EECS210 **HW#4** **Due: 10-12-17**

1. (20) Prove or disprove that the following relation R defined on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive.
   (a) $(x, y) \in R$ if and only if $x + y = 0$.
   (b) $(x, y) \in R$ if and only if $x - y$ is a rational number.

2. (10) Let Z be the set of all integers. Consider the relation R defined on Z such that $(x,y) \in R$ if and only if $x + y$ is an even integer. Prove or disprove that R is an equivalence relation defined on Z. If R is an equivalence relation, find a partition of Z using equivalence classes.

3. (10) Let A = {a, b, c, d}. Consider the following relation R defined on A.
   R = {(a,a), (b,b), (c,c), (d,d), (a,d), (d,a), (b,c), (c,b)}.
   Construct the directed graph representation for R and then use it to determine that R is an equivalence relation. Compute a partition for A using equivalence classes.

4. (15) Prove or disprove that the following relation R defined on the set A = {1, 2, 3, 4} is a partial order. Justify your answer.
   (a) R = {(1,1), (2,2), (3,1), (3,3), (3,4), (4,4)}.
   (b) R = {(1,1), (2,2), (2,3), (3,3), (4,2), (4,4)}.
   (c) R = {(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}

5. (10) Let N = {1, 2, …} be the set of all positive integers. Consider the relation R defined on N such that $(x,y) \in R$ if and only $x|y$. Prove that (N,R) is a partially ordered set. Is (N,R) a linearly ordered set? Justify your answer.

6. (10) Let Z be the set of all integers. Determine whether each of the following functions from Z to Z is one-to-one. Justify your answer.
   (a) $f(n) = n^3$.
   (b) $f(n) = \lceil n/2 \rceil$.

7. (10) Let Z be the set of all integers. Determine whether each of the following functions from Z to Z is onto. Justify your answer.
   (a) $f(m, n) = m^2 - n^2$.
   (b) $f(m, n) = |m| - |n|$.

8. (15) Given a function $f: R-\{\frac{3}{5}\} \to R-\{0\}$ defined by $f(x) = \frac{2}{3-5x}$.
   Prove that f is a bijection and then compute the inverse function f^{-1} for f. Verify that f and f^{-1} are indeed inverse function to each other.

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