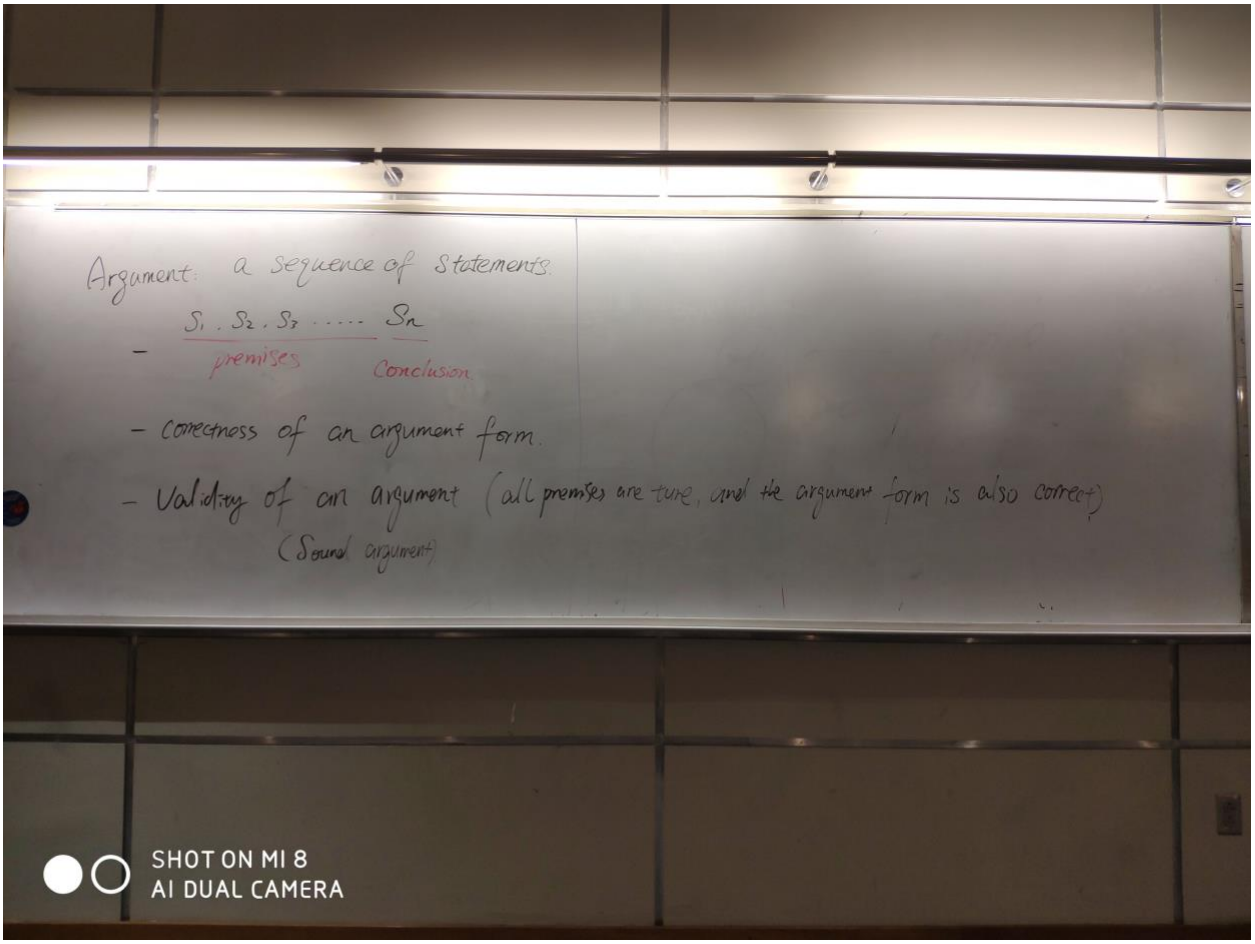
Argument: a sequence of statements.
- $S_1 . S_2 . S_3 . . . . . S_n$
premises
Conclusion
- correctness of an argument form.
- Validity of an argument (all premises are ture, and the argument form is also correct)
(Sound argument)
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Common Proof Techniques.

