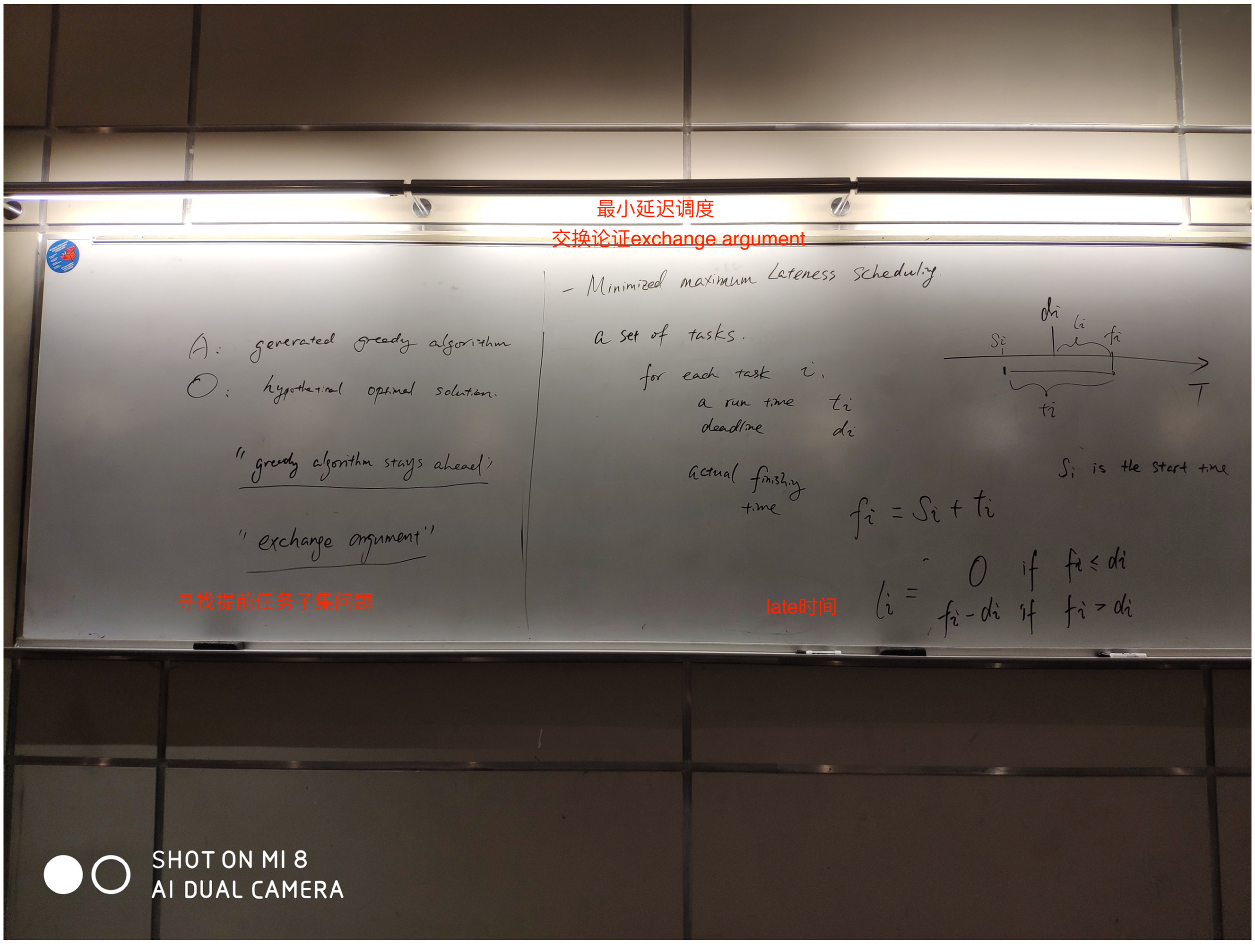
