

Qiusi Shen 004749315

1. $P \Rightarrow \neg Q, Q \Rightarrow \neg P$

P	Q	$\neg Q$	$\neg P$	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	T
F	F	T	T	T	T

From the last two columns $P \Rightarrow \neg Q, Q \Rightarrow \neg P$ are equivalent

$P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$	$P \Leftrightarrow \neg Q$
T	T	F	F	F	F	F	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	F	F	F

From the last two columns $P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent

2

In this question, use S for smoke, F for fire, H for heat

$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$

S	F	$\neg S$	$\neg F$	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	F	T	T	T	F	F

From the last column, I can see the statement is true under certain circumstances. So, the statement is Neither.

$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$

S	F	H	$S \vee H$	$S \Rightarrow F$	$(S \vee H) \Rightarrow F$	$(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
T	T	F	T	T	T	T
T	F	T	T	F	F	T
F	T	T	T	T	T	T
F	F	F	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	F	F

From the last column, I can see the statement is true under certain circumstances. So, the statement is Neither.

$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$

S	H	F	$S \wedge H$	$S \Rightarrow F$	$H \Rightarrow F$	$((S \Rightarrow F) \vee (H \Rightarrow F))$	$(S \wedge H) \Rightarrow F$	Statement
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

From the last column, I can see the statement is always true.

So, the statement is Valid.

3

In this question use Y for Mythical, I for Immortal, L for Mammal, H for Horned, G for Magical

a) Knowledge base

$$Y \Rightarrow I$$

$$\neg Y \Rightarrow (\neg I \wedge L)$$

$$(I \vee L) \Rightarrow H$$

$$H \Rightarrow G$$

b) Convert to CNF

$$Y \Rightarrow I \rightarrow \neg Y \vee I$$

$$\neg Y \Rightarrow (\neg I \wedge L) \rightarrow Y \vee (\neg I \wedge L) \rightarrow (Y \vee \neg I) \wedge (Y \vee L)$$

$$(I \vee L) \Rightarrow H \rightarrow \neg (I \vee L) \vee H \rightarrow (\neg I \wedge \neg L) \vee H \rightarrow (H \vee \neg I) \wedge (H \vee \neg L)$$

$$H \Rightarrow G \rightarrow \neg H \vee G$$

So, all the CNF are

1) $\neg Y \vee I$

2) $Y \vee \neg I$

3) $Y \vee L$

4) $H \vee \neg I$

5) $H \vee \neg L$

6) $\neg H \vee G$

c) Find if the unicorn is Mythical? Add $\neg Y$ to the knowledge base

7) $\neg Y$

8) L (resolve 3 and 7)

9) $\neg I$ (resolve 2 and 7)

10) H (resolve 5 and 8)

11) G (resolve 6 and 10)

12)

From the knowledge base I have, I can't prove the unicorn is Mythical

Find if the unicorn is Magical? Add $\neg G$ to the knowledge base

7) $\neg G$

8) $\neg H$ (resolve 6 and 7)

9) $\neg L$ (resolve 5 and 8)

10) $\neg I$ (resolve 8 and 4)

11) Y (resolve 3 and 9)

12) I (resolve 1 and 11)

13) contradiction (10 and 12)

So, I proved unicorn is magical.

Find if the unicorn is Horned? Add $\neg H$ to the knowledge base

7) $\neg H$

8) $\neg I$ (resolve 4 and 7)

9) $\neg L$ (resolve 5 and 7)

10) $\neg Y$ (resolve 1 and 8)

11) Y (resolve 3 and 9)

12) contradiction (10 and 11)

So, I proved unicorn is Horned.