

# Cardboard Comfortable When it Comes to Crashing 箱毀人爽

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## Abstract

A scene in an upcoming action movie requires a stuntman on a motorcycle to jump over an elephant. Cardboard boxes will be used to cushion his landing. To protect the nervous stuntman and the motorcycle, we investigate various configurations of the stunt. We formulate a model for the energy required to crush a box based on size, shape, and material. We also summarize the most readily available boxes on the market. We choose a maximum safe deceleration rate of 5 g based on comparison with airbag rigs used professionally for high fall stunts. To ensure that the stuntman lands on the box rig (rather than missing it completely) we analyze the uncertainty in his trajectory and extract the landing point uncertainty. We go on to construct a numerical simulation of the impact and motion through the boxes based on our earlier box crush energy calculations. After analyzing the sensitivity and stability of this simulation, we use it to examine the effectiveness of various configurations for the box stack (including different box sizes, types of boxes, and stacking patterns). Our findings indicate that 200 kg is the most desirable combined mass of the motorcycle and stuntman. A 300 kg mass is marginal and 400 kg is too heavy. We also conclude that a launch ramp angle of  $20^\circ$  is optimal when considering safety, camera angle, and clearance over the elephant. Our results show that a stack constructed of (30 inch)<sup>3</sup> boxes with vertical mattress walls spaced periodically is optimal when considering construction time, cost, and cushioning capacity. We recommend that this stack be constructed in dimensions 4 meters high, 4 meters wide, and 24 meters long. It will consist of approximately 1100 boxes and cost \$4300 in materials. The stuntman's wages are uncertain but fortunately the elephant works for peanuts.

即将上映的动作电影里的某一场景, 需要一位特技演员骑着摩托车飞跃一只大象. 在他着陆时, 需要用纸箱来进行缓冲. 为了保护紧张的特技演员以及他的摩托车, 我们研究了多种不同情况下的特技表演. 根据纸箱的尺寸, 形状和材料, 我们建立了一个模型来描述压碎一个纸箱所需要的能量. 同时, 我们也列出了市场上最容易买到的纸箱. 通过与高楼坠落特技中所用的安全气囊对比, 我们将最大安全减速度确定为 5g. 为了确保特技演员能够着陆在纸箱上 (而不是完全错过纸箱), 我们分析了其运动轨迹的不确定性, 并由此得出着陆点的不确定性. 接着, 基于先前对压碎纸箱所需能量的计算, 我们建立了一个数值仿真来模拟特技演员通过纸箱时的运动及所受冲击. 对模拟的灵敏度和稳定性进行分析之后, 我们用它来检验各种配置的纸箱堆效果 (包括不同的纸箱尺寸, 类型, 以及堆叠方式). 研究结果表明摩托车和特技演员的总质量最佳取值为 200kg. 300kg 的质量是临界值, 而 400kg 就过重了. 考虑到安全, 摄影视角, 以及大象上方间距, 我们得出最佳发射角为  $20^\circ$ . 我们的研究表明: 在综合考虑纸箱的堆放时间, 成本和缓冲效果下, 用边长 30 英寸的立方体纸箱, 周期性间隔地加入竖直的扁纸箱构造出的纸箱堆最佳. 我们推荐纸箱堆的尺寸为 4 米高, 4 米宽, 24 米长. 这大约需要 1100 个纸箱, 材料花费约 4300 美元. 特技演员的薪水还不确定, 不过好在大象只需吃点花生就行了.

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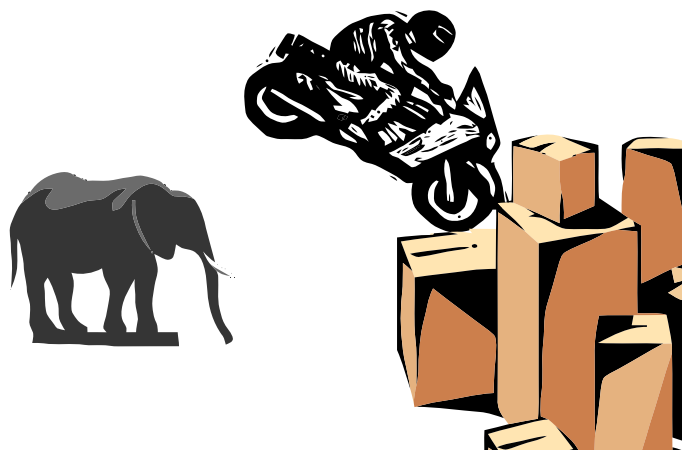


Figure 1: The death-defying leap over the elephant!

## 1 Introduction / 引言

The American movie-going public's appetite for action and danger on the screen steadily increases year by year. The trend is clear: big budget + big action = big payoff. Somewhere in the darker alleys of Hollywood a desperate director decides to make a name for himself by pulling off the biggest action sequence ever. He plans to film a motorcycle jumping over the biggest animal on land—the elephant!

Bureaucracy being what it is these days, Hollywood officials have hired us to ensure that neither the stuntman nor the elephant is injured (of course, they could care less about the director). The answer to keeping the elephant safe is simple: the elephant just stands in place and does nothing. The safety of the stuntman is a more difficult problem.

Airbag rigs are commonly rented for high fall stunts [1]. However, airbags are designed only to catch humans. Catching the motorcycle in the airbag

在美国，大众电影消费者对动作和心跳场景的口味一年比一年浓。这个趋势是显而易见的：高预算 + 大动作 = 巨额回报。这不，在好莱坞的某个僻暗的角落里，一位绝望的导演决定通过推出史上最大规模的系列动作电影来重拾名誉。他计划拍摄一个摩托车跃过陆地上最大动物 - 大象！

好莱坞官员这些日子似乎开始办正事了，他们聘用我们帮忙分析，确保这次的特技演员和大象均不会受伤（相比之下，他们当然没那么关心导演了）。要想保护大象安全，答案很简单：让它乖乖地站在那什么也别做就行。然而，特技演员的安全则是一个比较复杂的问题。

安全气囊最常被租用来拍摄高楼坠落特技 [1]。然而，气垫的设计只是用来接住特技演员的。用来接摩托



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would risk damaging it, and replacement costs might well exceed the budget of our unnamed director. The alternative is to use a cardboard box rig—a stack of boxes that will crush and absorb the impact of the motorcycle and stuntman.

**We are now faced with the following:**

1. The primary objective is to safely catch the stuntman and motorcycle.
2. As a secondary objective, we wish to minimize the cost and size of the box rig.

**Our approach is:**

1. First we investigate the relationship between the size/shape/material of a box and the work required to crush it. We call this quantity the *crush energy* of a box.
2. We review the most commonly available cardboard box types. We restrict our consideration to only these types because custom boxes are much more expensive than standard ones.
3. By comparison with an airbag rig, we estimate the maximum acceptable deceleration that the stuntman can experience during landing.
4. We analyze the trajectory of the motorcycle and the uncertainty in its landing location. This determines the proper placement of the box rig and how large an area it must cover to safely catch the motorcycle and stuntman.
5. Using the crush energy formula, we estimate the number of boxes that must be crushed in order to arrest the motion of the motorcycle and stuntman.
6. We formulate a numerical simulation of the motorcycle as it enters the

车会有损坏气垫的潜在危险，而且更换的费用可能会超出这位不知名导演的预算。一种替代方案就是用一个纸箱堆 - 一堆纸箱在被压碎的同时将吸收掉摩托车和演员带来的冲击。

**我们现在面临以下问题:**

1. 首要目标是要能安全地接住演员和摩托车。
2. 其次，我们希望花费的最少，以及纸箱堆尺寸最小。

**我们的解决途径是:**

1. 首先，我们研究纸箱的尺寸/形状/材料与其被压碎所需功之间的关系。我们称这个量为纸箱的损坏能量。
2. 我们查了几种最常用的纸箱类型。并把对问题的考虑仅仅限制在这些类型中，原因是专门定制的纸箱比标准的贵得多。
3. 类比气垫装置（高楼坠落特技），我们对特技演员在着陆时所能承受的最大加速度进行了估计。
4. 我们分析了摩托车的飞行轨迹，以及其着陆点的不确定性。这些可以帮助我们确定纸箱堆合适的放置位置和所需覆盖面积，以保证安全的接住摩托车和特技演员。
5. 利用纸箱的损坏能量公式，我们估算了为了制动摩托车和演员的运动所需损坏纸箱的数量。
6. 我们建立了一个针对摩托车进入纸箱堆时的数值



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box rig. Using this model we analyze the effectiveness of various types of boxes and stacking arrangements. We also compare low, medium, and high trajectory jumps.

7. As an alternative to catching the stuntman while he is still sitting on the motorcycle, we analyze the possibility of having the stuntman bail out in mid-air and land separately from the motorcycle.
8. Based on the results from our simulation, cost and construction considerations, and safety requirements, we make a set of strong recommendations regarding placement, size, construction, and stacking type of the box rig.

模拟模型. 利用这个模型, 我们分析了多种类型的纸箱, 堆叠方式的效果. 同时, 我们还对比了低, 中, 高三种轨迹的飞跃.

