit with probability  $p$ .  $v \rightarrow v + 1$

#### 5. Driving

After steps 1- 4 the each car is assigned a new position  $x$  based on its current velocity  $v$ .  $x = x + v$

#### 6. Exit Count and Car Cycling

If, after moving, a car is in a cell number above 100, having previously been in a cell number less than or equal to 100, the count of cars that have gone

本文的模型中, 虽然对不同的交通规则有不同的一系列操作, 但正如先前所述每一个操作都是对  $n$  行 3 列的矩阵进行修改, 本文中元胞自动机的交通模拟中一般操作如下:

#### 1. 加速

对于所有速度未达到最大值  $v_{\max}$ (道路的最大限速) 的车辆, 并且如果车辆前方有超过  $v + 1$  个空元胞, 则其速度增加一个单位.  $v \rightarrow v + 1$

#### 2. 换道

对于允许变道的交通规则, 如果一辆车的前方有  $d$  个空元胞, 经过第一个操作后速度大于  $d$ , 且附近没有车辆阻碍其变道, 那么这辆车就移到另一车道上, 然后重复这个步骤, 直到没有更好的车道可供移占. 对于靠右行驶的规则系统中, 在这一步操作中, 车辆只能往左边车道上移. 而对于无规则和居中行驶的系统, 车辆可以不受限地向左或右车道移.

#### 3. 减速以防止碰撞

如果一辆车的前方有  $d$  个空元胞, 且该车在下一时间步的速度超  $d$ , 则该车必需减速至  $d$ .  $v \rightarrow \min\{d, v\}$

#### 4. 随机化

对于任何速度小于  $v_{\max}$  的车辆, 其速度将以一定的概率  $p$  增加一个单位.  $v \rightarrow v + 1$

#### 5. 前进

对执行以上 1-4 步操作后, 根据当前速度  $v$  对所有车量指定一个新的位置  $x$ .  $x \rightarrow x + v$

#### 6. 经过出口的车辆数及车辆的循环

如果某辆车在前进后, 其元胞标号大于 100, 而在前进前, 元胞标号是小于或等于 100 的, 通过这个系统的车辆数将增加 1, 同时如果有车道标号为 1 的元胞为空,



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through the system will increase by 1, and if there is an open lane in cell 1 the car will be placed in it. If there is no open lane, the car is recycled at a later time step.  $\text{Throughput} \rightarrow \text{Throughput}, \text{Cell} \rightarrow 1$

#### 7. Check for and Respond to Accidents

If two cars are found to be in the same lane and cell, the count of accidents is increased by one, and their velocities are set to different negative numbers. As all steps other than “Acceleration” ignore cars with a negative value for speed, this will freeze the cars in their accident position for a fixed amount of time, taken to be the amount of time to clear the accident and return the cars to the road.  $\text{Accidents} \rightarrow \text{Accidents} + 1, v \rightarrow -1$  or  $-5$ .

#### 8. Return to Designated Start Lane

This step is only applied to the Right Hand Rule system and the Middle Lane rule system. For each car the model checks whether or not the car is in its designated start lane, 1 for Right Hand Rule and 2 or 3 for Middle Lane Rule, and if the car is in the wrong lane, and if there is no car adjacent to it in the direction of its start lane, the car will move one step toward its start lane. It will then repeat this step until it is no longer possible for it to move closer to its start lane.  $\text{Lane} \rightarrow \text{Lane} \pm 1$

The model was built in MATLAB as a series of scripts to perform the functions described above, with at least one script per step and multiple scripts for steps that varied in different rule systems, that could be inserted into a function in a modular fashion so as to simply create a function capable of running the model several times and averaging data for each of the four rule systems.

## 4 Results / 结果

### 4.1 Description of Parameters and Terminology / 参数及术语

All of the data presented are the result of averaging the results of ten simu-

那这么辆车将被移到这个车道标号为 1 的元胞的位置上. 如果不存在这样的车道, 这个车将在下一个时间步再回到起点.  $\text{Throughput} \rightarrow \text{Throughput}, \text{Cell} \rightarrow 1$

#### 7. 检查并处理事故

如果发现两辆车在同一个条车道且具有相同的元胞标号, 事故的计数就填加 1, 并且这两辆车的速度设为不同的两个负值. 对于具有负速度的车辆, 跳过了“加速”操作外的所有其它操作, 这将使涉事车辆在一段固定的时间内被冻结在事故发生的位置, 以模拟清理事故现场并使涉事车辆重回到道路所需的时间.  $\text{Accidents} \rightarrow \text{Accidents} + 1, v \rightarrow -1$  或  $-5$ .

#### 8. 回到最初指定的车道

本操作步只在靠右行驶和居中行驶的规则系统中执行. 对于每一辆车, 都会检查这辆车是否在其初始所指定的车道上, 1 表示靠右行驶规则, 2 或 3 则表示居中行驶规则, 如果某辆车不在其指定的车道上, 并且在朝着其指定车道方向上的侧边车道上没有车辆, 那么该车将向着其指定车道方向移动一个车道. 这个操作将被重复直到不能再靠近指定车道为止.  $\text{Lane} \rightarrow \text{Lane} \pm 1$

这个模型是在 MATLAB 环境下, 通过执行上述功能的一系列脚本建立的, 每一个时间步至少执行一个脚本, 在不同的交通规则系统中, 执行不同组合的脚本, 这些脚本可以以模块的方式插入到一个函数中, 以方便建立一个能运行多次并对四种交通规则系统中的每种的结果数据求平均的函数.

所有展示的数据都是本文对每种规则的模型模拟 10 次



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lations of the model for each rule system. A single simulation is defined as being 250 time steps of the model evaluated on a 100-cell region. The required reaction time for a driver to prevent an accident was fixed such that any given driver would succeed 30% of the time. All other parameters were considered variable, and tests were performed with every possible combination of conditions.

