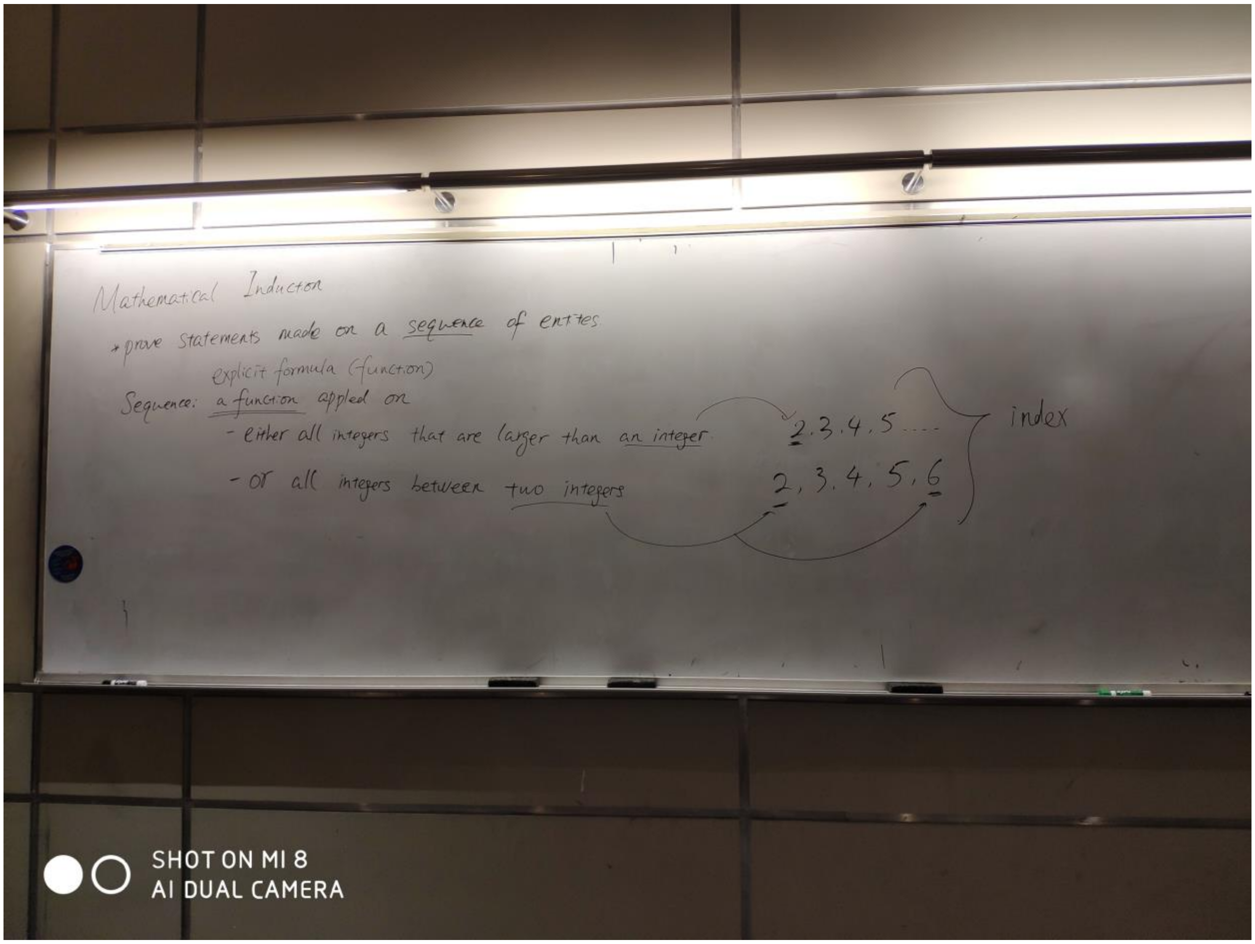

Mathematical Induction
* prove statements made on a sequence of entites
explicit formula (function)
Sequence: a function applied on
- either all integers that are larger than an integer
- or all integers between two integers
2, 3, 4, 5 ....
2, 3, 4, 5, 6
index
SHOT ON MI 8
AI DUAL CAMERA

Example (a sequence)

