HW2: Basic Logic

Q1: How to construct an argument from predicates?
A1: We can construct statements from predicates by either specifying the values of the parameters or applying quantifiers. Once we have the statements, we can construct compound statements through recursive applications of the negation, conjunction, disjunction, or condition operations. Then, we list all relevant statements as premises under a valid argument form, which leads to an argument.

Q2: How to tell if an argument is sound?
A2: An argument is sound when all of its premises are true and the argument form is valid.

Q3: Show the absorption law $p \lor (p \land q) \equiv p$ using a truth table.
A3: Construct the truth table:

p	q	$p \land q$	$p \lor (p \land q)$	p
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F

We see from the last two columns that, for any combinations of the values of the elementary statements p and q, the truth values of $p \lor (p \land q)$ and q are always identical. Therefore, by definition $p \lor (p \land q) \equiv p$.

Q4: Show that the following argument form is valid: $p \lor q$, $p \to r$, and $q \to r$, then r.
A4: Construct the truth table:

p	q	r	$p \lor q$	$p \to r$	$q \to r$	r
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T
T	T	F	T	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	F	T	T	F

We can see from the truth table that there are only three rows (the shaded rows) having all premises being true. And their corresponding conclusion is also true. Therefore, by definition, the argument form $p \lor q$, $p \to r$, and $q \to r$, then r is valid.