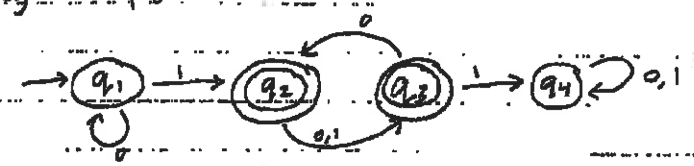
Homework assignment #4
EECS 510

1. Regular Expression for:



	k=0	k=1	k=2	k=3	k=4	
R_{11}^k	$0+\lambda$	0^*	0^*	0^*		Just need
R_{12}^k	1	0^*1	0^*1	$0^*1(00+10)^*$	$0^*1(00+10)^*$	← These two
R_{13}^k	$\emptyset$	$\emptyset$	$0^*1(0+1)$	$0^*1(0+1)(00+01)^*$	$0^*1(0+1)(00+01)^*$	← (1 → final state
R_{14}^k	$\emptyset$	$\emptyset$	$\emptyset$	$0^*1(0+1)(00+01)^*1$		
R_{21}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
R_{22}	λ	λ	λ	$\lambda+(00+10)^*$		
R_{23}	$0+1$	$0+1$	$0+1$	$(0+1)(00+01)^*$		
R_{24}	$\emptyset$	$\emptyset$	$\emptyset$	$(0+1)(00+01)^*1$		
R_{31}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
R_{32}	0	0	0	$(00+01)^*0$		
R_{33}	λ	λ	$\lambda+0(0+1)$	$(00+01)^*$		
R_{34}	1	1	1	$(00+01)^*1$		
R_{41}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
R_{42}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
R_{43}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$		
R_{44}	$0+1+\lambda$	$0+1+\lambda$	$0+1+\lambda$	$0+1+\lambda$		

$$R_{ij}^{k+1} = R_{ij}^{k} + R_{i\,k+1}^{k}\left(R_{k+1\,k+1}^{k}\right)^* R_{k+1\,j}^{k}$$

k=1

$$R_{11}^1 = R_{11}^0 + R_{11}^0 (R_{11}^0)^* R_{11}^0 = (0+\lambda)+(0+\lambda)(0+\lambda)^*(0+\lambda) = 0+\lambda+0^* = 0^*$$

$$R_{12}^1 = R_{12}^0 + R_{11}^0 (R_{11}^0)^* R_{12}^0 = 1+(0+\lambda)(0+\lambda)^*1 = 1+0^*1 = 0^*1$$

$$R_{13}^1 = R_{13}^0 + R_{11}^0 (R_{11}^0)^* R_{13}^0 = \emptyset+(0+\lambda)(0+\lambda)^*\emptyset = \emptyset$$

$$R_{14}^1 = R_{14}^0 + R_{11}^0 (R_{11}^0)^* R_{14}^0 = \emptyset+(0+\lambda)(0+\lambda)^*\emptyset = \emptyset$$

$$R_{21}^1 = R_{21}^0 + R_{21}^0 (R_{11}^0)^* R_{11} = \emptyset+\emptyset(0+\lambda)^*(0+\lambda) = \emptyset$$

$$R_{22}^1 = R_{22}^0 + R_{21}^0 (R_{11}^0)^* R_{12} = \lambda+\emptyset(0+\lambda)^*1 = \lambda$$

$$R_{23}^1 = R_{23}^0 + R_{21}^0 (R_{11}^0)^* R_{13} = (0+1)+\emptyset(0+\lambda)^*\emptyset = 0+1$$

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continued

$R_{24}^{1} = R_{24}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{14}^{0} = \emptyset + \emptyset(0+\lambda)^{*}\emptyset = \emptyset$

$R_{31}^{1} = R_{31}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{11}^{0} = \emptyset + \emptyset(0+\lambda)^{*}(0+\lambda) = \emptyset$

$R_{32}^{1} = R_{32}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{12}^{0} = 0 + \emptyset(0+\lambda)^{*}1 = 0$

$R_{33}^{1} = R_{33}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \lambda + \emptyset(0+\lambda)^{*}\emptyset = \lambda$

$R_{34}^{1} = R_{34}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{14}^{0} = 1 + \emptyset(0+\lambda)^{*}\emptyset = 1$

$R_{41}^{1} = R_{41}^{0} + R_{41}^{0}(R_{11}^{0})^{*}R_{11}^{0} = \emptyset + \emptyset\cdot(0+\lambda)^{*}(0+\lambda) = \emptyset$

$R_{42}^{1} = R_{42}^{0} + R_{41}^{0}(R_{11}^{0})^{*}R_{12}^{0} = \emptyset + \emptyset(0+\lambda)^{*}1 = \emptyset$

