

**Instruction:** You must show all your work clearly for credit. Partial credit will only be given to meaningful answers.

1. (30) Given a set  $S$  of 10 records with priorities  $\{15, 7, 10, 5, 18, 20, 15, 12, 6, 15\}$ .
  - (a) Construct the BST  $T_1$  for  $S$  by inserting the records, in the given order, into an initially empty binary search tree. When done, delete 15 and then 18 from  $T_1$ .
  - (b) Construct the BST  $T_2$  for  $S$  by inserting the records, in the reversed given order, into an initially empty binary search tree. When done, delete 15 and then 18 from  $T_2$ .
  - (c) Illustrate the data structure for the final BST for  $T_1$  and  $T_2$ .

*Remark:* You must show your BST clearly after each insert/delete operation.
2. Given a set  $R$  of 4 records with keys  $x_i$ ,  $x_1 < x_2 < \dots < x_4$ , a key  $x$ , and the probability function  $\Pr(x = x_i) = p_i$ , with  $p_1 = 0.35$ ,  $p_2 = 0.2$ ,  $p_3 = 0.15$ ,  $p_4 = 0.3$ ,  $1 \leq i \leq 4$ .
  - (a) (10) Construct a greedy optimal BST  $T_3$  for  $R$  using greedy approach as discussed in class. Compute the average number of comparisons in finding  $x$  in  $T_3$ .
  - (b) (20) Construct an optimal BST  $T_4$  for  $R$  using dynamic programming technique such that the average number of comparisons in finding  $x$  in  $T_4$  is minimized.

*Remark:* You must show your computations for all  $c_{i,j}$ ,  $t_{i,j}$ , and the reconstruction of the optimal BST clearly as discussed in class. Also, use smallest  $k$  to resolve ambiguity.
3. Given a set  $H$  of 15 records with priorities  $\{10, 5, 2, 15, 8, 12, 4, 1, 18, 6, 16, 11, 22, 36, 7\}$ .
  - (a) (15) Construct the 2-3 tree  $T_5$  for  $H$  by inserting the records, in the given order, into an initially empty 2-3 tree. When done, delete 1, 15, and then 36 from  $T_5$ .
  - (b) (15) Construct the 2-3 tree  $T_6$  for  $H$  by inserting the records, in the reversed given order, into an initially empty 2-3 tree. When done, delete 1, 15, and then 36 from  $T_6$ .
  - (d) (10) Illustrate the data structure for the final 2-3 tree for  $T_5$  and  $T_6$ .

*Remark:* You must show your 2-3 tree, including the content of each interior node clearly after each insert/delete operation.