7. 除了特技演员和摩托车一起着陆的情况外, 我们还分析了另一种可能: 特技演员与摩托车在半空中分离, 各自着陆.
8. 基于数值仿真的结果, 纸箱堆的费用和搭建因素、以及安全性要求, 我们给出一套关于纸箱堆的堆放位置, 尺寸, 构造和堆叠方式的建议.

## 2 The energy absorbed by crushing cardboard / 撞毁纸箱被吸收的能量

To calculate the ability of a cardboard box to absorb the impact of the stuntman and motorcycle we estimate the energy required to crush the box. This estimate is based on a combination of physical considerations and experimental box crushing.

- **Assumption:** The primary source of energy absorption is in the breakdown of the box walls due to edge compressive forces.

Commercial cardboard is rated by Edge Crush Test (ECT), which reports the pounds per inch of edge compressive force parallel to the flute (The flute of corrugated cardboard is the wavy layer between the two wall layers. Here, ‘parallel to the flute’ means the direction parallel to the flute wavefronts.) which the cardboard can withstand before breaking. This can be interpreted as the force against the edge per unit length of crease created [2, 5]. Note that once a crease has formed, very little work is required to further bend the cardboard.

To understand how the formation of wall creases relates to the process of crushing a box, we conducted several experiments by dropping a crush-test dummy on a box. See Fig. 2 for illustration of experimental setup.

We observed that:

为了计算纸箱吸收来自摩托车和特技演员冲击的能力, 我们首先估计撞毁纸箱所需要的能量. 这个估算是基于物理学理论和纸箱撞毁实验来实现的.

- **假设:** 纸箱被损毁时能量吸收的主要部分是源自侧面的压缩力.

商用纸箱是按照边压强度 (ETC) 来排等级的, 边压强度是指每英寸宽度的棱面纸板毁坏前在平行于凹槽方向 (瓦楞纸板的凹槽是指两层纸板间的波浪形纸芯夹层. 这里, 平行于凹槽 指的是平行于波浪形纸芯的波阵面.) 所能承受的压力 (单位: 磅). 这可以被理解为纸箱侧面为反抗形成单位长度折皱的支撑力. 值得注意的是, 一旦纸箱上形成了一个折皱, 要压弯整个纸箱只需要做很小的功.

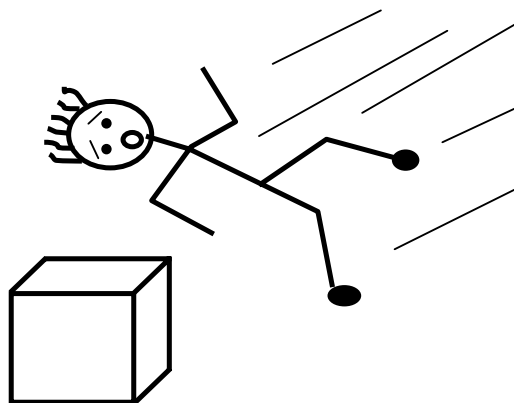
为了了解在压碎纸箱的过程中纸箱壁的折皱是如何形成的, 我们进行了多次实验, 实验是往纸箱上投下假人来测试. 实验的装置如图2所示.

我们观察到:



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**Figure 2: Experimental apparatus for crushing boxes.** We dropped a crush-test dummy (i.e. team member) onto several boxes and observed how the structure broke down each time.

- The first wall-creases typically form in the first %15 of the stroke distance.
- These creases extend across two faces of the box (A schematic of one such crease is illustrated in Fig. 3).
- Once these have formed the box deforms further with comparatively little resistance because additional creases are created by torque forces rather than edge compressive forces.
- The primary creases each have length approximately equal to the diagonal length of the face.

The work done in crushing the box is given by the average force applied times the distance through which it was applied. This and the above experimental qualitative results lead us to write the following equation for energy absorbed by a box of dimension  $l_x \times l_y \times l_z$  being crushed in the  $z$ -direction.

- 纸箱最初形成的折皱通常在首次下压约 15% 的距离时出现.
- 纸箱的折皱会延伸至两个对着的面 (图3中显示了这样一道折皱的示意图).
- 一旦折皱形成, 继续使纸箱变形、压垮时受到的阻力就相当小了: 因为新增的折皱主要由扭力产生, 而不再是对侧面的压力.
- 纸箱最初的两道折皱长度都近似等于其上表面对角线长.

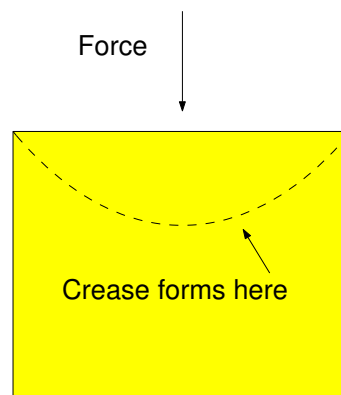
压碎纸箱过程中所做的功由平均施力乘上施力距离给出. 由此并结合上面实验的定性结果, 我们引出如下方程, 来计算三维尺寸为  $l_x \times l_y \times l_z$  的纸箱在  $z$  轴正方向上被压时吸收的能量.

$$\underbrace{E}_{\text{功或能量}} = \underbrace{ECT \times \left(2\sqrt{l_x^2 + l_y^2}\right)}_{\text{作用力}} \times \underbrace{l_z \times 0.15}_{\text{作用距离}} \quad (2.1)$$



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**Figure 3:** The first crease forms in a curve across the side faces as the box is compressed from above.

**As a reality check**, we compute the crush energy for a standard 8.5 in  $\times$  17 in  $\times$  11 in box with ECT 20 lbs/in and C-flute (the type commonly used to store paper). With these numerical values, Eq. 2.1 gives an energy of 187 Joules. This corresponds roughly to a 140 lbs person sitting on the box and nearly flattening it.

- **Comparison with experiment:** Crush-test dummy results confirm that this is indeed a good estimate.

In addition to the crush energy, energy can also be absorbed in the process of flattening the flute within the cardboard walls. However, the pressure required to do this is  $\sim 150$  kPa [10] and the surface area involved is more than a square meter, so a quick calculation show that the stuntman would decelerate too quickly if his kinetic energy were to be transferred into flattening boxes. We therefore ignore this additional flattening effect.

- Any successful box rig configuration must dissipate all of the kinetic energy of the stuntman and motorcycle through box crushing alone.

为了做一个实例验证，我们计算了尺寸为 8.5 英寸  $\times$  17 英寸  $\times$  11 英寸，ECT 为 20 磅/英寸的 C-楞型标准纸箱（常用作储存纸张）的损毁能量。将这些参数代入式 (2.1) 中，得出能量为 187 焦耳。这个大致相当于一个 140 磅的人坐在该纸箱上，并几乎将它压平。

- 实验对比：假人撞压测试的结果证实这是个很好的估算。

除了纸箱损毁能量外，把纸板的波浪形纸芯夹层彻底压平也会吸收能量。然而彻底压平纸板需要的压力值高达 150kPa，且受力表面积会大于 1 平方米，因此通过一个简单的计算就能得出：如果特技演员的动能被用来彻底压平纸箱，这种情况下特技演员将具有过大的减速度。因此，我们忽略这个彻底压平纸箱而额外吸收的能量。

- 任何成功的纸箱堆的结构都必须使特技演员和摩托车的所有动能仅被消耗在撞毁纸箱的过程中。



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### 3 Commonly available box types / 常用的纸箱类型

Minimization of costs is an important concern in this stunt. The cardboard box rig will consist of perhaps hundreds of boxes, and wholesale box prices can range up to \$10 or \$20 per unit (for larger boxes), so we therefore restrict our attention to only those box types which are most commonly available. We investigate the box types listed in table 1.

在这个特技表演中，如何把成本花费降到最低是一个重要问题。纸箱堆也许会包含数百个纸箱，纸箱的批发单价能高达 10 到 20 美元 (较大的纸箱)，因此我们将注意力只集中在那些最常用的纸箱类型上。我们将查到的纸箱类型结果列于表1中。

**Table 1:** Commonly available box types. Information from [4, 3].

Type	Size (in)	ECT rating (lbs/in)	Price per box
A	$10 \times 10 \times 10$	32	\$0.40
B	$20 \times 20 \times 20$	32	\$1.50
C	$20 \times 20 \times 20$	48	\$3.50
D	$30 \times 30 \times 30$	32	\$5.00
E	$44 \times 12 \times 12$	32	\$1.75
F	$80 \times 60 \times 7$	32	\$10.00

We will use this table later to evaluate the cost of various box rig configurations.

后面我们会用该表来评估不同结构的纸箱堆的花费。

### 4 Some quick estimates / 一些快速的估计

We now make a few rough calculations and estimates. We will use these results to set safety tolerances and as a guide in working with the more complex numerical simulation that we later formulate.

