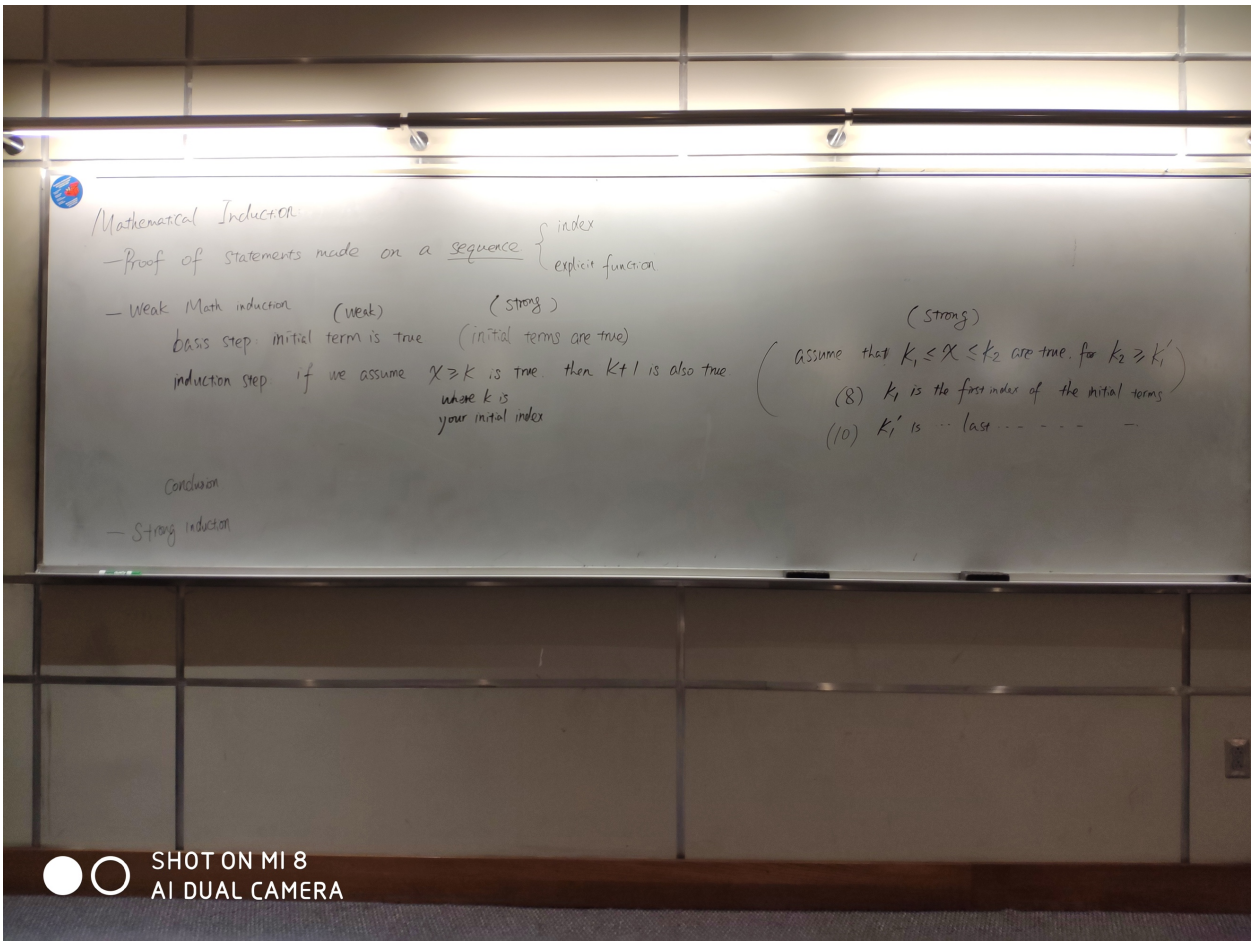
Mathematical Induction
—Proof of statements made on a sequence { index / explicit function
— Weak Math induction (weak) (strong)
basis step: initial term is true (initial terms are true)
induction step: if we assume $x \geq k$ is true then $k+1$ is also true
where k is your initial index
(strong)
(assume that $k_1 \leq x \leq k_2$ are true. for $k_2 \geq k_1'$)
(8) k_1 is the first index of the initial terms
(10) k_1' is ... last ... - - - - -
Conclusion
— Strong induction
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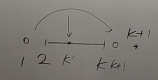
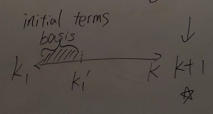
Proposition: for any integer $k \geq 8$, we can obtain k-cents using only 3- and 5-cent coins

