

## HW7: Stable Matching

**Q1: Consider the following preferences**

	favorite ↓			least favorite ↓			
	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Xavier	Amy	Bertha	Clare		Amy	Yancey	Xavier
Yancey	Bertha	Amy	Clare		Bertha	Xavier	Yancey
Zeus	Amy	Bertha	Clare		Clare	Xavier	Yancey

Men's Preference Profile      Women's Preference Profile

If you choose to index the men (x,y,z) as (0,1,2), and the women (a,b,c) as (0,1,2), and begin with an initial linked list of x->y->z, write down how the arrays M, W, and WR are constructed, and how the linked list and arrays RK and WP change over time.

**Linked list:** a list of free men

**M:** Men's preference list

**W:** Women's preference list

**WR:** Women's ranking array, where  $WR[i][j]$  stores the ranking of men  $j$  in woman  $i$ 's preference list

**RK:** Men's next proposal array, where  $RK[i]$  stores the next woman the man  $i$  is going to propose in the next round.

**WP:** Women's partner array, where  $WP[i]$  stores the current partner of woman  $i$ ,  $WP[i] = -1$  is woman  $i$  is unpaired.

**A1:**

*Initial setup:*

$X = 0; Y = 1; Z = 2;$

$A = 0; B = 1; C = 2;$

$M = [ \begin{matrix} [1,2,3] \\ [2,1,3] \\ [1,2,3] \end{matrix} ]$

$W = [ \begin{matrix} [2,1,3] \\ [1,2,3] \\ [1,2,3] \end{matrix} ]$

$WR = [ \begin{matrix} [2,1,3] \\ [1,2,3] \\ [1,2,3] \end{matrix} ]$

*Linked list:* X->Y->Z;

$RK[0]=0, RK[1]=0, RK[2]=0;$

$WP[0]=-1, WP[1]=-1, WP[2]=-1;$

*X proposes to A:*

*Linked list:* **Y->Z;**

**$RK[0]=1, RK[1]=0, RK[2]=0;$**

**$WP[0]=0, WP[1]=-1, WP[2]=-1;$**

*Y proposes to B:*

*Linked list: Z;*

$RK[0]=1$ ,  $RK[1]=1$ ,  $RK[2]=0$ ;

$WP[0]=0$ ,  $WP[1]=1$ ,  $WP[2]=-1$ ;

*Z Y proposes to A:*

*Linked list: Z;*

$RK[0]=1$ ,  $RK[1]=1$ ,  $RK[2]=1$ ;

$WP[0]=0$ ,  $WP[1]=1$ ,  $WP[2]=-1$ ;

(Check  $WR[0][0] = 2$ ,  $WR[0][2] = 3$ ;  $WR[0][0] < WR[0][2]$ , A rejects Z)

*Z Y proposes to B:*

*Linked list: Z;*

$RK[0]=1$ ,  $RK[1]=1$ ,  $RK[2]=2$ ;

$WP[0]=0$ ,  $WP[1]=1$ ,  $WP[2]=-1$ ;

(Check  $WR[1][1] = 2$ ,  $WR[1][2] = 3$ ;  $WR[1][1] < WR[1][2]$ , B rejects Z)

*Z Y proposes to C:*

*Linked list: N/A;*

$RK[0]=1$ ,  $RK[1]=1$ ,  $RK[2]=3$ ;

$WP[0]=0$ ,  $WP[1]=1$ ,  $WP[2]=2$ ;