Some Useful Laws of Logical Equivalences

Table 6: Basic Logical Equivalences (P.27)

$$p \wedge T \equiv p$$

Identity laws

$$p \lor \mathbf{F} \equiv p$$

$$p \lor T \equiv T$$

Domination laws

$$p \wedge \mathbf{F} \equiv \mathbf{F}$$

$$p \lor p \equiv p$$

Idempotent laws

$$p \wedge p \equiv p$$

$$\neg (\neg p) \equiv p$$

Double negation law (Involution law)

$$p \lor q \equiv q \lor p$$

Commutative laws

$$p \wedge q \equiv q \wedge p$$

$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$

Associative laws

$$(p \land q) \land r \equiv p \land (q \land r)$$

$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

Distributive laws

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$

$$\neg (p \land q) \equiv \neg p \lor \neg q$$

De Morgan's laws

$$\neg (p \lor q) \equiv \neg p \land \neg q$$

$$p \lor (p \land q) \equiv p$$

Absorption laws

$$p \land (p \lor q) \equiv p$$

$$p \lor \neg p \equiv T$$

Negation laws (Complement laws)

$$p \land \neg p \equiv \mathbf{F}$$

Table 7: Logical Equivalences Involving Conditional Statements (P.28)

$$p \rightarrow q \equiv \neg p \lor q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \lor q \equiv \neg p \rightarrow q$$

$$p \land q \equiv \neg (p \rightarrow \neg q)$$

$$\neg (p \rightarrow q) \equiv p \land \neg q$$

$$(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$$

$$(p \rightarrow r) \land (q \rightarrow r) \equiv (p \lor q) \rightarrow r$$

$$(p \rightarrow q) \lor (p \rightarrow r) \equiv p \rightarrow (q \lor r)$$

$$(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r$$

Table 8: Logical Equivalences Involving Biconditional Statements (P.28)

$$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$$

$$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

From "Discrete Mathematics and Its Applications," 7/e, K. Rosen