EECS210 **Written HW#6** **Due: 11-7-17**

1. (20) When an algorithm A is used to compute a problem Π with input S, it requires 0.2ms (10^{-3}s) to execute A when $|S| = 10^4$. If the complexity of the algorithm A is given by the following closed-form expressions, compute the time required to execute algorithm A when $|S| = 10^8$. Simplify your solution if possible.
   $(a)\ T(n) = 168n.$
   (b) $T(n) = 268n\log_{10} n.$
   $(c)\ T(n) = 368n^2.$
   $(d)\ T(n) = 2^n.$

2. (20) Let f(n) and g(n) be any arbitrary positive functions defined from N to R^+. Determine the value of the following propositions. You must answer either ***True or False*** but not both. Justify your answer.
   (a) If f(n) ≠ O(g(n)), then g(n) = Ω(f(n)).
   (b) If f(n) = O(g(n)), then g(n) = Ω(f(n)).
   (c) If f(n) = Θ(g(n)) and g(n) = Θ(f(n)), then f(n) = g(n).
   (d) If f(n) = o(g(n)), then f(n) ≠ Θ(g(n)).
   (e) If f(n) = O(g(n)), then $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$, where c is a constant with $0 \le c < \infty$.

3. (10) Given two algorithms A_1 and A_2 with $T_1(n) = 2n^2 + n + 210$ and $T_2(n) = 422n$. Compute smallest integer k such that algorithm A_2 outperforms algorithm A_1 for all $n \ge k$. You must show your computation for k clearly for credits.

4. (10) Using the definition of big-O to prove that $\frac{n^3 - 3n^2 + 210}{3n^2 - 12n + 15} = O(n)$.

5. (10) Using Proof by Contradiction technique to prove that $3n^3 - 168n^2 + 210n - 121 \ne \Omega(n^4)$.

6. (10) Using the definition of big-Θ to prove that $3n^3 - 168n^2 + 268n\lg n - 660 = \Theta(n^3)$.

7. (20) Using the Limit Ratio Theorem to prove or disprove the following statements.
   (a) $\frac{2n^3 + 21n - 18}{28n + 3^{10}} = \Theta(n^2).$

   (b) $5^n = \Omega(2^{n+10}).$