现在，我们要做一些粗略计算和估计。这些结果将用来设置安全性容差，并作为后面建立更复杂的数值仿真的向导。

#### 4.1 Maximum safe acceleration / 最大安全加速度

To determine acceptable forces and accelerations for the stuntman as he enters the cardboard box rig, we compare the box rig with other cushioning devices. In the stunt rigging business it is common practice to use an air bag for high-falls of up to 30 meters. Airbags rated for falls of up to 30 meters

为了确定杂技演员进入纸箱堆时可以承受的力和加速度大小，我们把纸箱堆与其他缓冲装置做一下对比。在特技表演的防具中，常用气垫保护从高达 30 米处跳下的演员。承受 30 米高空坠落的气垫大约需要 4 米厚。



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are approximately 4 meters deep.

Assume a stuntman falls from 30 meters above the airbag. Gravity accelerates him from rest to a velocity  $v$ . At this point he strikes the airbag and is decelerated completely, so we have

$$\sqrt{2gd_{\text{fall}}} = \sqrt{2a_{\text{bag}}h_{\text{bag}}}$$

where  $d_{\text{fall}}$  is the fall distance,  $a_{\text{bag}}$  is the deceleration rate the stuntman experiences in the airbag,  $h_{\text{bag}}$  is the height of the airbag, and  $g$  is the acceleration due to gravity. Thus

$$a_{\text{bag}} = \frac{d_{\text{fall}}}{h_{\text{bag}}}g = \frac{30\text{m}}{4\text{m}}g = 7.5g$$

We therefore conclude:

- **When using an airbag, the stuntman experiences an average acceleration of at most  $7.5g$ .** This provides an upper bound on the maximum acceleration that a person can safely withstand.
- **However, with the airbag stunt the stuntman is able to land in a position that distributes forces evenly across his body.** In our stunt the stuntman will be landing in the box rig while still on the motorcycle. This will result in greater chance for injury under high deceleration.
- **We choose  $5g$  as our maximum safe deceleration.** A box rig configuration which results in a higher acceleration will be rejected as unsafe.

## 4.2 Displacement and energy estimates, a reality check / 制动距离及能量的估计, 一个实例验证

If the deceleration of the stuntman and motorcycle is constant through the boxes then we can estimate the distance required to bring him to rest. Since any deviation from constant acceleration will increase either the stopping distance or the peak deceleration, this will give us a lower bound on the stopping distance and hence a lower bound on the required dimensions of the box rig.

Suppose the stuntman enters the rig at time  $t = 0$  with velocity  $v_0$

设想一位特技演员从 30 米高度坠落到气垫上时, 重力的作用令他从静止加速到速度  $v$ . 此时, 他撞上气垫, 然后完全减速至 0, 因此我们有:

其中  $d_{\text{fall}}$  为下落距离,  $a_{\text{bag}}$  为特技人在气垫上承受的减速度,  $h_{\text{bag}}$  为气垫的高度,  $g$  为重力加速度. 因此

因此我们得到以下结论:

- **当使用气垫时, 特技演员承受的最大平均 (负) 加速度为  $7.5g$ .** 这提供了一个人所能承受的安全加速度的上限.
- **但是, 借助气垫能够让演员在着陆时全身受力均匀.** 在我们的特技表演中, 特技演员将骑着摩托车着陆到纸箱堆. 在比较大的加速度下, 这将会增加特技演员受伤的风险.
- **我们选取  $5g$  作为最大安全加速度.** 任何会导致更大加速度的纸箱堆结构都是不安全的, 将不会被使用.

如果特技演员和摩托车在撞击纸箱的减速过程中, 加速度是一个常数的话, 我们可以估算出演员停下所需的制动距离. 然尔任何偏离恒定加速度的扰动都会增加制动距离或者加速度的最大值, 但这可以为我们提供制动距离的下限, 并由此得到所需纸箱堆在尺寸上的下限.

假设这位特技演员在  $t = 0$  时刻飞入纸箱堆, 速



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and experiences a constant deceleration  $a$  until he is brought to rest at time  $t = t_f$ . The stuntman's velocity  $v(t)$  is then given by  $v(t) = v_0 - at$ . Since the stuntman is at rest at time  $t_f$ , we have

$$t_f = v_0/a$$

Let  $x(t)$  be the displacement from the point of entry as a function of time. Since  $x(0) = 0$ ,

$$x(t) = v_0 t - (1/2)at^2$$

and so the total distance traveled through the boxes is

$$\Delta x = x(t_f) = \frac{v_0^2}{a} - \left(\frac{1}{2}\right)(a)\left(\frac{v_0}{a}\right)^2 = \frac{v_0^2}{2a}$$

Therefore we arrive at:

- Given an impact velocity  $v_0 \approx 20m/s$  and deceleration bounded by 5 g, the stuntman will require *at least* 4 meters to come to rest.

Conversely, if we instead have an idea of what the stopping distance should be, we can easily compute that the constant deceleration required to stop in a distance  $\Delta x$  is

$$\frac{v_0^2}{2g\Delta x}.$$

Using the calculation for the energy dissipated by crushing a box we can estimate how many boxes must be crushed to dissipate the energy of the incoming projectiles, i.e.: stuntman and motorcycle. The energy that must be dissipated in the boxes is roughly equal to the kinetic energy that the motorcycle and stuntman enter with. (Since the box rig should only be three or four meters high, the potential energy is a much smaller fraction of the total energy.) Thus for  $v_0 = 20m/s$  and a mass of 200 kilograms, the change in energy is 40,000 Joules. From Eq. 2.1 we calculate that the crush energy of a standard (30 inch)<sup>3</sup> box is 633 Joules.

度为  $v_0$ , 加速度恒定为  $a$ (加速度为负), 直到  $t = t_f$  时减速到静止. 特技演员任一时刻的速度  $v(t)$  由公式  $v(t) = v_0 - at$  给出. 由于特技演员在  $t = t_f$  时刻达到静止, 我们有

令  $x(t)$  为从飞入纸箱堆后前进的距离, 是时间的函数. 由于  $x(0) = 0$ ,

所以在纸箱堆中运动的总距离为

因此我们得出:

- 在给定冲击速度  $v_0 \approx 20m/s$ , 减速度限定为 5g 的条件下, 特技演员需要至少 4 米的距离制动.

反过来, 如果我们知道是制动距离应该为多少, 我们就能很容易计算出在距离  $\Delta x$  中所需要的制动加速度常量为

利用对撞毁一个纸箱耗能的计算, 我们可以估计一共需要多少纸箱来消耗掉即将冲出的“导弹”, 也就是特技演员和摩托车. 在纸箱堆中必须消耗的能量可以粗略地认为等于摩托车和演员刚进入纸箱堆时的初始动能. (由于纸箱堆只能高至 3 到 4 米, 因此势能只占总能量很少一部分.) 因此对于初速度  $v_0 = 20m/s$ , 总质量 200 千克的情况, 能量的变化为 40,000 焦耳. 从式 (2.1) 我们计算出一个尺寸为 30 立方英寸, 标准规格的纸箱的损毁能量为 633 焦耳.



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$$\frac{40,000 \text{ Joules}}{633 \text{ Joules/box}} \approx 60 \text{ boxes}$$

- **Conclusion:** The incoming stuntman and motorcycle must crush about 60 boxes to come to a stop.

- **结论:** 正在飞来的特技演员和摩托车必需撞坏大约 60 个纸箱后, 才能停下来.

## 5 Trajectory Analysis and Cushion Location 轨迹分析及着陆范围

Regardless of how much energy it takes to crush the cardboard boxes, they won't dissipate any of the stuntman's energy unless he actually lands on them. It is therefore important to consider the trajectory of the stuntman and motorcycle through the air so we know where the box rig should be placed and what the uncertainty in landing location is.

We calculate trajectories by solving the following differential equation, where  $v$  is the speed,  $k$  is the drag coefficient, and  $\vec{x}$  is the position:

$$(\vec{x})'' = -g\hat{z} - \frac{k}{m}|v|^2\hat{v}$$

MATLAB's ODE45 function was used to solve an equivalent system of first order equations. We use an air drag coefficient of  $k = 1.0$  (this value is from Ref. [6]). We see immediately from figure 4 that it would be unwise to ignore air resistance.

- **Air drag effects alter the stuntman's landing position by up to several meters.** We therefore incorporate air resistance into all simulations.

It is unreasonable to expect the stuntman to leave the ramp with exactly the same initial velocity and angle every jump. We therefore need to allow for some uncertainty in the resulting trajectory and ensure the cardboard cushion is large enough to support a wider range of possible landing locations. Since the ramp angle  $\phi$  is constant, we assume the initial angle between the direction of the stuntman and the ground is also  $\phi$  with no uncertainty. However, there is the possibility that the motorcycle might be moving slightly

无论有多少能量能撞毁多少纸箱, 除非这位演员能正好着陆在纸箱堆上, 否则特技演员的任何能量也不会被消耗. 所以说, 关键在于分析特技演员和摩托车在空中的运动轨迹, 我们才能知道纸箱堆应该放置在什么地方, 以及演员着陆位置的不确定性范围.

我们通过求解下面的微分方程来计算轨迹, 其中  $v$  为速度,  $k$  为阻力系数,  $\vec{x}$  空间位置:

本文使用 MATLAB 的 ODE45 函数来求解等效系统的一阶微分方程 (组). 我们取空气阻力系数  $k = 1.0$  (此值来自参考文献 [6]). 从图4中我们可以明显看出, 忽略空气阻力是非常不明智的做法.

