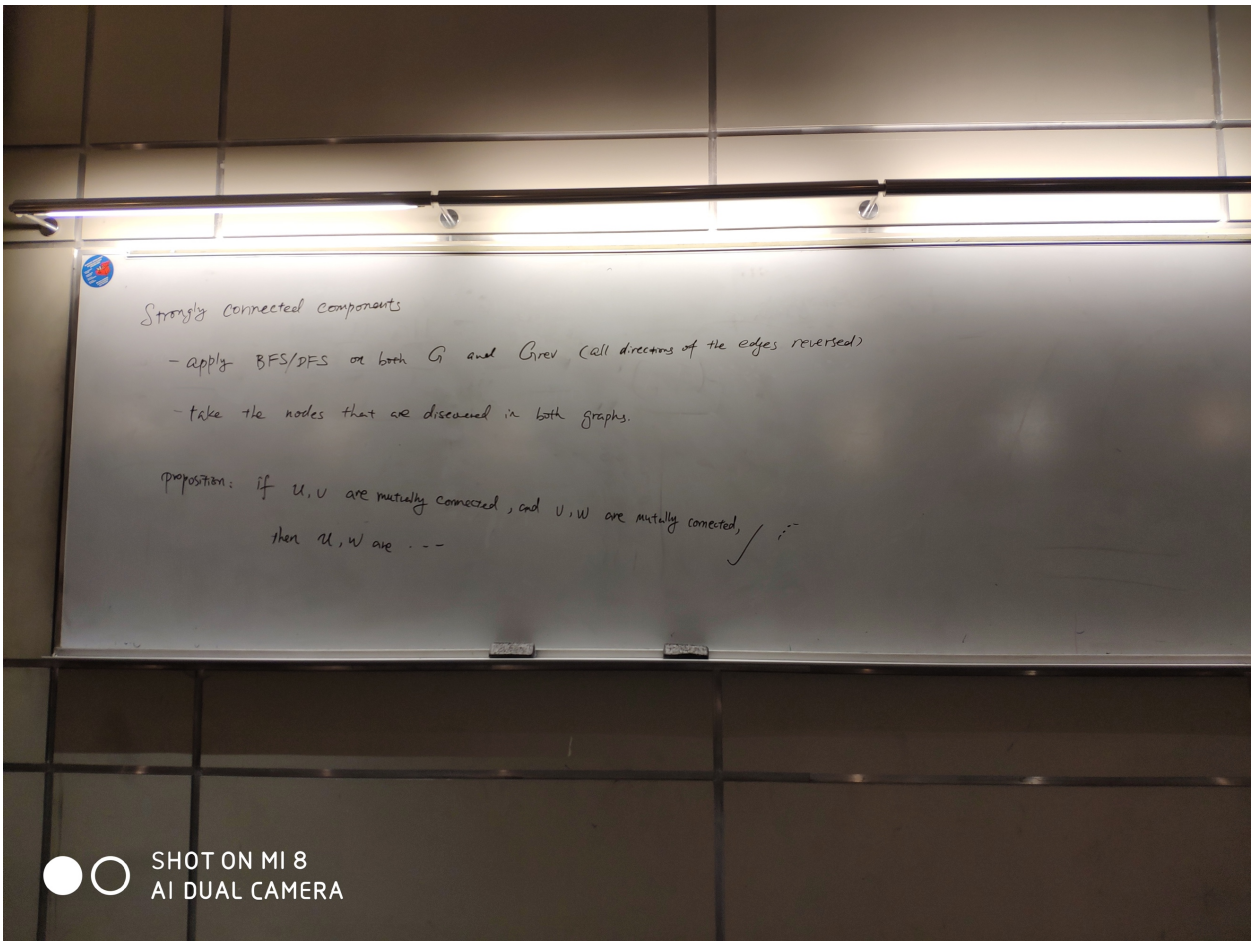
