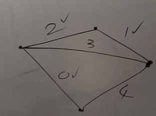
* Minimum Spanning Tree

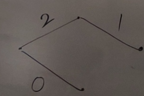
Given a graph $G = (V, E, W)$ (undirected graph)

find a set of edges $E' \subseteq E$

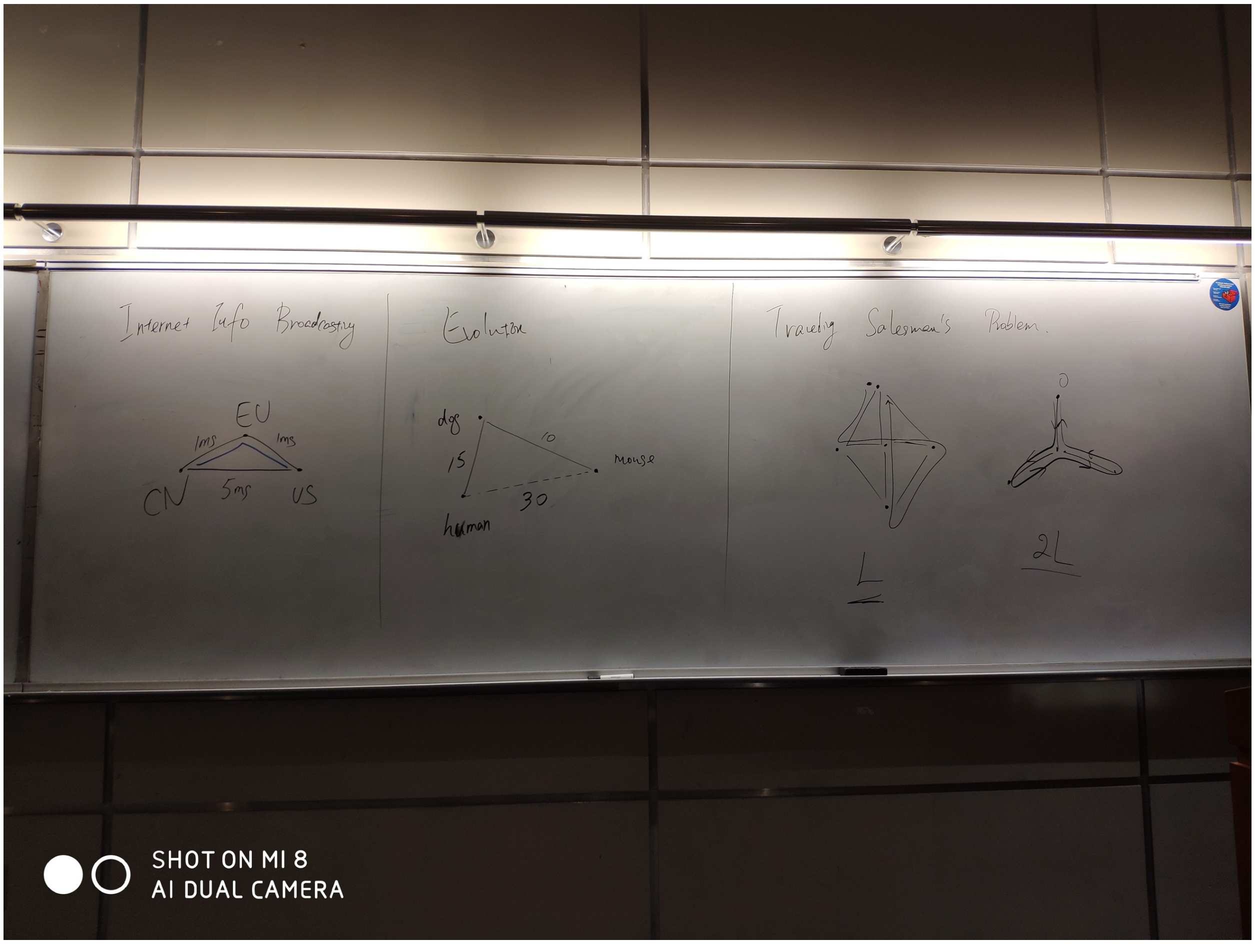
to connect all vertices

and $\min_{e \in E'} W_e$





$1+2+0=3$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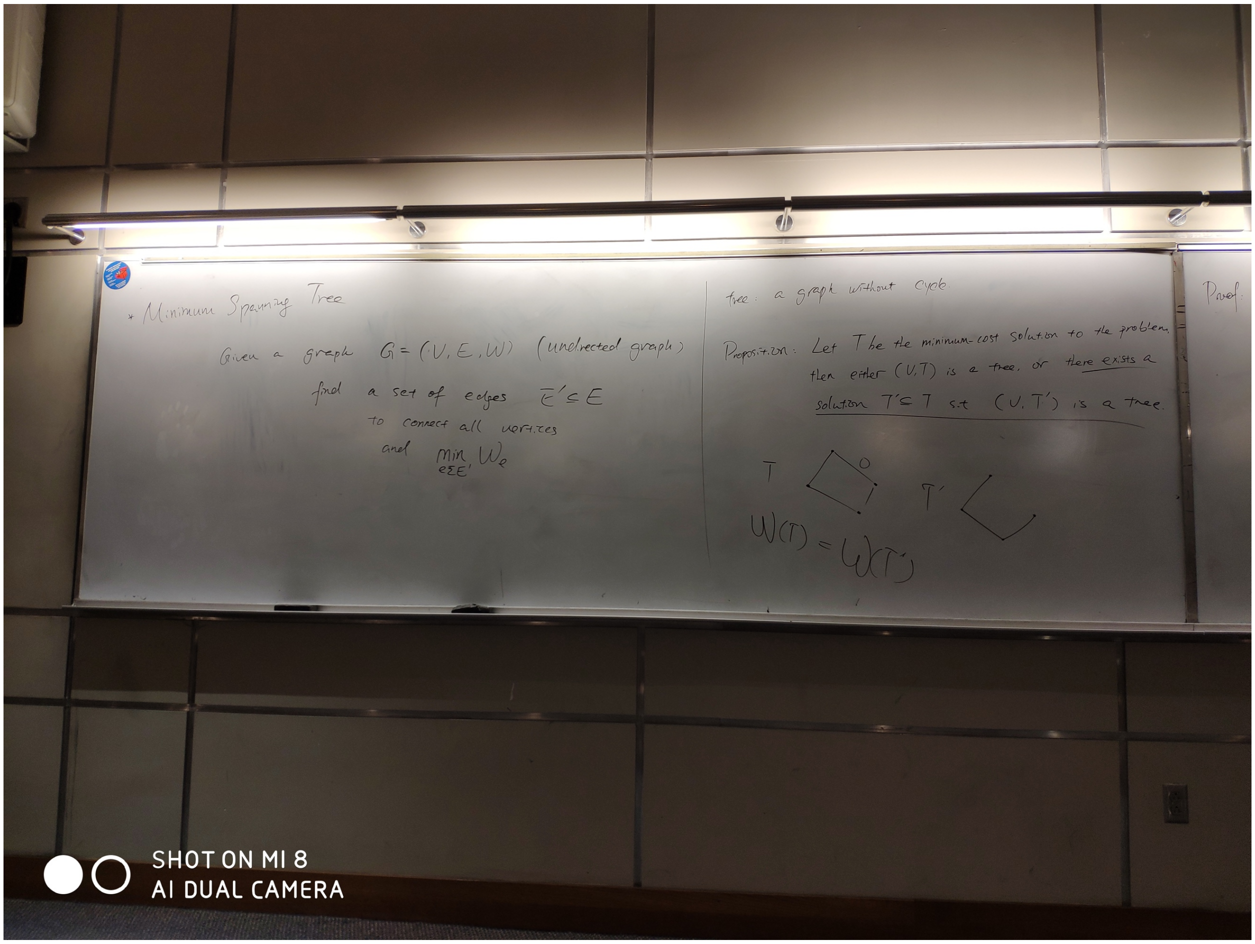

* Minimum Spanning Tree
Given a graph G = (V, E, W) (undirected graph)
find a set of edges E' ⊆ E
to connect all vertices
and min_{e∈E'} W_e
tree: a graph without cycle.
Proposition: Let T be the minimum-cost solution to the problem,
then either (V,T) is a tree, or there exists a
solution T' ⊆ T s.t (V, T') is a tree.
T
0
1
T'
W(T) = W(T')
Proof:
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Proof: if T is not a tree.

that means T contains at one cycle.

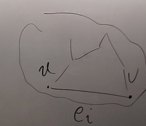
We can break the cycle by removing one of its edges, it does not disconnect any nodes.

The remain graph is still connected, but with (if the edge is positively weighted) less total weight, which contradicts with the assumption that T is a minimum weighted solution.

(if the edge is 0-weighted) it is T'

Greedy: progressively include the minimum-weighted edges.

Kruskal's algorithm:



Sort edges s.t. $w_1 \le w_2 \le \dots \le w_m$.

```
for each edge i = 1 ... m.
    (u, v) = e_i
    if (u and v are in different connected component)
        include e_i;
        merge the connected components for u, v into one
    endif.
endfor.
```

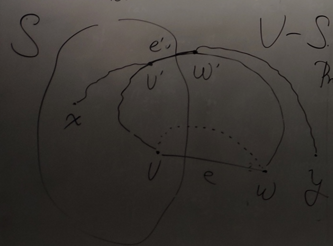
Cont. if we exchange e and e'. s.t $T' = \{T - e\} \cup e$.