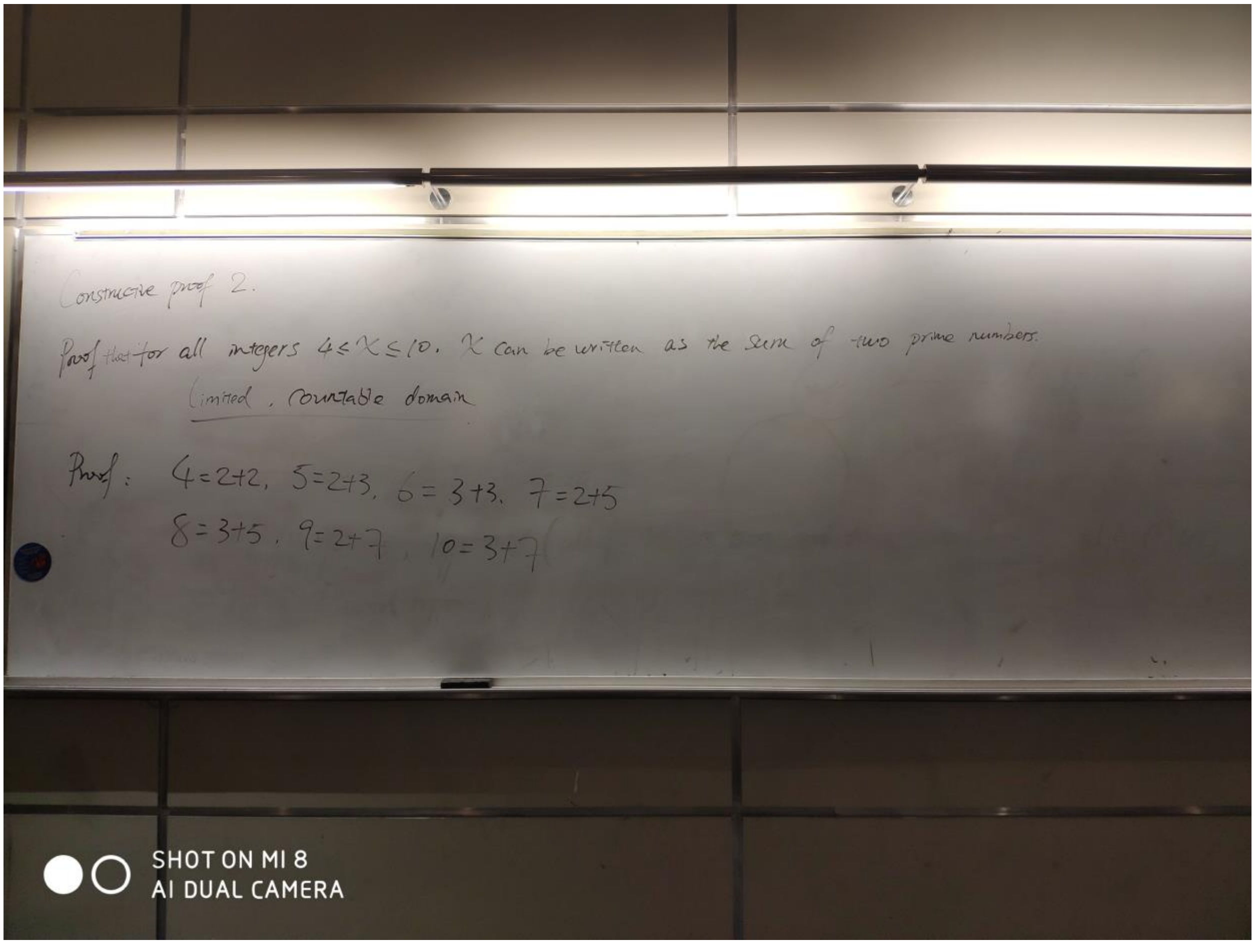
- Direct proof
  - Constructive proof (providing evidences)
    - Show by one example (proving ∃, disproving ∀)
    - proof by exhaustion. (limited, countable domain)
    - Generalization of generic particular (unlimited or uncountable domain)
- non-Constructive proof.

Constructive proof (1).

Proof that "for all integers a and b, if $a^2 = b^2$, then $a = b$" is false.

