Total ______

Test # 3

April 19, 2018

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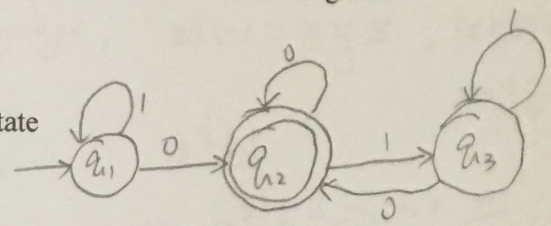
30 pts. 1. Using formulas for $r^k_{i,j}$ find a regular expression for the following dfa:

$1^*0(\lambda+(0+\lambda)^*(0+\lambda))$

$\emptyset^*$

	0	1	
q_1	q_2	q_1	initial, non-final state
q_2	q_2	q_3	final state
q_3	q_2	q_3	non-final state



Present your solution in the simplest form. **Show your work.**

	$k=0$	$k=1$	$k=2$
r^k_{11}	$1+\lambda$	1^*	1^*
r^k_{12}	0	1^*0	$1^*00^*=1^*0^+$
r^k_{13}	$\emptyset$	$\emptyset$	$1^*00^*1=1^*0^+1$
r^k_{21}	$\emptyset$	$\emptyset$	$\emptyset$
r^k_{22}	$0+\lambda$	$0+\lambda$	0^*
r^k_{23}	1	1	0^*1
r^k_{31}	$\emptyset$	$\emptyset$	$\emptyset$
r^k_{32}	0	0	$00^*=0^+$
r^k_{33}	$1+\lambda$	$1+\lambda$	$\lambda+0^*1$

$r^1_{23}=1+\emptyset$

$r^1_{31}=\emptyset+\emptyset$

$r^1_{32}=0+\emptyset$

$r^1_{33}=1+\lambda+\emptyset$

$r^2_{11}=1^*+1^*0(0+\lambda)^*(\emptyset)$

$r^2_{12}=1^*0+1^*0(0+\lambda)^*(0+\lambda)$

$r^2_{13}=\emptyset+1^*0(0+\lambda)^*1$

$r^2_{21}=\emptyset+\emptyset$

$r^2_{22}=(0+\lambda)+(0+\lambda)(0+\lambda)^*(0+\lambda)$

$r^2_{23}=1+(0+\lambda)(0+\lambda)^*1$

$r^2_{31}=\emptyset+0(0+\lambda)^*(\emptyset)$

$r^2_{32}=0+0(0+\lambda)^*(0+\lambda)$

$r^2_{33}=(1+\lambda)+0(0+\lambda)^*1$

$=1+\lambda+00^*1=1+\lambda+0^+1=(0^++\lambda)1+\lambda$

$r^1_{11}=(1+\lambda)+(1+\lambda)(1+\lambda)^*(1+\lambda)=1^*$

$r^1_{12}=0+(1+\lambda)(1+\lambda)^*0=$

$r^1_{13}=\emptyset+\emptyset.$

$r^1_{21}=\emptyset+\emptyset$

$r^1_{22}=0+\lambda+\emptyset$

$r^3_{12}=1^*0^++(1^*0^+1)(\lambda+0^*1)^*(0^+)$

$=1^*0^++(1^*0^+1\,0^*1\,0^+$

-3

$\boxed{=1^*0(0^*1^*)^*}$

20 pts. 2. Is the following language regular?

$$L = \{a^i b^j : i \geq 1, i = 2j\}$$

For a regular language find the DFA that accepts L. If L is not regular, prove it by using the pumping lemma.

Assume L is regular language, $\exists w = xyz$, $|xyz| \leq m$

$|xy| \geq 1$

$a^i b^j$

$= a^{2j} b^j \Rightarrow$ pick $\underline{m}$

$$\overbrace{a \cdots\cdots a}^{2m}\ \overbrace{b \cdots\cdots b}^{m}$$

$\underbrace{x \quad y}$ $\quad \underbrace{\quad z \quad}$

Case 1 : $i = 0$ $\qquad a^{2m-k} b^m \notin L \qquad k \geq 1$

It is not regular language!

