10703 Deep Reinforcement Learning and Control

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Slides borrowed from
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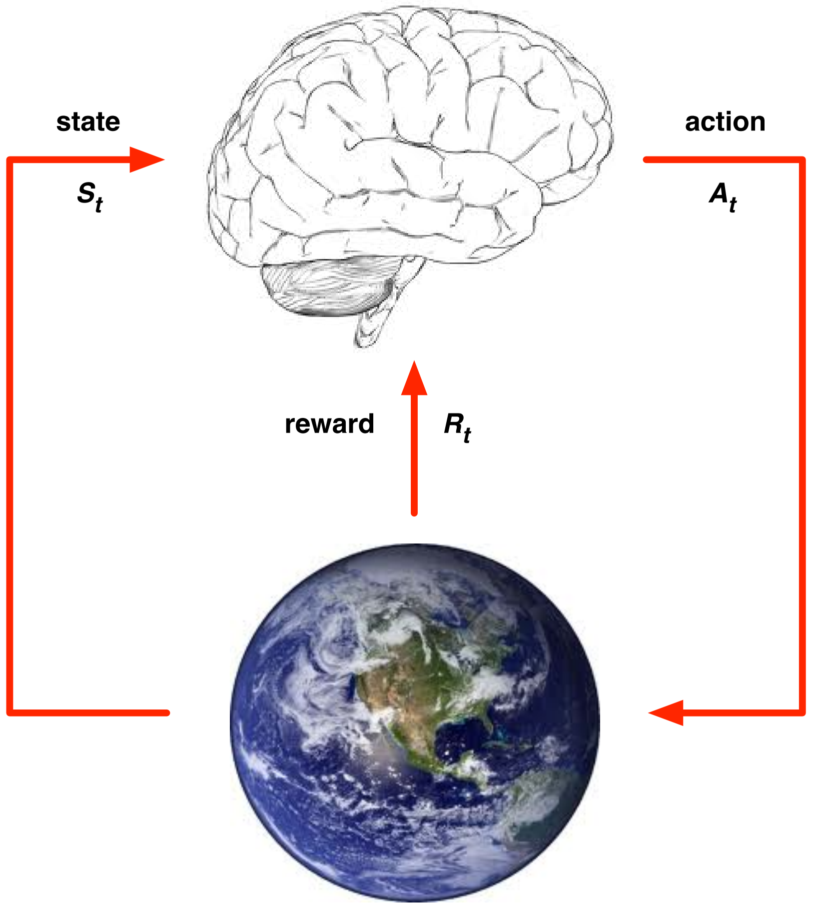
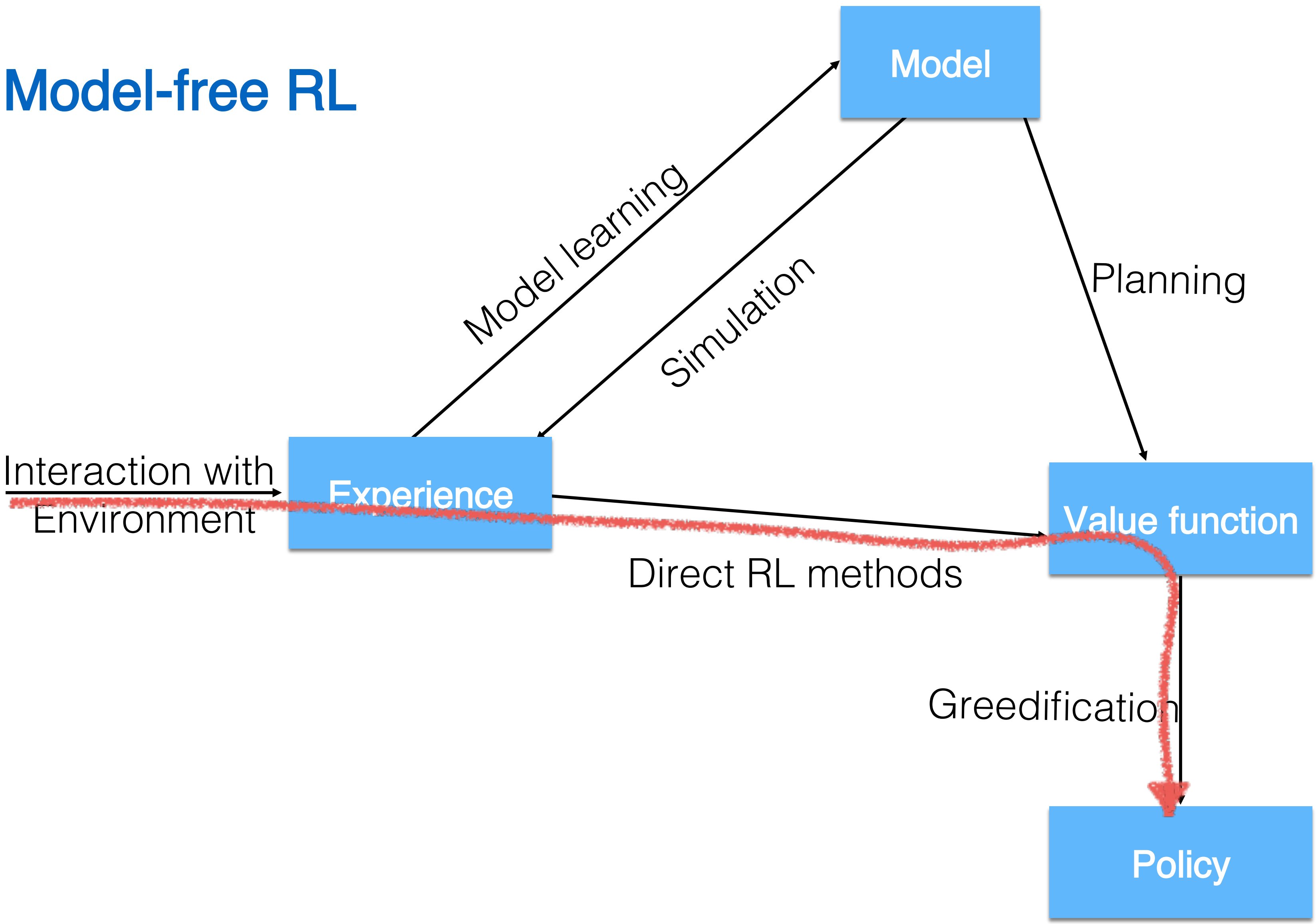
Learning and Planning with Tabular Methods