- Maximum velocity,  $v_{\max}$  (speed limit for the road).
  - *Fast* - For a fast speed limit,  $v_{\max}$  is set equal to 6 cells per time step.
  - *Slow* - For a slow speed limit,  $v_{\max}$  is set equal to 2 cells per time step.
- Rate of Random Speed Increase Potentially Resulting in Accidents
  - *Computer* - The probability of random speed changes being manually set to zero, which also mean the probability of an accident is zero.
  - *Accident Prone* - The probability of any given car speeding up in any given time step is 0.3.
  - *Normal* - The probability of any given car speeding up in any given time step is 0.01.
- Traffic Density
  - *Heavy* - Large number of cars in the region, which we have defined as 100 cars.
  - *Light* - Small number of cars in the region, which we have defined as 30 cars.

For each of the 5 lane numbers in each of the four rule systems, the “Average Flow Rate” is calculated by averaging the total number of cars that leave the region in a simulated time interval, across the ten simulations performed, and then dividing that value by the number of time steps in each simulation, which was fixed at 250. The “Average Accidents” is merely the total number of accidents in each simulation, averaged across the 10 simulations.

## 4.2 Figures / 图

The figures presented on the following pages summarize the data that were collected for each of the four conventions across all of the different speed limits, accident rates, traffic densities, and lane numbers used for simulations. They are presented by convention, along with a brief summary of the conditions of the con-

的平均结果. 每一次模拟都是在 100 个元胞的区域上, 250 个时间步长内. 驾驶员避免事故所需的反应时间是固定的, 使得每个驾驶员在某次潜在事故中成功避免的可能为 30%. 所有其它的参数都被认为是可以变化的, 测试所有可能的条件组合的情形.

- 最大速度,  $v_{\max}$ (公路的限速).
  - **快**-对大限速情况, 设置  $v_{\max}$  等于 6 元胞/时间步.
  - **慢**-对小限速情况, 设置  $v_{\max}$  等于 2 元胞/时间步.
- 引起事故的随机加速概率
  - 智能 - 速度随机改变的概率被人为的设置为 0, 这意味着发生事故的概率也为 0.
  - 冲动 - 任何一辆给定的车的在任何给定的时间步中随机加速的概率被设置为 0.3.
  - 正常 - 任何一辆给定的车的在任何给定的时间步中随机加速的概率被设置为 0.01.
- 交通密度
  - **高** - 在模拟区域放置有大量的车辆, 这里我们定义为 100 辆.
  - **低** - 在模拟区域放置有少量的车辆, 这里我们定义为 30 辆.

对于 4 种交通通规则系统中的任一种, 5 条车道中的任一条的“平均流量”的计算都是通过统计模拟期间通过模拟区域的车辆总数 (模拟十次求平均), 然后这个总数除以模拟的总时间步数, 即 250 步. “平均事故量”为一次模拟中事故发的总数, 也是 10 次模拟的平均值.

接下来的几页中展示的图汇总了分别从四总交通规则的模拟中获得的数据, 每一种交通规则模拟都涵盖了不同的限速, 事故发生率, 交通密度, 车道数情况. 这些图是按照交通规则分别展示的, 每一张图都附有该规则模拟条件 (这里



vention.

#### 4.2.1 Right Hand Rule / 靠右行驶

The Right Hand Rule Algorithms represent the standard convention of driving on the right, and moving left to overtake a car in front. All of the cars start in the right most lane, as if all the cars have all just entered the freeway. The cars then disperse to overtake slower cars but always try to move back into a spot in the right most lane possible.

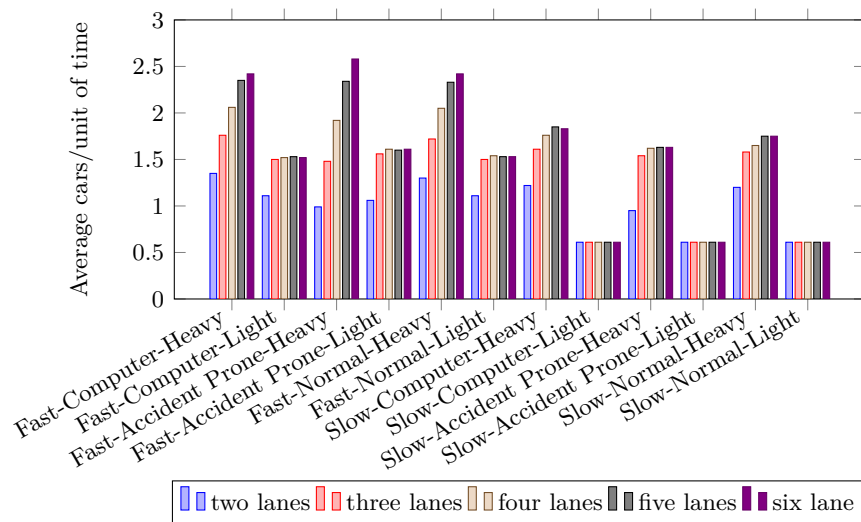


Figure 1: Average flow rates of right hand rule

#### 4.2.2 No Rules / 无规则

The No Rules method is referring to algorithms that do not prioritize moving to the left lane over moving the right lane. The Cars on the freeway start in random lanes in these simulations, and only move into different lanes if they are trying to overtake a car in front of it.

的条件指限速, 事故发生率, 交通密度, 车道数) 的简要说明.

靠右行驶规则表示右车道用于行驶, 左车道用于超车. 所有的车辆都是从最右边的车道上开始行驶的, 就像所有的车都刚刚进入高速公路一样. 然后这些车为了超过慢车而分散开来, 但是一有机会这些车就会试图回到最右边的车道上.