- **空气阻力的作用改变了演员的着陆位置, 造成了数米以上的变动.** 因此我们将空气阻力纳入所有的仿真计算中.

期望特技演员每次飞跃都精确的以相同的初始速度和角度离开坡道是不切实际的. 因此我们需要允许轨迹有一定的不确定性, 同时确保用来缓冲的纸箱堆足够大, 以支持更大的着陆范围. 鉴于坡道的倾角是一个常数, 我们假设演员运动方向与地面的初始夹角也确切地等于  $\phi$ , 没有不确定性. 然而在离开坡道时, 摩托车有可能稍微向左或向右轻微移动. 令  $\theta$  为坡道轴与演员速度矢



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to the left or right as it leaves the ramp. Let  $\theta$  be the azimuthal angle between the ramp axis and the stuntman's velocity vector. Ideally  $\theta$  should be zero, but small variations may occur. The other uncertain initial condition is the initial velocity  $v_0$ .

- In modeling the stuntman's possible trajectories, we assume the following uncertainties:
  - Initial velocity:  $v_0 = v_{\text{intended}} \pm 1 \text{ m/s}$
  - Azimuthal angle:  $\theta = 0 \pm 2 \text{ degrees}$

We use this to identify the range of possible landing locations by plotting the trajectories that result from the worst possible launches. These are shown in Fig. 5.

If the intended initial velocity is 22 m/s, the ramp angle is  $20^\circ$  and the mass of the rider plus motorcycle is 200 kg, then

- **Distance variation is:**  $\pm 2.5 \text{ meters}$ .
- **Lateral variation is:**  $\pm 1.5 \text{ meters}$ .

## 6 Impact simulation / 撞击模拟

In order to evaluate the effectiveness of various box rig configurations, we construct a numerical simulation of the motion of the stuntman and motorcycle through the box rig.

### 6.1 Assumptions / 假设

The full physics of the box rig is far too complex to model accurately. We therefore make the following assumptions to approximate and simplify the problem:

- **The problem is two dimensional.** We restrict our attention to the plane of motion of the stuntman. There is no reason for his trajectory to ever be significantly bent out of this plane. Making this simplification does remove the possibility of observing some box stacking effects. However, we will later show that these effects are negligible in most cases.

量间的夹解 (方位角). 理想状态下,  $\theta$  应该为 0, 但小的偏差是可能存在的. 此外, 另一个不确定的初始条件是初始速度  $v_0$ .

- 在模拟特技演员可能的运动轨迹时, 我们假设有如下的不确定性:
  - 初速度:  $v_0 = v_{\text{intended}} \pm 1 \text{ m/s}$
  - 方位角:  $\theta = 0 \pm 2 \text{ 度}$

基于这些假设, 我们通过画出在最坏发射情况下的轨迹图, 来确定可能的着陆范围. 这些结果见图5.

如果初始速度是 22 米/秒, 坡道倾角为  $20^\circ$ , 人车总质量为 200 千克, 那么着陆点的范围

- **前进方向位移偏差:**  $\pm 2.5 \text{ meters}$ .
- **侧向位移偏差:**  $\pm 1.5 \text{ meters}$ .

为了评价不同结构的纸箱堆效果, 我们构造了一个数值仿真模型来研究特技演员和摩托车在纸箱堆中的运动.

整个纸箱堆的物理机制太过复杂, 以致于难以精确地建模. 因此我们做了如下的近似假设来简化问题:

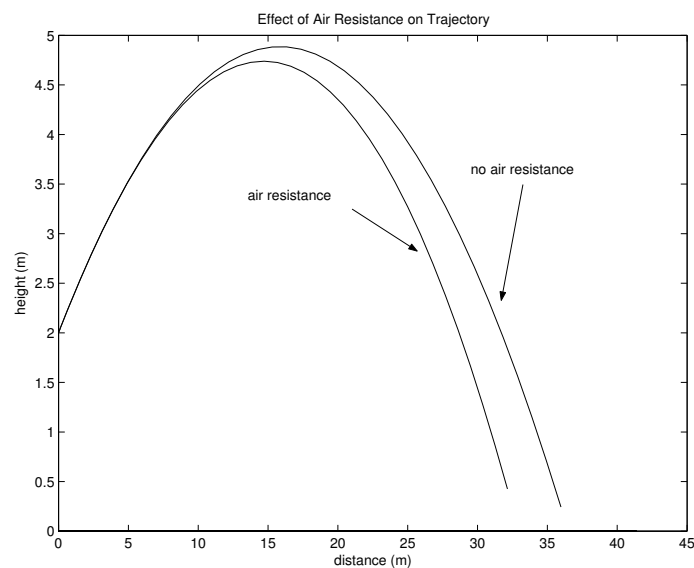
- **本文的问题是二维的.** 我们将研究局限于特技演员运动所在的平面. 特技演员和摩托车的运动轨迹如果明显地偏离出该平面, 这是没有任何理由的. 做这样一个简化假设会失去某些可以观察到某些纸箱堆叠效应的可能. 不过, 我们稍后会说明这些影响在大多数情况下是可以忽略不计.



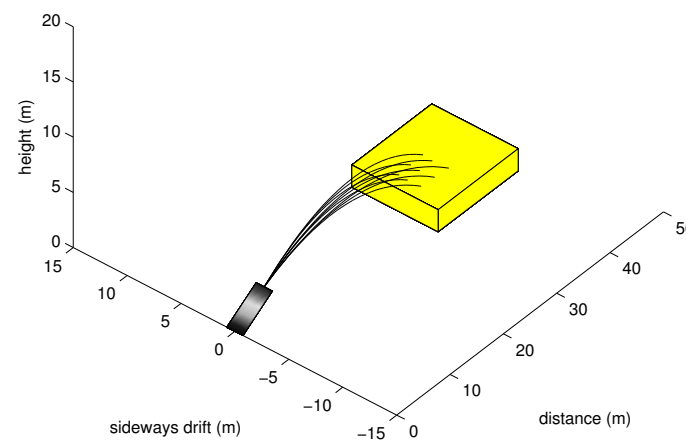
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**Figure 4:** Air resistance significantly changes the trajectory.



**Figure 5:** Trajectory Uncertainty Due to Launch Uncertainties. Note that the box rig depicted here is not to scale.

- **As the motorcycle plows through the boxes, a thick layer of crushed boxes accumulates against its front and lower surfaces.** These layers increase the effective size of the motorcycle and cause it to strike a larger number of boxes as it moves. We assume that this captures the effects of internal friction and viscosity within the boxes. A illustration of this effect is shown in Fig. 6.
- **In the average of striking a large number of boxes, the velocity magnitude is reduced but the direction is unchanged.** A collision is as likely to deflect the trajectory upward as downward.
- **Boxes are crushed rather than pushed out of the way.** In practice, this can be ensured by placing a strong netting around the three sides of the box rig that face away from the incoming stuntman.
- **Boxes are crushed to a uniform level.** In reality some boxes may

- 随着摩托车进入纸箱堆中, 在它的前表面和下表面会出现由一些压碎的纸箱积累而成的厚度层. 这些厚度层会增加摩托车的有效尺寸, 导致在运动过程中撞到更多的纸箱. 我们认为这种粘附效应是由于纸箱间内部摩擦和粘性引起的. 图6为这个吸附效应的示意图.
- 在撞击大量纸箱的平均过程中, 速度的大小会逐渐减少, 速度的方向不变. 碰撞很大程度上会改变运动轨迹, 使之向上或向下偏转方向.
- 尽管纸箱被压毁, 但不会飞出去. 在实际中, 我们可以通过在纸箱堆背向特技演员飞来方向的三侧放置强韧的网, 来确保纸箱不会飞出去.
- 所有纸箱被挤压到同一程度. 实际中有些纸箱可

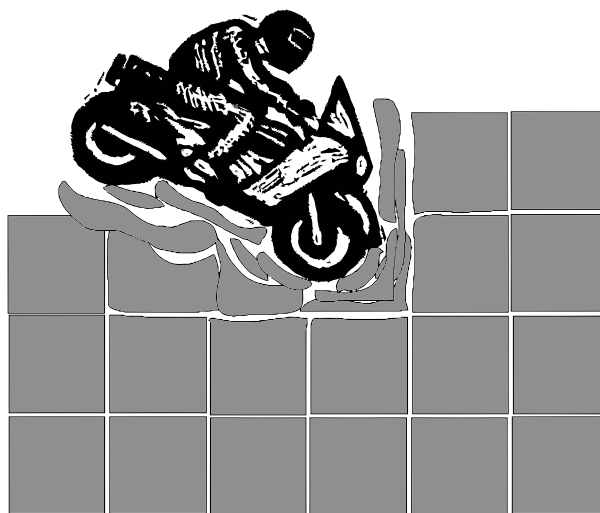


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be crushed only slightly, while others are completely flattened, but these effects disappear when we average over a large number of box collisions.

