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# Introduction to Decision Making Studies 02

Balloon Analog Risk Task (BART) 01

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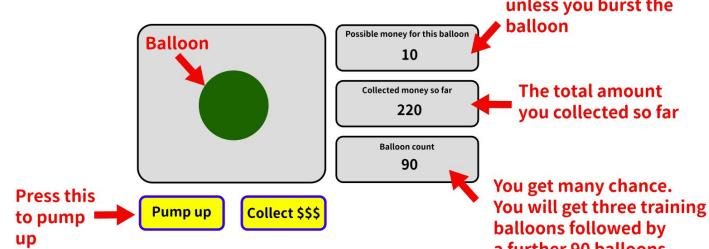
# Balloon Analog Risk Task (BART)

- Measuring risk-taking tendencies and the identification of individuals who are prone to take risks - impulsivity and sensation-seeking
- Participants are **not** informed about the probability of the balloon exploding.
- Typically, the degree of risk-taking on the BART is measured by the **adjusted BART score**, which is the average number of pumps for unexploded balloons. *The adjusted BART score is preferable because it is not directly affected by the explosion probability.*

## Balloon task instructions (page one of two)

In this task, you need to inflate a balloon (shown as a growing circle, here green).

Below you see what the game will look like. If you click the "pump up" button, your balloon will get bigger. The larger the balloon, the more money you can win. But the bigger the balloon gets, the more likely it might burst.



Use mouse or tap

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## Four parameters: $\alpha$ , $\mu$ , $\gamma$ , $T$

**Assumption 1:** Prior probability/belief + update per trial

- update the belief about the **probability (p) of the balloon exploding** after each trial.
- initially has a **prior belief** about the probability of the balloon exploding and **updates** the prior belief based on observation on each trial
- **$\alpha$  and  $\mu$  (RL):** the degree of learning from observations

$$p_k^{burst} = 1 - \frac{\alpha + \sum_{i=0}^{k-1} n_i^{success}}{\mu + \sum_{i=0}^{k-1} n_i^{pumps}} \quad \text{with } 0 < \alpha < \mu$$

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# Four parameters: $\alpha$ , $\mu$ , $\gamma$ , $\tau$

**Assumption 2: Participants decide the optimal number of pumps before each trial (expode)**

- **prospect theory**
- $r$ : is the amount of reward per successful pump
- $\gamma$ : risk-taking propensity

Expected utility (value) after  $l$  pumps on trial  $k$        $U_{kl} = (1 - p_k^{burst})^l (lr)^\gamma$

Optimal number of pumps     $v_k = \frac{-\gamma}{In(1 - p_k^{burst})}$  with  $\gamma \geq 0$

The probability that the participant will pump the:     $p_{kl}^{pump} = \frac{1}{1 + e^{\tau(l - v_k)}}$  with  $\tau \geq 0$

Softmax:     $p(D|\alpha, \mu, \gamma, \tau) = \prod_{k=1}^{k^{last}} \prod_{l=1}^{l^{last}} p_{kl}^{pump} (1 - p_{kl}^{pump})^{k^{last} - l}$

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# Four-parameter Model: reparametrized

Emphasizing the asymmetry between the gain side and the loss side, use rho instead.

$\phi$ : prior belief of success

$\eta$ : updating coefficient

$\rho^+$ : risk preference for gain

$\rho^-$ : risk preference for loss

$\tau$ : inverse temperature

$\lambda$ : loss aversion

$$U_{kl}^{pump} = (1 - p_k^{burst})r^{\rho^+} - p_k^{burst}\lambda \{(l-1)r\}^{\rho^-} \text{ with } 0 < \rho^+, \rho^- < 2, \lambda > 0$$

$$U_{kl}^{transfer} = 0$$

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## Non-learning version (Par3 model): $\theta$ , $\gamma$ , $\tau$

Participants **do not learn** (update their belief of the probability of the balloon exploding) during the BART  
-  $v$  &  $p$  will be fixed value.

$$v = \frac{-\gamma}{In(1 - \theta)} \text{ with } \gamma \geq 0$$

$$p_l^{pump} = \frac{1}{1 + e^{\tau(l-v)}} \text{ with } \tau \geq 0$$

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## Reparametrized version (Par4 model): reparametrizing $\alpha$ & $\mu$

- Reparametrize  $\alpha, \mu \rightarrow \Phi, \eta$  to make each parameter more **independent** and interpretable  $\phi = \frac{\alpha}{\mu} \quad \eta = \frac{1}{\mu}$
- Remove the  $\alpha < \mu$  constraint to enhance the efficiency of Bayesian estimation (especially MCMC)
- $\Phi$ : initial belief that pumping will not make the balloon explode
- $\eta$ : an updating coefficient of the participant's belief by the observed data; =0 -> not affected by the observed data
- The participant determines the optimal number of pumps before each trial may be unjustified: the participant may decide whether to pump the balloon or not just before each pump.

$$p_k^{burst} = 1 - \frac{\phi + \eta \sum_{i=0}^{k-1} n_i^{success}}{1 + \eta \sum_{i=0}^{k-1} n_i^{pumps}} \text{ with } 0 < \phi < 1, \eta > 0$$

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# The exponential-weight model (EW model)

Define:  $\psi = 1 - \phi$

$\psi$ : prior belief of burst

$$p_k^{burst} = \omega_{k-1}\psi + (1 - \omega_{k-1})P_{k-1} \text{ with } 0 < \psi < 1, \eta > 0$$

$\xi$ : updating exponent

$\rho$ : risk preference

$$U_{kl}^{pump} = (1 - p_k^{burst})r^\rho - p_k^{burst} \lambda \{(l-1)r^\rho\}$$

$\tau$ : inverse temperature

$$\text{with } 0 < \rho < 2, \lambda > 0, U_{kl}^{transfer} = 0$$

$\lambda$ : loss aversion

$$p_{kl}^{pump} = \frac{1}{1 + e^{\tau(U_{kl}^{transfer} - U_{kl}^{pump})}} \text{ with } \tau \geq 0$$

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# The Exponential-Weight Mean–Variance model (EWMV model)

- Mean–variance analysis instead of prospect theory
- Subjective utility of an option can be formulated by a linear combination of the expected value and the variance of potential outcomes.
- Risk preference is defined as a **coefficient of the variance term**, which is a proxy for risk.

$$U_{kl}^{pump} = (1 - p_k^{burst})r - p_k^{burst} \lambda(l-1)r + \rho p_k^{burst} (1 - p_k^{burst}) \{r + \lambda(l-1)r\}^2 \text{ with } \lambda > 0, U_{kl}^{transfer} = 0$$



# Reference & Resource

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