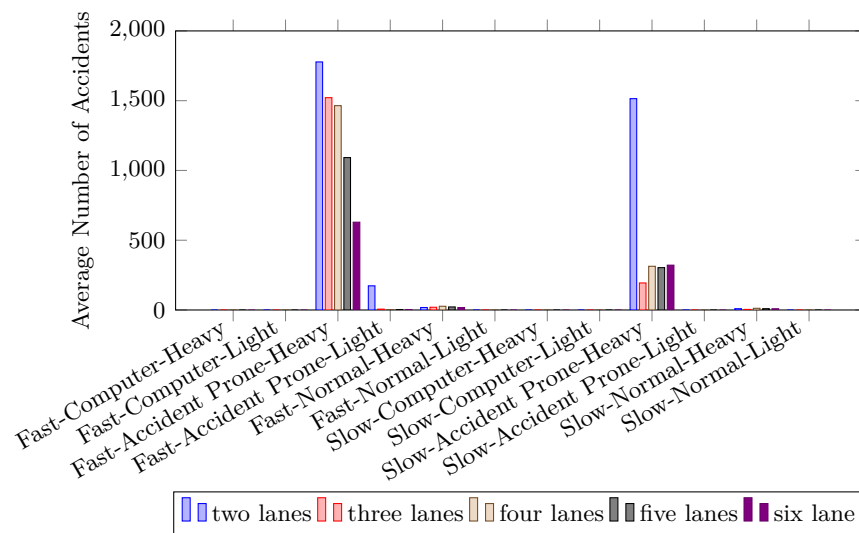


Figure 2: Average accidents of right hand rule



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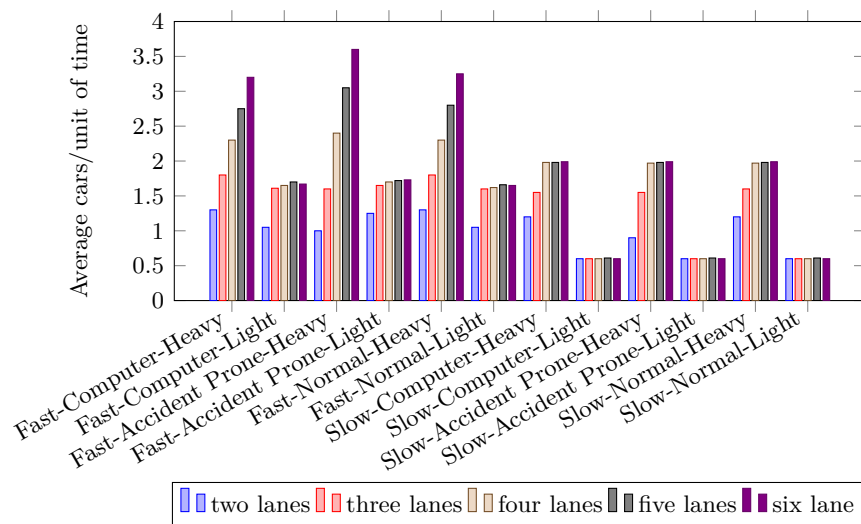


Figure 3: Average flow rates of no rules

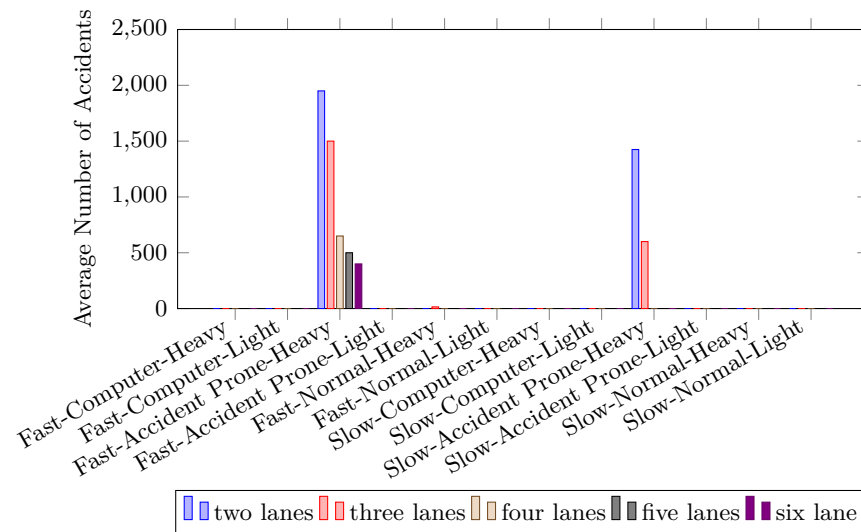


Figure 4: Average accidents of no rules

#### 4.2.3 No Lane Changes / 禁止换道

No lane changes refers to algorithms that do not allow cars to change lanes at all. There is no overtaking, all cars in this model have their initial starting lane generated randomly, and it is as if they entered the free-way, moved to a random lane, and stayed there until they would decide to leave the freeway. All lane-changing algorithms were removed.

#### 4.2.4 Middle Lane Rule / 居中进行

The Algorithms from Middle Lane Rule start all of the cars in the middle most lane of the freeway, and they disperse out to overtake one another. The overtaking mechanism is similar to “No Rules” in that they can over take on the left or on the right, but the cars always try to find an open spot in the middle lane of the freeway.