Proof: a counter example:

$a = 1$, $b = -1$


Constructive proof 2.
Prove that for all integers 4≤x≤10, x can be written as the sum of two prime numbers.
limited, countable domain
Proof: 4=2+2, 5=2+3, 6=3+3, 7=2+5
8=3+5, 9=2+7, 10=3+7
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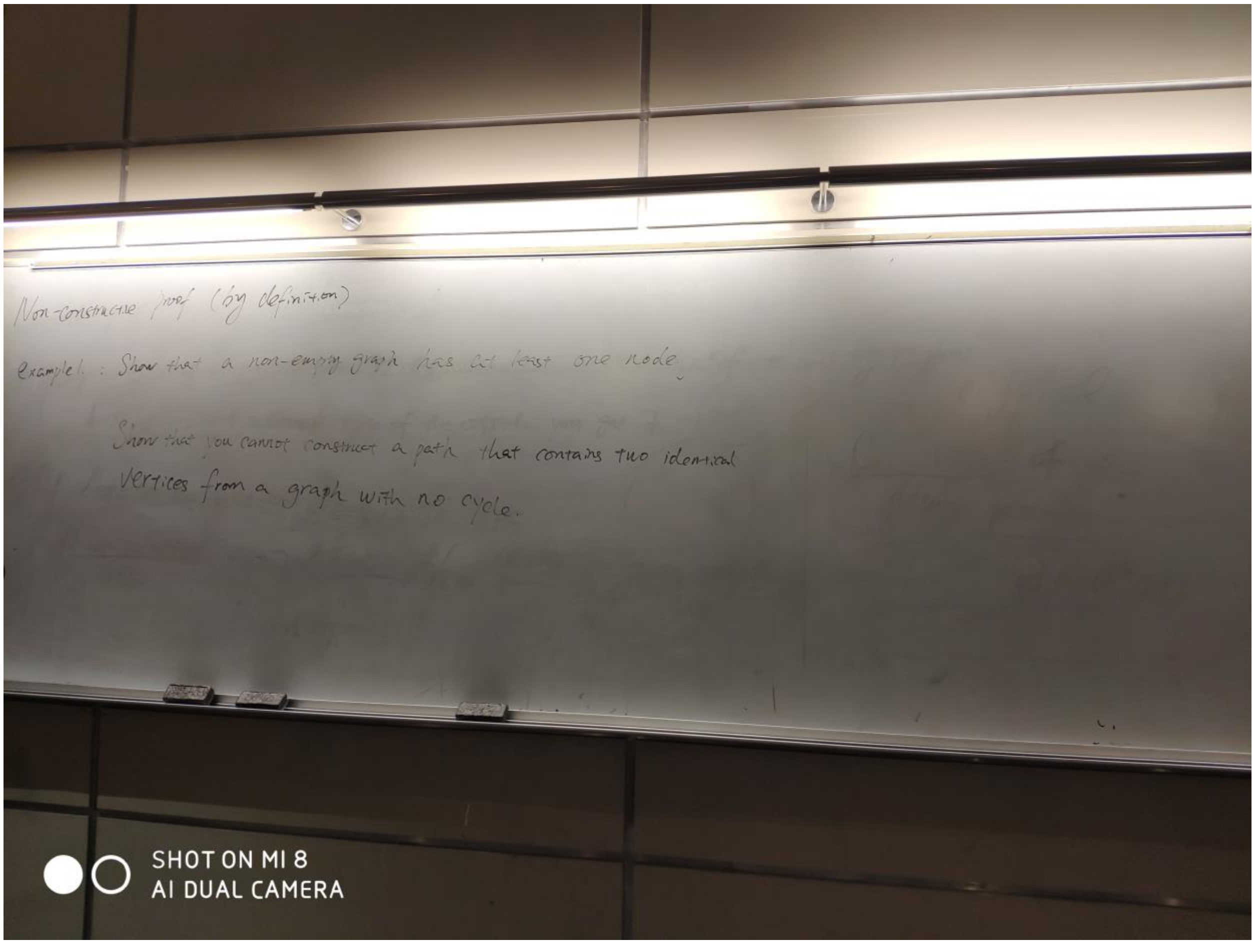
Constructive proof 3 (Unlimited, uncountable domain)

Proof that for any real number, if you add the number by 5, multiply by 4, subtract by 6, divide by 2, and subtract twice of the original, you get 7.

Proof: Let x be an arbitrary real number.

$$\frac{4(x+5)-6}{2}-2x=\frac{4x+20-6}{2}-2x=\frac{4x+14}{2}-2x=2x+7-2x=7$$

Generalization of a generic particular.


