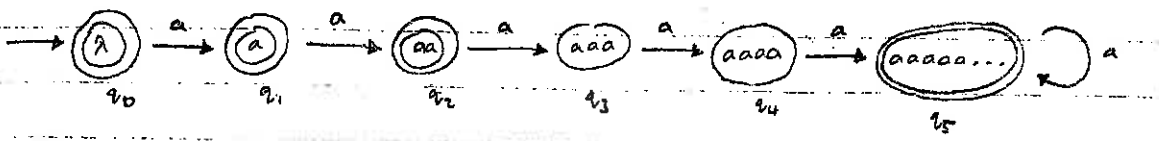
HW #3

2.4

3d. $L = \{ a^n : n \neq 3 \text{ and } n \neq 4 \}$

Hint:Using k-equivalence to minimum the result. The tables represent k-equivalence algorithm. According to language, draw a DFA.



	a
q_0	q_1 a
q_1	q_2 a
q_2	q_3 b
q_5	q_5 a
q_3	q_4 b
q_4	q_5 a

	a
q_0	q_1 a
q_1	q_2 b
q_5	q_5 a
q_2	q_3 c
q_3	q_4 d
q_4	q_5 a

	a
q_0	q_1 b
q_5	q_5 a
q_1	q_2 c
q_2	q_3 d
q_3	q_4 e
q_4	q_5 a

	a
q_0	q_1 c
q_5	q_5 b
q_1	q_2 d
q_2	q_3 e
q_3	q_4 f
q_4	q_5 b

∴ above diagram is minimized.

According to language, write regular expression.