禁止换道是指完全不允许任何车道改变其车道的规则。这种规则下不存在超车, 所有车的初始的车道都是随机生成的, 就如同它们进入高速公路中的一条随机的车道, 并且它们必需留在这一条车道上直到它们决定离开高速公路。所有换道的操作步骤都被忽略。

居中进行规则中, 所有车道初始都被放置在高速公路的最中间车道上, 然后他们散开以超过别的车辆。超车的机制类似于“无规则”, 车辆可以从左车道或者右车道超车, 但超车完后, 这些车辆始终会尝试寻找高速公路上中间车道上的空位 (换回中间车道)。



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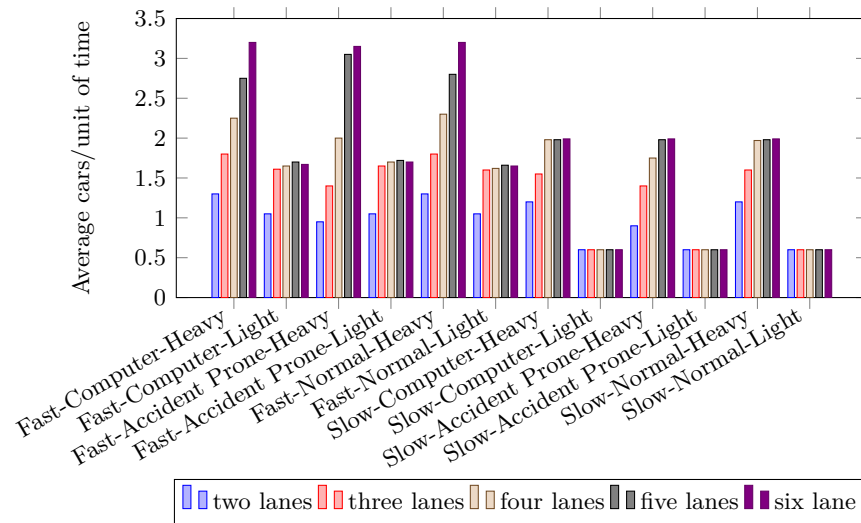


Figure 5: Average flow rates of no change rule

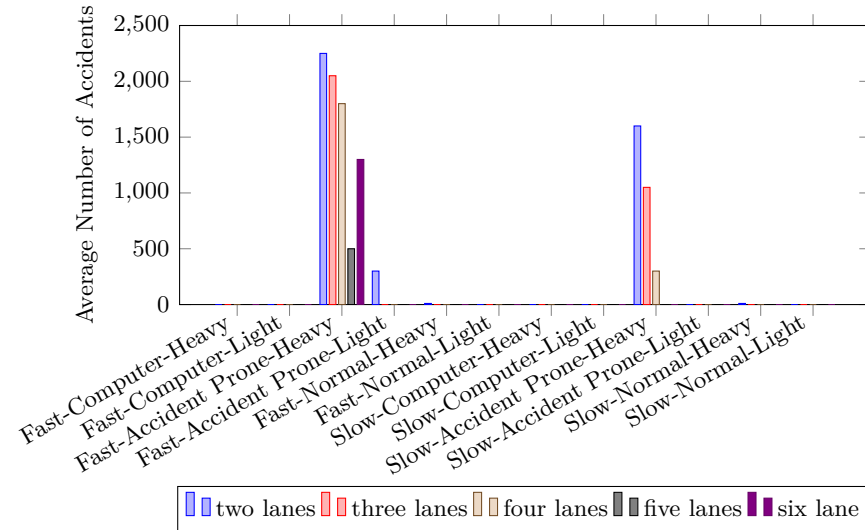


Figure 6: Average accidents of no change rule

### 4.3 Comparison of Rule Systems / 不同交通规则的比较

There were three different rule conventions that were used to compare against the right hand rule convention, which is the standard convention in America.

- *No Rules Convention*

The No Rules Convention was a simulation that allowed for drivers to overtake on either side of the road, and to stay in whatever lane they find themselves. This model provided the fastest flow rate out of any of the models but was slightly less safe for low lane numbers than the RHR. The No Rules Convention having the fastest flow rate was not entirely surprising, because the cars are all permitted to do anything they can to increase their speeds, and are able, unlike in the No Lane Changes convention, to switch lanes to move quickly past an accident. The Cars in the No Rules Model did not try to reposition themselves at the end of each iteration, which allowed them to spread themselves across all available lanes, causing particularly high flow rates for simulations with high lane counts. This convention led to slightly

靠右行驶规则是美国标准的交通规则, 与之相比较的规则有以下三种不同的交通规则.

- *无规则*

无规则模拟允许驾驶员从公路左右两边的任一条车道上超车, 前且可以留在任意车道上. 与其它所有规则相比, 这个规则给出了最大交通流量, 不过在车道数较少时, 相比于靠右行驶规则 (RHR) 有那么一点点不安全. 无规则具有最大的流量并不完全出人意料, 这因为无规则并不像禁止换道规则, 它允许所有车辆做任何可以使自己加速的事, 并且允许换道以越过一个交通事故. 无规则模型中的车辆不会在任何循环的最后尝试回到最初的车道上, 这允许它们占用任何可用的车道以获得更快的速度, 这也是该规则在车道数较多的模拟中能够获得较大的交通流量的原因. 在车道数较少时, 相比于