Proof:

Basis step: when $k=8$, we can obtain 8 using 3+5.
$k=9$, 9 using 3+3+3
$k=10$, 10 using 5+5

Induction Step: assume that $8 \leq x \leq k$, where $k \geq 10$.
for $k+1$, we know $k-2$ cents can be obtained using 3- and 5-cent coins. So we can get $k+1$ by adding another 3-cent coin





Conclusion

Proposition: Prove that any integer greater than 1 is divisible by a prime number.

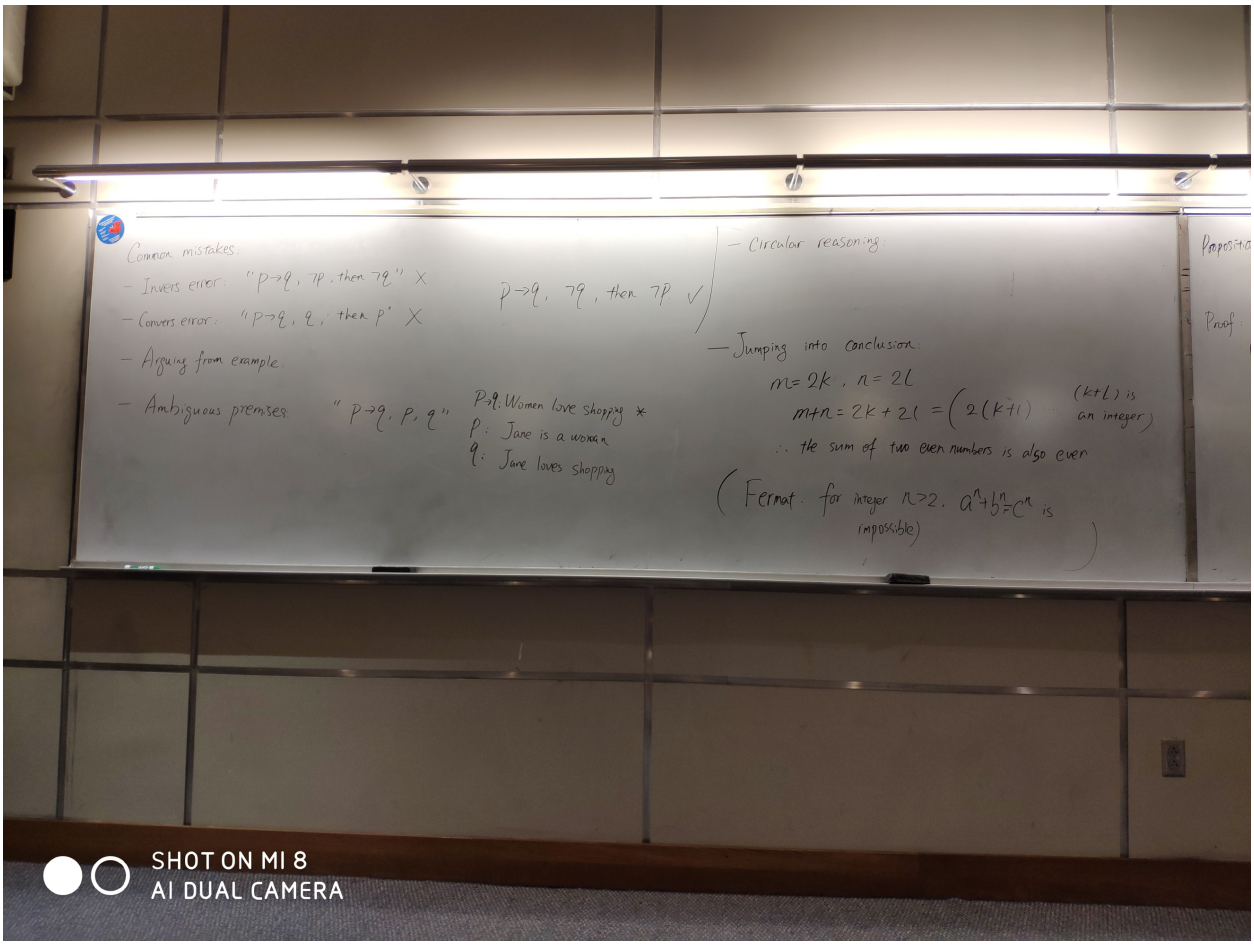
Proof:

Basis step: 2 is divisible by 2, a prime number.

Induction step: assume $2 \leq x \leq k$ is true, for $k \geq 2$
$(k_1 = k_1' = 2)$

To prove that $k+1$ is divisible by a prime

- if $k+1$ is prime, proved
- if $k+1$ is not a prime number, $k+1$ can be divided by some integer that is not 1 nor itself. Let that integer be k'.
$1 < k' < k+1$, so k' is divisible by a prime number, so is $k+1$.


Common mistakes:
- Invers error: "$p \to q$, $\neg p$, then $\neg q$" X
- Convers error: "$p \to q$, q, then p" X
- Arguing from example:
- Ambiguous premises: "$p \to q$, p, q"
$p \to q$, $\neg q$, then $\neg p$ ✓
$P \to q$: Women love shopping *
p: Jane is a woman
q: Jane loves shopping
- Circular reasoning:
- Jumping into conclusion:
$m = 2k$, $n = 2l$
$m+n = 2k + 2l = (2(k+l))$ ($k+l$) is an integer)
$\therefore$ the sum of two even numbers is also even
(Fermat: for integer $n>2$, $a^n+b^n=c^n$ is impossible)
Proposition
Proof:
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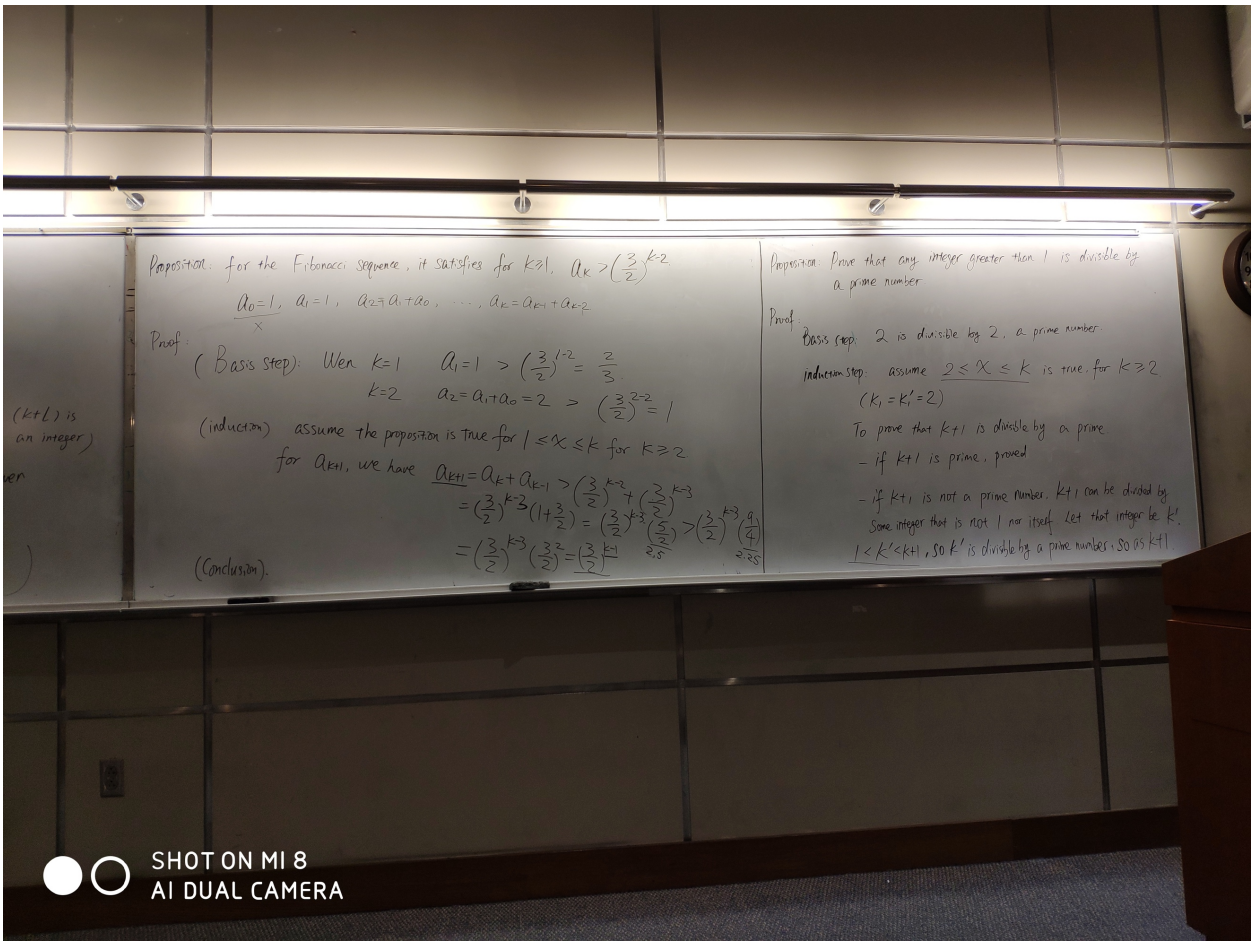
Proposition: for the Fibonacci sequence, it satisfies for $k \geq 1$, $a_k > \left(\frac{3}{2}\right)^{k-2}$
$a_0 = 1$, $a_1 = 1$, $a_2 = a_1 + a_0$, ..., $a_k = a_{k-1} + a_{k-2}$