能只被轻微压到，而另一些则完全被碾平，但当我们对大量受撞击纸箱做平均时，这些影响就消失了。



**Figure 6:** A wall of debris forms on the front and bottom surfaces of the motorcycle as it enters the boxes.

## 6.2 Formulation / 模型设置

We formulate the simulation as follows:

- The motorcycle and stuntman is represented by a bounding rectangle that is initially 1.2 meters long, 1.2 meters high, and 0.7 meters wide (though the width is irrelevant for most of the simulation).
- The box rig is represented by a 2-dimensional stack of boxes. We will consider several different stacking configurations.
- We numerically integrate the motion in discrete time steps of 0.05 seconds. The only object in motion throughout the simulation is the stuntman and motorcycle—all of the boxes are stationary.
- When the bounding rectangle intersects a box, the box is considered

我们对仿真设置如下：

- 摩托车和演员用一个矩形表示，这个矩形的初始长为 1.2m，宽为 0.7m，高为 1.2m 的（尽管大多模拟中宽度都是无关紧要）。
- 使用二维纸箱堆来代替现实的纸箱堆。我们将考虑多种不同的堆叠方式。
- 我们取离散的时间步长为 0.05s，来对运动方程做数值积分。仿真中唯一运动的是特技演员和摩托车 - 也就是说，所有纸箱都是静止的。
- 当一个表示特技演员和摩托车的边界矩形与一个



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crushed. We modify the stuntman's velocity according to the kinematics described in the following section (see Eq. 6.1) and ignore further interactions with the crushed box.

- For each box crushed we add a layer of additional thickness to either the front or the bottom (for horizontal and vertical collisions resp.) of the motorcycle bounding rectangle. We assume that boxes are crushed to %20 of their length or height for horizontal and vertical collisions respectively. We allow the front layer to extend above and below the original bounding rectangle (and likewise for the bottom layer) so that the force of the motorcycle striking a tall box will effectively be distributed along the length of the box. These debris layers increase the effective size of the motorcycle and therefore cause it to strike a larger number of boxes as it moves. We use this process to account for the effects of friction.
- The vertical component of the velocity is set to zero when the bounding rectangle strikes the ground.

### 6.3 Kinetics / 动能

As the stunt person falls into the rig with the motorcycle each box he collides with will collapse and absorb a small amount of his kinetic energy, thereby slowing his descent.

- Upon collision with a box, the box crushes and absorbs an amount  $\Delta E$  of energy from the stuntman's kinetic energy.
- The crushed box is then pinned against the forward moving face of the stuntman and motorcycle and must move with him. This contributes an additional mass of  $m_{box}$ .

We calculate the change in his velocity using conservation of energy and assuming that the velocity direction remains unchanged (this is a good approximation in the average of a large number of collisions).

$$\frac{1}{2}(m_0 + m_{box})v_{new}^2 = \max\left(\frac{1}{2}m_0v_0^2 - \Delta E, 0\right)$$

and we are taking the maximum here to avoid imparting more energy

纸箱相交时, 这个纸箱就被认为压碎了. 我们根据下一小节中的动能表达式 (见式 (6.1)) 来更新特技演员的速度, 忽略已被压碎纸箱的影响.

- 对于每个被压坏的纸箱, 我们在摩托车边界矩形前部或底部 (分别对应水平, 竖直方向的碰撞) 添加一个厚度. 我们假设纸箱在被水平或竖直碰撞时的长和高被压缩至原来的%20. 我们允许边界矩形 (表示特技人和摩托车的) 在初始尺寸的基础上, 前部厚度层向前或向后延展 (底部厚度层也如此), 因此表示摩托车变成一个更大矩形时, 它的力就会很有效的分布在这个更大的矩形上. 这些厚度层会增加摩托车的有效尺寸, 从而在运动中撞到更多的纸箱. 我们用这个过程解释摩擦力的影响.
- 当摩托车边界矩形撞到地面时, 其速度的竖直分量设为 0.

随着特技演员和摩托车一同飞入纸箱堆中, 它们碰撞的每个纸箱都会吸收掉它们的一小部分动能, 进而减缓他的下降速度.

- 碰撞一个纸箱时, 纸箱损坏的同时, 会从特技演员和摩托车的动能中吸收一份大小为  $\Delta E$  的能量.
- 此后, 被碰撞的纸箱被吸附在特技演员和摩托车前进的一面, 一同继续移动. 这将为系统额外增加质量  $m_{box}$ .

我们利用能量守恒定律来计算特技演员和摩托车的速度变化, 并假设速度方向保持不变 (在大量碰撞时, 这是一个不错的近似平均).

这里我们取最大值, 这是为了避免摩托车给予纸箱



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into the box than the motorcycle has. Solving for  $v_{new}$  yields

能量超过摩托车所拥有的能量. 求解新的速度  $v_{new}$  可得:

$$v_{new} = \sqrt{\max\left(\frac{m_0 v_0^2 - 2\Delta E}{m_0 + m_{box}}, 0\right)} \quad (6.1)$$

We use this equation to calculate the new velocity after each collision.

我们就用此式来计算每次碰撞后的新速度.

## 6.4 Stability and sensitivity analysis / 稳定性与灵敏度分析

Given the crude nature of our collision detection, there is the danger of finding results that depend sensitively on the initial location of the motorcycle relative to the phase of the box rig periodicity (rig periodicity is typically less than 1.5 meters). To show that these phase alignment effects are negligible we vary the initial location of the motorcycle by 0.4 meters (%37 of the rig periodicity) either direction.

**Result:** Deceleration rates and stopping distance vary by less than %5. The simulation is therefore insensitive to where the motorcycle lands relative to the period of the box rig.

As a second check, we vary the time step size from .025 to 0.1 seconds (0.05 is our standard value).

**Result:** No distinguishable changes in results with variation in time step size. This verifies that the simulation is highly insensitive to the size of the time steps.

鉴于我们对碰撞判断的粗略特点, 如果发现模型结果对摩托车的着陆起点与纸箱堆叠周期 (纸箱堆叠周期一般小于 1.5 米) 相位的相对位置非常敏感, 这是很危险的. 为了证明这个相位差效应影响很小, 我们将摩托车的起始着陆位置前后各移动 0.4 米 (为堆叠周期的 37%).

**结果:** 减速度和制动距离的改变都不到 5%. 因此本文的仿真模型对摩托车的着陆起点与纸箱堆叠周期相位的相对位置并不敏感.

作为第二项检验, 我们把时间步长从 0.025 变化到 0.1 秒 (0.05 秒为我们的标准时间步长).

**结果:** 在改变时间步长过程中, 结果没有显著变化. 这证实了本文的仿真模型对时间步长的大小非常不敏感.

## 6.5 Configurations considered / 相关参数的考虑: 运动轨迹, 堆叠方式, 人车质量

We consider the following configurations for the stunt:

- **Three flight trajectories for the motorcycle and stuntman: low, medium, and high.** These provide three different entry angles and velocities for the simulation. Each trajectory is also designed to clear an elephant that is roughly three meters tall [7]. Details of these trajectories are given in table 3 and they are shown to scale in

我们考虑如下配置的特技表演:

- **三种不同的特技演员和摩托车的飞行轨迹: 低, 中和高** 这为仿真提供了三种不同的飞入角度及速度. 每种轨迹都是为一只约 3 米高的大象 [7] 所设计. 更多关于轨迹的细节见表3, 这些轨迹也被呈现在图8中.

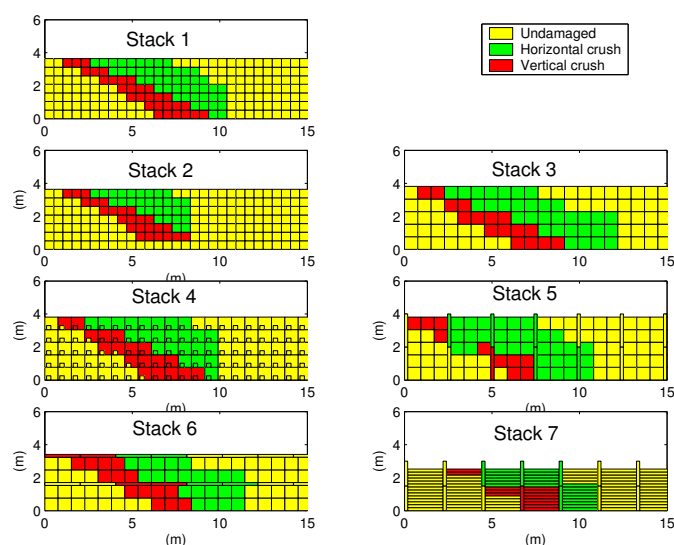


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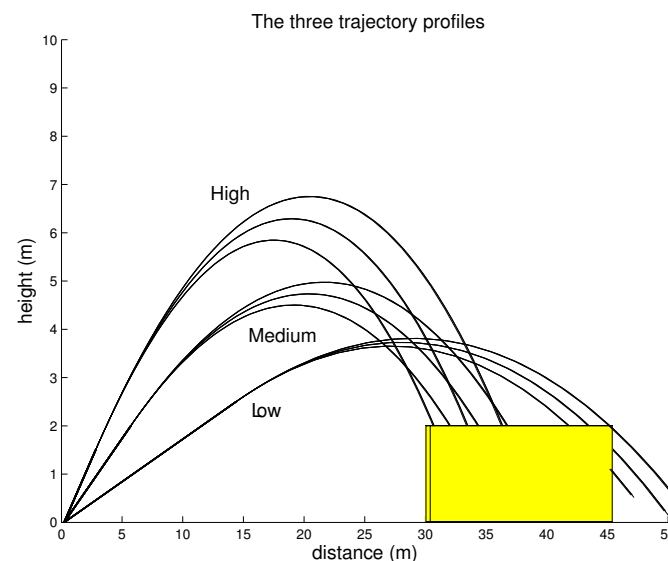
Fig. 8.

