EECS210 **Written HW2** **Due: 9-19-17**

Instruction: You must show your work clearly for credit.

1. (10) Let S(x) be the statement "x is a student in EECS210," let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let H(x) be the statement "x has a hamster." Assume that the domain of x is the set of all people, express each of the following statements in terms of S(x), C(x), D(x), F(x), quantifiers, and logical connectives.
   (a) A student in EECS210 has a cat, a dog, and a hamster.
   (b) All students in EECS210 have a cat, a dog, or a hamster.
   (c) Some student in EECS210 has a cat and a hamster, but not a dog.
   (d) No student in EECS210 has a cat, a dog, and a hamster.
   (e) For each of the three animals, cats, dogs, and hamster, there is a student in EECS210 who has this animal as a pet.

2. (20) A discrete mathematics class consists of 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior.
   Let P(x,y,z) be the statement that student x has class standing y and is majoring in z, where the variable x ranges over students in the class, the variable y ranges over the four class standings, and the variable z ranges over all possible majors.
   Express each of the following statements in terms of P(x,y,z), quantifiers, and logical connectives and then determine its truth value. Justify your answer.
   (a) There is a student in the class who is a junior.
   (b) Every student in the class is a computer science major.
   (c) There is a student in the class who is neither a mathematics major nor a junior.
   (d) Every student in the class is either a sophomore or a computer science major.
   (e) There is a major such that there is a student in the class in every year of study with that major.

3. (10) Do Problem 6 on Page 78.
   Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained." *Remark:* You must indicate the reason/rule used in your argument.

4. (10) Use direct proof technique to prove that for any integers x, y, and z, if x + y and y + z are both even, then x + z must also be even.

5. (10) Use proof by contradiction technique to prove that for any integers x, y, and z, if x + y and y + z are both even, then x + z must also be even.

6. (10) Use proof by contraposition technique to prove that for any given integer n, if $5n^2 + 210$ is odd, then n must also be odd.

7. (5) Without evaluating the numbers $65^{1000} - 8^{2001} + 3^{177}$, $79^{1212} - 9^{2399} + 2^{2001}$, and $24^{4493} - 5^{8192} + 7^{1777}$, prove that the product of two of these numbers is nonnegative.

8. (5) Use a counter-example to disprove the statement that for any rational numbers x and y, x^y must be a rational.

9. (10) Use a proof by contradiction to prove that the product of a nonzero rational number and an irrational number is irrational.

10. (10) Use proof by contradiction technique to prove that $\sqrt{3}$ is irrational.

9/8/17