Finding out how different baking chemicals react under certain conditions by linear regression model

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Code and data supporting this analysis is available: https://github.com/QiushuZhou/Sta304-Final-Project.git

Abstract

This study is based on a 2^3 factorial experiment, and the goal of this study is to explore the relationship between the predictor variables baking chemicals, vinegar, water temperature and the estimation variable, the height of the foam produced by acid-base chemical reaction. The corresponding analysis is based on a linear regression model. This study shows that the types of baking chemicals, vinegar, water temperature are all valuable predictors for foaming reaction.

Keywords

baking chemical, factorial design, linear regression, interactions, main effect, cube plot

Introduction

As everyone knows, pancake is a very classical pastry and a lot of people consider pancakes as their fast breakfast. The traditional pancake recipe includes water, one type of base (baking soda) and one type of acid (normally buttermilk). The basic concept is that using the foam reaction between base and acid to make the structure of pancakes fluffy and soft. Nowadays, we have a more modern recipe which uses baking powder instead of adding acid and base separately. The second recipe seems to be easier, however, I am still curious to know that if baking powder can make foam reaction as good as baking soda, and how the difference between them when they react with acid and water. (for baking power, it is extra acid) Therefore, the purpose of this experiment is to find out what are the effects of two baking chemicals, vinegar(acid), and water temperature on foaming reaction.

Table 1: An Overview of the Design in this Study

Factors	Level 1	Level 2
chemical	baking soda(+1)	baking powder(-1)
vinegar	add(+1)	not add(-1)
water temperature	hot(+1)	cold(-1)

Data

To complete this experiment, I will collect heights of the foam caused by chemical reaction while varying baking chemicals, vinegar, and water temperature at two different levels each. The two different types of baking chemicals provided in this experiment are: baking soda and baking powder. Simultaneously, I will randomly add or not add vinegar in each run. There will be eight different measurements in total and two repetitions in each of the measurements, thus, it is a 2^3 factorial design with 16 observations in total.

The source of the data is from me since I completed this experiment on my own. The following table shows

the heights of the foam in sixteen different runs, and this is basically my dataset. If Quercus accepts csv. and mp4. file, I will also upload my dataset there with a video which records the entire process of my experiment. Otherwise, I will try to upload them onto GitHub.

Table 2: The Dataset/My Experiment Results

Foam Height	Chemical	Vinegar	Water_Temperature
3.1	1	1	1
0.6	1	1	-1
0	1	-1	1
0	1	-1	-1
0.7	-1	1	1
0.3	-1	1	-1
0.2	-1	-1	1
0	-1	-1	-1
3.2	1	1	1
0.6	1	1	-1
0	1	-1	1
0	1	-1	-1
0.8	-1	1	1
0.3	-1	1	-1
0.1	-1	-1	1
0	-1	-1	-1

Model

Statistical Analysis Plan

To complete the analysis of this factorial experiment, I will firstly set up a linear regression model to determine how significant each variable is by analyzing the p-values:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

, where y represents the outcome variable which is the height of foam; x1 = chemical = baking powder or baking soda; x2 = vinegar = add or not; x3 = water temperature = cold or hot

The function that I used to run this model is lm() function in RStudio.

After setting up this model, three interaction plots will be provided as well, for analyzing the relationship between variables.

In addition, I will use a cube plot to find out the main effect of each variable.

Results

From the linear regression output (Figure 1) we get the fitted model is:

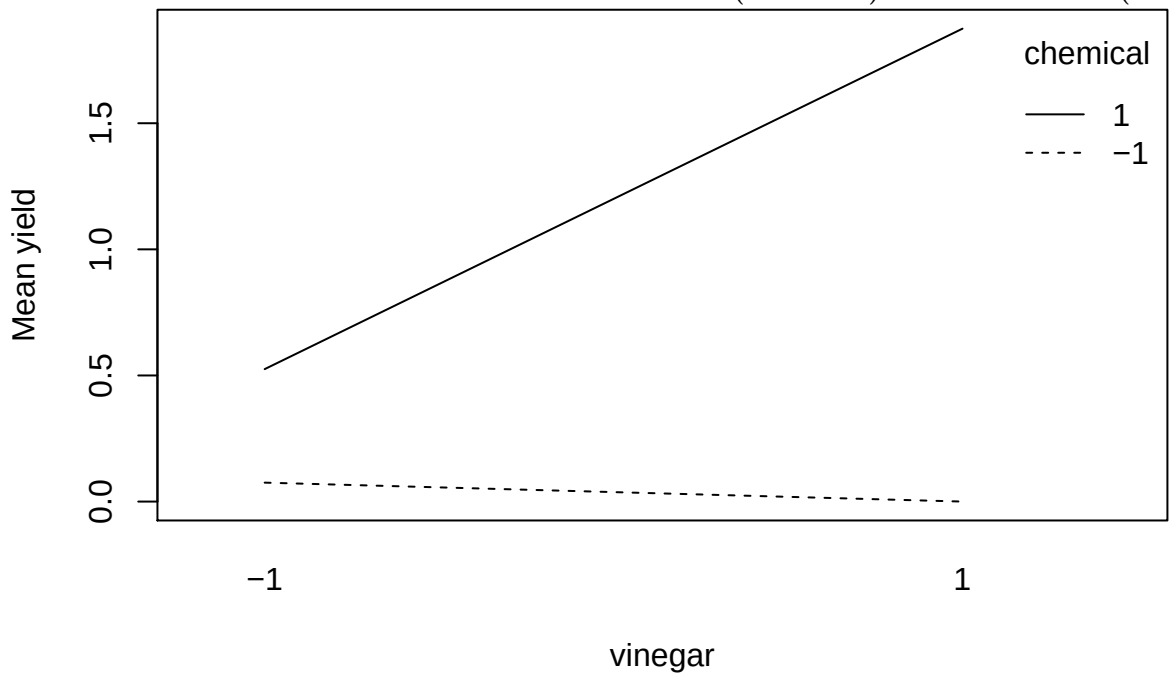
$$y = 0.619 + 0.319 x_1 + 0.581 x_2 + 0.394 x_3$$

