

Find the closest pair of points



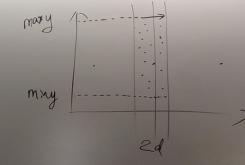
$$d' = \min(d^Q, d^R)$$

(upper bound)

+ any two between-subproblem points that could have a distance shorter than d'
must both locate within the band L .

\Rightarrow We only check all pairs of points within the band.

$$= O(n)$$

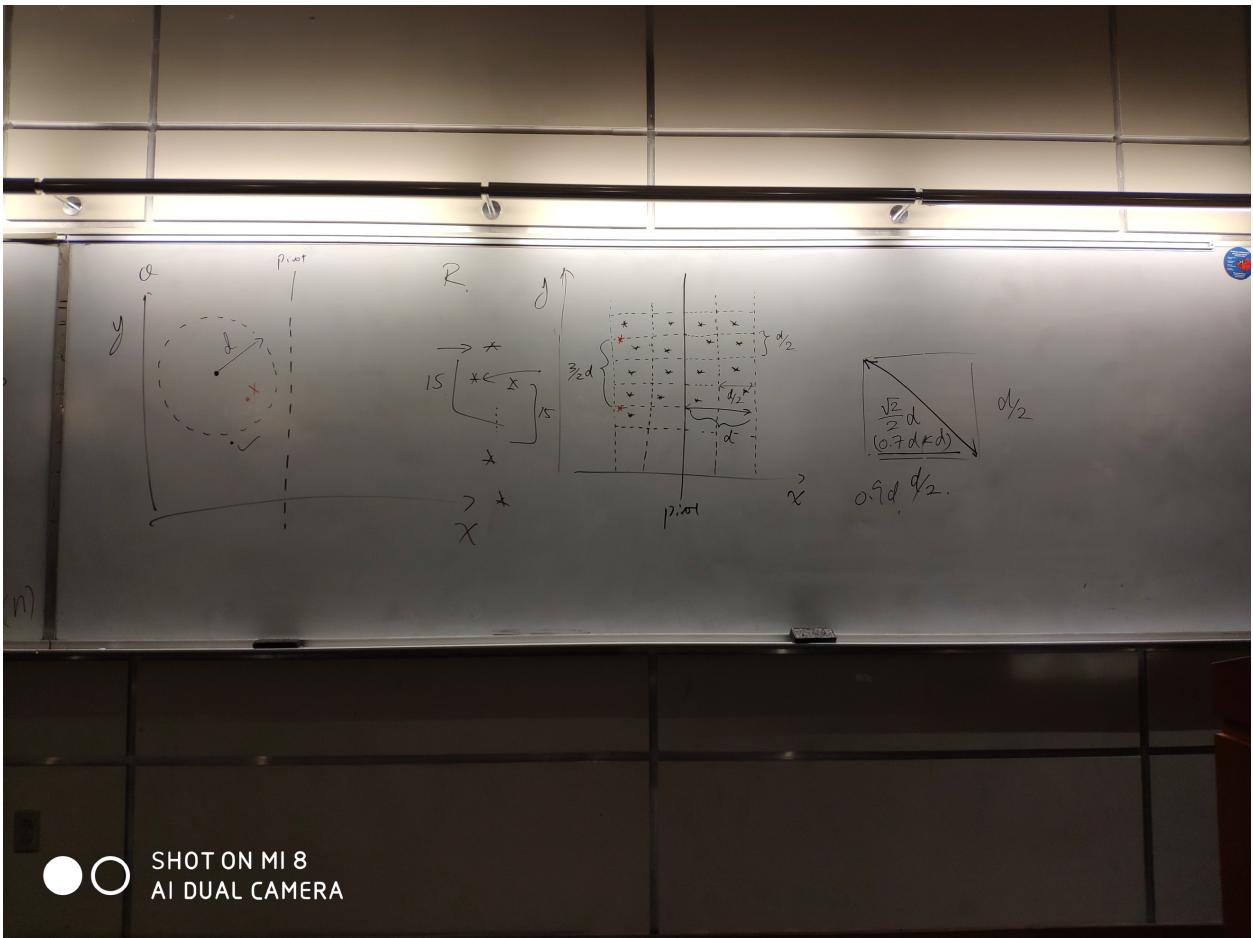


expected $O(n)$

$$T = 2T\left(\frac{n}{2}\right) + ?(n)$$



SHOT ON MI 8
AI DUAL CAMERA



SHOT ON MI 8
AI DUAL CAMERA

$\text{CPP}(P_x, P_y) \quad O(n \log n)$

if $|P_x| \leq 3$.
find the closest pair of points (x, y)
return (x, y)

Construct Q_x, Q_y, R_x, R_y (Linear time $O(n)$)

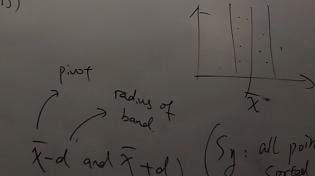
$T\left(\frac{n}{2}\right)$ $d^Q = \text{CPP}(Q_x, Q_y)$

$d^R = \text{CPP}(R_x, R_y)$

$d = \min(d^Q, d^R)$

Construct S_y (Comparing all points in P_y with $\bar{x} - d$ and $\bar{x} + d$) $(S_y: \text{all points within the band and sorted by } y\text{-axis})$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$
$$\Rightarrow O(n \log n)$$



SHOT ON MI 8
AI DUAL CAMERA

```
for i in |Sy| - 15  
    for j in 1 ... 15  
        find distance between Sy[i] and Sy[i+j]  
        if the distance is less than d,  
            record;  
    endfor  
return the recorded min distance.
```

{ O(n)



