A conditional approach for joint estimation of wind speed and direction

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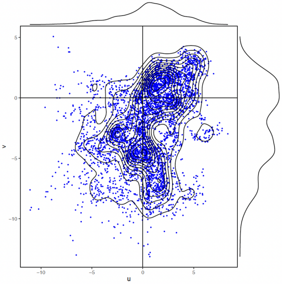
Introduction

How to accurately model wind speed and wind direction?

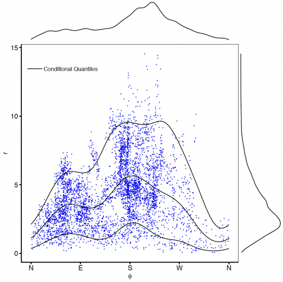
- Weather forecasting
- Aviation
- Renewable energy
- Air quality monitoring

Background and related work

Representation of the wind vector in terms of Cartesian coordinates (u, v) (left) and polar coordinates (r, ϕ) (right) with the corresponding marginal distributions:



(a) (u, v) representation



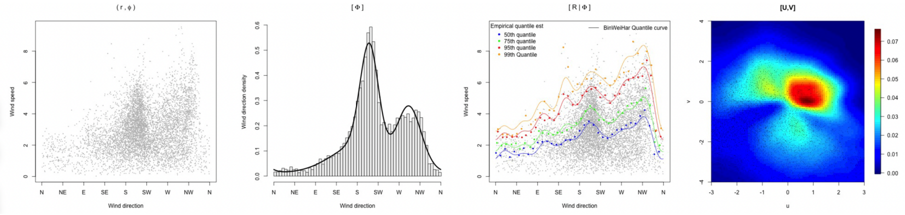
(b) (r, ϕ) representation

- Joint model of Cartesian components with a bivariate normal distribution is not always appropriate.
- Under polar coordinates, most studies only foucus on the wind speed.

Joint Distribution of Wind Speed and Wind Direction

$$[R, \Phi] = [\Phi] \times [R|\Phi]$$

- $[\Phi]$: the distribution of wind direction
- $[R|\Phi]$:the conditional distribution of wind speed given wind direction



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Note

- Estimate joint distribution using the "divide and conquer" strategy
- Avoid the direct modeling of the complex two-dimension in one step
- Achieve flexible modeling of wind speed and wind direction that preserves the intrinsic characteristics of the two variables, namely non-negativity and circularity.

Wind Direction: Von Mises distribution

Model Wind Direction Distribution [Φ]

The wind direction is modeled by a mixture of von Mises distribution:

$$f_\Phi(\phi|\mu,\kappa,\omega) = \sum_{j=1}^{N_\Phi} \omega_j f(\phi;\mu_j,\kappa_j) = \sum_{j=1}^{N_\Phi} \omega_j \frac{1}{2\pi I_0(\kappa_j)} \exp[\kappa_j \cos(\phi-\mu_j)]$$

- ω_j: weights of mixture von mises distribution
- μ_j: directional mean of the distribution
- κ_j: concentration of the distribution

Directional Wind Speed: Weibull Distribution

Model Conditional Distribution of Wind Speed $[R|\Phi]$

Weibull distribution is assumed to allow the parameters of the distribution be smooth yet flexible periodic functions of the wind direction.

$$f_{R|\Phi}(r|\phi,\alpha,\beta) = \left(\frac{\alpha(\phi)}{\beta(\phi)}\right)\left(\frac{r}{\beta(\phi)}\right)^{\alpha(\phi)-1}\exp\left[\left(-\frac{r}{\beta(\phi)}\right)^{\alpha(\phi)}\right]$$

The parameters are modeled by Fourier series:

- $\alpha(\phi) = b_{\alpha,0} + \sum_{k=1}^{K_\alpha}[a_{\alpha,k}\cos(k\phi) + b_{\alpha,k}\sin(k\phi)]$
- $\beta(\phi) = b_{\beta,0} + \sum_{k=1}^{K_\beta}[a_{\beta,k}\cos(k\phi) + b_{\beta,k}\sin(k\phi)]$

Binned Weibull Harmonic Regression (BWHR)