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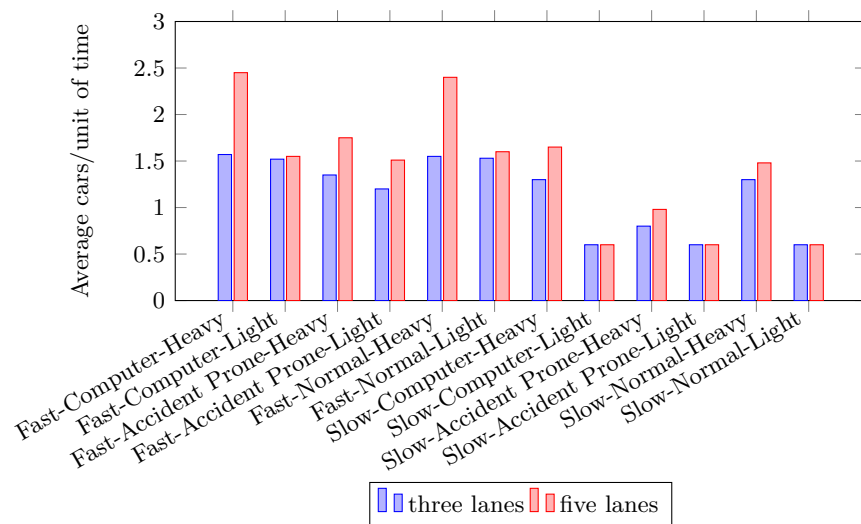


Figure 7: Average flow rates of middle lane rule

more accidents compared to the RHR when lanes were low, likely due to cars frequently moving in and out of lanes directly in front of each other. However, when lane counts were high, the No Rules convention exhibit much better safety than the RHR convention, with the cars in the RHR convention confining themselves to be as far to the right as possible, thereby increasing the likelihood that accidents can occur.

- *No Change Convention*

The simplest of all the derivatives is the No Change Convention, which generated a fixed number of cars into random lanes where the cars would remain throughout the entire simulation. This is essentially the 1-dimensional Nagel-Schreckenberg model running in multiple lanes at the same time. This model provided the second fastest results, but also was the most dangerous.

The simulation moves in discrete steps, one of them being the random speed up routine. When an accident occurred in the No Change Model, the cars behind the accident were unable to change lanes to move around the accident,

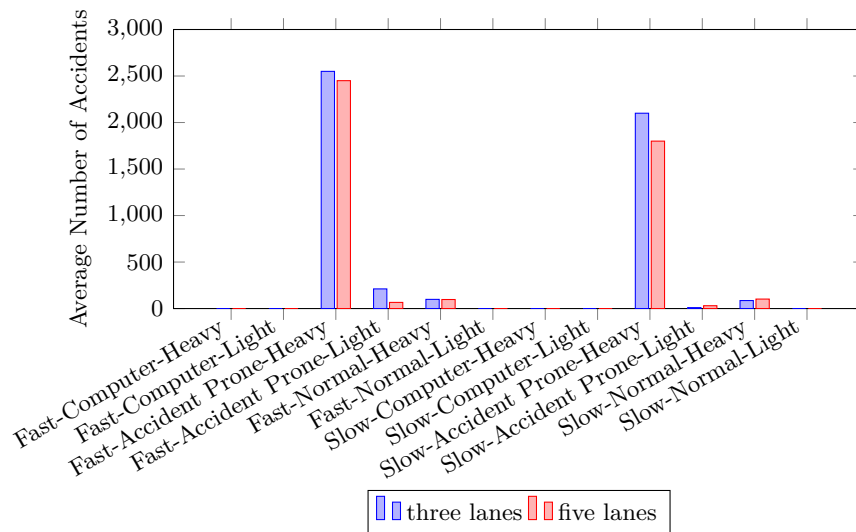


Figure 8: Average accidents of middle lane rule

靠右行驶规则 (RHR) 会多引起那么一点点事故, 这可能是因为车辆频繁地在其它车辆前面换来换去车道导致的。不过, 当车道数较多时, 相比于靠右行驶规则 (RHR), 无规则会表现的更安全, 在靠右行驶规则中, 车辆强制自己尽可能的靠右行驶, 因此增加了事故发生的可能。

- **禁止换道**

禁止换道规则是所有规则中最简单的, 这个规则在初始时生成一个固定数量的车辆随机分布在各条车道上, 在整个模拟中车辆都将保持在自己的车道上 (行驶)。这实质是同时运行了多条一维的 NS 模型。这个模型给出了交通流量第二大的结果, 但却也是最危险的规则。

模拟中车辆的运动是分解为离散操作步骤完成的, 其中有一个步骤是随机加速。当一个事故发生成禁止换道的模型中, 事故后方的车辆不能换到以越过事故, 而



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as they could in all other models. The result of this fact is that when the probability of random speed ups and the car density were both high, a single accident resulted in many cars becoming stuck at zero velocity behind it for five time steps. Due to the high accident probability, one of these cars eventually experiences a random speed up, at which time it plows into the car in front of it, causing an accident, resulting in accident chain reactions. This can be seen in the second figure of Section 4.2.4, where the “Accident-Prone Heavy Traffic” simulations show extremely high number of accidents. In the same figure, the simulations that had a 0 or 0.01 probability of a speed up showed little or no accidents.

Interestingly, simulations run by Alexander Lobkovsky Meitiv of the National Center for Biotechnology Information for his personal blog[7] using his own adaptation of the Nagel-Schreckenberg model, indicated that a “stay in lane” rule system yields higher average car velocities than if cars switch into faster moving lanes. Those are the only two rule sets he discusses, the “switch into faster lanes” rule system is not directly analogous to our “No Rules” system, his model operates on a truly circular track, and he does not attempt to account for accident rates. Nonetheless, his conclusions provide an interesting second look at the performance “No Lane Changes” rule system against fast switching rule systems in preserving a consistent high rate of traffic flow for the car system as a whole. Despite its many differences from our model, his model lends support to the conclusion suggested by our own data, that a system of never changing lanes leads to fast movement of all cars relative to certain other lane changing systems.

The No Change Convention was the most unrealistic out of all provided because it didn’t allow drivers to change lanes at all, even in case of accidents, but was still useful as a way to measure the effectiveness of models, and estimate the general effect of many cars changing lanes on the traffic flow of the system as a whole.