$R_{43}^{1} = R_{43}^{0} + R_{41}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \emptyset + \emptyset(0+\lambda)^{*}\emptyset = \emptyset$

$R_{44}^{1} = R_{44}^{0} + R_{41}^{0}(R_{11}^{0})^{*}R_{14}^{0} = (0+1+\lambda) + \emptyset(0+\lambda)^{*}\emptyset = 0+1+\lambda$

$k=2$

$R_{11}^{2} = R_{11}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{21}^{1} = 0^{*} + 0^{*}1(\lambda)^{*}\emptyset = 0^{*}$

$R_{12}^{2} = R_{12}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{22}^{1} = 0^{*}1 + 0^{*}1(\lambda)^{*}\lambda = 0^{*}1 + 0^{*}1 = 0^{*}1$

$R_{13}^{2} = R_{13}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{23}^{1} = \emptyset + 0^{*}1(\lambda)^{*}(0+1) = 0^{*}10 + 0^{*}11$

$R_{14}^{2} = R_{14}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{24}^{1} = \emptyset + 0^{*}1(\lambda)^{*}\emptyset = \emptyset$

$R_{21}^{2} = R_{21}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{21}^{1} = \emptyset + \lambda(\lambda)^{*}\emptyset = \emptyset$

$R_{22}^{2} = R_{22}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{22}^{1} = \lambda + \lambda(\lambda)^{*}\lambda = \lambda$

$R_{23}^{2} = R_{23}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{23}^{1} = (0+1) + \lambda(\lambda)^{*}(0+1) = 0+1$

$R_{24}^{2} = R_{24}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{24}^{1} = \emptyset + \lambda(\lambda)^{*}\emptyset = \emptyset$

$R_{31}^{2} = R_{31}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{21}^{1} = \emptyset + 0(\lambda)^{*}\emptyset = \emptyset$

$R_{32}^{2} = R_{32}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{22}^{1} = 0 + 0(\lambda)^{*}\lambda = 0$

$R_{33}^{2} = R_{33}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{23}^{1} = \lambda + 0(\lambda)^{*}(0+1) = \lambda + 0(0+1)$

$R_{34}^{2} = R_{34}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{24}^{1} = 1 + 0(\lambda)^{*}\emptyset = 1$

$R_{41}^{2} = R_{41}^{1} + R_{42}^{1}(R_{22}^{1})^{*}R_{21}^{1} = \emptyset + \emptyset(\lambda)^{*}\emptyset = \emptyset$

$R_{42}^{2} = R_{42}^{1} + R_{42}^{1}(R_{22}^{1})^{*}R_{22}^{1} = \emptyset + \emptyset(\lambda)^{*}\lambda = \emptyset$

$R_{43}^{2} = R_{43}^{1} + R_{42}^{1}(R_{22}^{1})^{*}R_{23}^{1} = \emptyset + \emptyset(\lambda)^{*}(0+1) = \emptyset$

$R_{44}^{2} = R_{44}^{1} + R_{42}^{1}(R_{22}^{1})^{*}R_{24}^{1} = 0+1+\lambda + \emptyset(\lambda)^{*}\emptyset = 0+1+\lambda$

$k=3$

$R_{11}^{3} = R_{11}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{31}^{2} = 0^{*} + 0^{*}1(0+1)[\lambda+0(0+1)]^{*}\emptyset = 0^{*}$

$R_{12}^{3} = R_{12}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{32}^{2} = 0^{*}1 + 0^{*}1(0+1)[\lambda+0(0+1)]^{*}0 = 0^{*}1(00+10)^{*}$

$R_{13}^{3} = R_{13}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{33}^{2} = 0^{*}1(0+1) + 0^{*}1(0+1)[\lambda+0(0+1)]^{*}(\lambda+0(0+1)) = 0^{*}1(0+1)(00+01)^{*}$

$R_{14}^{3} = R_{14}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{34}^{2} = \emptyset + 0^{*}1(0+1)[\lambda+0(0+1)]^{*}1 = 0^{*}1(0+1)(00+01)^{*}1$

$R_{21}^{3} = R_{21}^{2} + R_{23}^{2}(R_{33}^{2})^{*}R_{31}^{2} = \emptyset + (0+1)[\lambda+0(0+1)]^{*}\emptyset = \emptyset$

Homework 4
continued

$R_{22}^3 = R_{22}^2 + R_{23}^2 (R_{33}^2)^* R_{32}^2 = \lambda + (0+1)[\lambda + 0(0+1)]^* 0 = \lambda + (00+10)^*$