What can I learn by interacting with the world?

Past classes: the agent learned to estimate value functions and optimal policies from experience.



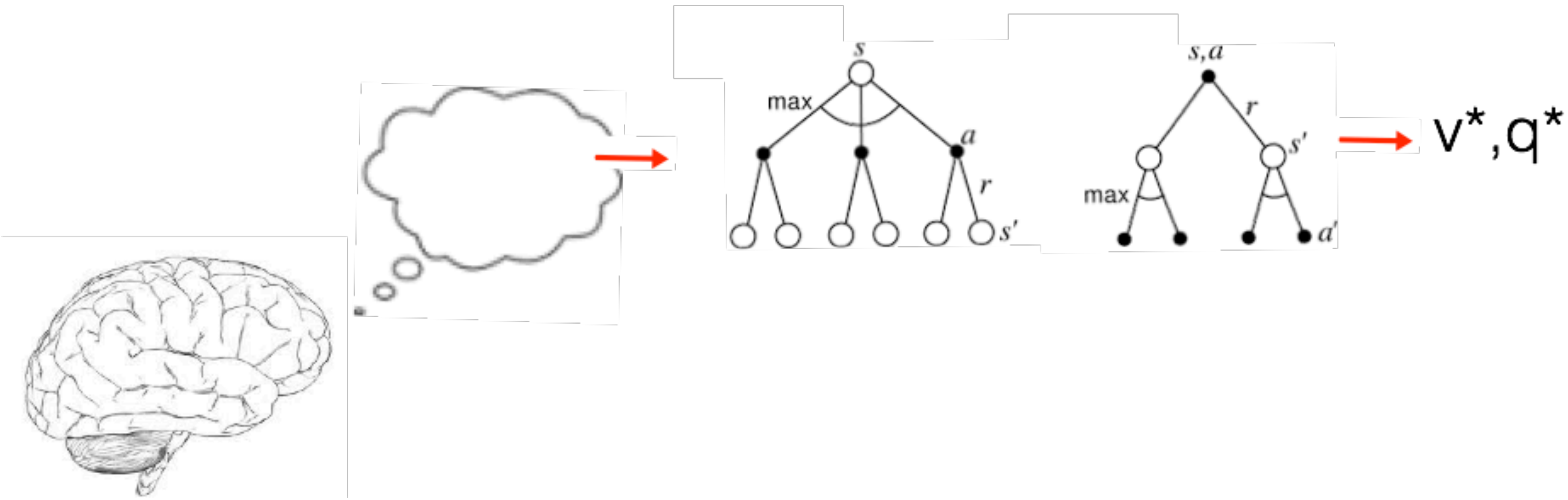
Model-free RL



What can I learn by interacting with the world?

