

## Graphs

\* Vertices / nodes

\* edges. directed / undirected.

$$\begin{array}{ll} (u, v) & \{u, v\} \\ \not\models & \equiv \\ (v, u) & \{v, u\} \end{array}$$

\* degree: # of edges incident from a node.  $\nwarrow$  directed graphs

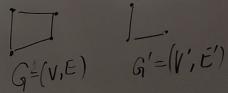
\* dense / sparse graphs      dense :  $|E| = O(|V|^2)$       in-degree      out-degree

sparse :  $|E| = O(|V|)$  ✓

\* power-law: # of nodes having a certain degree decreases exponentially with the degree.

\* weights: weight nodes or edges, or both.

\* subgraph:  $V' \subseteq V$ , and  $G' = (V', E')$



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两种表示方法:

- Representations.

$G = (V, E)$        $|V|$ : # of vertices.  $|E|$ : # of edges

\* adjacency list      Space:  $O(|E| + |V|)$  : time (whether two nodes are connected?)  $O(|V|)$       time: (testing all incident edges) :  $O(|V|)$

\* matrix      Space:  $O(|V|^2)$  : time ( - - - - - )  $O(1)$       time: ( - - - - - ) :  $O(|V|)$

complexity: build, search connected nodes or all incident edges

all nodes:  $O(|V|^2)$   
one node:  $O(|V|)$

Try all incident edges for all nodes.

$O(|E|)$       each node:  $\frac{O(|E|)}{O(|V|)} = O(1)$

$|E| = O(|V|)$



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### Paths and Graph Connectivity.

\* path: a sequence of vertices  $v_1, v_2, \dots, v_k$  where for e.g.  $v_i, v_{i+1}$ , a  $(v_i, v_{i+1})$  edge exists.

$v_1 \rightarrow v_2 \dots v_i, v_{i+1}, \dots, v_k$

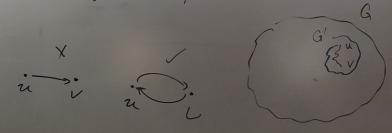
\* Simple path: a path without cycle.

Cycle:  $v_1, v_2, \dots, v_k$  where  $v_1, v_2, \dots, v_{k-1}$  are distinct and  $v_1 = v_k$

### Connected components:

a subgraph  $G'$  s.t. for every  $u \in V$  and  $v \in V'$

an  $u \dots v$  path exists.

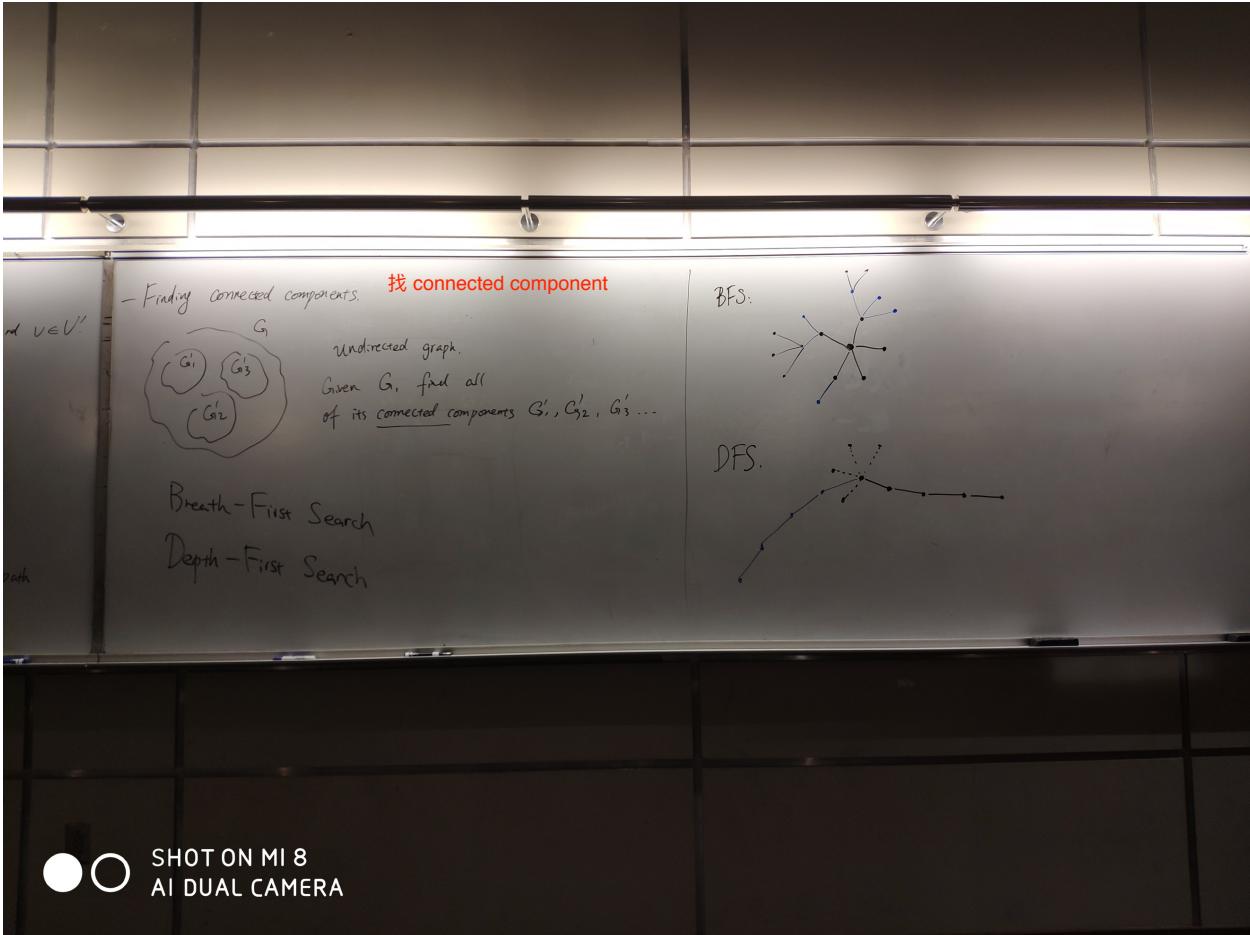


### Strongly connected component:

...  $u \dots v$  path and  $v \dots u$  path exist



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BFS:  $(s, t)$  connectivity.

Create an empty set  $S$ .  
Create empty queue  $Q$ .

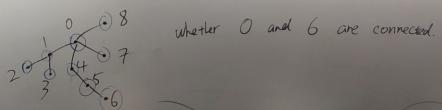
$Q.$  enqueue ( $s$ )

while  $Q$  is not empty  
  Current  $\leftarrow Q.$  dequeue()  
  if current =  $t$ :  
    return True  
  endif  
  foreach  $k$  that is adjacent to current  
    if  $k$  is not in  $S$ :  
      add  $k$  to  $S$   
       $Q.$  enqueue ( $k$ )

endwhile.  
return False.

$S: 0, 1, 4, 7, 8, 2, 3, 5, 6.$

Queue:



whether 0 and 6 are connected.



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