$R_{23}^3 = R_{23}^2 + R_{23}^2 (R_{33}^2)^* R_{33}^2 = (0+1) + (0+1)[\lambda + 0(0+1)]^* [\lambda+0(0+1)]^* = (0+1)(00+01)^*$

$R_{24}^3 = R_{24}^2 + R_{23}^2 (R_{33}^2)^* R_{34}^2 = \emptyset + (0+1)[\lambda+0(0+1)]^* 1 = (0+1)(00+01)^* 1$

$R_{31}^3 = R_{31}^2 + R_{33}^2 (R_{33}^2)^* R_{31}^2 = \emptyset + [\lambda+0(0+1)]^* [\lambda+0(0+1)]^* \emptyset = \emptyset$

$R_{32}^3 = R_{32}^2 + R_{33}^2 (R_{33}^2)^* R_{32}^2 = 0 + [\lambda+0(0+1)]^* [\lambda+0(0+1)]^* 0 = (00+01)^* 0$

$R_{33}^3 = R_{33}^2 + R_{33}^2 (R_{33}^2)^* R_{33}^2 = [\lambda+0(0+1)]^* + [\lambda+0(0+1)]^* = (00+01)^*$

$R_{34}^3 = R_{34}^2 + R_{33}^2 (R_{33}^2)^* R_{34}^2 = 1 + [\lambda+0(0+1)]^* (\lambda+0(0+1))^* 1 = (00+01)^* 1$

$R_{41}^3 = R_{41}^2 + R_{43}^2 (R_{33}^2)^* R_{31}^2 = \emptyset + \emptyset (\lambda + 0(0+1))^* \emptyset = \emptyset$

$R_{42}^3 = R_{42}^2 + R_{43}^2 (R_{33}^2)^* R_{32}^2 = \emptyset + \emptyset(\lambda+0(0+1))^* 0 = \emptyset$

$R_{43}^3 = R_{43}^2 + R_{43}^2 (R_{33}^2)^* R_{33}^2 = \emptyset + \emptyset (\lambda + 0(0+1))^* (\lambda+0(0+1))^* = \emptyset$

$R_{44}^3 = R_{44}^2 + R_{43}^2 (R_{33}^2)^* R_{34}^2 = 0+1+\lambda + \emptyset(\lambda+0(0+1))^* 1 = 0+1+\lambda$

$k=4$

$R_{12}^4 = R_{12}^3 + R_{14}^3 (R_{44}^3)^* R_{42}^3 = 0^*1(00+10)^* + 0^*1(0+1)(00+01)^* 1 (0+1+\lambda)^* \emptyset = 0^*1(00+10)^*$

$R_{13}^4 = R_{13}^3 + R_{14}^3 (R_{44}^3)^* R_{43}^3 = 0^*1(0+1)(00+01)^* + 0^*1(0+1)(00+01)^* 1 (0+1+\lambda)^* \emptyset =$

$= 0^*1(0+1)(00+01)^*$

Final Regular Expression $= R_{12}^4 + R_{13}^4$

$= 0^*1(00+10)^* + 0^*1(0+1)(00+01)^* =$

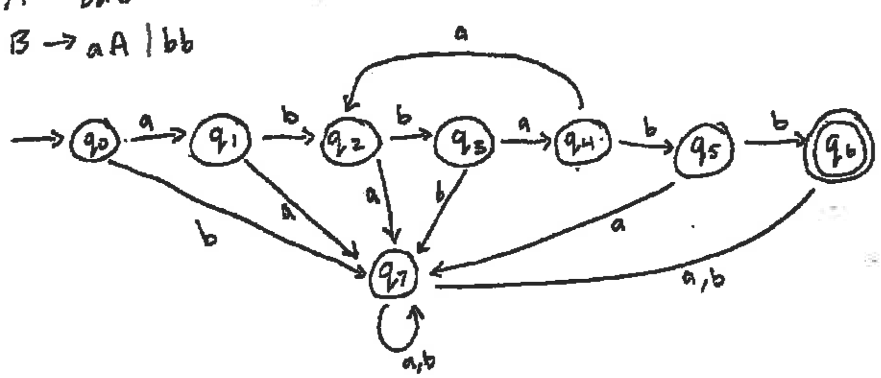
$= 0^*1\left((00+10)^* + (0+1)(00+01)^*\right)$