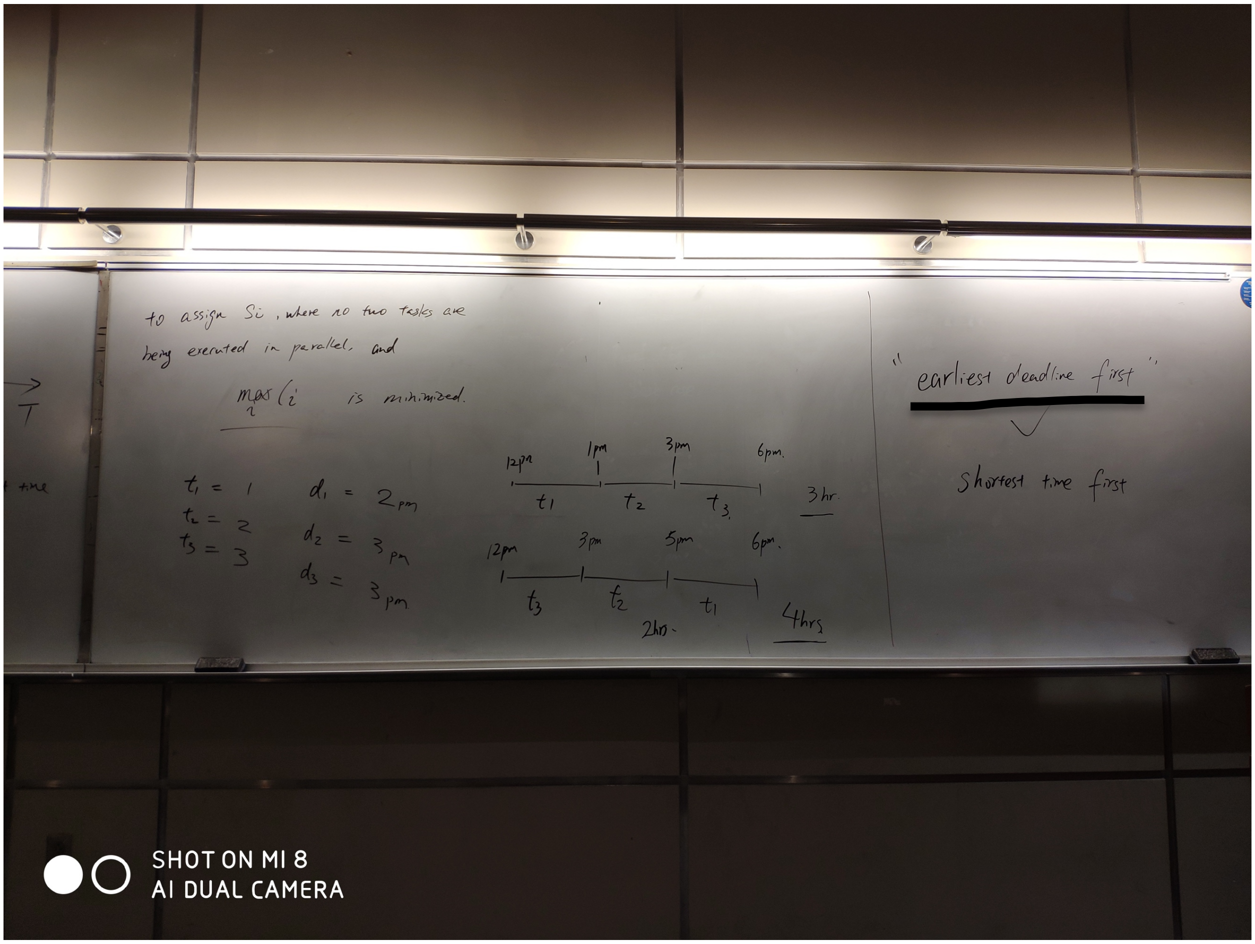
最小延迟调度