- **Seven different stacking arrangements.** Details of these arrangements are shown in Tab. 2 and Fig. 7.
- **Three values for the total mass of the motorcycle and stunt-man:** 200kg, 300kg, and 400kg. These masses are reasonable for mid-range to large motorcycles.



**Figure 7:** Box stacking configurations. Crush patterns are the result of simulated impacts of a 200 kg combined weight coming in from the low trajectory.

- **七种不同的纸箱堆叠方式.** 具体堆叠方式见表2和图7.
- **三种不同的特技演员和摩托车总质量:** 200kg, 300kg, 和 400kg. 这些质量能够合理地对应中型到大型的摩托车.



**Figure 8:** The three trajectory profiles that we examine in our simulations.

## 6.6 Data analysis / 数值分析

The simulation provides us with a record of the velocity as a function of time. These velocity plots appear jagged and step-like because of the discrete way in which our simulation handles collisions. We obtain the acceleration by fitting a straight line to the velocity vs. time plot and measuring the

仿真可以为我们记录下运动的速度 (作为时间的函数) 这些速度的点连线呈现出锯齿, 阶梯状, 这是由于在我们的仿真中是以离散的方式来处理碰撞的. 通过将速度和时间拟合 (线性拟合) 成一条直线并求其斜率, 我们



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**Table 2:** The seven box rig configurations that we examine. See Fig. 7 for illustrations, and refer to Tab. 1 for box type data.

Stack type	Cost (per sq. meter)	Comments
1	\$40.40	Standard rig, made of box type <i>B</i> (20in cube).
2	\$94.20	Standard rig, made of heavy duty box type <i>C</i> (20in cube, ECT 48).
3	\$43.10	Standard rig, made of box type <i>D</i> (30in cube).
4	\$46.5	Like 3, but type <i>A</i> boxes placed inside the <i>D</i> boxes.
5	\$46.3	Modification of 3—additional vertical walls of type <i>F</i> mattress boxes.
6	\$40.90	Like 5, but horizontal mattress box walls.
7	\$46.1	Mattress boxes (type <i>F</i> ) stacked horizontally, with periodic vertical walls

**Table 3:** We simulate the stunt with the following three different trajectories.

Jump type	Initial velocity	Ramp angle	Jump distance
Low	29 m/s	10°	30.0 meters
Medium	22 m/s	20°	28.5 meters
High	20 m/s	30°	30.4 meters

slope. An example of this is shown in Fig. 9.

In examining the velocity plots for each simulation run, we looked at:

1. Deceleration experience by the stuntman, averaged over the *entire* time

可以得到加速度大小. 图9显示了这样一个例子.

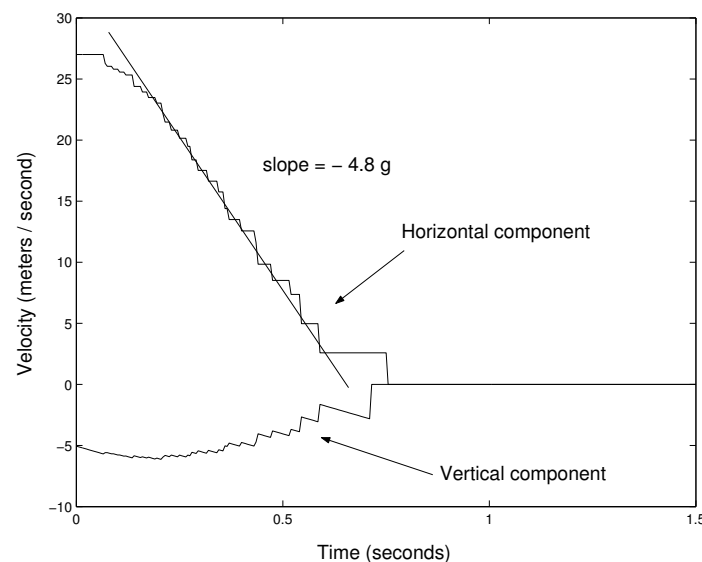
在研究每次仿真的速度点连线图时, 我们关注:

1. 特技演员从碰撞到静止整个时间内所承受的减速



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**Figure 9:** Velocity vs time; plotted for 200 kg low trajectory impact on box stack type 1 (20 inch cubes, stacked in standard style).

from impact to rest.

2. Maximum of deceleration averaged over 0.1 second intervals.
3. Whether or not the boxes completely arrested vertical motion before the stuntman hit the ground.

If either (1) or (2) ever exceeds the maximum safe deceleration threshold of 5 g, we consider that configuration to be unsafe. When condition (3) is not met then the stuntman may experience a severe and dangerous jolt as he strikes the ground. This is bad, but we will propose some work arounds shortly.

## 6.7 Results / 结果

**Regarding the mass,** 400 kg is too heavy—the boxes give way beneath the incoming motorcycle too easily. It would require a stack of boxes nearly

度平均值.

2. 减速度在以 0.1 秒为间隔取平均时的最大值.
3. 演员触及地面前, 纸箱是否已完全制动了竖直方向的运动.

如果 (1) 或 (2) 明显超过了最大安全加速度 5g, 我们认为该方案是不安全的. 如果条件 (3) 不满足, 演员在落地时可能承受强烈, 危险的冲击. 这显然不妙, 不过我们马上会提出相关的解决方案.

**关于总质量,** 400kg 太沉: 纸箱堆很容易就会给迎面而来的摩托车让路. 这时将需要一个近 6 米高的纸箱堆



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6 meters high to safely catch this hot potato. 300 kg is marginal, and 200 kg is optimal for using a box rig.

**Stacking types:** (refer to Fig. 7 for illustration)

1. Made from the cheapest and most common boxes, this stack resulted in 4.8 g deceleration, which is within safety margins (but just barely). It stopped the motorcycle in 11 meters<sup>†</sup>.
  2. Rejected because is resulted in deceleration of over 6 g, but brings the motorcycle to rest<sup>†</sup> in only 7 meters.
  3. Very soft deceleration of about 3.6 to 4.1 g. The only problem is that this stack did not completely stop the vertical motion. Also, it took 13 meters to bring the motorcycle to rest<sup>†</sup>.
  4. Marginally safe deceleration from 4.8 to 5.1 g, but this stack is the best at arresting the vertical motion. Stopping distance of 9 meters<sup>†</sup>.
  5. Behavior is similar to type (3), but stopping distance is reduced by 2 meters to 11<sup>†</sup>.
  6. The extra horizontal mattress boxes make very little difference. Deceleration is 4.1 g, and vertical motion is not slowed enough to prevent hitting the ground hard.
  7. Rejected because deceleration at 5.2 to 5.7 is considered unsafe.
- (†: Stopping distances are reported for the medium trajectory and are measured from the point of impact to the furthest box damaged. The motorcycle actually comes to rest in a significantly shorter distance, but it pushes a wall of debris several meters ahead of it.)

才能接住这个烫手的山芋. 300kg 则是一个临界值, 而 200kg 就是最佳的选择了.

**堆叠种类:** (一一对应于图 7)

1. 由最便宜, 最常用的纸箱组成, 这个纸条堆能使减速度降至 4.8g, 这是在安全临界范围内 (但也是勉强安全而已). 它让摩托车在 11 米<sup>†</sup>处停止.
  2. 不可用, 因为这个纸条堆导致减速度超过了 6g, 但它却能让摩托车仅在 7 米<sup>†</sup>处就停止了.
  3. 提供了非常柔和的减速度, 在 3.6 到 4.1g 之间. 它唯一的问题在于: 纸箱堆没能完全制动竖直方向的运动. 同时, 它还需要 13 米<sup>†</sup>制动距离才能让车停止.
  4. 减速度在 4.8 到 5.1g 之间, 处于安全临界值的附近, 但是此纸箱堆能最好地阻止竖直方向的运动. 它的制动距离为 9 米<sup>†</sup>.
  5. 表现情况与第 (3) 种相似, 但是制动距离短了 2 米, 为 11 米<sup>†</sup>.
  6. 水平方向额外增加的垫式纸箱几乎没有什么贡献. 减速度为 4.1g, 同时竖直方向减速度太小, 以至于不能阻止摩托车撞击到地面.
  7. 不可用, 因为减速度达到了 5.2 到 5.7g, 不安全.
- (†: 以上报告中的制动距离是针对中型运动轨迹的而言的, 制动距离测量是从撞击点到最远被损坏纸箱的距离. 实际上, 摩托车显然会在更短制动距离内静止, 但它还是会推倒了前方几米内的一些纸箱.)



