Name: Key EECS 662 / Spring 2019 / Quiz 6

Consider a language with the following typing rules

$$\frac{}{\Gamma \vdash x : \Gamma(x)} \quad \frac{}{\Gamma \vdash z : \texttt{Int}} \quad \frac{}{\Gamma \vdash b : \texttt{Bool}}\ (b \in \{\texttt{True}, \texttt{False}\})$$

$$\frac{\Gamma[x \mapsto T_1] \vdash t : T_2}{\Gamma \vdash \lambda x.t : T_1 \to T_2} \quad \frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1\, t_2 : T_2} \quad \frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : (T_1, T_2)} \quad \frac{\Gamma \vdash t : (T_1, T_2)}{\Gamma \vdash \texttt{fst}\, t : T_1}$$

$$\frac{\Gamma \vdash t : (T_1, T_2)}{\Gamma \vdash \texttt{snd}\, t : T_2} \quad \frac{\Gamma \vdash t_1 : \sigma \quad \Gamma[x \mapsto \sigma] \vdash t_2 : T}{\Gamma \vdash \texttt{let}\ x = t_1\ \texttt{in}\ t_2 : T} \quad \frac{\Gamma \vdash t : \forall \alpha.\sigma}{\Gamma \vdash t : \sigma[\tau/\alpha]} \quad \frac{\Gamma \vdash t : \sigma}{\Gamma \vdash t : \forall \alpha.\sigma}\ (\alpha \notin fv(\Gamma))$$

1. For each of the following pairs of types, give the *most specific* type scheme that includes both types. For example, the most specific type scheme that includes both $\texttt{Int}$ and $\texttt{Int} \to \texttt{Int}$ is $\forall \alpha.\alpha$, but the most specific type scheme that includes $\texttt{Int} \to \texttt{Int}$ and $\texttt{Int} \to Bool$ is $\forall \alpha.\texttt{Int} \to \alpha$.

   (a) $\texttt{Int} \to \texttt{Int}$, $\texttt{Bool} \to \texttt{Int}$

   $\forall \alpha.\alpha \to \texttt{Int}$

   (b) $(\texttt{Int}, \texttt{Bool}) \to \texttt{Int}$, $(\texttt{Bool}, \texttt{Int}) \to \texttt{Bool}$

   $\forall \alpha.\forall \beta.(\alpha, \beta) \to \alpha$

   (c) $\texttt{Int} \to \texttt{Int}$, $(\texttt{Int} \to \texttt{Int}) \to (\texttt{Int} \to \texttt{Int})$.

   $\forall \alpha.\alpha \to \alpha$

2. Derive the typing assertion $\emptyset \vdash \lambda a.\texttt{snd}\, a : \forall \alpha.\forall \beta.(\alpha, \beta) \to \beta$.

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{}{\{a \mapsto (\alpha, \beta)\} \vdash a : (\alpha, \beta)}}{\{a \mapsto (\alpha, \beta)\} \vdash \texttt{snd}\, a : \beta}}{\emptyset \vdash \lambda a.\texttt{snd}\, a : (\alpha, \beta) \to \beta}}{\emptyset \vdash \lambda a.\texttt{snd}\, a : \forall \beta.(\alpha, \beta) \to \beta}\ \text{消任意变量}}{\emptyset \vdash \lambda a.\texttt{snd}\, a : \forall \alpha.\forall \beta.(\alpha, \beta) \to \beta}$$