8.1

9b. $L_2 = \{ a^n b^m : n < 4, m \leq 4 \}$

$r = (\lambda + a + aa + aaa)(\lambda + b + bb + bbb + bbbb)$

10. $(\Phi^*)^* = \Phi^* = \{\lambda\}$

$a\Phi = \emptyset$

$r + \emptyset = r$,
$r\emptyset = \emptyset$,
$\emptyset^* = \lambda$,

number of w is not divided by 3

21. (a) $L = \{ w : |w| \bmod 3 \neq 0 \}$

$r = ((a+b)(a+b)(a+b))^*((a+b) + (a+b)(a+b))$

(b) $L = \{ w : n_a(w) \bmod 3 = 0 \}$

number of a is divided by 3

$r = b + (b^* a b^* a b^* a b^*)^*$

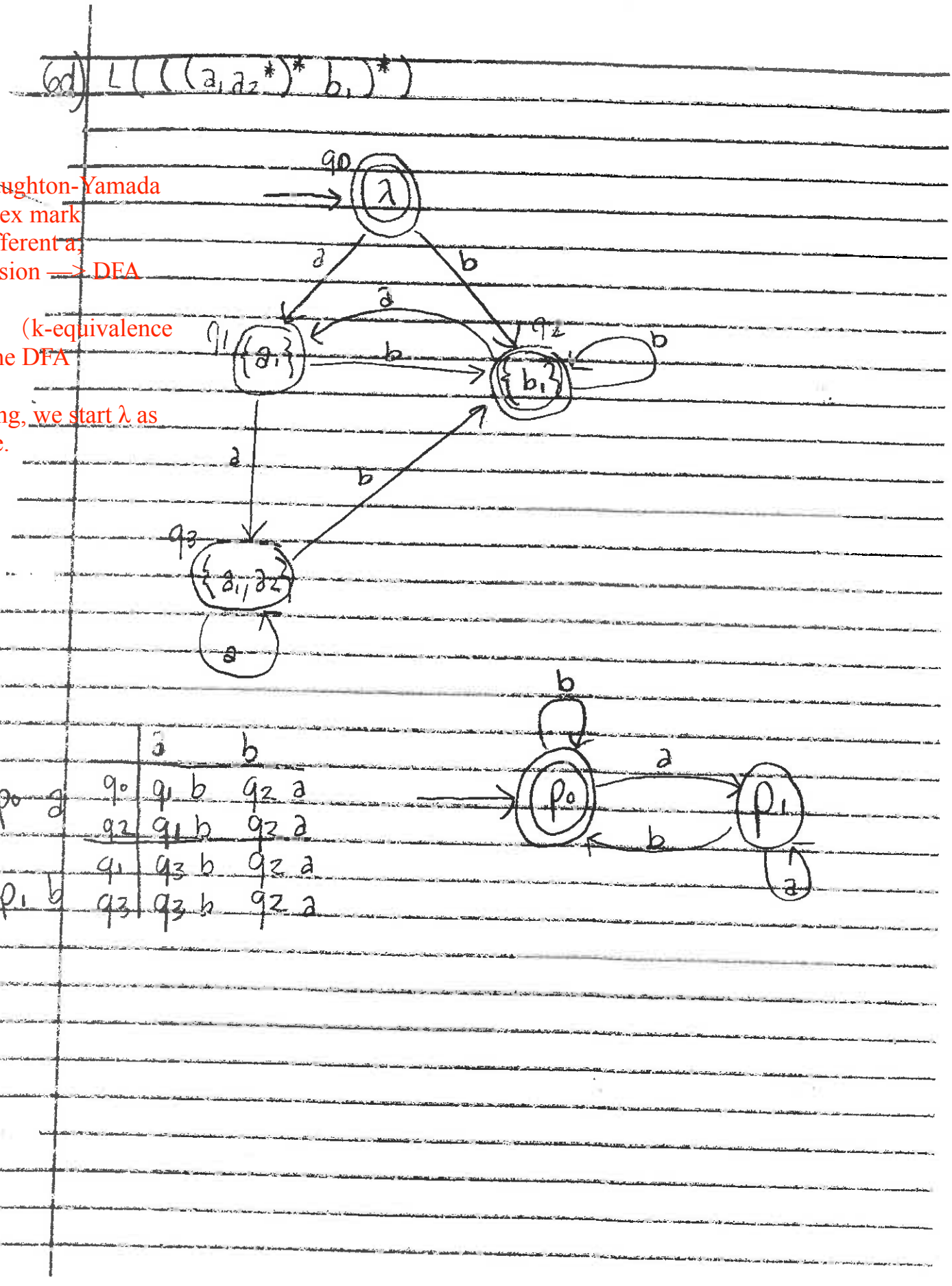
(c) $L = \{ w : n_a(w) \bmod 5 > 0 \}$

$r = (b^* a b^* a b^* a b^* a b^* a)^* (b^* a b^* + b a b^* a b^* + b a b^* a b^* a b^* + b a b^* a b^* a b^* a b^*)$

Use the McNaughton-Yamada algorithm, Index mark distinguish different a; regular expression —> DFA

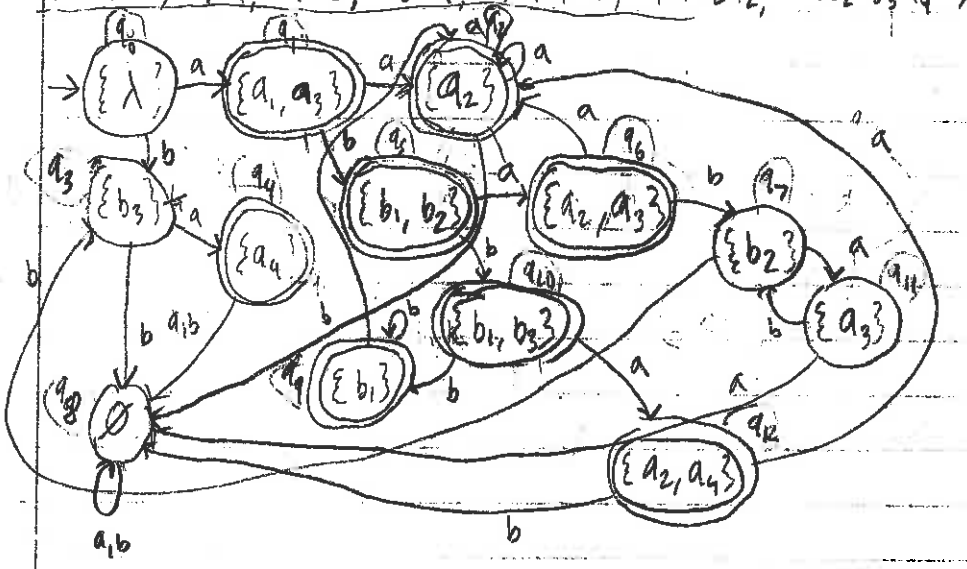
then minimize （k-equivalence algorithm） the DFA

In the beginning, we start λ as the initial state.



7a. $L(ab^*a^*) \cup L((ab)^* ba) = L((a_1 b_1^* a_2^*) + ((ab)^* b_3 a_4))$

$L = \{ a_1, a_1 b_1, a_1 a_2, b_3 a_4, a_1 b_1 b_1 a_2, a_1 b_1 a_2 a_2, a_1 b_2 b_3 a_4, \dots \}$



	a	b
q_0	q_1	q_3
q_1	q_2	q_5
q_2	q_2	q_8
q_3	q_4	q_8
q_4	q_8	q_8
q_5	q_6	q_{10}
q_6	q_2	q_7
q_7	q_{11}	q_3
q_8	q_8	q_8
q_9	q_7	q_9
q_{10}	q_{12}	q_9
q_{11}	q_8	q_7
q_{12}	q_2	q_8