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- **Hypothesis:** the difficulty of slowing the vertical motion enough might be overcome by stacking the box rig on top of a landing ramp.
- **Conclusion:** These results indicate that type (1) stacking is optimal without a ramp. However, with a landing ramp under the boxes, type (3) or type (5) stacking may be used to achieve a much softer deceleration.

In light of these results, we tried additional variations on the type (5) stack. We conclude that the 30 inch boxes (type *D*) with mattress box walls spaced every 4 boxes is optimal.

- **假设:** 充分减缓竖直方向运动的困难, 也许能通过将纸箱堆堆在一个着陆斜坡上来解决.
- **结论:** 以上这些结果表明: 在没有着陆斜坡的情况下类型 (1) 的纸箱堆是最优的. 然而, 若将纸箱堆一个着陆斜坡上, 类型 (3) 或 (5) 也许就获得更缓和的减速度了.

鉴于以上结果, 我们尝试在第 (5) 种纸箱堆上基础上进行一些改动. 我们得出: 用 30 英寸的纸箱 (种类 *D*), 每隔 4 个纸箱竖直放置一个垫式纸箱是最佳的选择.

## 7 An alternative idea: the stuntman could bail out mid-flight / 一种可能情形: 人车空中分离

So far our goal in this model has been to safely decelerate the total combined mass of the stuntman and his motorcycle. However, it is possible that they may separate before impacting the box rig. In fact, it may even be desirable for this separation to occur because it would reduce the chance of injury resulting from the stuntman being pinned against the motorcycle. We would therefore like to model how far apart the stunt person and the motorcycle could land. We assume they separate after clearing the elephant and allowing for a clear camera shot. This corresponds to a distance of about 25 meters. We then run the same simulation as before but alter the vertical velocity at the point of separation and then follow separately the two different trajectories. An estimate of the change of momentum is necessary to figure out the corresponding changes in velocity. If the stuntman jumps vertically away from the motorcycle, it makes sense to consider the analogy of a person jumping on the ground. A decent jump corresponds to about half a meter. Since the initial velocity is given by  $v_0 = \sqrt{2gd}$  where  $d$  is the height, we find that  $v_0$  is roughly 3 m/s. Accordingly, we increase the stuntman's vertical velocity by 3 m/s. Then the corresponding change in velocity for the motorcycle is given by conservation of momentum. The resulting trajectories are plotted in figure 10.

When the trajectory is medium or high, stuntman and motorcycle are only separated by about six meters at the point of landing in the cardboard

到目前为止, 我们的目标都是要安全地让特技演员和摩托车这个整体系统减速下来. 然而, 也有可能是一种可能是特技演员和摩托车在碰撞到纸箱堆之前就分离了. 事实上, 我们甚至希望这样的分离发生, 因为它能避免特技演员被他身下的摩托车压伤的可能, 从而减少演员受伤的几率. 因此, 我们希望能够模拟出特技演员和摩托车在着陆时能够分离多远. 假设特技演员和摩托车在飞过大象, 并拍完要拍的影片后允许分离. 这大概对应着 25 米的位置 (水平方向离起跳点 25m). 然后我们运行之前的仿真程序, 但在特技演员和摩托车分离点处改变竖直方向速度, 然后特技演员和摩托车沿着两条不同的轨迹分离开来. 这里有必要估算一下动量的变化量, 来计算出相应的速度变化. 如果特技演员从摩托车上竖直跳出, 类比考虑人在地面上跳起的情形. 正常跳跃大约有半米高. 由于初速度由公式给出  $v_0 = \sqrt{2gd}$ , 其中  $d$  为跳跃高度, 从而我们推导出  $v_0$  大约为 3m/s. 因此, 我们将特技演员的在竖直方向上的速度增加 3m/s. 同时根据动量守恒得出摩托车相应的速度的变化量. 这个过程的运动轨迹显示在图10中.

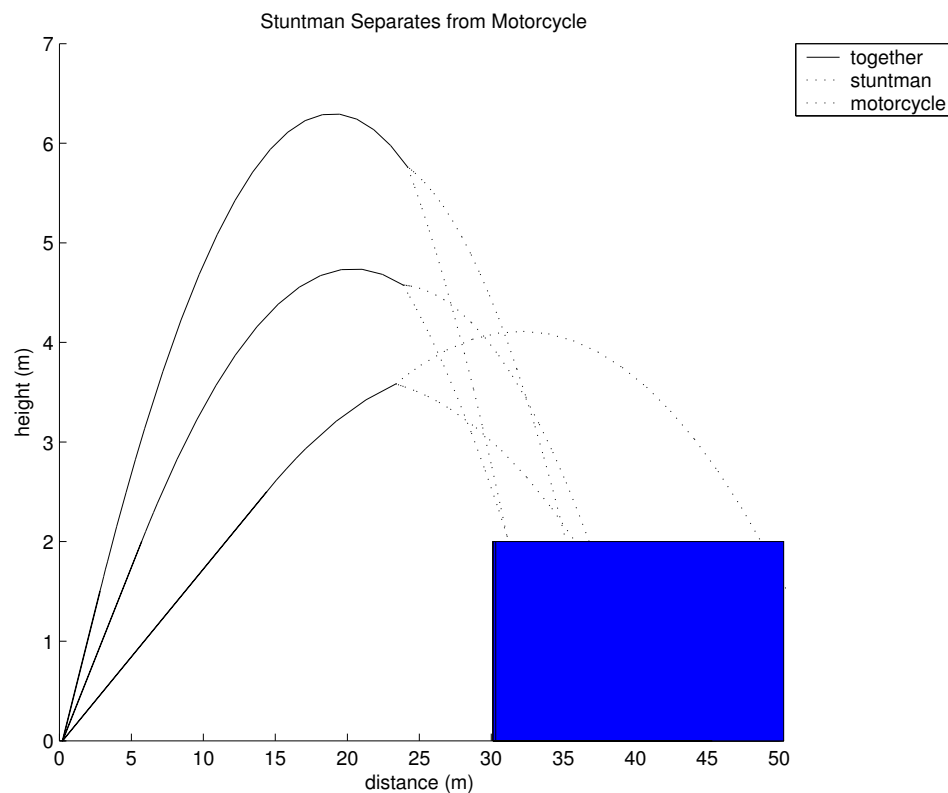
当飞行轨迹为中型或高型时, 特技演员和摩托车在着陆纸箱堆时的距离相差只有 6 米. 然而, 当轨迹为低



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**Figure 10:** stuntman separating from motorcycle in three possible trajectories.

boxes. When the trajectory is low, however, this separation increases to around 15 meters. This presents a problem if we want to protect both the motorcycle and the stuntman. Naturally, the safety of the person is the most important. It is simple to extend the box rig to the projected landing location of the stuntman. Unfortunately, simulations show that a box configuration designed to smoothly decelerate the combined mass of motorcycle and stuntman doesn't work as well when there is just the mass of the person to contend with. In fact, it's possible that the stuntman will decelerate so

型时, 这个分离距离就会增加到 15 米左右. 如果我们要同时保护好摩托车和特技演员, 这就会摆出一个问题. 当然, 人的安全是最重要的. 要将纸箱堆延伸到特技演员着陆的位置是非常容易的. 不过非常不幸的是, 仿真结果显示: 一个能够使特技演员和摩托车整体质量平稳地减速的堆叠设计, 当只制动一个人的质量时表现的并没有之前好. 事实上, 这可能会使演员过快地减速而超出我们的重力加速度安全容限. 我们的仿真显示: 这种



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quickly that our g-force tolerance is exceeded. Our simulations show that this is in fact the case for *all* of the box stacks that we considered. For the heights and speeds considered, a box rig is unsafe. However, if the boxes are stacked loosely enough with some spacing between the boxes as in figure 11, then it is still possible to decelerate the stuntman at a reasonable rate.

Therefore the best solution for the safety of the stuntman is to re-design the box rig, using a softer material and/or looser stacking, so that it accounts for his mass alone if he does indeed intend to separate from the motorcycle.