-**So far:** We know the true environment (dynamics and rewards) and just use it to ***plan and estimate value functions*** (value iteration, policy iteration using exhaustive state sweeps of Bellman back-up operations).

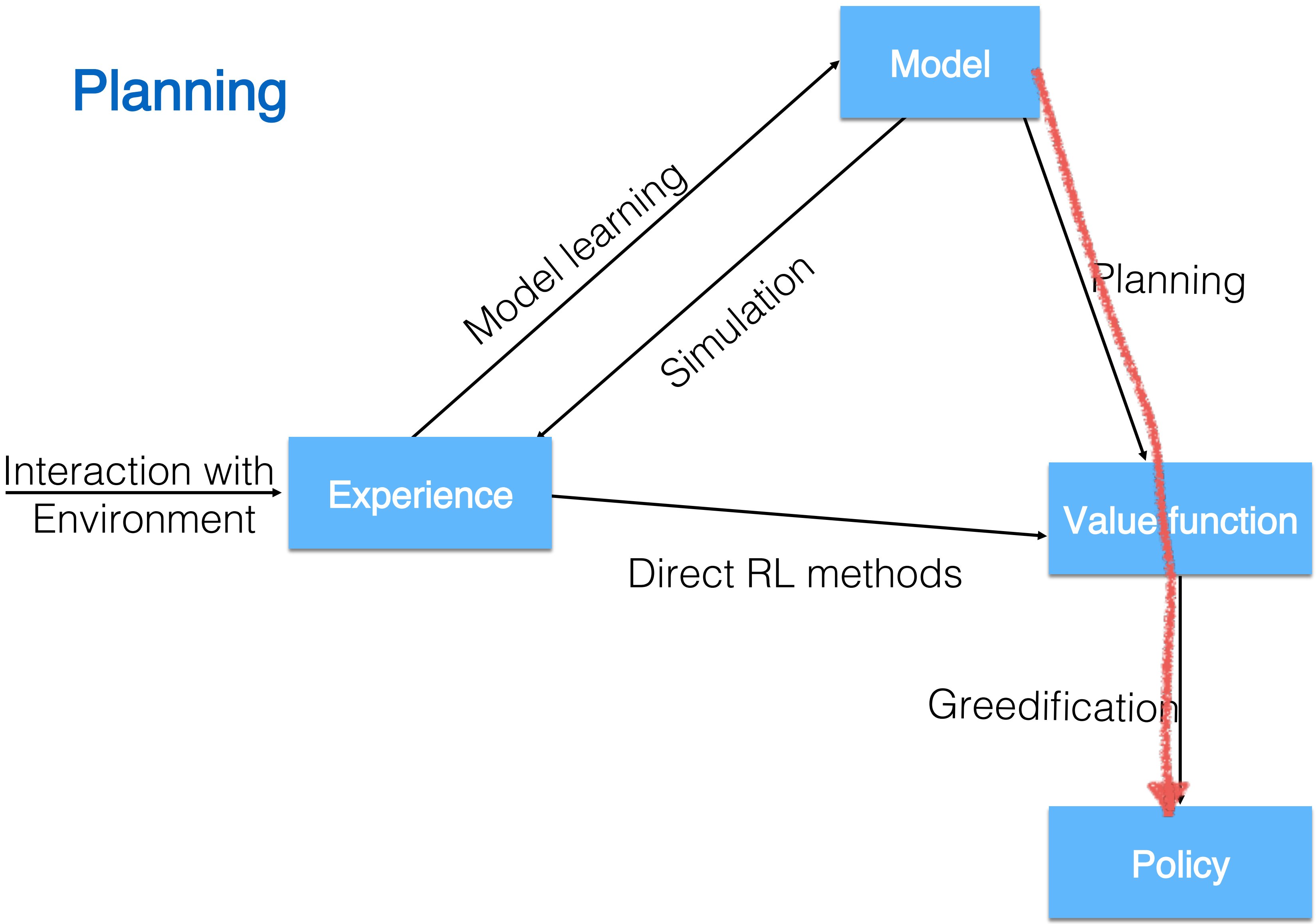
- Very slow when many states.



