

SHOT ON MI 8  
AI DUAL CAMERA

\* Can I reach  $t$  from  $s$  in  $K$  steps? (BFS)

\* Find a path that contains  $s$ .

$O(|E| + |V|)$  adjacency list

Proposition: For every node that is ever placed in  $Q$ , it is reachable from  $s$ .

Proof by induction.

Proof: (Basis step):  $s$  is placed in  $Q$ , and  $s$  is reachable from  $s$ .  
So the proposition is correct.

(Induction step): Assume that any node  $v$  that is placed in the queue after  $K$  iterations, is also reachable from  $s$ .

For a node  $w$  that is visited in the  $K+1$  th step, we can reach its predecessor  $w'$  in the  $K$ th step. By induction,  $w'$  is reachable from  $s$  and there exists an  $s-w'$  path. So, we can construct  $s-w'-w$  path st.  $w$  is reachable from  $s$ .

(Conclusion) ...



SHOT ON MI 8  
AI DUAL CAMERA

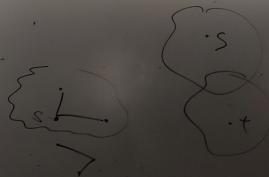
Proposition: All nodes that are reachable from  $S$  will be placed in  $\mathcal{Q}$  sometime.

Prof. (Basis)  $S$  will be placed in  $\mathcal{Q}$ .

(induction) All nodes that is  $k$ -step away from  $S$  will be placed in  $\mathcal{Q}$ .

Since we try all neighbors, all nodes that are reachable in  $k+1$ th iterations will be placed in  $\mathcal{Q}$ .

(Conclusion) --

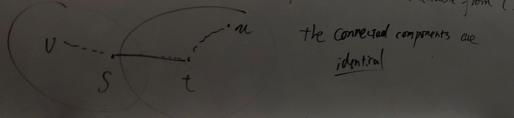


SHOT ON MI 8  
AI DUAL CAMERA

Proposition: For two vertices  $s$  and  $t$ , their connected components are either identical or disjoint.

(1) if  $s$  and  $t$  are connected  $\Rightarrow$  there is an  $s-t$  path.  
for every node  $u$  that is reachable from  $t$ , we can construct  
 $s-t-u$  path, s.e.  $u$  is reachable from  $s$ .

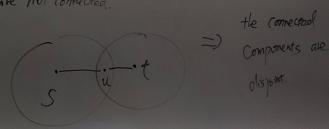
Similarly, all nodes  $v$  that is reachable from  $s$  is also reachable from  $t$ .



(2), if  $s$  and  $t$  are not connected.

by contradiction. We assume that the connected components of  $s$  and  $t$  overlap, at node  $u$ .

because  $s-u$  are connected, and  $u-t$  are connected.  
We can construct  $s-u-t$  path to connect  
 $s, t$ . Contradicting with the fact that  $s$  and  $t$   
are not connected.



SHOT ON MI 8  
AI DUAL CAMERA

+ Strongly connected component.

$u, v$  is a Strongly connected component,

there exists a  $u \rightarrow v$  path }  
and a  $v \rightarrow u$  path } Mutually reachable

Proposition: if  $u, v$  are mutually reachable, and  $v, w$  are mutually reachable,  
then  $u, w$  are mutually reachable.



→ Construct a  $u \rightarrow w$  path and a  $w \rightarrow u$  path.

$u \rightarrow w : u \rightarrow v \rightarrow w$

$w \rightarrow u : w \rightarrow v \rightarrow u$ .

$G^r$  - Construct  $G^r$

pick a node  $S_r$  and run BFS/DFS on  $G^r$   
 $S_r \dots$  on  $G^r$

take the intersection of the nodes that are detected  $\Rightarrow$  Strongly connected component.



SHOT ON MI 8  
AI DUAL CAMERA