Name: Key

Recall the evaluation rules for state:

$$\frac{1}{z \mid s \Downarrow z \mid s} \quad \frac{t_1 \mid s_1 \Downarrow z_1 \mid s_2 \quad t_2 \mid s_2 \Downarrow z_2 \mid s_3}{t_1 + t_2 \mid s_1 \Downarrow z_1 + z_2 \mid s_3} \quad \frac{\lambda x.t \mid s \Downarrow \lambda x.t \mid s}{\lambda x.t \mid s \Downarrow \lambda x.t \mid s}$$

$$\frac{t_1 \mid s_1 \Downarrow \lambda x.t \mid t_2 \quad t_2 \mid s_2 \Downarrow w \mid s_3 \quad t[w/x] \mid s_3 \Downarrow v \mid s_4}{t_1 t_2 \mid s_1 \Downarrow v \mid s_4} \quad \frac{t \mid s_1 \Downarrow v \mid s_2}{\text{get} \mid s \Downarrow s \mid s} \quad \frac{t \mid s_1 \Downarrow v \mid s_2}{\text{put} t \mid s_1 \Downarrow v \mid v}$$

Note that put t evaluates to the value of t.

1. The above rules use *left-to-right* evaluation. Give the result of evaluating each of the following terms (just the value, not the state) using both left-to-right and *right-to-left* evaluation orders.

Expression	Initial state	Result in ltr	Result in rtl
$\operatorname{put} 3 + \operatorname{get}$	1	6	4
$(\lambda a.put(get+a))get$	1	2	2
$(\lambda a.\lambda b.a + b) \operatorname{get} (\operatorname{put} 3)$	1	4	6

2. Give the derivation tree for $(\lambda a.put a)$ get $| 3 \downarrow 3 | 3$.

$$\frac{\overline{3\mid 3\Downarrow 3\mid 3}}{\lambda a.\mathsf{put}\, a\mid 3\Downarrow \lambda a.\mathsf{put}\, a\mid 3} \quad \frac{\overline{3\mid 3\Downarrow 3\mid 3}}{\gcd\mid 3\Downarrow 3\mid 3} \quad \frac{}{(\mathsf{put}\, a)[3/a]\mid 3\Downarrow 3\mid 3}$$