- T' has the same or even lower total weight by algorithm.

- T' is a spanning tree (it still connect all vertices)
  for any vertices $x \in S$ and $y \in V-S$ that are previously connected using $e' = (v', w')$. We know that paths $\underline{(x \ldots v')}$ and $\underline{(w' \ldots y)}$ exist. Also, because v and w are connected through v', w'. there exis paths $\underline{(v', v)}$ and $\underline{(w', w)}$.
  So, we can connect x, y by $x \ldots v' \ldots \underbrace{v, w}_{e} \ldots w' \ldots y$.

- T' does not contain cycle.
  in T, there exist only one v, w path, i.e $v \ldots v', w' \ldots w$.
  otherwise there would be a cycle in T, contradicting with the fac
  T is an MST.
  adding e' will create a cycle. $v \ldots \underbrace{v', w'}_{e'} \ldots \underbrace{w, v}_{e}$. this cycle is unique because of the above argument. We have broken the cycle by remo
  e'. So no cycle exists in T'.

Proposition: Assume that all edges have distinct weights. Let S be any subset of vertices that is neither empty nor the entire set, and let $e=(v,w)$ be the minimum-cost edge picked by the greedy algorithm and has one end in S and another end in $V-S$. Then every MST (minimum spanning tree) contains the edge e. (cut property)



Proof: (by contradiction) if we have an MST that does not contain e. Let this MST be T.

Since T is an MST $\Rightarrow$ all vertices are connected $\Rightarrow$ v and w are connected.

It must exists a path between v and w. Because $v \in S$ and $w \in V-S$ $\Rightarrow$ there exists an edge e' (v', w') in the path s.t. $v' \in S$ and $w' \in V-S$.