- *Middle Lane Convention*

The Middle Lane convention showed the lowest flow rate of all models, as well as showing the highest accident rate of all the models. While data for this rule was only collected for a three-lane and a five-lane freeway, owing

其它所有规则的模型却可以. 这个事实导致的结果是: 当随机加速的概率和车辆密度同时较大时, 单个交通事故将导致其后大量的车辆卡住并在五个时间步长内保持速度为 0. 由于较大的事故发生率, 某一辆车终于经过随机加速后, 爆了前方车辆的菊, 引起了事故, 最终导致了一连串连索反应. 这一点可以从第4.2.4节中的第二幅图中看出, “Accident-Prone Heavy Traffic”的模拟结果显示出事数量非常大. 在同一张图中, 随机加速概率为 0 或者 0.01 的模拟表现出较少或没有事故.

有趣的是, 国家生物技术信息中心的 Alexander 在其博客 [7] 里用自己改进的 NS 模型的模拟结果显示: “待在自己现在的车道”的规则系统比车辆往速度快的车道上换能获得更大的平均速度. 他只讨论了这两种方式, “换到快速的车道”的规则系统并不直接与本文的“无规则”系统类似, 他的模型是在一个真正的环道上实施的, 但他并没有尝试计算事故的发生率. 但是, 他的结论提供了一个有趣的对“禁止换道”规则性能的重新审视: 相比于换到更快车道的规则系统, “禁止换道”规则系统总体上为交通系统保持了一个相对较大交通流量. 尽管他的模型与本文的有很多不同, 但是他的模型仍然支持了由本文的数据得到的结论, 相对于某些其它的换道系统, 一个从来不换车道的交通系统会使所有车辆都较快的行驶.

禁止换道是本文所提供的最不切实际的规则, 这是因为它根本不允許驾驶员换道, 甚至在发生交通事故时, 但这一规则仍然是有用的, 它可以作为一个参照来评价其它模型的性能, 估计其它换道规则对系统的交通流量的总体效果.

- *居中行驶*

在所有模型中, 居中行驶规则表现为流量最小, 同时也表现为最大的事故率. 由于偶数车道的高速公路并没有单个的最中间的车道, 这个规则的数据是从三条和五



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to the fact that even-laned freeways do not have a single middle lane, the data collected give strong evidence that the model leads to high accident rates and low numbers of cars flowing through the region than any other rule convention studied. Much like the RHR convention, the increase from three lanes to five lanes did not result in a decrease in accident rates on the scale of that seen for the No Rules and No Lane Changes conventions. Of the four conventions studied, this one is considered to be by far the worst.

## 5 Discussion / 讨论

### 5.1 Limitations of the model / 模型的局限性

In our model there are factors that were not properly taken into account due to our assumptions and the way that the algorithms were developed. The model works in steps, which means that every car in the simulation is looked at individually, and cars don't move simultaneously. The model presented is all built off of the Nagel-Schreckenburg model which used uniform undefined time steps, and this is carried over into our model's designs. One flaw of the model is that each of the steps of the algorithms are carried out individually (cars will one by one move left, then one by one do safety checks etc.) the only time that accidents occur is in the speed up phase, which would cause all accidents to be rear-end accident, where one car hits the car directly in front of it, and no accidents are caused as cars move into other lanes. In reality this doesn't happen, accidents can come from any direction, but this all comes from the assumption that all accidents occur randomly.

由于算法中的每一操作步骤都是逐个执行的, 计算机循环是有方向的, 因此换道也是优先次序, 这也是本文的一大缺点, 不过这完全可以用很简单的方法解决: 对要换道的车辆进行随机排序, 再按随机顺序依次换道。

In our model the way that we accounted for accidents was through random number generation. Cars would speed up or slow down depending on whether a number was less than a value that is specified in the parameters of the model, and an accident would occur if the driver's randomly generated response time value was not low enough. We created simulations were a perfect drivers (i.e. computer driven vehicles) were used and thus we assumed that they would have a

条车道的高速公路的模拟中收集到的, 收集到的数据强有力证明: 这个规则引起较大的事故发生率, 并且相比于所研究的其它交通规则, 该规则下通过整个区域的车辆总数最小. 居中行驶和靠右行规则非常相似, 从三条车道增加至五条车道并不会像无规则和禁止换道规则那样大幅度地降低事故发生率. 对于被研究的四种交通规则, 这个被认为是目前为止最差的一个.

由于本文的假设和算法建立的方式, 在本文的模型中很多因素无法正确的考虑. 本文的模型是分步执行的, 这意味着模拟中所有的车都是独立的, 车辆并不同时移动. 本文展示模型是完全是基于 NS 模型建立的, 而 NS 模型的等时间步长的不确定也被带到了我们的模型设计中. 本文模型的一个缺点就是算法中的每一操作步骤都是逐个执行的 (例如车辆是一个一个的移到左边, 然后一个一个的检查是否安全). 只有在加速操作过程中, 当一辆车爆了前方车辆的菊时才会发生事故, 这意味着将所有的交通事故都归为追尾事故, 且车辆在换道时不会发生任何事故. 事实上并不是这样, 事故可以发生在任何方向上, 但是这些都源自所有事故都是随机发生的假设.

