

Divide-and-conquer algo.

- Merge-Sort.

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

# of  
Subproblems

other time complexity

time complexity  
to merge  
the results

Important!

Size of  
each subproblem

$$\begin{aligned} & \underline{O(n \log n)} \\ & O(n^2) \checkmark \\ & O(n) \times \end{aligned}$$

\* unrolling.

\* Substitution.

- "guess"

- induction: there exists a constant  $C$   
that upper-bounds the  
running time.



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"Showing  $O(n)$  guess does not work".

(Basis step)

assume  $T(n) = O(n)$

$$T(2) \leq 2C \quad \checkmark$$

$$O(n) \Rightarrow T(n) \leq cn$$

(induction)  $T(k) \leq ck$  for  $k \geq 2$ .

$$T(k+1) = 2T\left(\frac{k+1}{2}\right) + O(k+1) \leq 2 \cdot c \left(\frac{k+1}{2}\right) + \alpha(k+1) = c(k+1) + \underline{\alpha(k+1)}$$

Purpose: to show  $\underline{T(k+1)} \leq c(k+1)$   $\times$

$c > 0$   
 $k+1 > 0$



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Median Finding 中位数查找

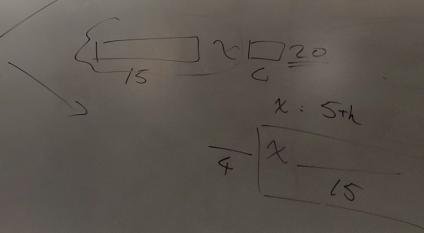
give a list of unsorted numbers, find the  
Median of them.

$$\mathcal{O}(n \log n)$$

$$\mathcal{O}(n)$$

$$1, 2, 3, 4, 5$$

$$5, 3, 2, 1, 4$$



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Median Finding. 限制条件

SELECT (A, i)

A: the list of unsorted numbers.  
i: the ranking of the number to search.

$$\bar{c} = |A|/2$$

Divide A into n groups of 5 ( $n = |A|/5$ )

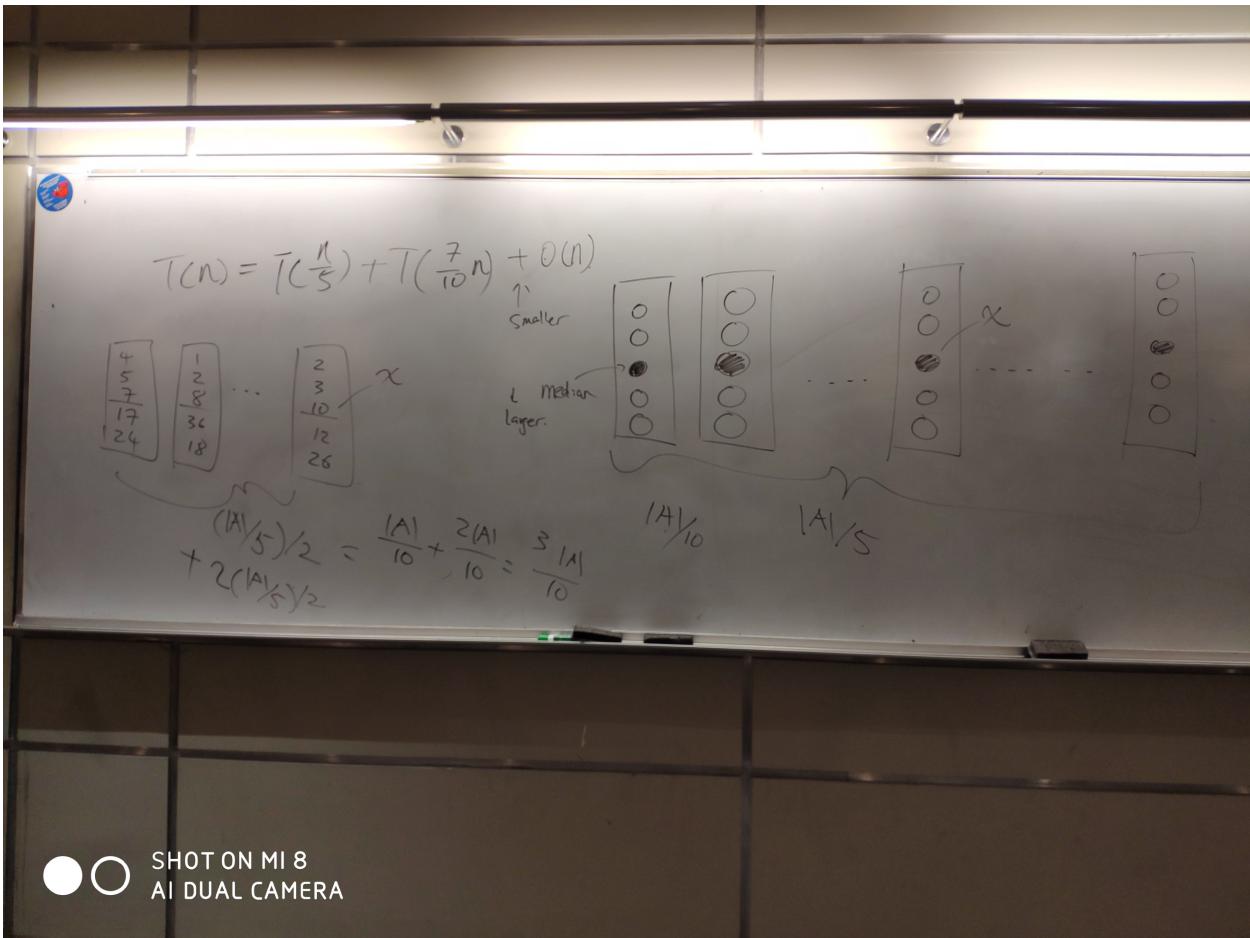
Find the median for each of these groups; Let the set of  $n$  medians be  $A'$   
 SELECT  $(A', |A'|/2)$ , and let the median of  $A'$  be  $a'$ .

SELECT (A, A/2), and let the median of A' be x.  
Perhaps all angles are less than

$$x = 8.$$



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Median Finding.

SELECT(A, i)

A: the list of unsorted numbers.  
i: the ranking of the number to search.

$$i = |A|/2$$

[3 8 13] A'

① Divide A into n groups of  $\leq 5$  ( $n = |A|/5$ )

② Find the median for each of these group; Let the set of n medians be A'

\* SELECT(A', |A'|/2), and let the median of A' be X.

③ Partition all numbers around X, and let K be the ranking of X

\* if  $k=i$ : return k.

\* elseif  $i < k$ : SELECT(A[1...k-1], i) (1-5)

else : SELECT(A[k+1...i], i-k)

endif

$$X = 8$$

1, 2, ..., 8  
n=3  
K-1 K

15 [A]

(6-10) (11-15)



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$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7}{10}n\right) + O(n)$$

induction:  $T(k) \leq ck$  for  $k \geq 5$

$$T(5) \leq 5c$$

$$\begin{aligned} \text{To show } T(k+1) &\leq c\frac{(k+1)}{5} + c\frac{7(k+1)}{10} + o(k+1) = \frac{2c(k+1)}{10} + \frac{7c(k+1)}{10} + o(k+1) \\ &= \frac{9c}{10}(k+1) + o(k+1) = \underline{c(k+1)} + \underline{\left(a - \frac{c}{10}\right)(k+1)} \end{aligned}$$

*if  $c \geq 10a$*



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