Non-constructive proof (by definition)
example1: Show that a non-empty graph has at least one node.
Show that you cannot construct a path that contains two identical vertices from a graph with no cycle.
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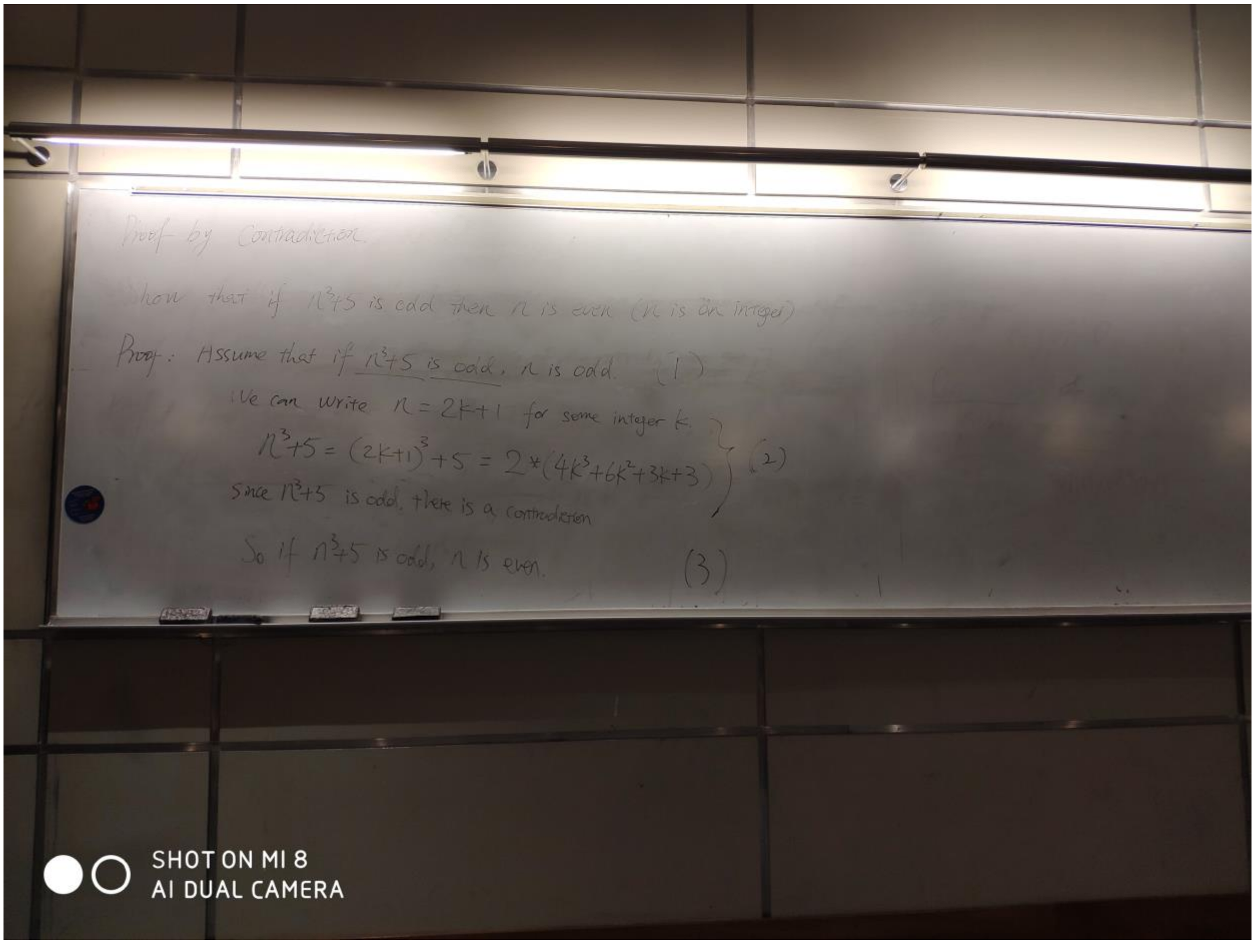
Proof by Contradiction