Figure 1: Linear Regression Output from R

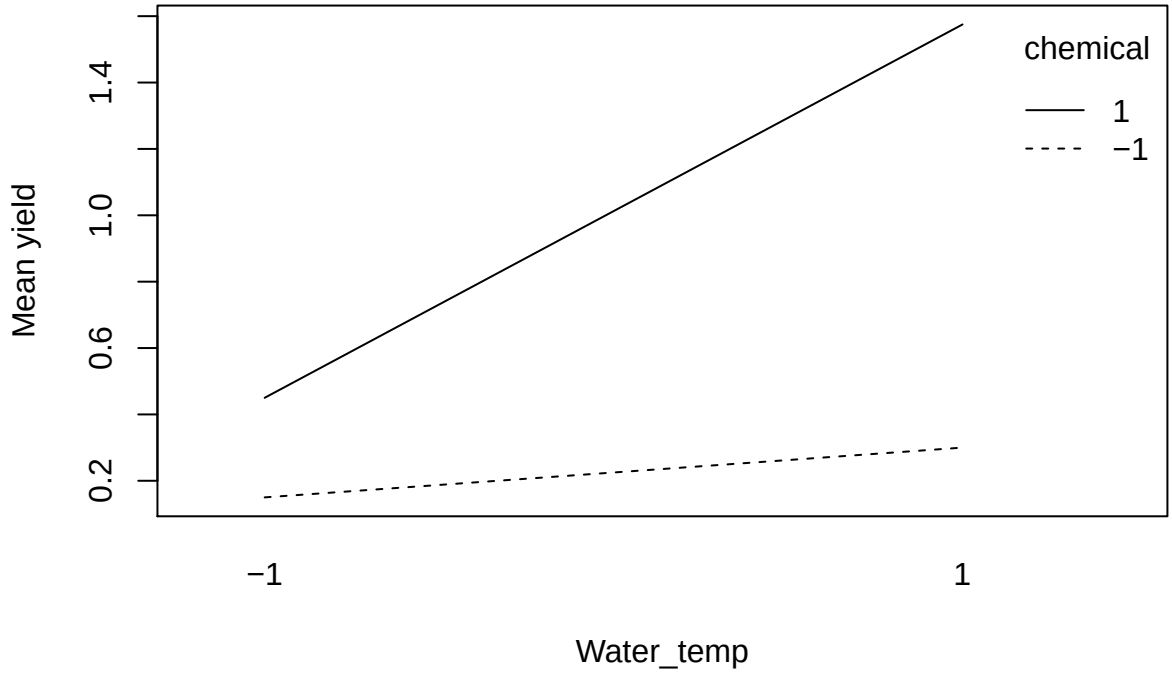
```
## # A tibble: 8 x 5
##   term                              estimate std.error statistic  p.value
##   <chr>                                <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)                          0.619    0.0108      57.2 9.75e-12
```

```
## 2 chemical                          0.319    0.0108      29.4 1.92e- 9
## 3 vinegar                           0.581    0.0108      53.7 1.61e-11
## 4 water_temperature                 0.394    0.0108      36.4 3.58e-10
## 5 chemical:vinegar                  0.356    0.0108      32.9 7.93e-10
## 6 chemical:water_temperature        0.244    0.0108      22.5 1.60e- 8
## 7 vinegar:water_temperature         0.356    0.0108      32.9 7.93e-10
## 8 chemical:vinegar:water_temperature 0.281   0.0108      26.0 5.17e- 9
```

