

Instruction: You must show your work clearly for credit.

1. (10) Let n and m be any integers. Prove that if the product nm is even, then either n is even or m is even.
2. (12) Prove that for any odd integer n , there exist integers a and b such that $n = a^2 - b^2$.
3. (12) Prove that $\sqrt[3]{2}$ is irrational.
4. (10) **Prove, or disprove**, that there exist two sets A and B such that $A \in B$ and $A \subseteq B$.
5. (10) Let A and B be any sets. Prove, or disprove, that if A and B have the same power set, then $A = B$.
6. (10) Let A , B , and C be any sets. Prove that $(B - A) \cup (C - A) = (B \cup C) - A$ using a membership table.
7. (12) Let A , B , and C be any sets. Use **direct proof technique** to prove that $(B - A) \cup (C - A) = (B \cup C) - A$.
8. (12) Let A , B , and C be any sets. Use **direct proof technique** to prove that $(A - B) - C = (A - C) - (B - C)$.
9. (12) Let A , B , and C be sets. Using Set Identities to prove that
$$\overline{(A \cap B) \cup C} = \overline{(A \cup C)} \cap \overline{(B \cup C)}.$$