Two-step Procedure

1. Divide the wind direction into N bins, and then fit a Weibull distribution to the wind speed data within each bin via maximum likelihood method to obtain the estimates $\{\hat{\alpha}_j, \hat{\beta}_j\}_{j=1}^{N}$ and their standard error $\{se(\hat{\alpha}_j), se(\hat{\beta}_j)\}_{j=1}^{N}$
2. Estimate $\alpha(\phi)$ and $\beta(\phi)$ using a harmonic regression via WLS
   - $\alpha(\phi) = b_{\alpha,0} + \sum_{k=1}^{K_\alpha}[a_{\alpha,k}\cos(k\phi) + b_{\alpha,k}\sin(k\phi)]$
   - $\beta(\phi) = b_{\beta,0} + \sum_{k=1}^{K_\beta}[a_{\beta,k}\cos(k\phi) + b_{\beta,k}\sin(k\phi)]$

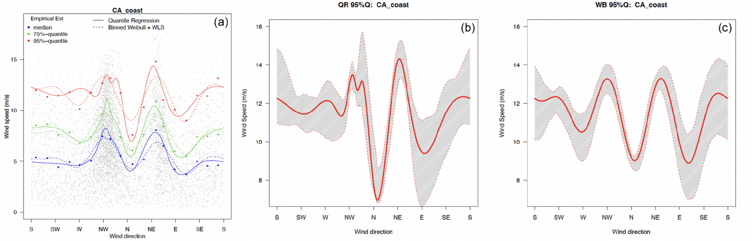
   The estimated standard errors $\{se(\hat{\alpha}_j), se(\hat{\beta}_j)\}_{j=1}^{N}$ are incorporated to enable a weighted version of harmonic regression. More specifically, the weights are the squares of the inverses of the standard errors of the MLEs.

Periodic B-spline Quantile Regression (BPQR)

- Quantile regression is a general method for estimating conditional quantiles of response variable, i.e. $Q_Y(\tau) = F_Y^{-1} = \inf\{y : F(y \geqslant \tau)\}$. One can approximate the underlying conditional distribution by estimating a set of conditional quantile levels.
- In this study, we model the τ-quantile of the directional wind speed distribution, ($[R|\Phi]$) using QR with the directional quantile curve as a periodic B spline.
$$\hat{Q}_{R|\Phi}(\tau|\phi) = B(\phi)^\top \hat{\beta}(\tau)$$
- $B(\phi)$ is a periodic B-spline. It is used to preserve the circular property of the directional wind speed distribution while providing modeling flexibility in terms of quantile functions.

Bootstrap Uncertainty Quantification

- For both BWHR and BPQR, we used bootstrap to quantify the uncertainty in parameter estimation. The block bootstrap is used in order to preserve the interannual temporal dependence presented in the climate application.
- We draw 500 block bootstrap samples, where a block represents one year. The $100 \times (1-\alpha)$ % bootstrap percentile interval is constructed using the $\alpha/2$ upper and lower percentiles.



Performance Matrix

It is important to incorporate wind direction distribution when quantifying the estimation performance.

Weighted Integrated Mean Relative Error (WIMRE)

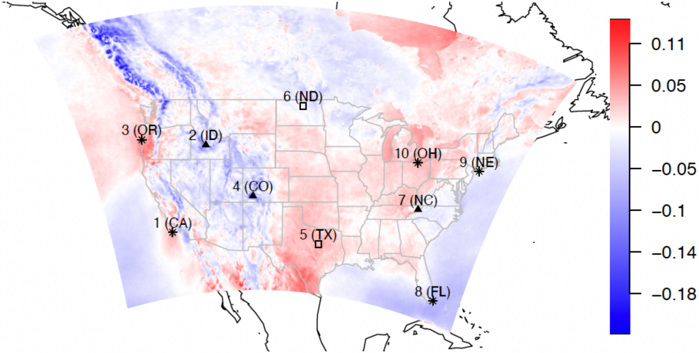
$$\text{WIMRE} = \frac{\int_0^{2\pi} \left|\frac{\hat{g}(\phi)-g(\phi)}{g(\phi)}\right| f(\phi) d\phi}{\int_0^{2\pi} f(\phi) d\phi}$$

- The weight is based on the wind direction density $f(\phi)$
- $g(\phi)$ is the the quantity of interest as a function of direction (e.g.: quantile of wind speed, quantile difference etc.)

Data Sources

Goal

Estimate how the joint distribution of wind speed and direction may change from present to a possible future climate condition.



