The following set of inference rules attempts to capture the "less-than" relation for natural numbers.

$$\frac{}{m < m+1} \qquad \frac{m < n \quad n < p}{m < p}$$

1. Which of the following are *valid* derivations given the rules above? (That is, which of these derivations follow the rules above, not just which ones have "true" conclusions, based on your intuition for the meaning of $<$.) Circle "valid" or "invalid" for each derivation.

(a) $\dfrac{\dfrac{}{1<3} \quad \dfrac{}{3<4}}{1<4}$ Valid **Invalid**

(b) $\dfrac{\dfrac{}{1<2} \quad \dfrac{\dfrac{}{2<3} \quad \dfrac{}{3<4}}{2<4}}{1<4}$ **Valid** Invalid

(c) $\dfrac{\dfrac{}{1<2} \quad \dfrac{}{3<4}}{1<4}$ Valid **Invalid**

(d) $\dfrac{\dfrac{}{2<3} \quad \dfrac{}{3<4}}{1<4}$ Valid **Invalid**

2. Give two *different* (but valid) derivations of "$0 < 3$", using the rules above.

$$\dfrac{\dfrac{\dfrac{}{0<1} \quad \dfrac{}{1<2}}{0<2} \quad \dfrac{}{2<3}}{0<3} \qquad \dfrac{\dfrac{}{0<1} \quad \dfrac{\dfrac{}{1<2} \quad \dfrac{}{2<3}}{1<3}}{0<3}$$