3. Are the following languages context-free?

10 pts. (a) $\{a^i b^j c^k : j, k \geq 0, i = j + k\}$,

10 pts. (b) $\{a^i b^j c^k : i \geq 0, i = j = k\}$.

For a context-free language find a grammar. If a language is not context-free, prove it by using the pumping lemma.

a)

$S \to aBbC$

$B \to aBb \mid aBC$

$C \to c \mid cC$

aBbC

aaBb

end?,

-3

$$\boxed{\begin{array}{l} S_0 \to aS_0c \mid S_1 \\ S_1 \to aS_1b \mid \lambda \end{array}}$$

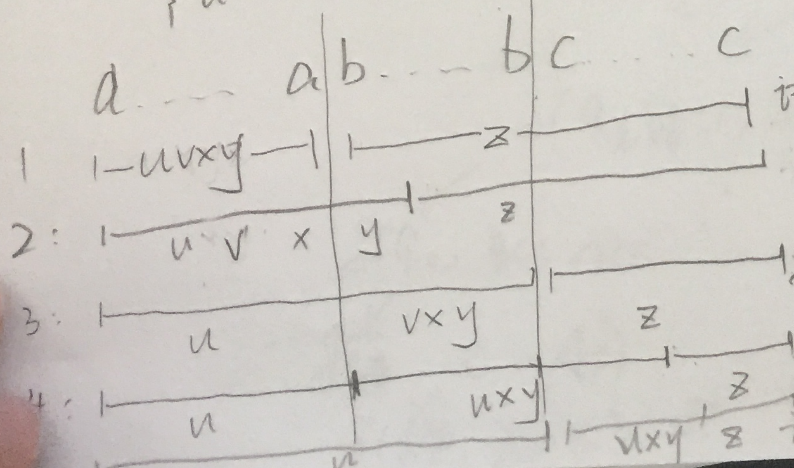
b) Assume L is a context-free language, $w = uvxyz$

$1 \leq |vxy| \leq m$

$i = j = k$

$\{a^m b^m c^m\}$

$W = uv^ixy^iz$



$i=0,\ a^{m-k}b^mc^m \notin L \quad 1 \leq k \leq m$

$,\ a^{m-r}b^{m-s}c^m \notin L \quad 1 \leq r+s \leq m$

$i=0,\ a^mb^{m-k}c^m \notin L \quad 1 \leq k \leq m$

$a^mb^{m-r}c^{m-s} \quad 1 \leq r+s \leq m$

$i=0\ a^mb^mc^{m-k} \quad 1 \leq k \leq m$

30 pts. 4. Design a pushdown automaton that accepts the following language

$$\{ab^ic^{2i}b^{2j}c^j : i, j \geq 1\}.$$

Do not use transition graph.

beginning top stack $= \{z\}$

control unit $= \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Gamma = \{A\}$

$\Sigma = \{a, b, c\}$

$F = \{q_5\}$

$\delta(q_0, a, z) = \{q_1, z\}$

$\delta(q_1, b, z) = \{q_1, AAz\}$

$\delta(q_1, b, A) = \{q_1, AAA\}$

$\delta(q_1, c, A) = \{q_2, \lambda\}$

$\delta(q_2, c, A) = \{q_2, \lambda\}$

$\delta(q_2, b, \cancel{z}) = \{q_3, Az\}$

$\delta(q_2, b, A) = \{q_3, AA\}$

$\delta(q_3, b, A) = \{q_2, A\}$

$\delta(q_2, c, A) = \{q_4, \lambda\}$

$\delta(q_4, c, A) = \{q_4, \lambda\}$

$\delta(q_4, \lambda, z) = \{q_5, \lambda\}$

check

$\delta(q_0, abccbbbbcc, z)$

$\downarrow$

$\delta(q_1, bccbbbbcc, z)$

$\downarrow$

$\delta(q_1, ccbbbbcc, AAz)$

$\downarrow$

$\delta(q_2, cbbbbcc, Az)$

$\delta(q_2, bbbbcc, z)$

$\delta(q_3, bbbcc, Az)$

$\delta(q_3, bbcc, AAz)$

$\delta(q_2, bbcc, Az)$

$\delta(q_2, bcc, AAz)$

$\delta(q_4, c, Az)$

$\delta(q_4, \lambda, z) \Rightarrow \{q_5, \lambda\}$