

1. (10) Using **induction**, prove that  $2^n > n^2$ ,  $\forall n \in \mathbb{N}, n > 4$ .
2. (10) Do Problem 6 on Page 329.
3. (15) Do Problem 8 on Page 329.
4. (15) Do Problem 16 on Page 330.
5. (15) Do Problem 12 on Page 342.

6. (15) Recall that in defining the Fibonacci Sequence Number  $\{f_i\}_{i=0}^{\infty}$ , we have

$$f_0 = 0, f_1 = 1, f_n = f_{n-1} + f_{n-2}, \forall n > 1.$$

Prove by using induction on  $n$  that

$$\begin{bmatrix} f_n \\ f_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \forall n \geq 1.$$

7. (20) Given the following recurrence equation  
 $T(9) = 1$ ,

$$T(n) = 9T\left(\frac{n}{3}\right) + n - 8, n = 3^k > 9.$$

- (a) Using the method of **repeated substitutions**, compute  $T(n)$  in closed-form.  
Remark: You must show at least three substitutions and the general pattern for  $T(n)$  clearly for credit.
- (b) Verify the correctness of your solution  $T(n)$  using induction.