本文的模型中计算事故数量的方式依赖于随机数的产生. 车辆的加速或减速取决于产生的随机数是否小于模型中某一指定的参数, 如果随机生成的驾驶员反应时间不是足够的短就会发生事故. 我们构造了一种完美的驾驶方式 (即智能驾车) 用于模拟, 因此我们假设驾驶员完全掌握同围的交通信息, 近而他们随机加速的概率为 0. 对于“正常”情况的模拟,



perfect knowledge of the surrounding system, and thus their probability of speeding up would be 0. For “normal” simulations we put the probability of an accident occurring at less than .01. This assumes that drivers would have control of their vehicles and would slam on the breaks less, and or not be distracted while driving. In our simulations of accident prone drivers, the probability of a speeding up is set to .3, this was to cover all situations where drivers had significantly less control over their vehicles. We did not however alter the response time needed to avoid an accident in any of these situations. Different weather conditions would call for different response times, and different chances of speeding ups. Ice on the roads would be more serious than higher wind speeds. Instead accident prone simulations were run to cover any of those situations instead of looking into the matter with more detail.

Throughout our model, an accident was caused when two cars in the same lane had the same position value in Car\_Matrix. This would cause cars in the same lane that were behind the accident to lag and help simulate slowed down traffic. However, in some of the simulations certain accidents could lead to more accidents, in a domino effect. This was caused by the random speed ups that we put in, when cars were lined up in consecutive cells in their respective lanes, if there were multiple cars behind each other waiting for an accident to clear, and a speed up occurred (which could be quite high in accident prone simulations), then another accident would continue to block that lane. This posed as quite a problem in the No Change simulation as cars could not leave their lanes when an accident occurred, and thus there were more accidents in the No Change simulations.

Cars and their actions were simulated by the matrix Car\_Matrix, and one major limitation of the matrix, by virtue of our programming in MATLAB, was that the number of cars in the matrix could not exceed the number of rows of cells being used to represent the lanes of the highway. Thus, we couldn't simulate proper grid lock, or true high density traffic patterns. This becomes increasingly clear as we look at the data for highways with more lanes. Since the number of cars was held constant for the number of lanes being used in the simulation, the car density of the grid drops as we add more lanes, this means that traffic gets lighter and lighter even though the simulation are technically being looked at as “heavy traffic” and accidents drop and the flow rate increases because cars have

我们设置事故发生的概率低至 0.01. 这假设驾驶员会控制他们的车辆并且会踩刹, 很少或者完全不会在驾车时分心. 在本文冲动驾车模式的模拟中, 随机加速的概率被设置为 0.3, 在任何情况下, (这么大的概率都表明) 驾驶员明显很少控制他们的车辆. 在任何情况下, 我们都没有更改为避免事故所需的反应时间. 不同的天气状况需要不同的反应时间, 以及不同的随机加速概率. 路面有冰的情况要比大风时候更危险. 我们并不研究有关天气对交通影响的更多细节, 而用冲动模式的模拟来涵盖所有这些情形.

在我们的模型中, 当矩阵 Car\_Matrix 中出现两辆车具有相同车道和相同位置值时表示发生了事故. 这将引起同一条车道上事故后方的车辆停滞, 以模拟整个交通的减慢. 然而, 在一些模拟中, 某些确定的交通事故会连锁反应地引起更多的交通事故. 这是由本文添加的随机加速引起的, 当车辆被连续地排列在各自的车道上, 如果事故后方存在多辆等待事故现场清理的车, 并且发生了随机加速 (这种事在 accident prone 模拟中很常见), 随之而来的就是另一场事故并继续阻碍车道上的交通. 在禁止换道的模拟中, 当发生事故后车辆不能离开自己的车道, 这确实是一个相当大的问题, 这导致了更多的事出出现在了禁止换道的模拟中.

车辆及其行为是由矩阵 Car\_Matrix 来模拟的, 借助 MATLAB 编程, 用矩阵来模拟的一个主要限制是: 矩阵中车辆数不能超过表示高速公路车道元胞的行数. 因此, 我们不能正确地模拟完全阻塞的网格, 以及真实的交通车量密度. 这个限制在多条车道的高速公路的模拟结果中显得更为显著. 对于不同车道数的模拟中, 车辆数保持为常数, 格子里的车辆密度随着我们增加车道数而减小, 即便是在被认为“交通堵塞”的情况下, 这也意味着交通越来越畅通, 由于车辆之间的相互作用越来越少使得通行无阻, 因此事故的发生量减小, 并且车流量增加. 尽管如此, 这个模型仍然是可用的, 因





less and less interactions with one another and therefore cars can move with less obstruction. The model can still be used because as the iterations increase, the cars will move closer and closer together since they all want to move forward, and thus at the top of the grid of cells the cars will bunch together which creates a form of pseudo-grid lock. Also, since the rate of change of the density decreases at the same rate in all of the models, all of the data experiences the same shift throughout all the simulations.

Finally, cars also do not enter or leave the freeway. In addition, cars also either come onto the freeway all in the same lane, or randomly depending on which simulation was used. Then the cars all stay on the freeway and only move forward. In reality cars enter and exit all the time, and this means that cars would have reasons to cross over lanes other than in compulsion to the regulations of the road. If cars would have crossed over to exit then there would be more chances for accidents to occur, and cars would disperse over the grid of cells a lot more than they did in any of our simulations.

## 5.2 Symmetry of model / 模型的总结

All of the algorithms of our model would easily transition over to any driving systems that implemented left hand side driving. In countries where the left side of the road is the norm, cars place the driver on the other side, thus the visibility for each driver is the same and our algorithms would still hold because they all use integers to describe the different lanes, positions and, velocities, in essence our algorithms are all symmetric. So we could model cars that drove on the left by renumbering the lanes of Car\_Matrix backwards,  $n:1$  lanes instead of  $1:n$ , and the model's algorithms would still hold true.

## 5.3 Sensitivity / 灵敏性分析

The model appears to be very sensitive to the number of lanes in any given simulation, due to the fact that the function that propagates the freeway region with automobiles cannot add more vehicles than cells, regardless of the number of lanes in the simulation. This limitation was built into MATLAB's randperm function, and could not be circumvented in the scope of the project. The result of

为随着循环的增加, 由于要前进, 车辆间将会越来越近, 因此在元胞组成的网格的顶部会出现车辆聚集, 这形成了一种伪堵塞. 另外, 由于在所有模型中车辆密度的都是以相同的速率减少, 所有模拟中得到的所有数据都经过了同样条件的变化.

