

1

a) Prove $\Pr(A, B | K) = \Pr(A | B, K) \Pr(B | K)$

$$\Pr(A, B, K) = \Pr(A, B | K) * \Pr(K)$$

$$\begin{aligned} \Pr(A, B, K) &= \Pr(A | B, K) * \Pr(B, K) && \text{since } \Pr(B, K) = \Pr(B | K) * \Pr(K) \\ &= \Pr(A | B, K) * \Pr(B | K) * \Pr(K) \end{aligned}$$

So $\Pr(A, B | K) * \Pr(K) = \Pr(A | B, K) * \Pr(B | K) * \Pr(K)$ divide both sides by $\Pr(K)$
Then I proved $\Pr(A, B | K) = \Pr(A | B, K) \Pr(B | K)$

b) Prove $\Pr(A, B | K) = \Pr(B | A, K) \Pr(A | K) / \Pr(B | K)$

$$\Pr(A, B, K) = \Pr(A | B, K) * \Pr(B, K) = \Pr(A | B, K) * \Pr(B | K) * \Pr(K)$$

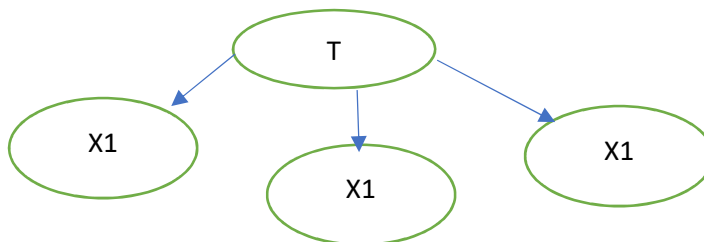
$$\Pr(A, B, K) = \Pr(B | A, K) * \Pr(A, K) = \Pr(B | A, K) * \Pr(A | K) * \Pr(K)$$

So $\Pr(A | B, K) * \Pr(B | K) * \Pr(K) = \Pr(B | A, K) * \Pr(A | K) * \Pr(K)$
I divide both sides by $\Pr(K)$ and $\Pr(B | K)$

Then I proved $\Pr(A, B | K) = \Pr(B | A, K) \Pr(A | K) / \Pr(B | K)$

2

We assume a random variable T denoting which coin {a, b, c} we drew



The CPT for the A is

T	P (T)
a	1/3
b	1/3
c	1/3

Since the X_i ($i = 1, 2, 3$) are independent from each other. The probability to get heads are the same for $i = 1, 2, 3$. The CPT for X_i give T is

T	X_i	P(T)
a	Heads	0.2
b	Heads	0.6
c	heads	0.8

3

black	square	one	Pr
1	1	1	2/13
1	1	0	4/13
1	0	1	1/13
1	0	0	2/13
0	1	1	1/13
0	1	0	1/13
0	0	1	1/13
0	0	0	1/13

$$A1: \Pr(\text{black}) = 2/13 + 4/13 + 1/13 + 2/13 = 9/13 = 0.692$$

$$A2: \Pr(\text{square}) = 2/13 + 4/13 + 1/13 + 1/13 = 8/13 = 0.615$$

$$A3: \Pr(\text{square}, (\text{one} \vee \text{black}) / \Pr(\text{one} \vee \text{black}) = (7/13) / (11/13) = 7/11$$

For the two sentences

$$A = \text{one}, \beta = \text{square}, \gamma = \text{black} \quad \Pr(\text{one} | \text{black}) = 1/3 \quad \Pr(\text{one} | \text{black}, \text{square}) = 1/3$$

$\Pr(\text{one} | \text{black}) = \Pr(\text{one} | \text{black}, \text{square})$ So it is Independent

$$A = \text{one}, \beta = \text{square}, \gamma = \neg\text{black} \quad \Pr(\text{one} | \neg\text{black}) = 1/3 \quad \Pr(\text{one} | \neg\text{black}, \text{square}) = 1/3$$

$\Pr(\text{one} | \neg\text{black}) = \Pr(\text{one} | \neg\text{black}, \text{square})$ So it is Independent

4

a)

$I(A, \emptyset, \{B, E\})$

$I(B, \emptyset, \{A, C\})$

$I(C, A, \{B, D, E\})$

$I(D, \{A, B\}, \{C, E\})$

$I(E, B, \{A, C, D, F, G\})$

$I(F, \{C, D\}, \{A, B, E\})$

$I(G, F, \{A, B, C, D, E, H\})$

$I(H, \{E, F\}, \{A, B, C, D, G\})$

b)

False: because path ACFHE is open.

False: because path GFHE is open.

False: because path BEH is open.

c)

$$P(a,b,c,d,e,f,g,h) = P(a|b,c,d,e,f,g,h)P(b|c,d,e,f,g,h)P(c|d,e,f,g,h)P(d|e,f,g,h)P(e|f,g,h)P(f|g,h)P(g|h)P(h)$$

d)

A and B are independent

$$P(A = 0, B = 0) = P(A = 0) * P(B = 0) = 0.8 * 0.3 = 0.24$$

A and E are independent

$$P(E = 1 | A = 1) = P(E = 1)$$

$$P(E = 1) = P(E = 1 | B = 0)P(B = 0) + P(E = 1 | B = 1)P(B = 1) = 0.9*0.3 + 0.1*0.7 = 0.34$$