Show that if n^3+5 is odd then n is even (n is an integer)
Proof: Assume that if n^3+5 is odd, n is odd. (1)
We can write n = 2k+1 for some integer k.
n^3+5 = (2k+1)^3+5 = 2*(4k^3+6k^2+3k+3)
(2)
Since n^3+5 is odd, there is a contradiction
So if n^3+5 is odd, n is even.
(3)
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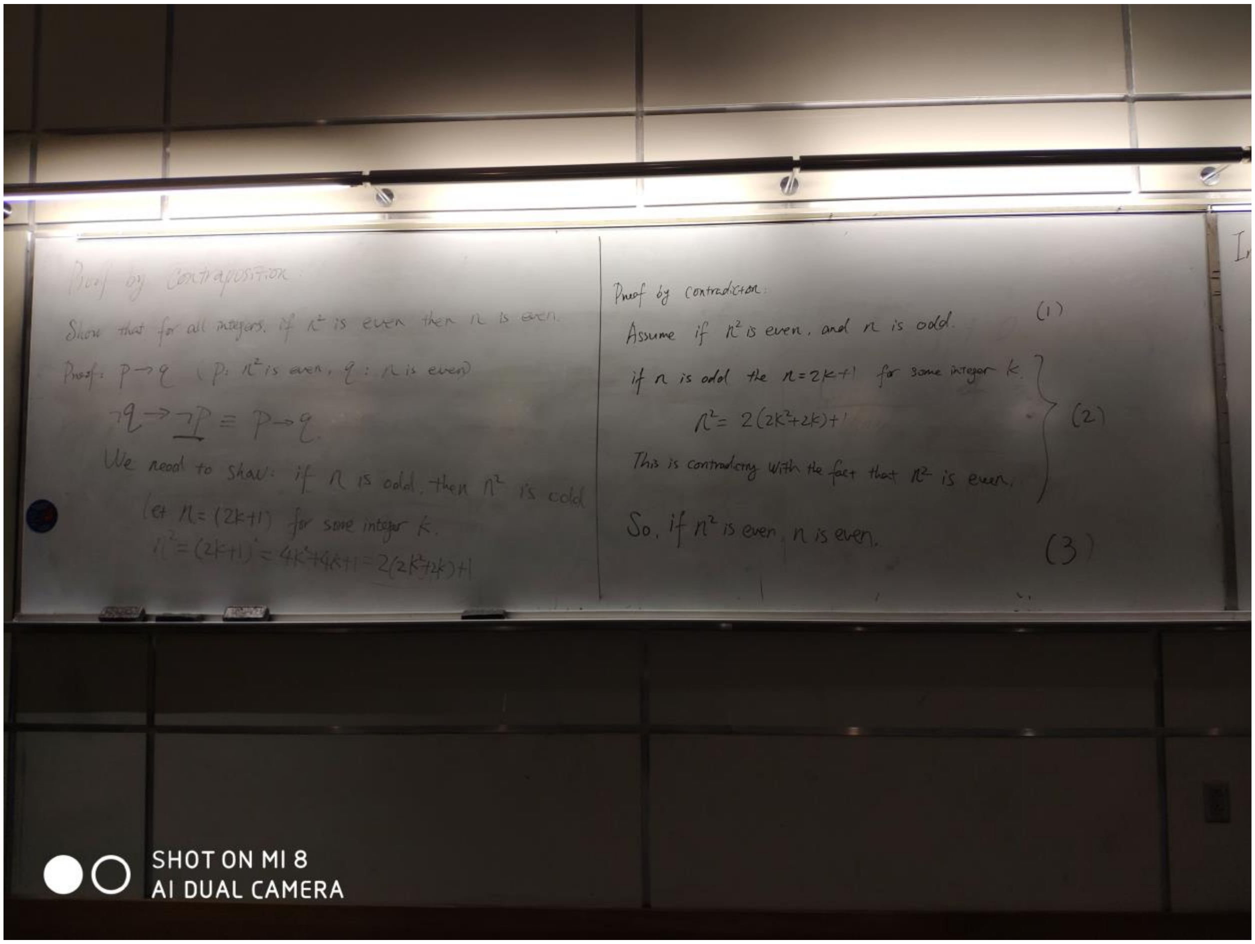