Proof
(Basis step): When $k=1$ $a_1 = 1 > \left(\frac{3}{2}\right)^{1-2} = \frac{2}{3}$
$k=2$ $a_2 = a_1 + a_0 = 2 > \left(\frac{3}{2}\right)^{2-2} = 1$
(induction) assume the proposition is true for $1 \leq \alpha \leq k$ for $k \geq 2$
for a_{k+1}, we have $a_{k+1} = a_k + a_{k-1} > \left(\frac{3}{2}\right)^{k-2} + \left(\frac{3}{2}\right)^{k-3}$
$= \left(\frac{3}{2}\right)^{k-3}\left(1+\frac{3}{2}\right) = \left(\frac{3}{2}\right)^{k-3}\left(\frac{5}{2}\right) > \left(\frac{3}{2}\right)^{k-3}\left(\frac{9}{4}\right)$
$= \left(\frac{3}{2}\right)^{k-3}\left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^{k+1}$
(Conclusion).
Proposition: Prove that any integer greater than 1 is divisible by a prime number
Proof:
Basis step: 2 is divisible by 2, a prime number.
induction step: assume $2 \leq \alpha \leq k$ is true for $k \geq 2$
($k_1 = k_1' = 2$)
To prove that $k+1$ is divisible by a prime
- if $k+1$ is prime, proved
- if $k+1$ is not a prime number, $k+1$ can be divided by some integer that is not 1 nor itself. Let that integer be k'. $1 < k' < k+1$, so k' is divisible by a prime numbers, so is $k+1$
$(k+1)$ is an integer)
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