不安全的现象在所有我们考虑的纸箱堆叠类型都会出现。从高度和速度上说，纸箱堆是不安全的。但是，倘若这些纸箱是足够稀疏地排列，纸箱间留出一定空隙，像图11所示那样，那么就仍有可能使特技演员以一个安全的减速度制动。

因此，保护演员安全最好的办法，就是重新设计纸箱堆，这时要考虑用更加柔软的材料及/或更稀疏的堆叠方式，这样就能将演员确实要跳出摩托车时，只有单人的质量时的情形考虑进去了。

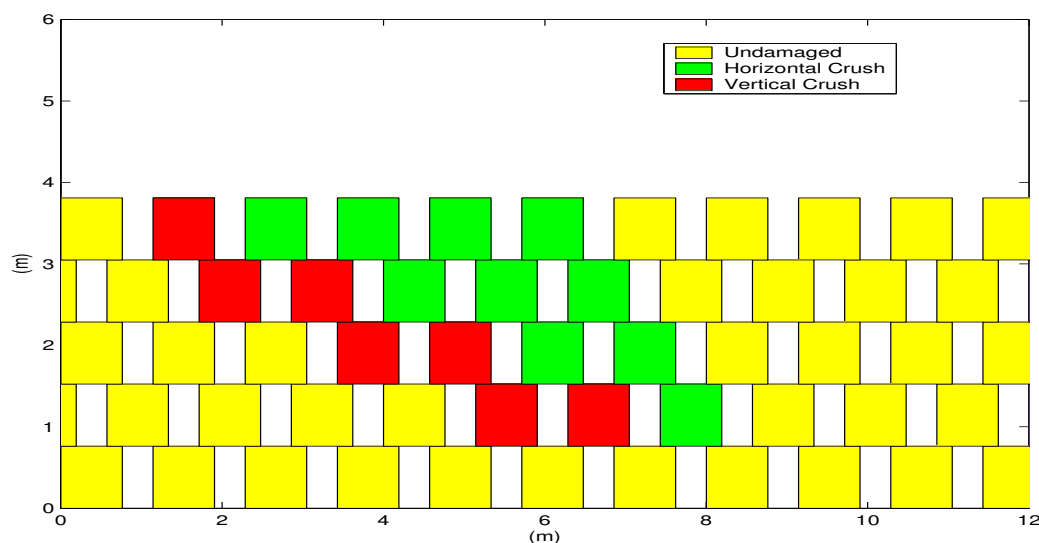


Figure 11: Box stacking arrangement that is suitable for catching a stuntman who is *not* on a motorcycle.

## 8 Recommendations / 建议

- **Which mass is best?** Our simulations show that a 400 kg mass is simply too heavy to be adequately slowed by a box stack less than 4
- **哪个质量最好？** 我们的仿真结果显示 400kg 的人车质量属于过重，以至于低于 4 米高的纸箱堆都



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meters high. The motorcycle invariably falls through the rig and hits the ground beneath at over 5 m/s. While this may not seem like much, the motorcycle could easily tumble over in the boxes and crush the stuntman along with the cardboard. The 300 kg mass was marginal, but the safest was 200 kg. Therefore, if there is any choice in filming the scene, we strongly recommend the use of a lighter motorcycle.

- **Which trajectory is best?** The low trajectory ( $10^\circ$ ) provides the least risk of coming down too hard. However, it allows only minimal clearance over the elephant (only 1 meter for a tall elephant) and requires the highest speed to make the jump successfully, which increases the risk.
- **Which type of boxes and stacking is best?** The type (1) stack, made of (20 inch)<sup>3</sup> boxes, is best for landing without a ramp. With a ramp under the rig, type (3), made from (30 inch)<sup>3</sup> boxes, and type (5), which is type (3) with added vertical mattress box walls, are optimal. The added walls of type (5) decreased the landing distance by 2 meters, so fewer boxes are required and the construction cost is reduced.
- **What size must the rig be?** With the 200 kg mass our simulation shows that 3 meters height are usually enough for the low trajectory, but 4 are necessary for the high trajectory. This can be reduced to as little as 2 if the rig is stacked on top of a landing ramp. Stopping distance is between 10 and 13 meters (as measured from point of entry to the front of the debris wall) depending on stack type, and we estimate in §6 from the trajectory variability calculation that the landing location uncertainty is 1 meter laterally and 3 meters forwards or backwards. We consider an additional %50 beyond these uncertainties to be necessary to suitably ensure safety. Therefore our recommendations are:
  - Height should be 4 meters without landing ramp, 2 meters with ramp.
  - Width should be 4 meters.
  - Length should 24 meters for type (1) or (5) stacking, and 29 meters for type (3) stacking.

难以制动. 摩托车总是穿过纸箱堆, 并以大于 5 的速度撞击在地面. 然而这还不是全部, 摩托车很容易会在纸箱堆中滚动, 撞到演员和硬纸板. 300kg 是临界安全值, 而 200kg 才是最安全. 因此, 在拍这一个特技时可以选择的话, 我们强烈推荐使用质量轻些的摩托车.

- **哪种轨迹最佳?** 低型轨迹 ( $10^\circ$ ) 提供了一种避免过硬着陆风险最低的方法. 然而, 它能在飞越大象时离大象上方的空隙是最小的 (对一个高大的大象, 仅有 1 米的空隙), 同时需要保证成功跳跃的速度又是最大的, 而这意味着风险的增加.
- **哪种纸箱类型, 堆叠方式最佳?** 由 20 立方英寸纸箱组成的类型 (1) 纸箱堆, 是无斜坡情况下最利于着陆的选择. 纸箱堆下是斜坡时, 由 30 立方英寸纸箱构成的类型 (3), 以及类型 (5) (类型 (3) 中添加了竖直的垫式纸箱) 是最优选择. 而类型 (5) 中添加的气垫墙将制动距离减少了 2 米, 因此只需要更少的纸箱了, 同时成本花费也可降低.
- **纸箱堆尺寸应该多大?** 以人车总质量 200kg, 我们的模拟显示: 纸箱堆 3 米高对于低型轨迹通常是足够的, 但对高型轨迹就必须有 4 米. 高度最小能减小到 2 米, 此时要求我们将纸箱堆放置在着陆斜坡上. 制动距离取决于堆叠的类型, 在 10 到 13 米之间 (测量从飞入的点至被压碎纸箱前端的距离), 同时我们在 §6 中从轨迹的变化中估算出, 着陆点的不确定范围为: 侧向 1 米, 前后 3 米. 我们认为有必要再多增加 50% 不确定范围, 以确保万无一失. 因此我们建议:
  - 无着陆斜坡时纸箱堆高应为 4 米, 有着陆斜坡时为 2 米.
  - 纸箱堆宽为 4 米.
  - 对于堆叠类型 (1) 或 (5), 纸箱堆长为 24 米; 对于堆叠类型 (3), 纸箱堆长为 29 米.



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- **How much does it cost?** The cost is between \$4300 for type (1) and \$5300, depending on the precise configuration. Note that this is approximately the same as the cost of renting an airbag rig for a day [1].
- **How many boxes is that?** Type (1) stack requires 2000 (20 inch)<sup>3</sup> boxes, and type (3) requires 1100 (30 inch)<sup>3</sup> boxes. Type (5) uses the same number as (3) and a few additional mattress boxes.

**Final recommendation:** The overall best type of box rig to use is (5)–(30 inch)<sup>3</sup> boxes stacked as usual, with vertical mattress box walls every couple meters to distribute the forces over a larger number of boxes. This configuration results in the softest deceleration while still effectively stopping the stuntman and motorcycle. It also requires the fewest boxes, so the cost is lower and the setup time is minimized. This configuration is shown in detail in Fig. 12

- **花费如何?** 堆叠类型 (1) 的花费在 4300 到 5300 美元之间, 取决于具体的堆叠结构。值得注意的是这大约相当于租借安全气囊一天的花费 [1].
- **共要使用多少个纸箱?** 堆叠类型 (1) 需要 2000 个 20 立方英寸的纸箱, 类型 (3) 需要 1100 个 30 立方英寸的纸箱。类型 (5) 只比 (3) 额外多一些垫型纸箱。

**最终建议:** 在所有堆叠方式当中, 最佳选择是堆叠类型 (5)-用 30 立方英寸的纸箱来堆, 同时每隔两米放置一个竖直垫式纸箱, 将力分散给大量纸箱。这个堆叠方式能以最柔和的减速, 同时依旧能有效地制动特技演员和摩托车。另外, 它所需要的纸箱数量最小, 因此花销也较低, 堆叠时间也很短。该堆叠方式的具体布局见图12.

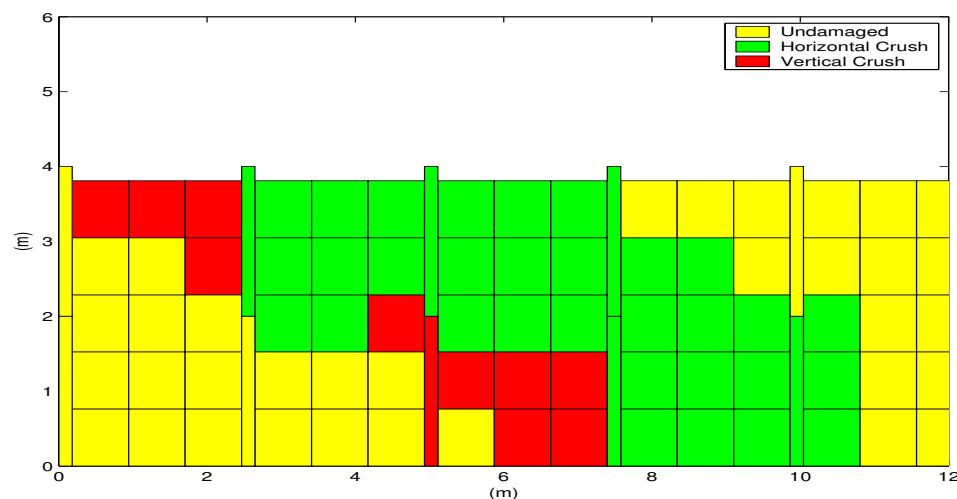


Figure 12: The best way to stack the boxes.



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