$O(k+1) \le c(k+1)$

$T(k+1) \le c(k+1)\log_2\left(\frac{k+1}{2}\right) + O(k+1)$

$\le c(k+1)(\log_2(k+1) - 1) + a(k+1)$

$= c(k+1)\log_2(k+1) + (a-c)(k+1)$

if we make $c \ge a$.

$\Rightarrow T(k+1) \le c(k+1)\log_2(k+1)$

Conclusion. ...

Substitution:

assume $T(n) = O(n\log n) \Rightarrow T(n) \le cn\log(n)$

this is true if we can prove that c always exists for large n

Induction (Base) $T(2) \le c \cdot 2 \cdot \log_2 2 = 2c$ this is always true because $T(2)$ is a constant

(induction). $T(k) \le ck\log k$ for $2 \le k$

$T(k+1) = 2T\left(\frac{k+1}{2}\right) + O(k+1) \le 2 \cdot c\frac{(k+1)}{2}\left(\log\left(\frac{(k+1)}{2}\right)\right) + O(k+$

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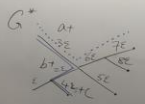
relax the constrain that G only has distinctly-weighted edges.



T (the hypothetical MST of G)

$W_G(T^*) = a+b+c$

T^*:

A: Kruskal's algorithm

$A(G^*) = T^*$ (MST: true)

$W_{G^*}(T^*) = (a+3\varepsilon)+(b+2\varepsilon)+(c+4\varepsilon)$

$W_{G^*}(T) = a+d+(3\varepsilon+7\varepsilon+6\varepsilon) \le a+d+(\varepsilon+2\varepsilon+3\varepsilon+\cdots+8\varepsilon) = a+d+\varepsilon\sum i$