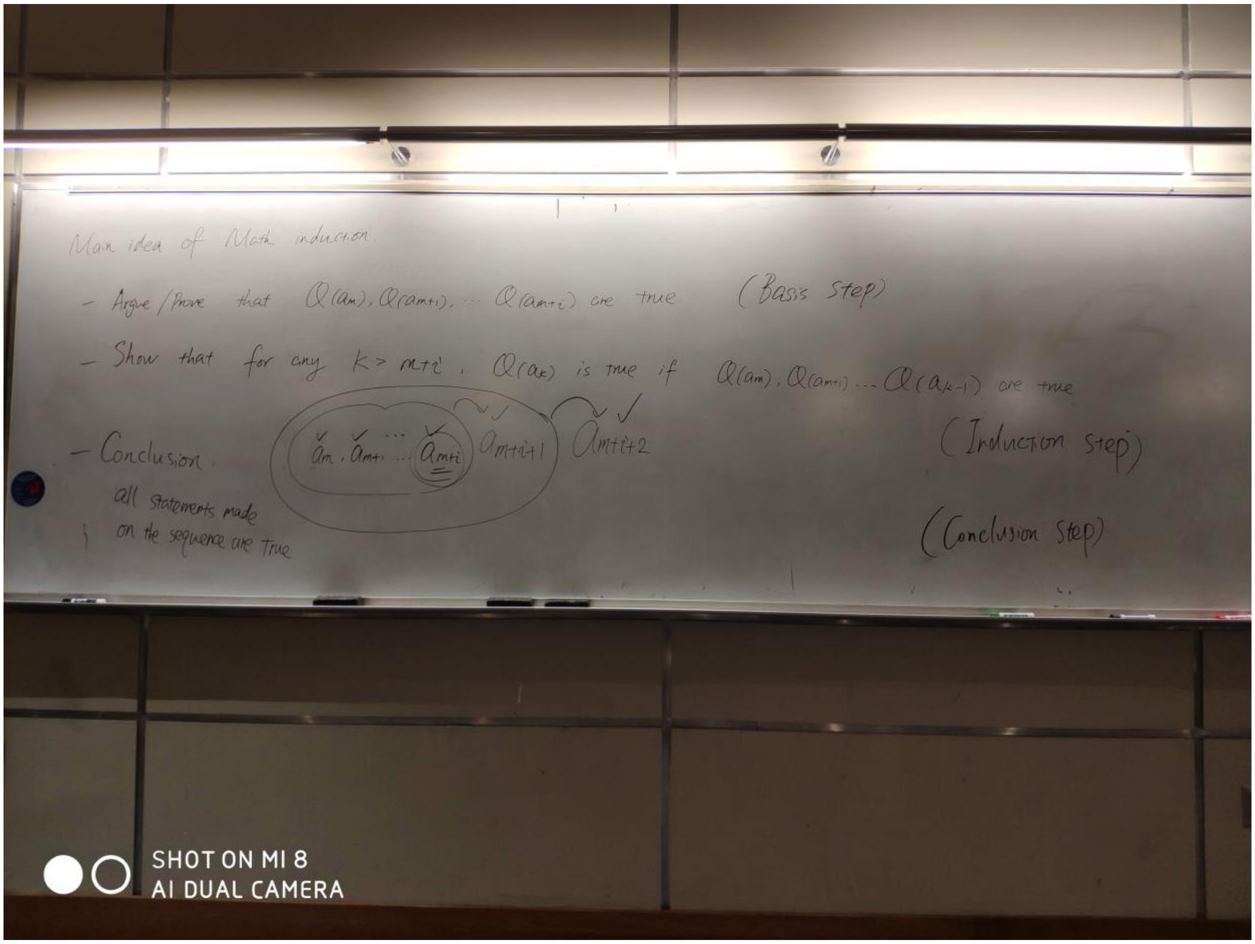
Let: $a_1, a_2, a_3 \ldots$ be a sequence of the Fibonacci numbers

the indices for the sequence are all integers greater or equal to 1

the explicit function is
$$F(n) = \begin{cases} F(n-1) + F(n-2) & n > 2 \\ 1 & \text{if } 1 \le n \le 2 \end{cases}$$

a_i is called a <u>term</u>

- Statement made on a sequence $\underline{Q(a_m)}, Q(a_{m+1}) \ldots$


Main idea of Math induction
- Argue / Prove that $Q(a_m), Q(a_{m+1}), \dots Q(a_{m+i})$ are true (Basis Step)
- Show that for any $k > m+i$, $Q(a_k)$ is true if $Q(a_m), Q(a_{m+1}) \dots Q(a_{k-1})$ are true
(Induction step)
- Conclusion
$a_m, a_{m+1} \dots a_{m+i}$ a_{m+i+1} a_{m+i+2}
all statements made on the sequence are True
(Conclusion step)
SHOT ON MI 8
AI DUAL CAMERA

- Weak induction

$\checkmark \quad \checkmark \quad \cdots \quad \checkmark \quad \checkmark$

$a_m, a_{m+1}, \cdots a_{m+i}\ \ a_{m+i+1}\ \ \ a_{m+i+2}$

Example (Weak induction)

* Prove that: for every positive integer n, 11^n-6 is divisible by 5.