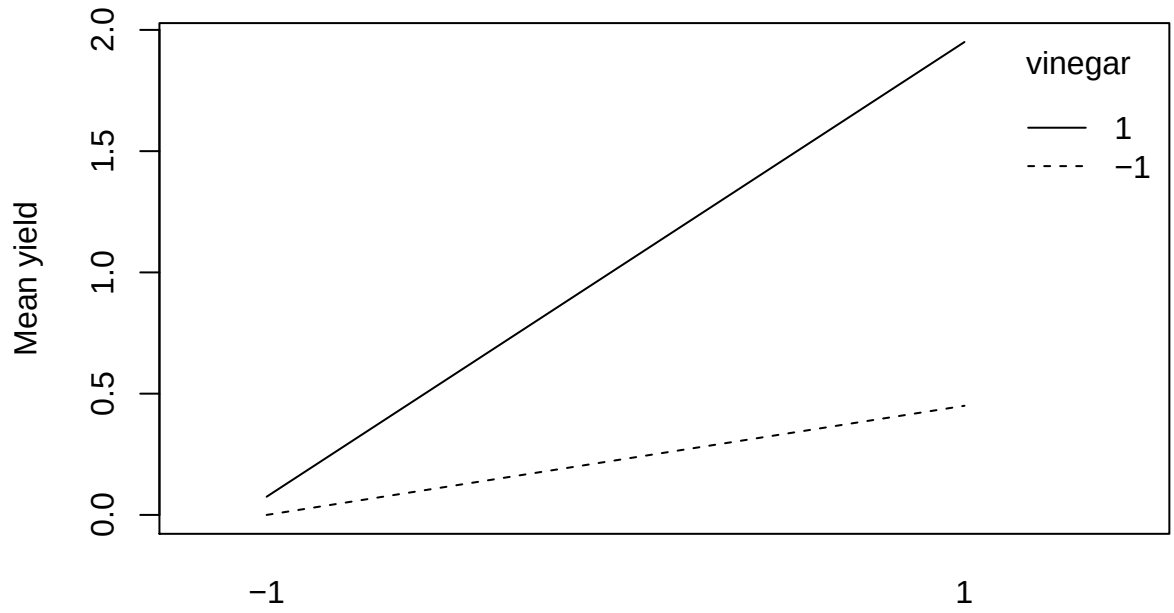
Plot 1: The Interaction Plot between Variable1 (Chemical) and Variable2 (Vinegar)