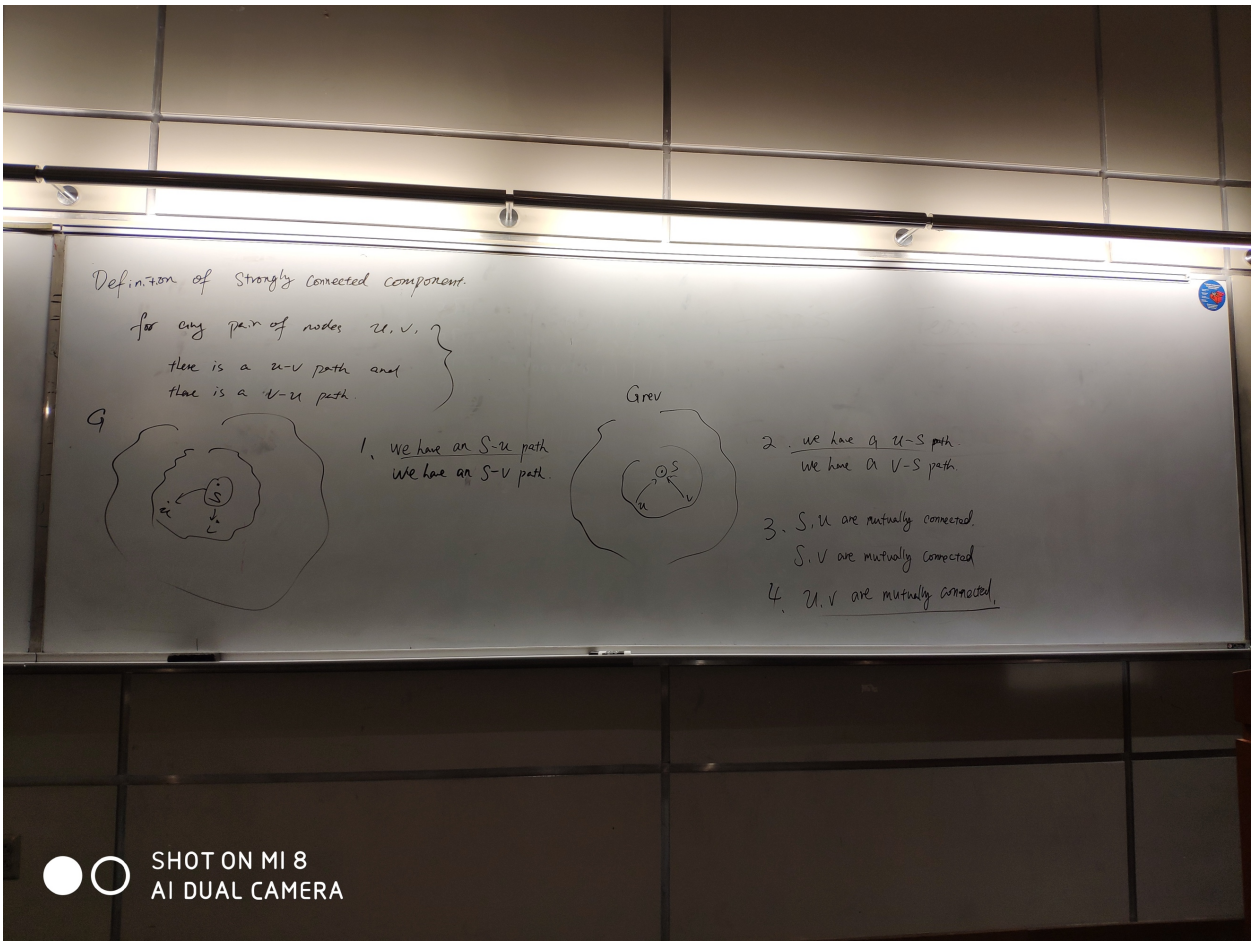