交换论证exchange argument

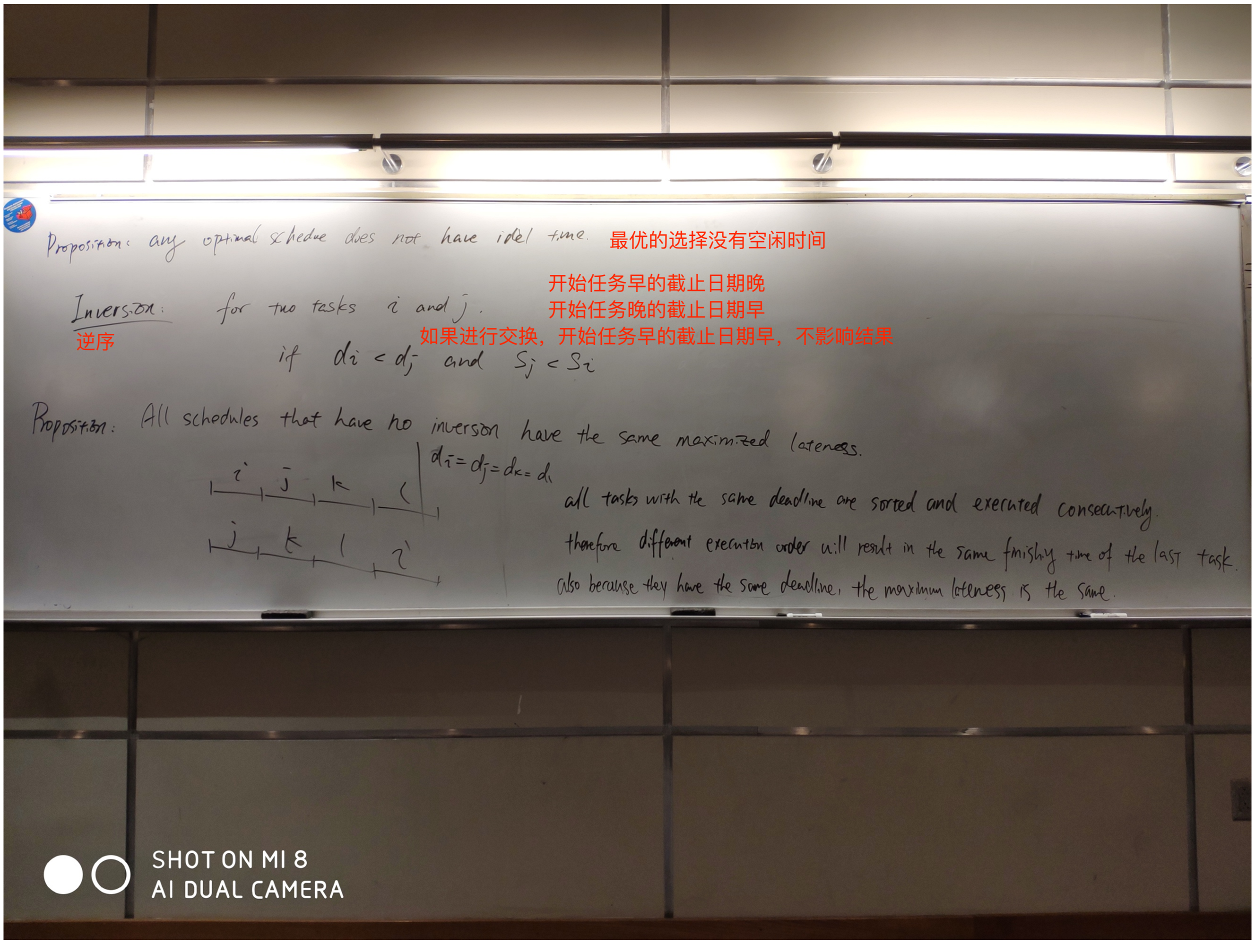


寻找提前任务子集问题

late时间



to assign Si , where no two tasks are
being executed in parallel, and
max Ci is minimized.
t1 = 1
t2 = 2
t3 = 3
d1 = 2pm
d2 = 3pm
d3 = 3pm
12pm
1pm
3pm
6pm.
t1
t2
t3
3hr.
12pm
3pm
5pm
6pm.
t3
t2
t1
2hrs.
4hrs
" earliest deadline first "
shortest time first
SHOT ON MI 8
AI DUAL CAMERA



