Test # 1

February 20, 2018

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20 pts. 1. For languages L_1 and L_2 over the same alphabet Σ, prove or disprove the following claim:

$\overline{L_1} \cap \overline{L_2} \cdot \overline{L_1 \cup L_2}$ $L_1 + L_2 = L_1 \cup L_2$

$$\Sigma^* - (L_1L_2)^* = (\Sigma^* - (L_1)^*) \subseteq (\Sigma^* - (L_2)^*).$$

① Prove: $\Sigma^* - (L_1L_2)^* \subseteq (\Sigma^* - (L_1)^*) \cap (\Sigma^* - (L_2)^*)$

$LHS = \Sigma^* - (L_1L_2)^* = \overline{L_1^* \cdot L_2^*} = w \in \overline{L_1^* \cap L_2^*}$

$= w \in \overline{L_1^*} \cup w \in \overline{L_2^*}$

$L_1 = \{ab$

$L_1L_2 = abc$

$= w \in ((\Sigma^* - L_1^*) \cup (\Sigma^* - L_2^*))$

$\not\subseteq (\Sigma^* - L_1)^*) \cap (\Sigma^* - L_2^*)$

② Prove $(\Sigma^* - L_1)^*) \cap (\Sigma^* - L_2^*) \subseteq \Sigma^* - L_1 L_2^*$

$w \in (\overline{L_1^*} \cap \overline{L_2^*}) = w \in \overline{L_1^* \cup L_2^*} = w \in \overline{L_1^* + L_2^*}$

$= w \in \Sigma^* - (L_1^* + L_2^*)$

disprove:

$\not\subseteq \Sigma^* - (L_1L_2)^*$

Example: $L_1 = \{a^n\} : n \geq 2\}$

$L_2 = \{b^m : m \geq 0\}$

$L_1L_2 = \{a^n b^m : n, m \geq 0\}$

$\Sigma^* - L_1^* L_2^* \neq (\Sigma^* - L_1^*) \cap \Sigma$

2. For a language L over $\Sigma = \{a, b\}$ find a grammar that generates it, where

20 pts. (a) $L = \{aa^m b^n a^n bab^m : m, n \geq 0\}$,

20 pts. (b) $L = \{a^m b^n : m \geq n + 2\}$.

a)

$S \to aA$

$A \to aCb \mid aAb \mid C$

$C \to DE$

$D \to bDa \mid \lambda$

$E \to ba$

b)

$S \to aaA$

$A \to aAb \mid \lambda \mid aA$

{ aa, aaa, aaaa, aaa b }

aba, aabab, ababa, aaabbaabb}

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20 pts. 3. Check whether the following two grammars $G_1 = (\{A, B, S\}, \{a, b\}, S, P_1)$ and $G_2 = (\{A, B, S\}, \{a, b\}, S, P_2)$ are equivalent, where P_1 is the following set of productions

$S \to AB \mid \lambda,$
$A \to aAbb \mid abb,$
$B \to aaBb \mid aab,$

and P_2 is the following set of productions

$S \to AB,$
$A \to aAbb \mid \lambda,$
$B \to aaBb \mid \lambda.$

Show your work. Either tell what differs the languages generated by these two grammars or provide a sketch of the proof that these grammars are equivalent.

① $S \to AB \mid \lambda \quad \Rightarrow \lambda$

① $S \to \lambda$

② $AB \Rightarrow aAbb\ aaBb \Rightarrow a(aAbb)bb\ aa(aaBb)b$

① abbaab.

② aabbbbaaaaabb

$L_{(1)} = \{ a^n b^{2n} a^{2m} b^m : n, m \geq 0 \}$

$abb \in P_1$

They are ~~different~~

② $S \to AB$

① $S \to \lambda\ \lambda = \lambda.$

② $AB \Rightarrow aAbb\ aaBb \Rightarrow a(aAbb)bb\ aa\ aaBb \Rightarrow a^n b^{2n} a^{2m} b^m$

① abbaab

② aabbbbaaaaabb

$L_{(2)} = \{ a^n b^{2n} a^m b^{2m} : n, m \geq 0 \}$

Because $L_{(1)} = L_{(2)}$, they are the same Language!

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3 x 3 x 3 = 27

20 pts. 4. Show a transition graph for a transducer that accepts strings from the alphabet $\Sigma = \{a, b, c\}$, splits every string into substrings of the length equal to four, and responds with the number of a's in the substring (of the length four). The transducer outputs 0's before evaluation of a substring. For example, the sequence

aaaaabcaa

should be converted into the sequence

000400020.

Show your work.