Strongly connected components
- apply BFS/DFS on both G and Grev (all directions of the edges reversed)
- take the nodes that are discovered in both graphs.
proposition: if u, v are mutually connected, and v, w are mutually connected,
then u, w are . . .
SHOT ON MI 8
AI DUAL CAMERA


Definition of Strongly connected component.
for any pair of nodes u, v,
there is a u-v path and
there is a v-u path
G
s
u
v
1. we have an S-u path
we have an S-v path.
Grev
s
u
v
2. we have a u-S path
we have a v-S path
3. S, u are mutually connected.
S, v are mutually connected
4. u, v are mutually connected.
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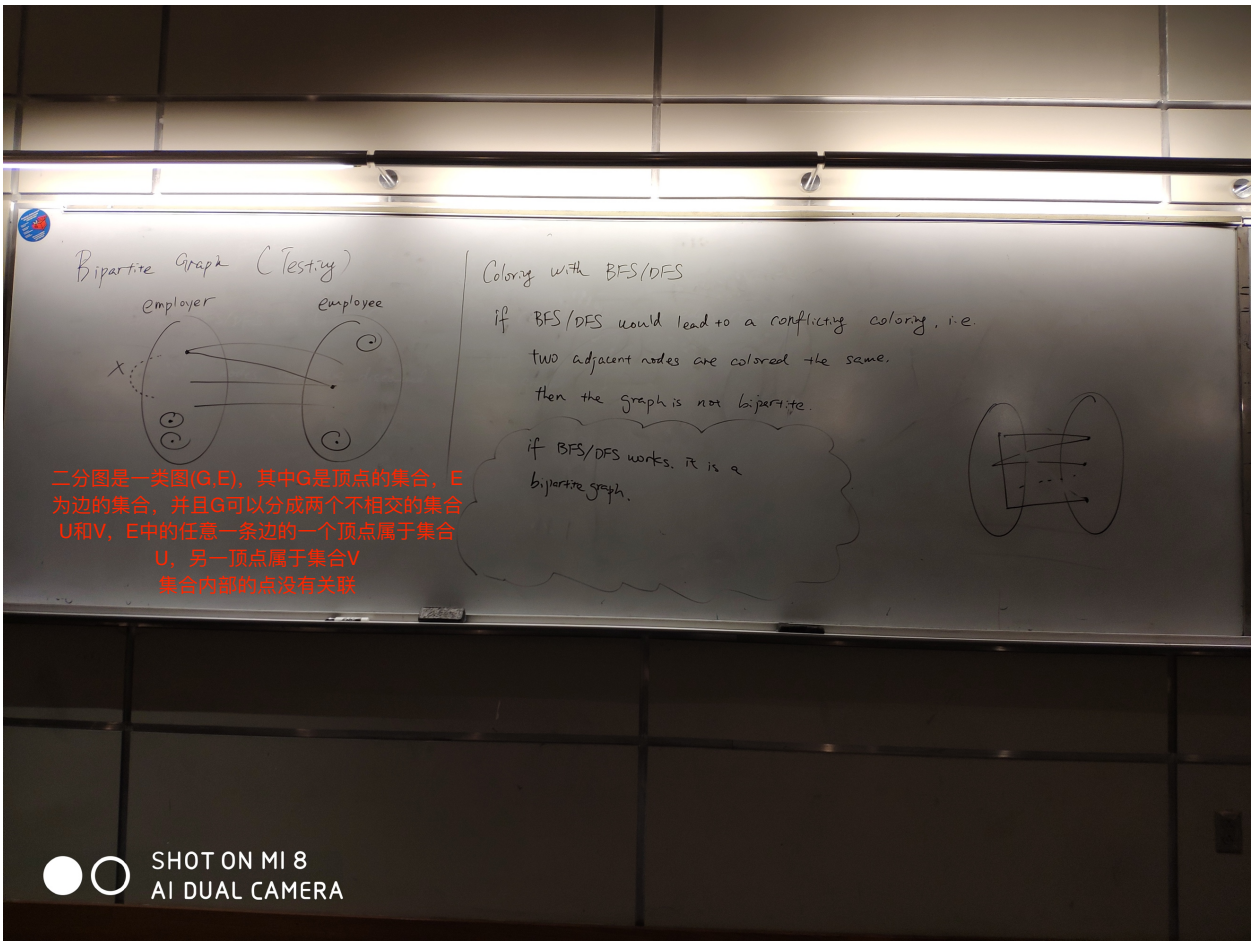
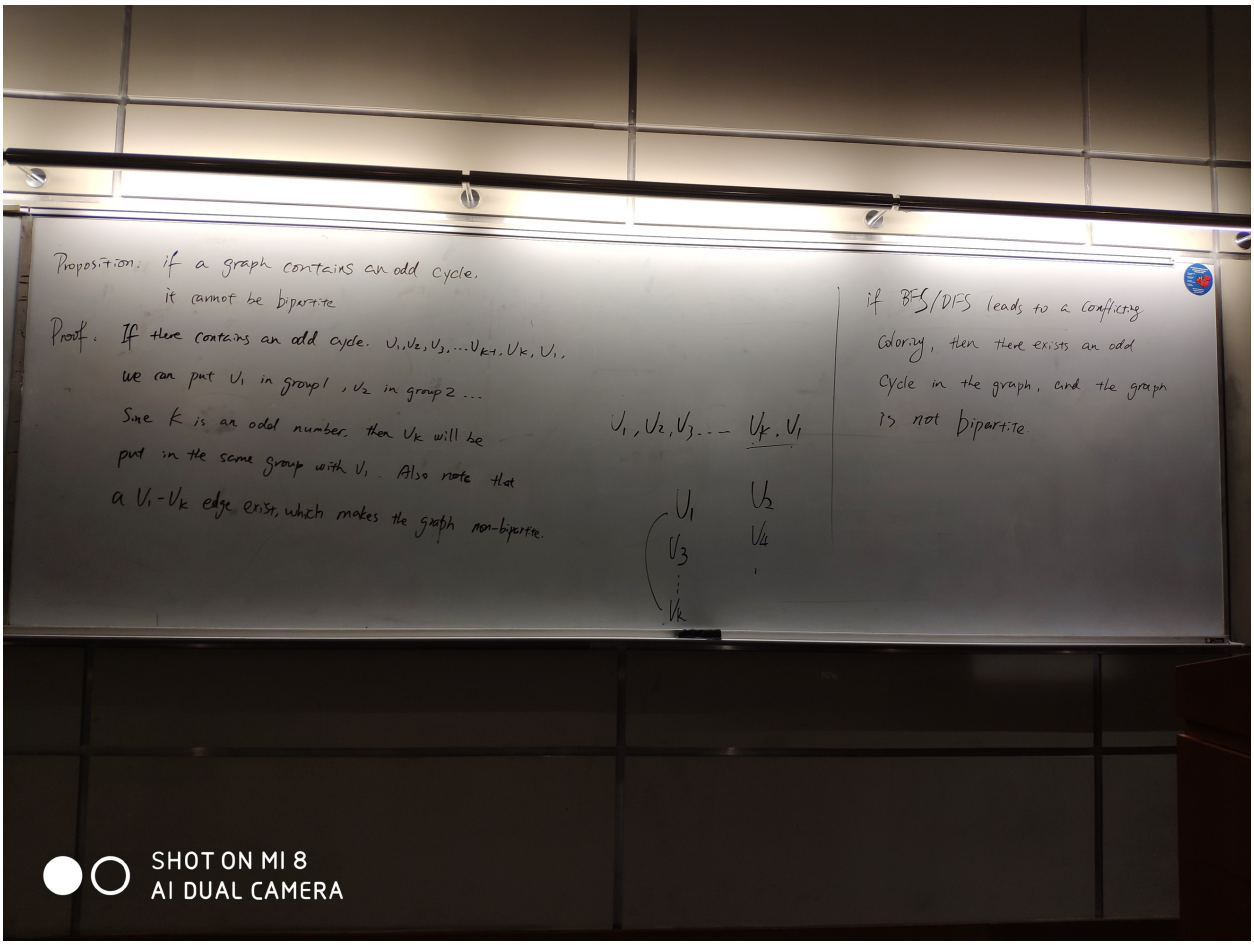