The map is colored by the ratio of changes in the 10-year mean 95th percentile of the wind speed distribution projected by WRF.

Data Sources

Model

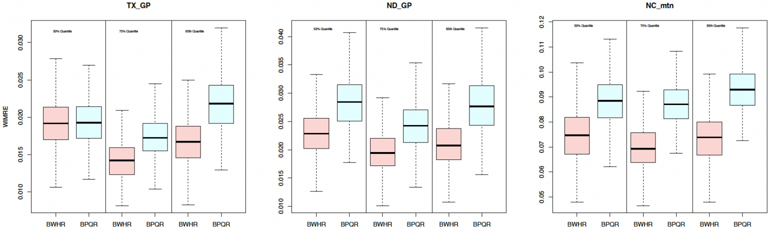
WRF (Weather Research and Forecasting Model): 3-hourly and 12 km temporal and spatial resolution (1995-2004, 2085-2094)

- WRF - GFDL
- WRF - CCSM
- WRF - HadGEM

Benchmark

- NARR (off-shore): 3-hourly and 32 km temporal and spatial resolution (1995-2004)
- NLDAS (inland): 3-hourly and 12 km temporal and spatial resolution (1995-2004)

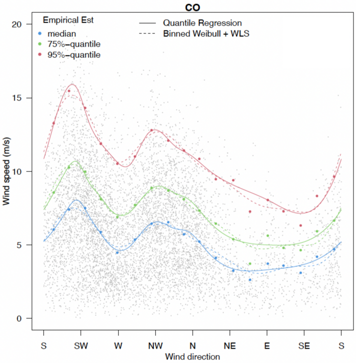
Model Comparison

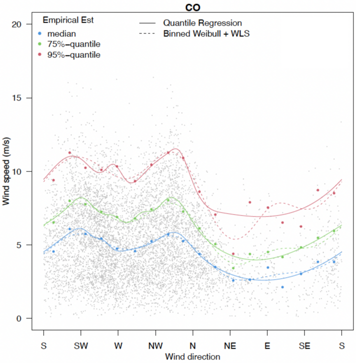


(m/s)	TX_GP		ND_GP		NC_mtn	
	PBQR	BWHR	PBQR	BWHR	PBQR	BWHR
$\text{WIMRE}_{q_\tau=0.95}$	0.022 (0.004)	0.017 (0.003)	0.028 (0.005)	0.021 (0.004)	0.093 (0.009)	0.074 (0.010)
$\text{WIMRE}_{q_\tau=0.75}$	0.017 (0.003)	0.014 (0.003)	0.024 (0.004)	0.020 (0.003)	0.093 (0.009)	0.070 (0.009)
$\text{WIMRE}_{q_\tau=0.50}$	0.020 (0.003)	0.019 (0.003)	0.029 (0.005)	0.023 (0.004)	0.088 (0.010)	0.075 (0.011)

- BWHR method outperforms BPQR method at all the locations and quantile scenarios for estimating the directional wind speed distribution.

Future Projections of Wind Conditions





- The estimated wind speed quantiles (50th, 75th, and 95th) conditioned on direction for WRF-HadGEM outputs at the CO mountain location in winter
- The overall wind speed tends to show a decrease in intensity (across all quantiles) in particular in the dominant direction mode.

Conclusion

Summary and Discussion

- We propose a conditional approach to jointly model wind speed and wind direction
- The method allows us to detect difference in present and future wind speed and direction

Limitation and Future Work

- All the analyses are performed pointwise. In order to assess the regional wind fields and their future predictability, it is critical to explore their spatiotemporal structures in future work.
- Future works may consider the spatial aggregates of different quantities to reduce the noise of climate variability and potentially observe stronger trends.

Thank you for you attention!

1 Wu, Qiuyi, Julie Bessac, Whitney Huang, Jiali Wang, and Rao Kotamarthi. "A conditional approach for joint estimation of wind speed and direction under future climates." Advances in Statistical Climatology, Meteorology and Oceanography 8, no. 2 (2022): 205-224.

2 Murphy, Eva, Whitney Huang, Julie Bessac, Jiali Wang, and Rao Kotamarthi. "Joint modeling of wind speed and wind direction through a conditional approach." arXiv preprint arXiv:2211.13612 (2022).