$= W_G(T)+\varepsilon\sum i$

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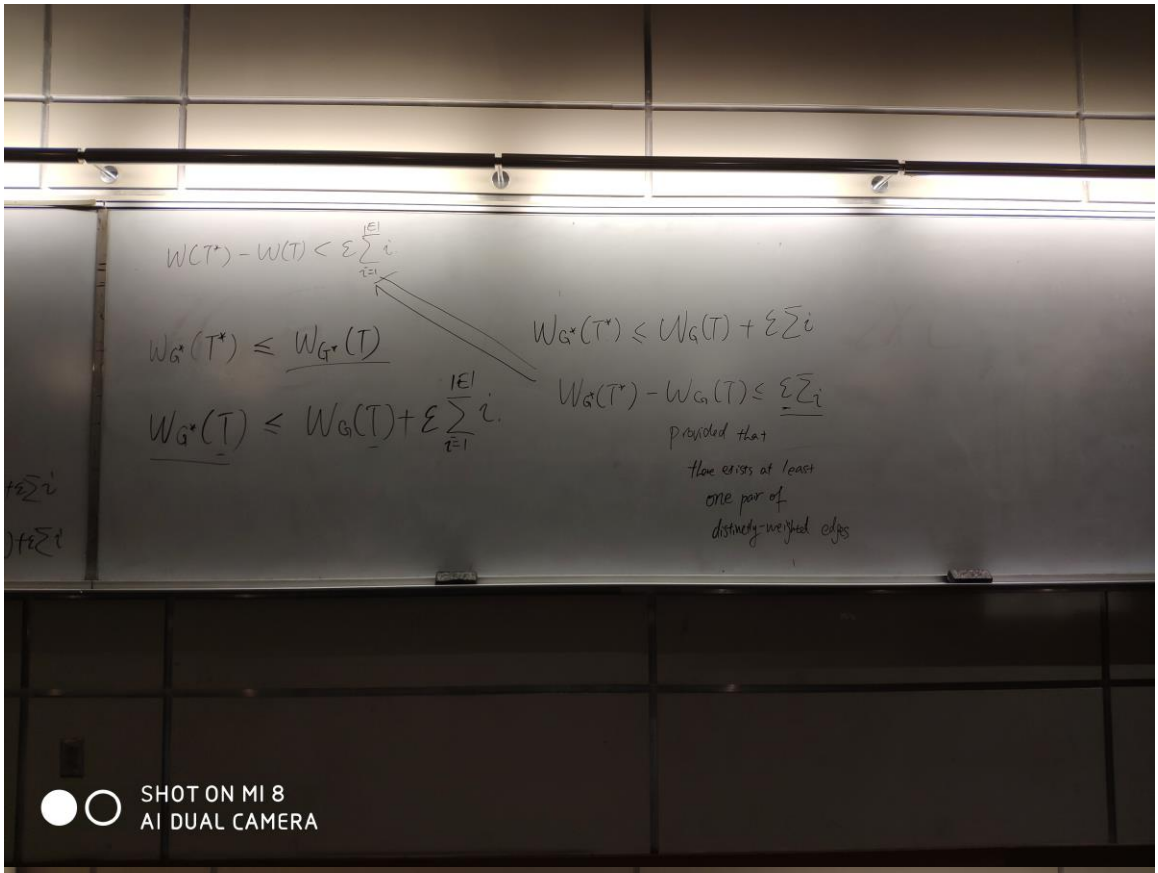
$W(T^*) - W(T) < \varepsilon \sum_{i=1}^{|E|} i$
$W_{G^*}(T^*) \leq W_{G^*}(T)$
$W_{G^*}(T) \leq W_G(T) + \varepsilon \sum_{i=1}^{|E|} i$
$W_{G^*}(T^*) \leq W_G(T) + \varepsilon \sum i$
$W_G(T^*) - W_{G_1}(T) \leq \varepsilon \sum i$
provided that
there exists at least
one pair of
distinct-weighted edges
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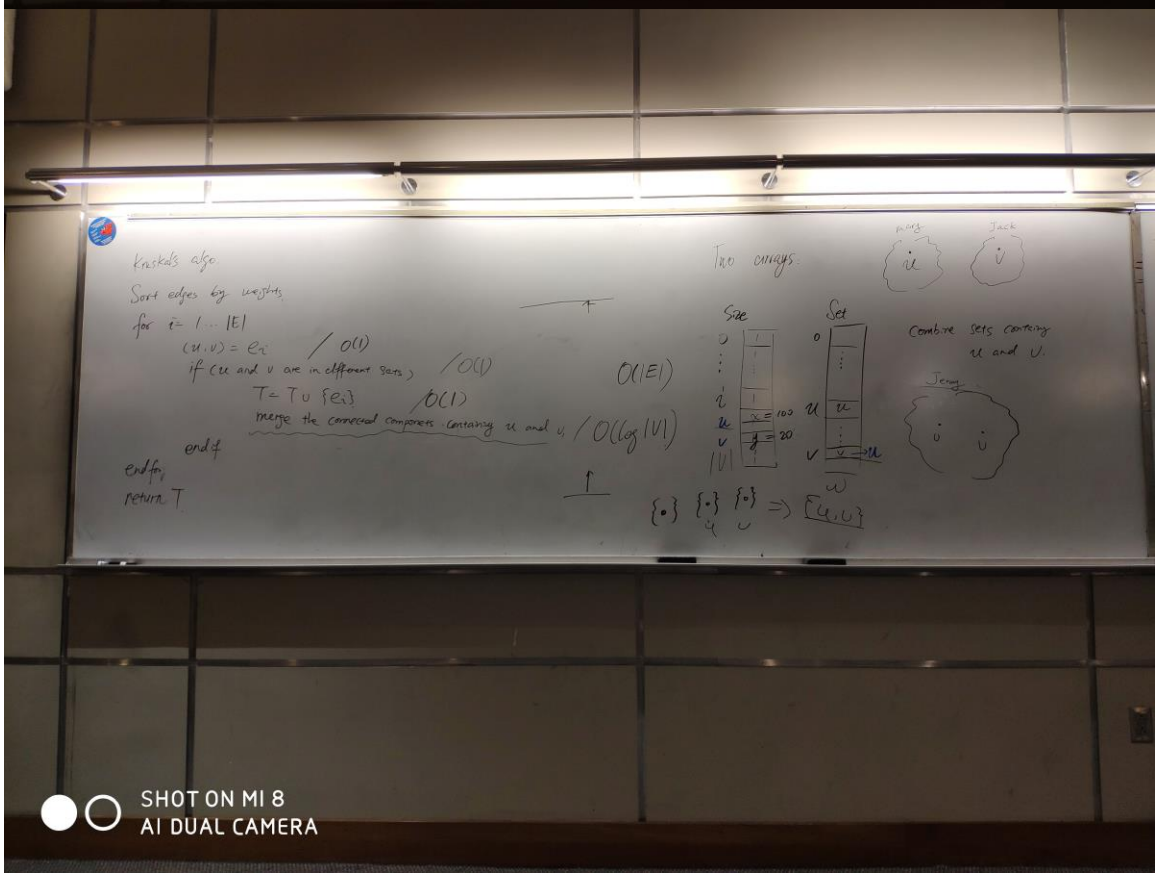
kruskals algo
Sort edges by weights
for i = 1 ... |E|
(u,v) = e_i / O(1)
if (u and v are in different sets) / O(1)
T = T ∪ {e_i} / O(1)
merge the connected components containing u and v / O(log|V|)
end if
end for
return T
O(|E|)
Two arrays
Size
Set
Jack
Combine sets containing u and v.
Jerry
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Proposition: Relabling can be done in $\log(|V|)$ time on average

Proof: We choose to rename the set with a smaller size

* for each vertex

each merge operation must at least double the size of set that contains it

because each set can contain at most $|V|$ elements, there are at most $\log_2|V|$ renaming for each vertex

* Total # of renaming: $|V| \cdot \log_2|V|$

Since there are at most $|V|$ merges, each merge takes $O(\log|V|)$ on average

$X = 100$

$y = 20$

$\Downarrow$ 20 renaming

$X+y = 120$

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Divide-and-Conquer Algorithms

Merge-Sort.

$T(1)$ { given a list of numbers

Seperate the list into two sets, each with a half size

$T(\frac{n}{2})$ MergeSort(S_1)

$T(\frac{n}{2})$ MergeSort(S_2)

$O(n)$ Combine S_1 and S_2 into S

return S

$S = 1, 2, 3, 5, 6$

$1, 5, 6$ $\quad 2, 3$ $\quad 5$ $\quad O(n)$

$1, 5$ $\quad 6$ $\quad 2$, 3 $\quad 5$ $\quad O(n)$

1 $\quad 5$

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