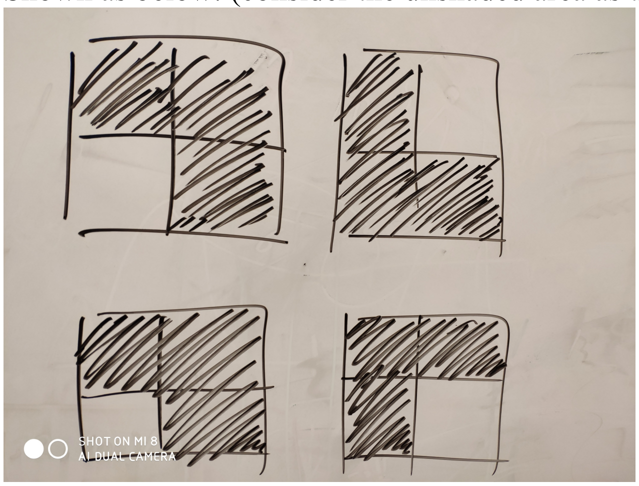
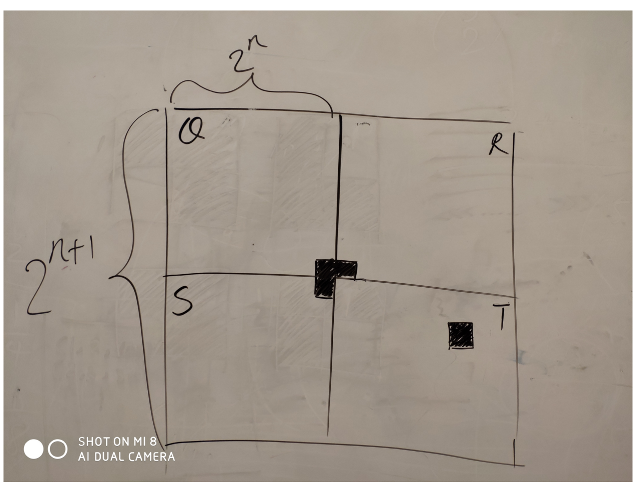
HW5: Math Induction (continue)

Q1: Prove that for any positive integer n, a $2^n \times 2^n$ checkerboard with any one square removed can be tiled using right triominos (which contains three squares). Note: this can be proved using weak induction.

A1: (Basis step) When $n = 1$, we can surely tile the 2×2 checkerboard with one triomino. Shown as below: (consider the unshaded area as the hole, and the shaded area as the trimino)



(Induction step) We assume for any $k \geq 1$, the proposition is true. To tile a $2^{k+1} \times 2^{k+1}$ checkerboard, we can partition it into four pieces, each one is $2^k \times 2^k$. Let them be Q, R, S, T. We know that at least one of them contains a hole, and that that be T. By induction, T can be tiled. Now, tile a right triomino at the center, using one corner-square each from Q, R, S. Q, R, S all become a $2^k \times 2^k$ checkerboard containing a hole, and by induction they can all be tiled using right triominos (see figure below).



(Conclusion) Therefore, for any positive integer n, a $2^n \times 2^n$ checkerboard with any one square removed can be tiled using right triominos.