Proof by Contraposition:
Show that for all integers, if n² is even then n is even.
Proof: p → q (p: n² is even, q: n is even)
¬q → ¬p ≡ p → q.
We need to show: if n is odd, then n² is odd
Let n = (2k+1) for some integer k.
n² = (2k+1)² = 4k²+4k+1 = 2(2k²+2k)+1
Proof by contradiction:
Assume if n² is even, and n is odd. (1)
if n is odd the n = 2k+1 for some integer k.
n² = 2(2k²+2k)+1
This is contradicting with the fact that n² is even. (2)
So, if n² is even, n is even. (3)
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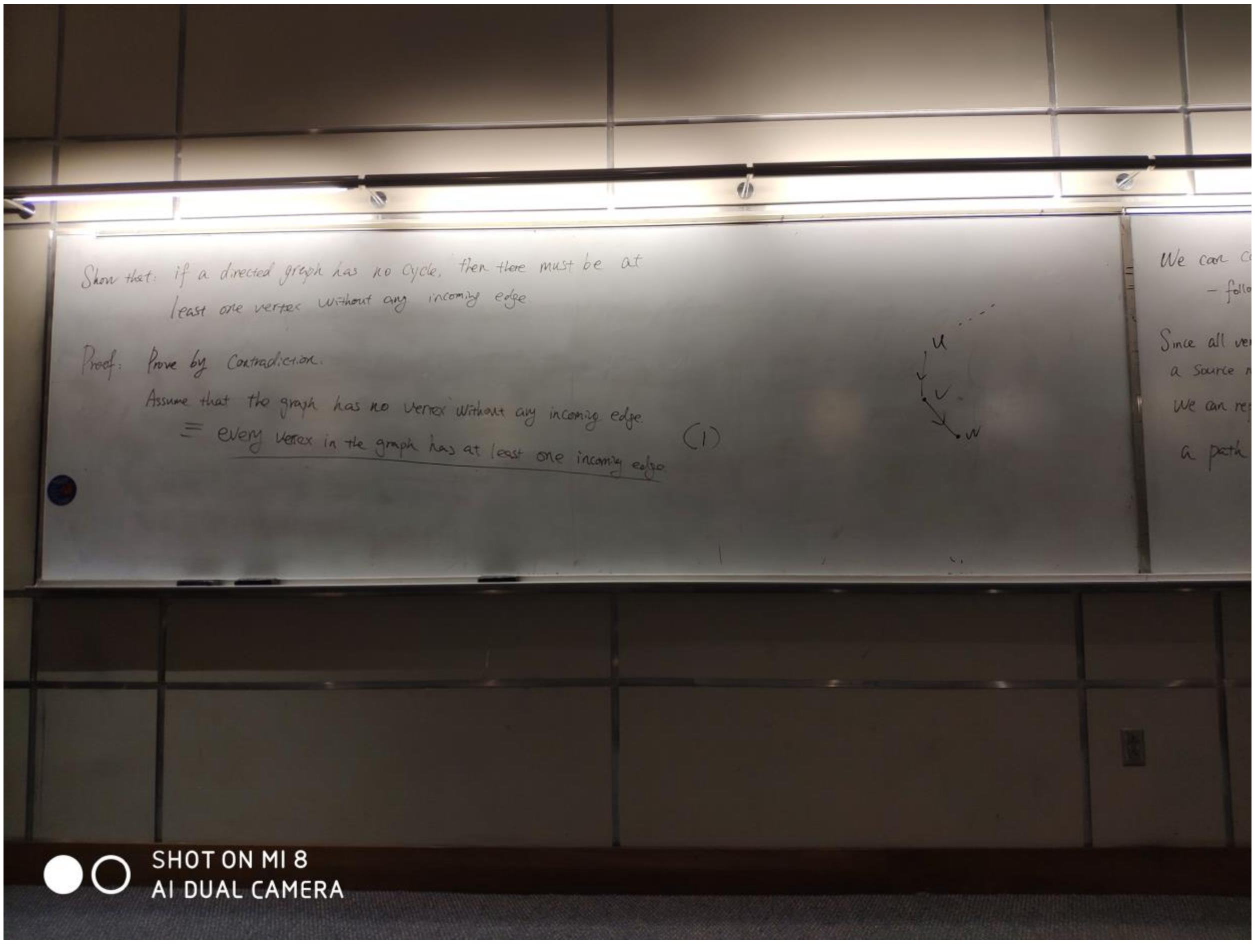
Show that: if a directed graph has no cycle, then there must be at least one vertex without any incoming edge.
Proof: Prove by Contradiction.
Assume that the graph has no vertex without any incoming edge.