- (Conclusion ⋯)

Proof:

- (Basis step): When $n=1$, $11^n-6=5$, which is divisible by 5

- (Induction step): Assume the statement is true for $\underline{\underline{k \geq 1}}$

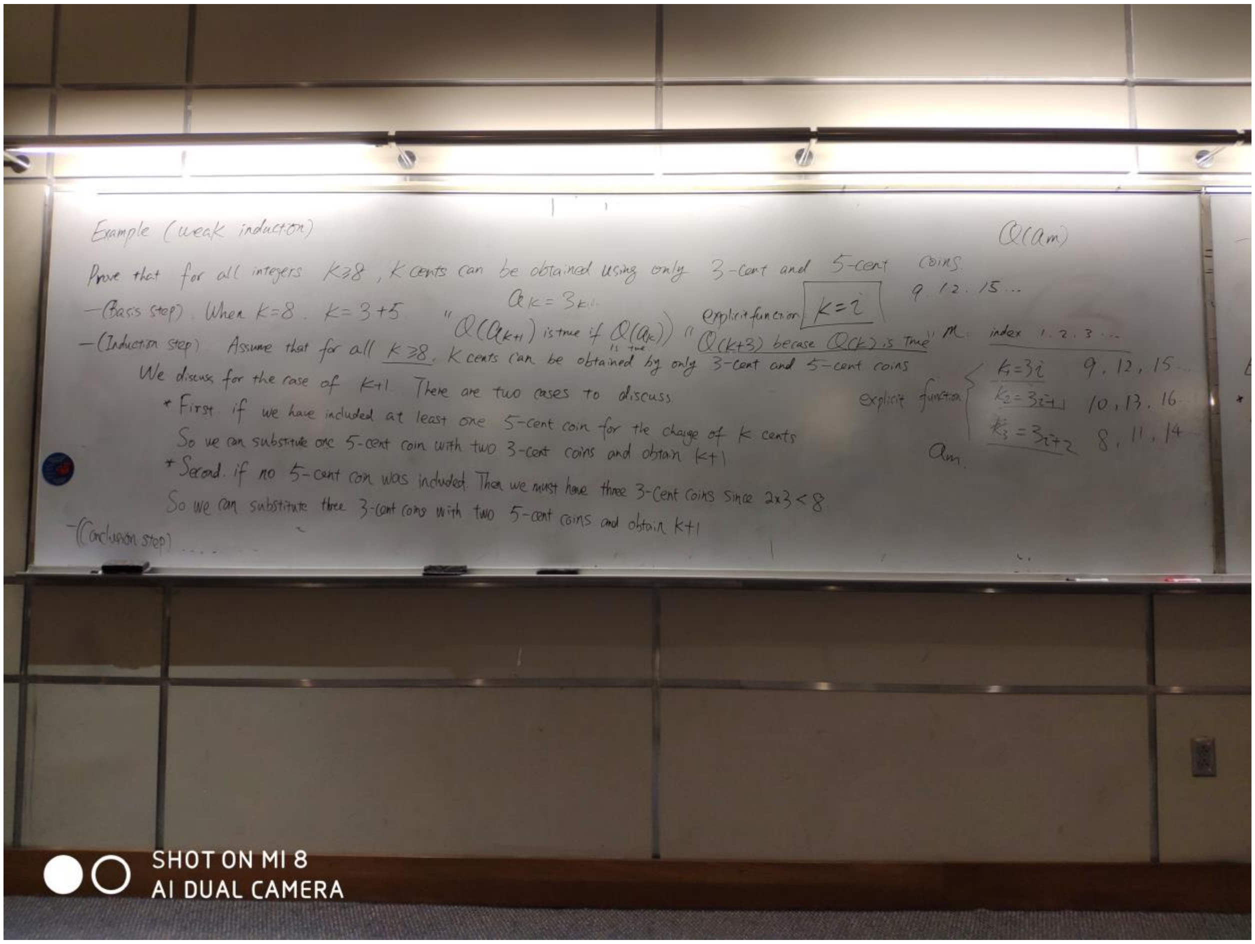
We want to show that $11^{k+1}-6$ is also divisible by 5