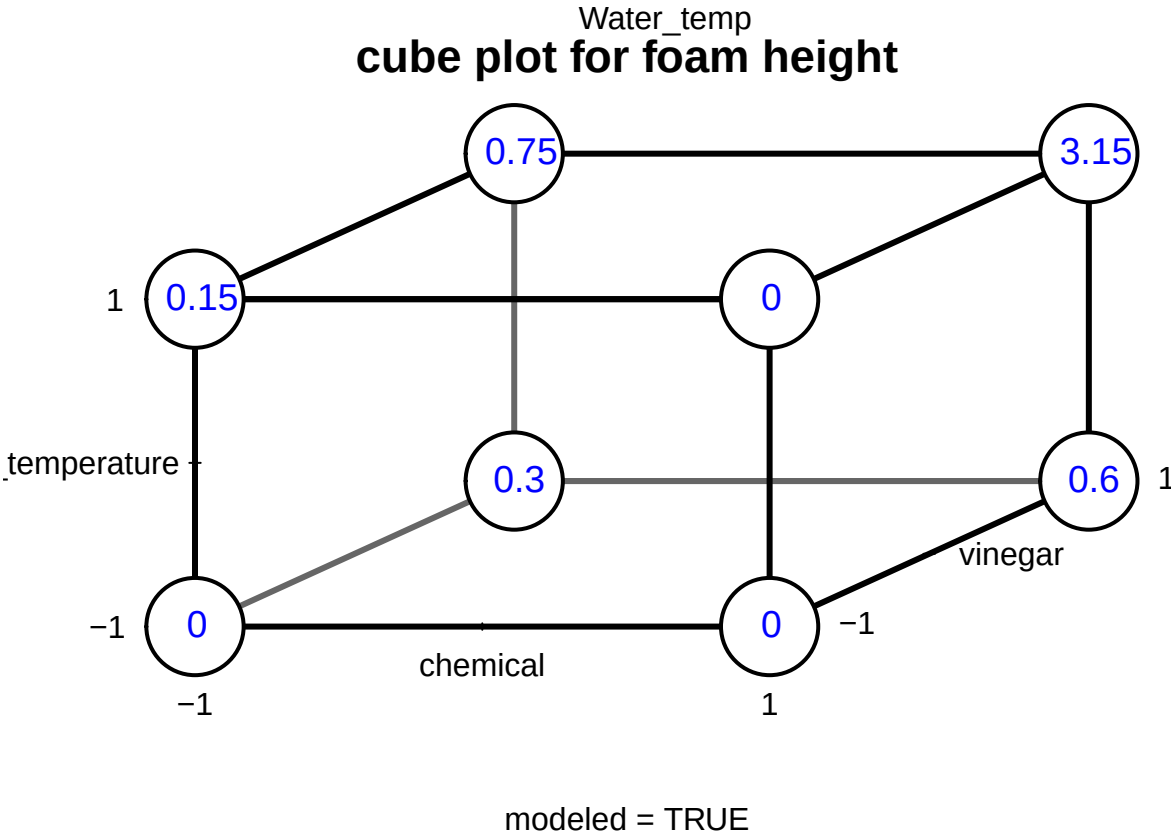


Plot 2: The Interaction Plot between Variable1 (Chemical) and Variable3 (Water_temperature)



Plot 3: The Interaction Plot between Variable2 (Vinegar) and Variable3 (Water_temperature)





Plot 4: Cube Plot for Foam Height

Main effect and Interactions:

Table 3: The Main Effects and Interactions Obtained by Multiplying the Regression Coeffcicents by 2

Factors	Effect
chemical	0.6375
vinegar	1.1625
water_temperature	0.7875
chemical:vinegar	0.7125
chemical:water_temperature	0.4875
vinegar:water temperature	0.7125
chemical:vinegar:water_temperature	0.5625

Confidence Interval:

Figure 2: 95% CI for true values of effects

```
##                                       2.5 %    97.5 %
## (Intercept)                       1.1875735 1.2874265
## chemical                          0.5875735 0.6874265
## vinegar                           1.1125735 1.2124265
## water_temperature                 0.7375735 0.8374265
## chemical:vinegar                  0.6625735 0.7624265
## chemical:water_temperature        0.4375735 0.5374265
## vinegar:water_temperature         0.6625735 0.7624265
## chemical:vinegar:water_temperature 0.5125735 0.6124265
```

Discussion

Interpretations of main effects and interactions:

0.6375 : The foam height increases by 0.6375 if the added chemical is baking soda. The p-value for estimated coefficient of chemical is less than 0.05, so we reject H0, and there is enough evidence that the adding baking soda affected the height of foaming reaction positively. 1.1625 : The p-value for estimated coefficient of vinegar is less than 0.05, so we reject H0, and there is enough evidence that the adding vinegar affected the height of foaming reaction positively. 0.7875 : The p-value for estimated coefficient of water temperature is less than 0.05, so we reject H0, and there is enough evidence that the adding hot water affected the height of foaming reaction positively. All p-values for the interaction effects were very small as well. Also, according to the interactions plots we drew, there's no parallel lines. Therefore, we can reject H0, and there is enough evidence of interaction among the effects of chemicals, vinegar and water temperature. The effects of chemicals, vinegar and water temperature on the heigth of foaming reaction are dependent of each other.

Estimated variance:

The estimated variance of each of 16 observation is

$$RSE^2 = s^2 = 0.0433^2 = 0.00187$$

The standard error for all factorial effects is the same, which is

$$0.01083 * 2 = 0.02166$$

And then the estimated variance is

$$0.02166^2 = 0.000469$$

Estimated variance of effect:

$$V(effect) = (\frac{1}{8} + \frac{1}{8})s^2 = 0.25(0.0433^2) = 0.000469$$

Comments on CI:

All 95% CIs we got above for all main effects and interaction effects do not include zero. This means that there is enought evidence to say baking soda, vinegar and hot water would influence the height of foaming reaction, and all factors are interactive with each other.

Conclusion:

I conducted an experiment to look for factors that influence the height of foaming reaction by using a 2^3 factorial design with a replication. From the data analysis, we know that baking chemicals, vinegar, and water temperature all have positive estimated coefficients and small p-values. So there is enough evidence to say that adding baking soda, adding vinegar(acid) and high temperature will improve the height of the foam reaction. Adding baking soda can raise 0.6375 scores in foam's height on average, adding vinegar can raise 1.1625 scores in foam's height on average, and high temperature water can raise 0.7875 scores in foam's height on average. Also, the effects of baking chemicals, vinegar, and water temperature are dependent of each other due to the interaction analysis we did above. From this factorial, I know that the combination of baking soda, acid, and hot water can make really high foam reaction, but it's also hard to control the dosage and amount. So for lightening the texture of baking goods, baking powder is more preffered as a kitchen leavening agent. Since this is a home-made experiment, thus there may be some errors in accuracy of measurements. For future purpose, replicating each run can possibly increase accuracy.

Reference

RStudio Team (2020). RStudio: Integrated Development for R. RStudio, PBC, Boston, MA URL http://www.rstudio.com/.