≡ every vertex in the graph has at least one incoming edge (1)
u
v
w
We can
Since all
a Source
We can
a path
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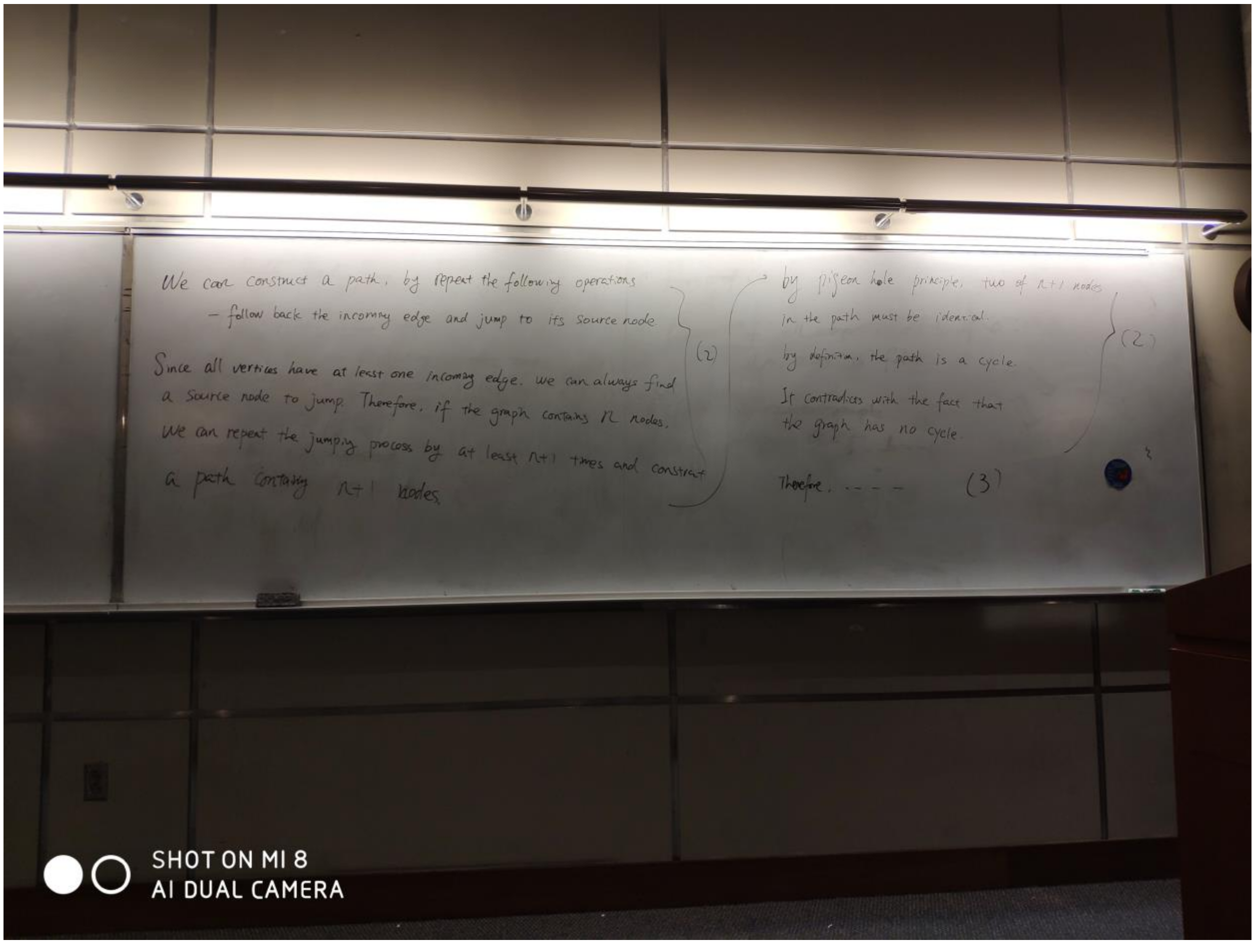

We can construct a path, by repeat the following operations
– follow back the incoming edge and jump to its source node
Since all vertices have at least one incoming edge. we can always find a source node to jump. Therefore, if the graph contains n nodes, We can repeat the jumping process by at least $n+1$ times and constract a path containing $n+1$ nodes.
(2)
by pigeon hole principle, two of $n+1$ nodes in the path must be identical.
by definition, the path is a cycle.
It contradicts with the fact that the graph has no cycle.
(2)
Therefore, ----
(3)
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