Planning: any computational process that uses a model to create or improve a policy

$$\text{Model} \xrightarrow{\text{Planning}} \text{Policy}$$

Planning


Model
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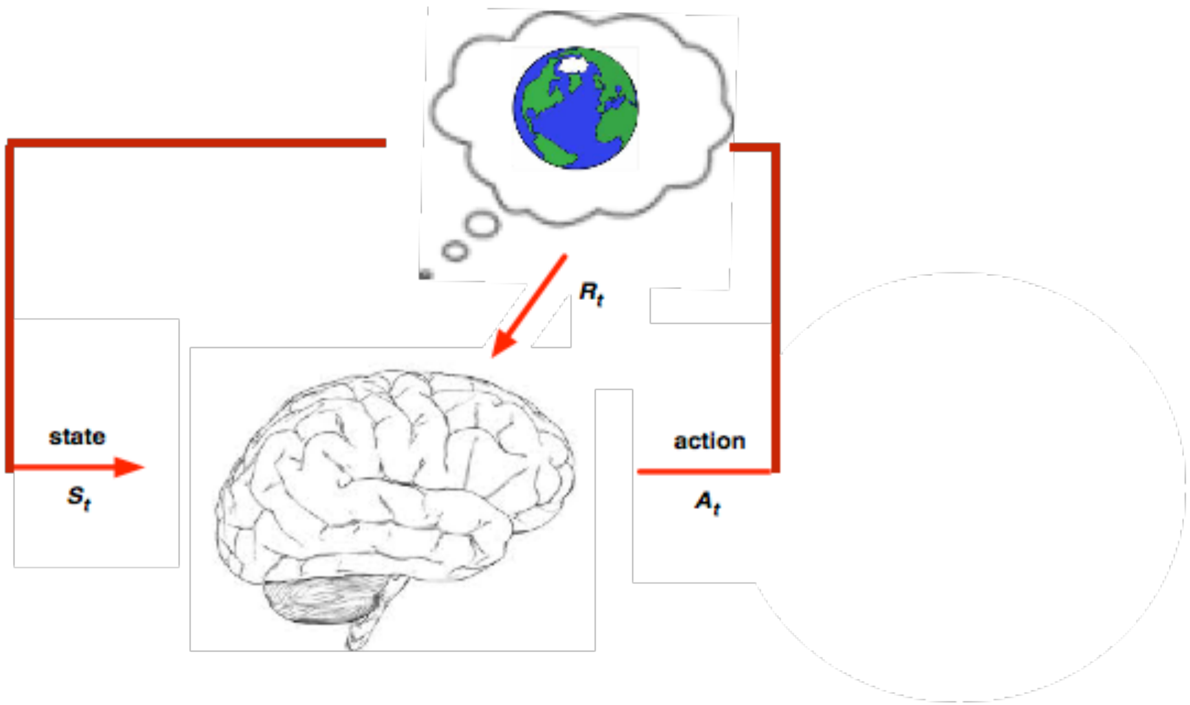
What can I learn by interacting with the world?

This lecture: we will combine both, learning from experience and planning:

- If the model is unknown, we will learn the model.