模拟过程中, 没有新的车辆进入或离开高速公路系统. 此外, 车辆初始进入高速公路时要么全部分布在同一条车道上, 要么根据模拟需要随机分布于每条车道上. 然后所有的车都待在高速公路系统中, 并且只向前移动. 实际上高速公路上不停的有车进入或离开, 这意味着车辆有理由越过车道, 而不是强制的服从交通规则. 如果汽车经过出口, 那么它将更可能发生事故, 相比于本文的任何模拟, 车辆会更大程度的分散在元胞网格上.

本文模型中所有的交通规则都可以容易的过渡到靠左行车的交通系统中. 在那些靠左行驶的国家, 车辆的驾驶员座位在另一边, 因此司机的能见度是一样的, 并且我们的规则同样适用, 因为我们的模型中完全只用了整数来表示不同的车道, 位置和速度, 而且本质上我们的算法是对称的 (即左右没差别). 因此我们可以通过重新反向给矩阵 Car\_Matrix 中的车道编号来模拟车辆靠左行驶, 用  $n:1$  取代  $1:n$ , 而模型中的算法仍然有效.

在任何一个特定的模拟中, 本文的模型似乎都对车道数非常灵敏, 这时由于在高速公路区域上生成车辆的功能不能生成超过元胞数量的车数, 即便在多条车道的模拟中. 这个限制由 MATLAB 的 randperm 函数 (randperm 是 matlab 函数, 功能是随机打乱一个数字序列) 造成, 并且无法在本



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this fact was that for high lane numbers, the two conventions that had no defined start lane appeared to be very safe, despite appearing less safe for low lane numbers. This is a result of the fact that for a six lane highway a car was only expected to appear every 6 cells, so for conventions that were able to spread to all lanes, the car density became extremely low, while remaining relatively high in systems that forced cars to return toward some specific lane.

项目范围内避免. 这也导致了在车道数较多时, 两种不指定初始车道的规则(无规则和禁止换道)表再为非常安全, 虽然在车道数较小时并不是太安全. 这也同样导致了对于六条车道的高速公路平均每 6 个元胞才出现一辆车, 因此对于那些能够将车辆分散在所有车道上的规则, 车辆密度变得非常小, 而对于那些强制车辆回到某个指定车道上的规则, 车辆密度则保持相对较高.

作者把模型的限制部分归结为 MATLAB 中的函数 `randperm`, 本人不以为然, 自己编程水平弱, 却怪人家 MATLAB 函数不好. `randperm` 是 matlab 中生成随机排列序列的函数, `randperm` 函数有两种用法:

- 第一种: 如 `randperm(6)` 则可生成 1 到 6 的随机排列, 结果可能为 [2 4 5 6 1 3];
- 第二种: 如 `randperm(6,3)` 则是从 1:6 个数中随机选三个, 结果可能为 [4 2 5].

本人猜想, 作者一定是 `randperm(100,n)` 来生成  $n$  个车的初始位置. 一条车道上格子的个数为 100, 因此  $n$  必然是小于 100. 因此对于多条车道, 系统中车量数最多也只能是 100. 这就是作者说的灵敏性的原因所在.

Another factor toward which the model appeared to be particularly sensitive was the probability governing the rate of accidents in a simulation. Though only three values for this parameter were tested, a non-linear relationship was observed between the number of accidents and the probability governing accident rate. It appeared that once a few accidents occurred in a simulation, they tended to result in many more accidents occurring as well. This presented itself as extremely high accident rates for all simulations of dense traffic with high speed limits and a high probability, far above what would be expected based on the values for other conditions. This might have been partially due to what the extremely high value we set for the probability of random speed ups in these simulations, which was intended to simulate a worst case scenario in terms of driving conditions, but that alone cannot account for the data seen, which must have been partially caused by accident chain reactions.

模型中另一个相对较为灵敏的因素是控制模拟中事故发生率的概率. 虽然这个参数只测试了三种值, 但仍可以观察出事故数量与控制事故发生率的概率呈非线性关系. 模拟中一但有少量的事故发生, 它们似乎还会导致更多的事故发生. 这种现象在车辆密度较大, 限速较高及随机加速概率较大的所有模拟中都会导致极高的事故率, 远高于基于其它条件值导致的事故量的期望. 其部分原因可能是: 为了模拟最坏交通行驶条件的情况, 在本文的这些模拟中我们设置了非常大的随机加速概率. 但是这不能完全解释我们所看到的数据结果, 这里面一定还有部分原因是事故的连索反应.

## 5.4 Conclusion / 结论

With all the data presented to us, we have concluded that the current convention of driving on the right is the safest method of driving. Our data showed us

基于程现在我们面前的所有数据, 本文可以得到这样一个结论: 现在使用的靠右行驶的交通规则是最安全的一种驾



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driving with the ability to perform lane changes in either direction, or not being able to change lanes at all would give us faster car flow, but was less safe than the right hand rule. As we are prioritizing driving safely and getting the driver to their destination, we decided that the increased flow rate of the No Rules Method, and the No Change Method was not worth the subsequent increase in accidents in each method.

驶方式. 我们的数据还向我们显示: 允许向两个方向换道, 或者完全不允许换道将带来更大的交通流量, 但比靠右行驶危险. 由于我们优先考虑安全驾驶并到达他们的目的地, 因此我们认为无规则和禁止换道以增加事故的数量为代价, 增加交通流量是不值得的.





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