SHOT ON MI 8
AI DUAL CAMERA

$$X \quad \begin{array}{c} Y_2 \\ \boxed{Y_0 X_0 X_1 \dots} \\ \hline X_0 & X_1 \end{array}$$

$$Y \quad \begin{array}{c} Y_2 \\ \boxed{Y_0 Y_1 Y_2 \dots} \\ \hline Y_0 & Y_1 \end{array}$$

$T(n)$
 $= 9T\left(\frac{n}{3}\right)$
 $\leq +O(n)$

$X_0 = x_0 x_1 \dots x_{\frac{n}{2}}$
 $X_1 = x_{\frac{n}{2}+1} x_{\frac{n}{2}+2} \dots x_n$

y_0
 y_1

$XY \Rightarrow (X_0 * 2^{\frac{n}{2}} + X_1)(Y_0 * 2^{\frac{n}{2}} + Y_1)$
 $= \underline{X_0 Y_0 * 2^{\frac{n}{2}}} + (X_0 Y_1 + X_1 Y_0) * 2^{\frac{n}{2}}$
 $+ \underline{X_1 Y_1}$

$X_0 Y_0 \checkmark$
 $X_0 Y_1$
 $X_1 Y_0$
 $X_1 Y_1 \checkmark$

$= \underline{(X_0 + X_1) * (Y_0 + Y_1)} - \underline{X_0 Y_0} - \underline{X_1 Y_1}$
 $\frac{n}{2} \quad \frac{n}{2} \quad \frac{n}{2}$
 $\frac{n+1}{2}$



SHOT ON MI 8
AI DUAL CAMERA

$$1,234 * 5,678$$

$$= \left(\begin{array}{c} 12 * 10^2 + 34 \\ \times_0 \end{array} \right) \left(\begin{array}{c} 56 * 10^2 + 78 \\ \times_0 \end{array} \right)$$

$$T(n) = 3T\left(\frac{n}{2}\right) + O(n) \Rightarrow O(n^{1.58})$$

$$O(n^2) = O\left(\log_2^3 n\right) \quad T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

2⁶⁴

$$\begin{array}{r} 12345 \quad n \\ 56789 \quad n \\ \hline 00000 \end{array}$$

$$O(1) \Rightarrow O(n^2)$$



SHOT ON MI 8
AI DUAL CAMERA