2. DFA for:

$S \to abA$

$A \to baB$

$B \to aA \mid bb$

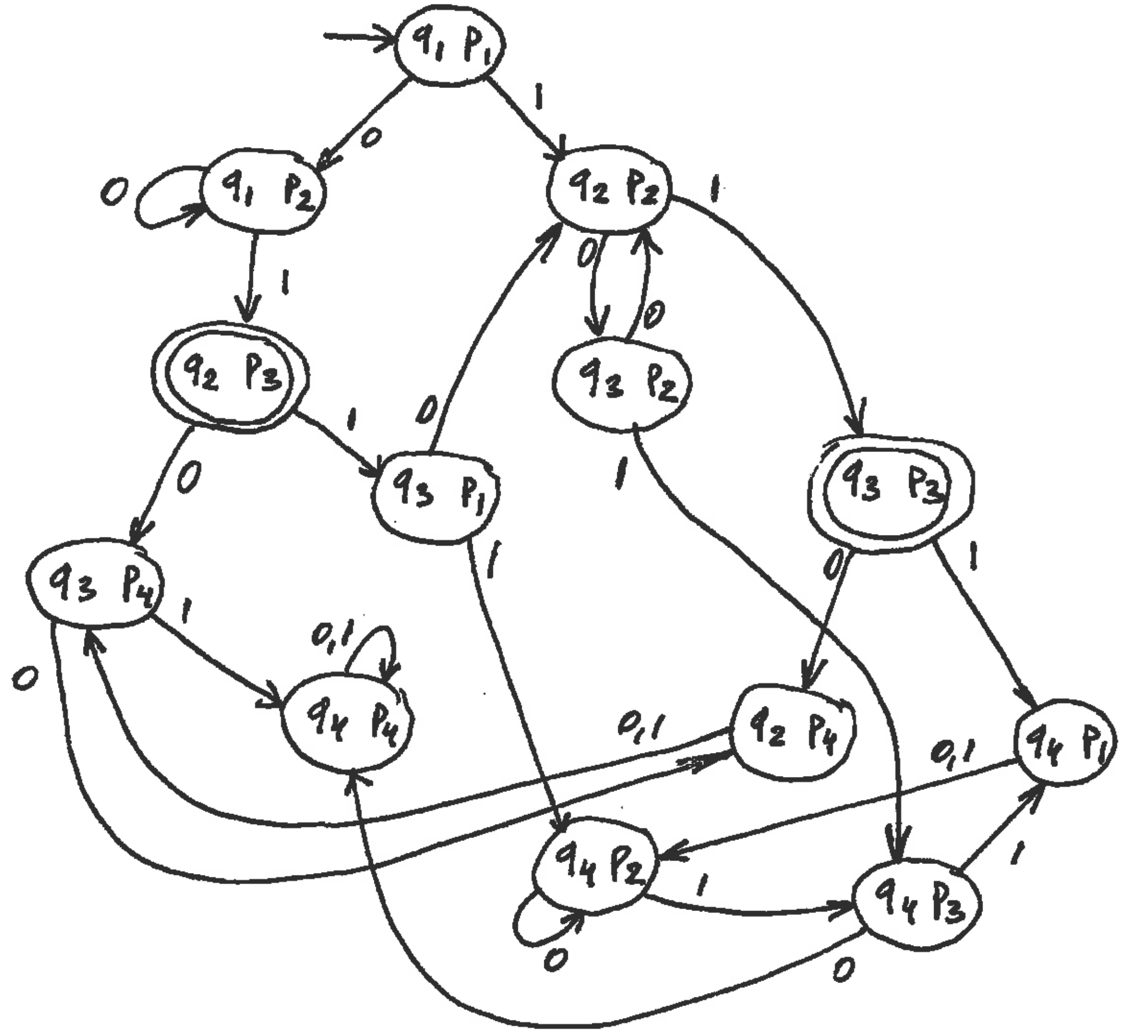


Homework 4
continued

3. $S \to Abb$
$A \to Aaba \mid B$
$B \to abba$

4. Right-linear grammar:
$S \to 0A \mid 1A$
$A \to 0A \mid 1B$
$B \to 1S \mid \lambda$

5. Intersection of #1 and #4:



6. Let M be a prime number, $M \geq m$.

$W_{M+1} = xy^{M+2}z$

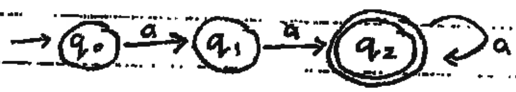
$|xy^{M+1}z| = |xyz| + |y| \cdot M = M + |y| \cdot M = M(1+|y|)$

so $W_{M+1} \notin L$

7. $L = \{a^n : n$ is prime or product of primes$\}$

All positive integers > 1 are prime or a product of primes so $L = \{a^n : n > 1\}$,

Since a DFA can be constructed that accepts this language:



The language is regular.