⇒

	a	b	
	q_1	q_2	q_5	$\alpha\alpha$
	q_2	q_2	q_8	$\alpha\beta$
	q_4	q_8	q_8	$\beta\beta$
α	q_5	q_6	q_{10}	$\alpha\alpha$
	q_6	q_2	q_7	$\alpha\beta$
	q_9	q_2	q_9	$\alpha\alpha$
	q_{10}	q_{12}	q_9	$\alpha\alpha$
	q_{12}	q_2	q_8	$\alpha\beta$
	q_0	q_1	q_3	$\alpha\beta$
β	q_3	q_4	q_8	$\alpha\beta$
	q_7	q_{11}	q_3	$\beta\beta$
	q_8	q_8	q_8	$\beta\beta$
	q_{11}	q_8	q_7	$\beta\beta$

Cannot combine

⇒

	a	b	
	q_1	q_2	q_5	$\beta\alpha$
α	q_5	q_6	q_{10}	$\beta\alpha$
	q_9	q_2	q_9	$\beta\alpha$
	q_{10}	q_{12}	q_9	$\beta\alpha$
	q_2	q_2	q_8	$\beta\theta$
β	q_6	q_2	q_7	$\beta\theta$
	q_{12}	q_2	q_8	$\beta\theta$
γ	q_4	q_8	q_8	
	q_0	q_1	q_3	$\alpha\Delta$
Δ	q_3	q_4	q_8	$\gamma\theta$
	q_7	q_{11}	q_3	$\theta\Delta$
θ	q_8	q_8	q_8	$\theta\theta$
	q_{11}	q_8	q_7	$\theta\theta$

1) separate finite state and no finite state. 0-equivalence
2) combine the same states. 1 equivalence
3) same 2) until no more combine
4) Cannot combine different field of same results，eg blue line!!
已经分离的两个区域中相同的组合不能在一起。看蓝线不能组合在一起
5) Draw a mininal DFA

7b cont

	a	b
α	q_1	q_2 q_5 $\beta\alpha$
	q_5	q_6 q_{10} $\beta\alpha$
	q_9	q_2 q_9 $\beta\alpha$
	q_{10}	q_{12} q_9 $\beta\alpha$
β	q_2	q_2 q_8 $\beta\gamma$
	q_6	q_2 q_7 βM
	q_{12}	q_2 q_8 $\beta\pi$
γ	q_4	q_8 q_8
Δ	q_0	q_1 q_3
θ	q_3	q_4 q_8
M	q_7	q_{11} q_3
π	q_8	q_8 q_8 $\pi\pi$
	q_{11}	q_8 q_7 πM

⇒

	a	b
	q_1	q_2 q_5 $\beta\alpha$
	q_5	q_6 q_{10} $\gamma\alpha$
α	q_9	q_2 q_9 $\beta\alpha$
	q_{10}	q_{12} q_9 $\beta\alpha$
β	q_2	q_2 q_8 $\beta\varphi$
	q_{12}	q_2 q_8 $\beta\varphi$
γ	q_6	q_2 q_7
Δ	q_4	q_8 q_8
θ	q_0	q_1 q_3
M	q_3	q_4 q_8
π	q_7	q_{11} q_3
φ	q_8	q_8 q_8
σ	q_{11}	q_8 q_7

⇒

	a	b
	q_1	q_2 q_5 $\gamma\beta$
α	q_9	q_2 q_9 $\gamma\alpha$
	q_{10}	q_{12} q_9 $\gamma\alpha$
β	q_5	q_6 q_{10}
γ	q_2	q_2 q_8 $\gamma\sigma$
	q_{12}	q_2 q_8 $\gamma\sigma$
Δ	q_6	q_2 q_7
θ	q_4	q_8 q_8
M	q_0	q_1 q_3
π	q_3	q_4 q_8
φ	q_7	q_{11} q_3
σ	q_8	q_8 q_8
ε	q_{11}	q_8 q_7

		a	b
F	α	q_1	q_2 q_5 /
F	β	q_9	q_2 q_9 $\Delta\beta$
		q_{10}	q_{12} q_9 $\Delta\beta$
F	γ	q_5	q_6 q_{10}
F	Δ	q_2	q_2 q_8 $\Delta\varepsilon$
		q_{12}	q_2 q_8 $\Delta\varepsilon$
F	θ	q_6	q_2 q_7
F	M	q_4	q_8 q_8
	π	q_0	q_1 q_3
	φ	q_3	q_4 q_8
	σ	q_7	q_{11} q_3
	ε	q_8	q_8 q_8
	Ω	q_{11}	q_8 q_7