

**Instruction:** You must show all your work clearly for credit. Partial credit will only be given to meaningful answers.

1. Use the definition of big-O to prove that  $n^4 - 8n^3 + 16n^2 - 3n + 560 \neq O(n^3)$ .
2. For any two functions  $f(n)$  and  $g(n)$ , by definition,  $f(n) = o(g(n))$  iff  $f(n) = O(g(n))$  and  $f(n) \neq \Theta(g(n))$ . Use the definition of little-o to prove or disprove that  $n^4 - 8n^3 + 16n^2 - 3n + 560 = o(n^5)$ .
3. Let  $f(n)$  and  $g(n)$  be any two positive functions. Prove or disprove the statement that if  $f(n) = O(g(n))$ , then  $2^{f(n)} = O(2^{g(n)})$ .
4. By assuming that all basic operations require the same constant cost  $K$ , compute the cost of the resource function  $R_w(n)$  in closed-form for the following program segment using the simplified approach as discussed in class. *Remark:* You must first set up a summation equation for  $R_w(n)$  and then evaluate the sum(s) clearly for credit.
 

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      x = 210;
      y = 560;
      for i = 1 to n*n do
        for j = i to n do
          y = x * y / 660 + 388;
        endfor;
      endfor;
      
```
5. Given a set of records with 7 keys  $S = \{35, 28, 43, 17, 39, 3, 46\}$ .
  - (a) By using the hash function  $h(x) = x \bmod m$  and chaining with singly linked list in constructing an open hash table  $H$  with  $m = 11$  buckets, insert the records in  $S$ , in the given order, into  $H$ . You must show your computations for locations and illustrate the final structure of your hash table  $H$  clearly for credit. *Remark:* Insertion must be done at the beginning of the list.
  - (b) By using the hash function  $h(x) = x \bmod m$  and quadratic probing in constructing a closed hash table  $H$  with  $m = 11$  buckets, insert the records in  $S$ , in the given order, into  $H$ . You must show your computations for locations and illustrate the final structure of your hash table  $H$  clearly for credit.
  - (c) Given two hash functions  $h(x) = x \bmod m$  and  $h^+(x) = p - x \bmod p$ . By using open addressing with  $f_i = i * h^+(x)$  and double hashing in constructing a closed hash table  $H$  with  $m = 11$  buckets and  $p = 5$ , insert the records in  $S$ , in the given order, into  $H$ . You must show your computations for locations and illustrate the final structure of your hash table  $H$  clearly for credit.

6. If a set of 4090 records is being stored using a binary tree T with 4090 nodes (one record per node), answer the following questions with *integer solution* if possible.
- What is the min height of T?
  - What is the max height of T?
  - What is the min number of leaves in T?
  - What is the max number of leaves in T?
  - If T is being implemented using the sequential array data structure, what is the size of an array A in order to store T?
7. Construct the (unique) binary tree corresponding to the given pair of tree traversals if possible.  
**Remark:** You must show all your steps clearly as illustrated in class for credit. If no such a tree is possible, you must justify your answer.
- Preorder: G B D F A I H J K L E C  
 Inorder: B I A F K J H L D E G C
  - Postorder: H I B C A K G E J D F  
 Inorder: I H C B K G J E F D A
  - Postorder: H G F B K L J I D A  
 Inorder: F G H B A D K J L I
8. Given a set S of 4 records with keys  $\{x_1, x_2, x_3, x_4\}$ ,  $x_1 < x_2 < x_3 < x_4$ . Construct all possible binary search trees (BST) that can be used to store S. Remark: You must illustrate all your BSTs clearly for credit.
9. Given a set of 10 records with priorities  $S = \{10, 15, 3, 8, 20, 5, 17, 27, 19, 12\}$ .
- Construct a BST T for S by inserting the records, in the given order, into an initially empty binary search tree. When done, delete 10, and then 20 from the tree.
  - Construct a BST T for S by inserting the records, in the reverse given order, into an initially empty binary search tree. When done, delete 5, and then 19 from the tree.
- Remark:** You must show your BST after each insertion/deletion for credits. For deletion, you must use deleteMin operations as discussed in class.