Proposition: any optimal schedue does not have idel time.
最优的选择没有空闲时间
Inversion: for two tasks i and j.
逆序
开始任务早的截止日期晚
开始任务晚的截止日期早
如果进行交换，开始任务早的截止日期早，不影响结果
if $d_i < d_j$ and $S_j < S_i$
Proposition: All schedules that have no inversion have the same maximized lateness.
$d_i = d_j = d_k = d_l$
i j k l
j k l i
all tasks with the same deadline are sorted and executed consecutively.
therefore different execution order will result in the same finishing time of the last task.
also because they have the same deadline, the maximum lateness is the same.
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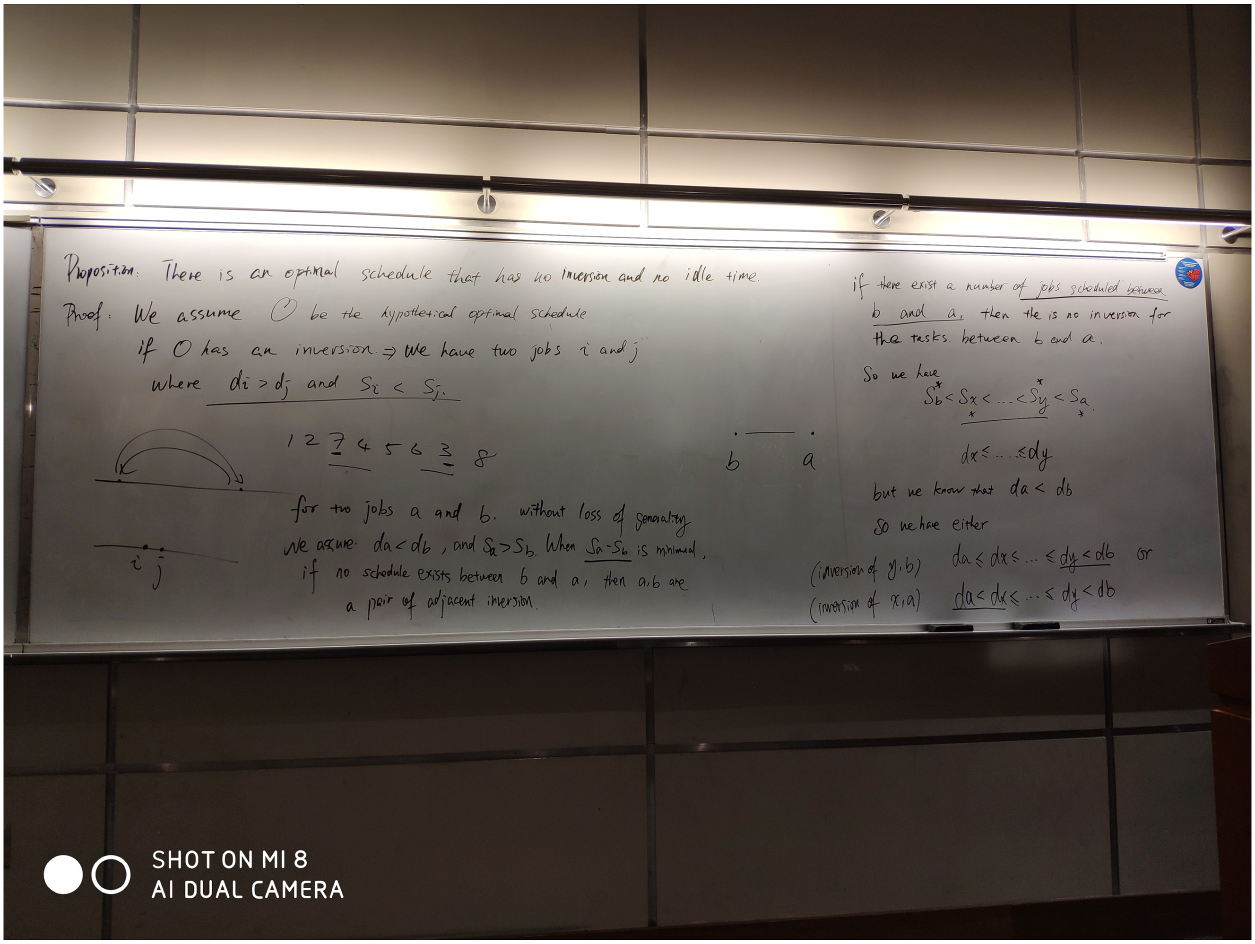
Proposition: There is an optimal schedule that has no inversion and no idle time.
Proof: We assume $\mathcal{O}$ be the hypothetical optimal schedule.
if $\mathcal{O}$ has an inversion. ⇒ We have two jobs i and j
where $d_i > d_j$ and $S_i < S_j$.
1 2 7 4 5 6 3 8
i j
for two jobs a and b. without loss of generality
We assume $d_a < d_b$, and $S_a > S_b$. When $S_a - S_b$ is minimal,
if no schedule exists between b and a, then a, b are
a pair of adjacent inversion.
b a
if there exist a number of jobs scheduled between
b and a, then the is no inversion for
the tasks. between b and a.
So we have
$S_b^* < S_x < \ldots < S_y^* < S_a$
$d_x \leq \ldots \leq d_y$
but we know that $d_a < d_b$
So we have either
(inversion of y, b) $d_a \leq d_x \leq \ldots \leq d_y < d_b$ or
(inversion of x, a) $d_a < d_x \leq \ldots \leq d_y < d_b$
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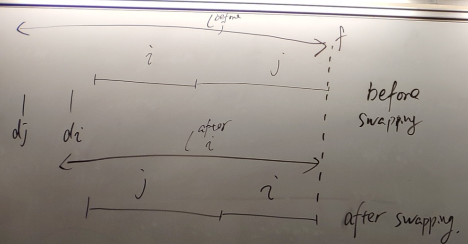
We can continuously swap adjacent inversions, until the schedule is inversion-free. It takes a finite number of steps.

does not increase lateness ...

O (optimal) $\longrightarrow \longrightarrow \longrightarrow \cdots \longrightarrow A$ (inversion-free) optimal.



for the lateness induced by task j

because we will execute j ealier, we do not increase its associated lateness.

for the lateness induced by task i

$L_i^{after} = f - d_i$

$L_j^{before} = f - d_j$

since $d_j < d_i$, it follows that $L_i^{after} < L_j^{before}$

any swapping of a pair of adjacent inversion will not increase maximum lateness.

So there exists an optimal schedule that is inversion-free and no idle time.

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