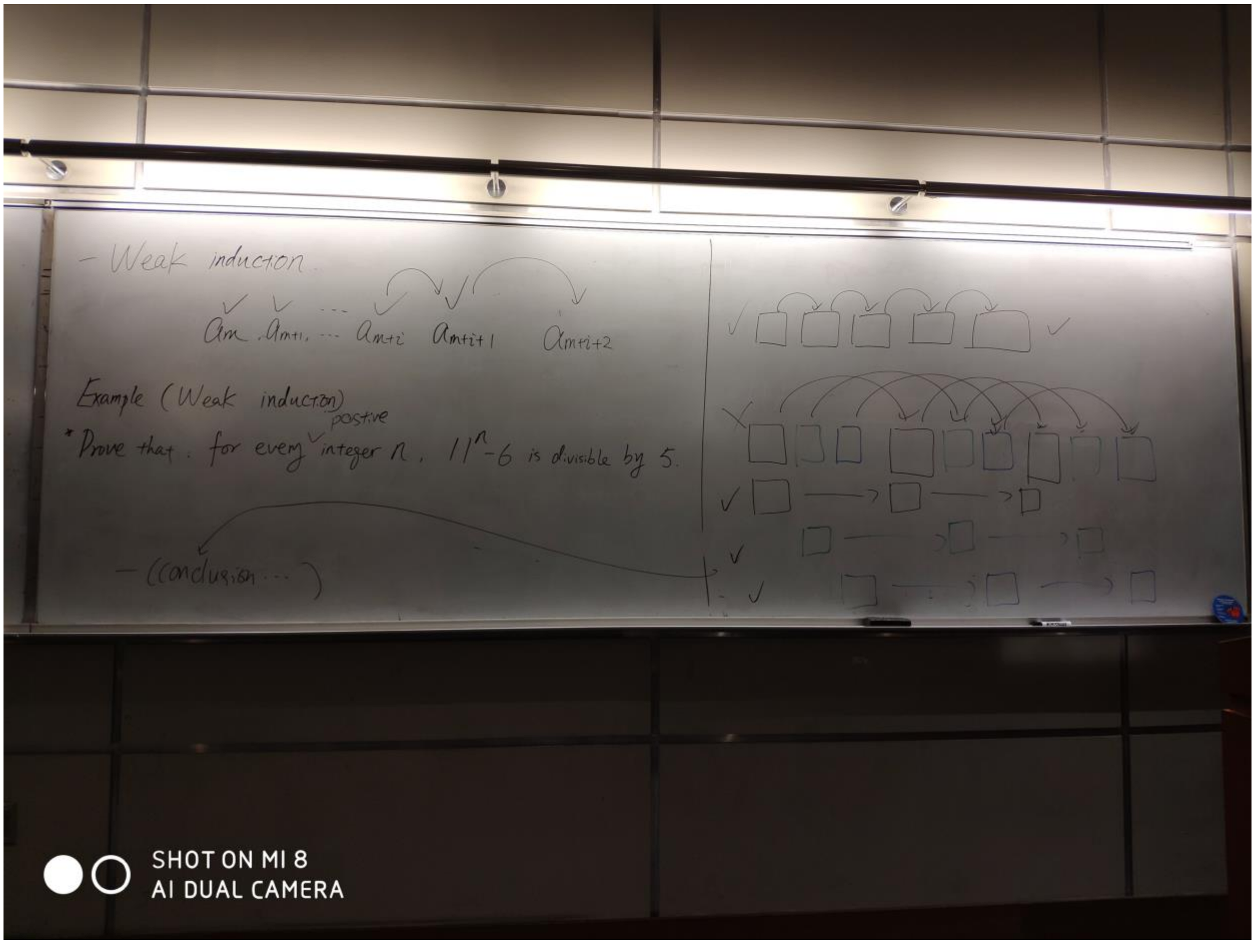
$$11^{k+1}-6 = 11*(11^k)-6 = 11*(\underline{11^k-6}+6)-6$$

Since 11^k-6 is divisible by 5. Let $11^k-6=5x$ for some integer x

$$11^{k+1}-6 = 11*(5x+6)-6 = 55x+66-6 = 55x+60 = 5(11x+12)$$

So $11^{k+1}-6$ is also divisible by 5


Example (weak induction)
Prove that for all integers K≥8, K cents can be obtained using only 3-cent and 5-cent coins.
Q(am)
9, 12, 15...
-(Basis step) When K=8. K=3+5
ak = 3k
"Q(ak+1) is true if Q(ak)) is true"
explicit function k=i
"Q(k+3) becase Q(k) is true"
m: index 1, 2, 3 ...
-(Induction step) Assume that for all K≥8, K cents can be obtained by only 3-cent and 5-cent coins
We discuss for the case of K+1 There are two cases to discuss.
* First if we have included at least one 5-cent coin for the charge of K cents
So we can substitute one 5-cent coin with two 3-cent coins and obtain K+1
* Second if no 5-cent coin was included. Then we must have three 3-cent coins since 2x3<8
So we can substitute three 3-cent coins with two 5-cent coins and obtain K+1
-(Conclusion step) .....
explicit function
k1=3i 9, 12, 15...
k2=3i+1 10, 13, 16
k3=3i+2 8, 11, 14
am
SHOT ON MI 8
AI DUAL CAMERA


- Weak induction
am , am+1 , ... am+i am+i+1 am+i+2
Example (Weak induction)
* Prove that, for every positive integer n, 11^n - 6 is divisible by 5.
- (conclusion ...)
SHOT ON MI 8
AI DUAL CAMERA