二分图是一类图(G,E)，其中G是顶点的集合，E为边的集合，并且G可以分成两个不相交的集合U和V，E中的任意一条边的一个顶点属于集合U，另一顶点属于集合V
集合内部的点没有关联


Proposition: if a graph contains an odd cycle, it cannot be bipartite
Proof: If there contains an odd cycle. $V_1, V_2, V_3, \ldots V_{k-1}, V_k, V_1$, we can put V_1 in group1, V_2 in group2 ...
Since k is an odd number. then V_k will be put in the same group with V_1. Also note that a V_1-V_k edge exist, which makes the graph non-bipartite.
$V_1, V_2, V_3 \ldots V_k, V_1$
V_1 V_2
V_3 V_4
V_k
if BFS/DFS leads to a conflicting coloring, then there exists an odd cycle in the graph, and the graph is not bipartite
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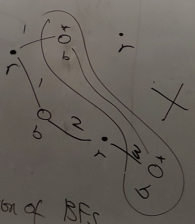
Proposition: If BFS fails, then there exists an odd cycle in the graph

Proof: Let the conflicting coloring, i.e. coloring of two adjacent nodes with the same color, happen on u and v.



Since u, v are adjacent.

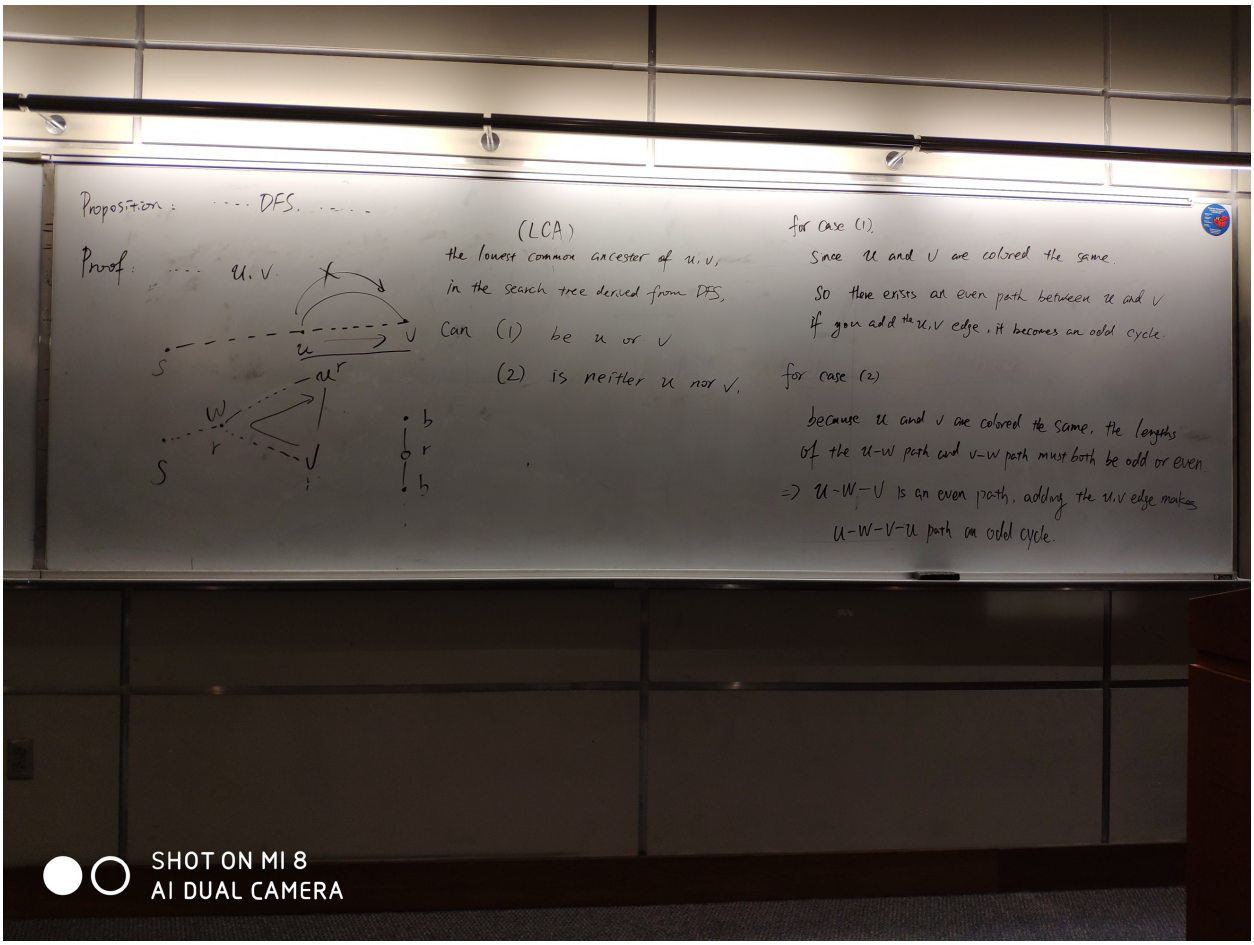
either u, v are colored at the same iteration of BFS or colored consecutively

It is impossible that u, v are colored consecutively, because they would be colored differently otherwise.

$\Rightarrow$ u, v must be colored in the same iteration.

Let the source be s, then s-u path and s-v path have the same length.

$\Rightarrow$ the u-s-v path is even. If you add the edge v-u, the path u-s-v-u becomes an odd cycle.


Proposition : ---- DFS. ----
Proof: ---- u, v
u
v
s
w
r
b
r
b
(LCA)
the lowest common ancester of u, v,
in the search tree derived from DFS,
Can (1) be u or v
(2) is neither u nor v.
for case (1).
Since u and v are colored the same.
So there exists an even path between u and v
If you add the u, v edge, it becomes an odd cycle.
for case (2)
because u and v are colored the same, the lengths
of the u-w path and v-w path must both be odd or even.
=> u-w-v is an even path, adding the u, v edge makes
u-w-v-u path an odd cycle.
SHOT ON MI 8
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