1. Suppose we have a system with integer and floating point types, with the following subtyping rules:

$$\frac{}{T <: T} \qquad \frac{}{\texttt{Pos} <: \texttt{Int}} \qquad \frac{}{\texttt{Int} <: \texttt{Float}} \qquad \frac{T_1 <: T_2 \quad T_2 <: T_3}{T_1 <: T_3} \qquad \frac{T_2 <: T_1 \quad U_1 <: U_2}{T_1 \to U_1 <: T_2 \to U_2}$$

Fill in the following boxes with the correct subtyping relationship. That is, if the left-hand type is a subtype of the right-hand type, write "<:"; if the right-hand type is a subtype of the left-hand type, write ":>"; if neither is true, write "neither".

$\texttt{Int} \to \texttt{Int}$	:>	$\texttt{Int} \to \texttt{Pos}$
$\texttt{Int} \to \texttt{Int}$	:>	$\texttt{Float} \to \texttt{Int}$
$(\texttt{Pos} \to \texttt{Int}) \to \texttt{Int}$	:>	$(\texttt{Int} \to \texttt{Int}) \to \texttt{Int}$
$\texttt{Int} \to (\texttt{Int} \to \texttt{Float})$	:>	$\texttt{Int} \to (\texttt{Int} \to \texttt{Int})$
$((\texttt{Int} \to \texttt{Int}) \to \texttt{Int}) \to \texttt{Int}$	:>	$((\texttt{Int} \to \texttt{Float}) \to \texttt{Int}) \to \texttt{Int}$

2. Given the same types and subtyping relation as the previous question, suppose we have the following terms and typing:

$$\frac{}{\Gamma \vdash z : \texttt{Pos}}\ (z > 0) \qquad \frac{}{\Gamma \vdash z : \texttt{Int}} \qquad \frac{}{\Gamma \vdash z.z : \texttt{Float}} \qquad \frac{\Gamma \vdash t_1 : T \quad \Gamma \vdash t_2 : T}{\Gamma \vdash t_1 + t_2 : T}\ (T \in \{\texttt{Pos}, \texttt{Int}, \texttt{Float}\})$$

$$\frac{\Gamma[x \mapsto T_1] \vdash t : T_2}{\Gamma \vdash \lambda x.t : T_1 \to T_2} \quad \frac{\Gamma \vdash t_1 : T_1 \to T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1\, t_2 : T_2} \quad \frac{\Gamma \vdash t : T_1 \quad T_1 <: T_2}{\Gamma \vdash t : T_2}$$

Derive the typing assertion $\emptyset \vdash ((\lambda a.a + 2)\, 1.2) + 3 : \texttt{Float}$.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{a \mapsto \texttt{Float}\} \vdash a : \texttt{Float}} \quad \dfrac{\dfrac{}{\{a \mapsto \texttt{Float}\} \vdash 2 : \texttt{Int}} \quad \dfrac{}{\texttt{Int} <: \texttt{Float}}}{\{a \mapsto \texttt{Float}\} \vdash 2 : \texttt{Float}}}{\dfrac{\{a \mapsto \texttt{Float}\} \vdash a + 2 : \texttt{Float}}{\emptyset \vdash \lambda a.a + 2 : \texttt{Float} \to \texttt{Float}}} \quad \dfrac{}{\emptyset \vdash 1.2 : \texttt{Float}}}{\emptyset \vdash (\lambda a.a + 2)\, 1.2 : \texttt{Float}} \quad \dfrac{\dfrac{}{\emptyset \vdash 3 : \texttt{Int}} \quad \dfrac{}{\texttt{Int} <: \texttt{Float}}}{\emptyset \vdash 3 : \texttt{Float}}}{\emptyset \vdash ((\lambda a.a + 2)\, 1.2) + 3 : \texttt{Float}}$$