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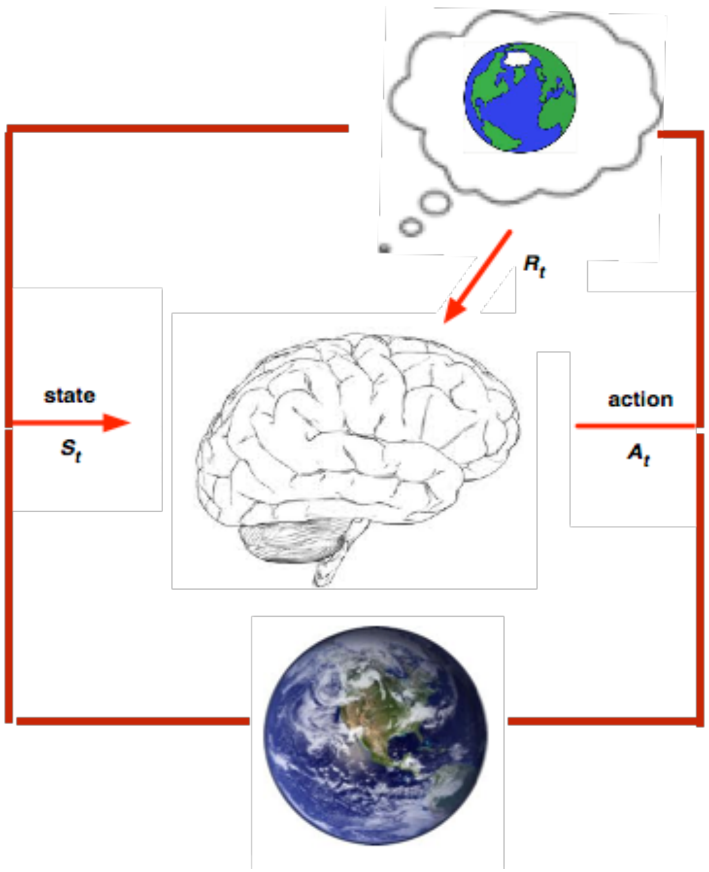
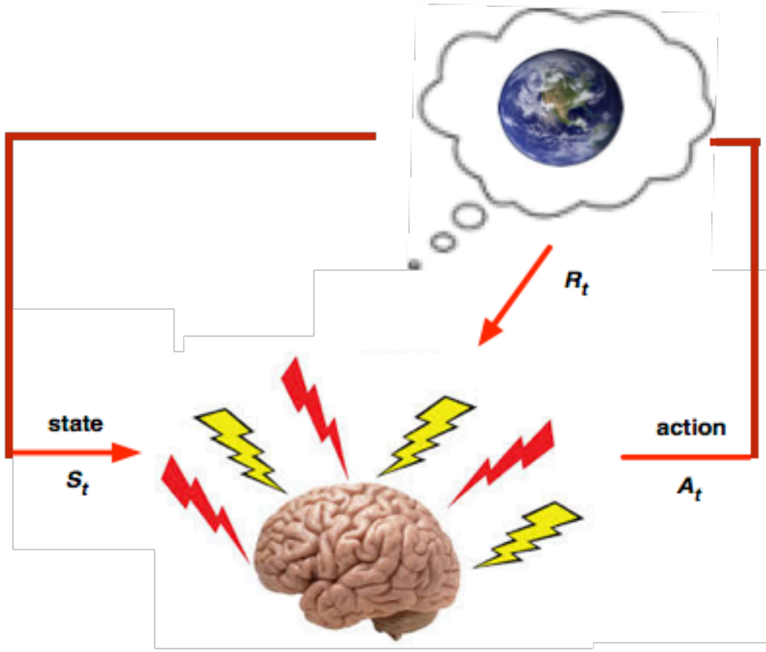
- Learn value functions using both real and simulated experience



What can I learn by interacting with the world?

This lecture: we will combine both, learning from experience and planning:

- If the model is unknown, we will learn the model
- Learn value functions using both real and simulated experience
- Learn value functions online using model-based look-ahead search



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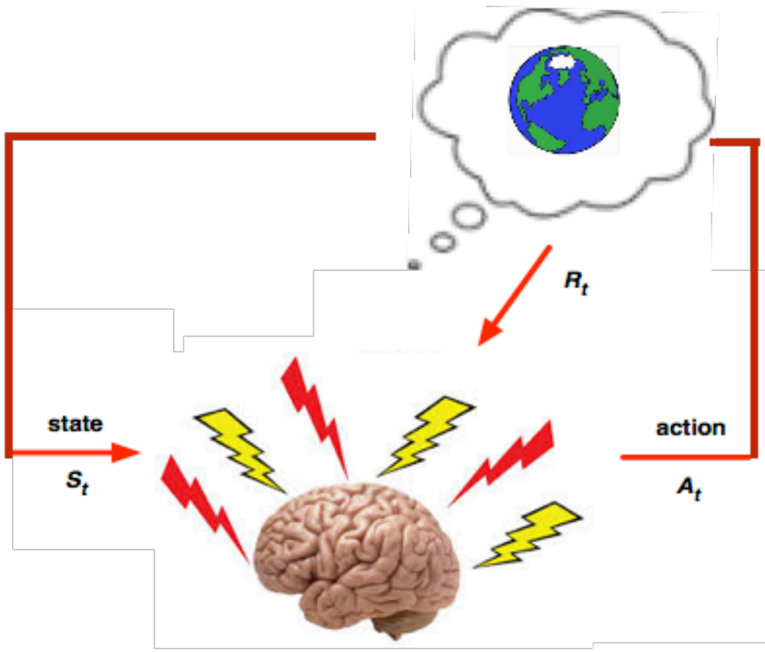
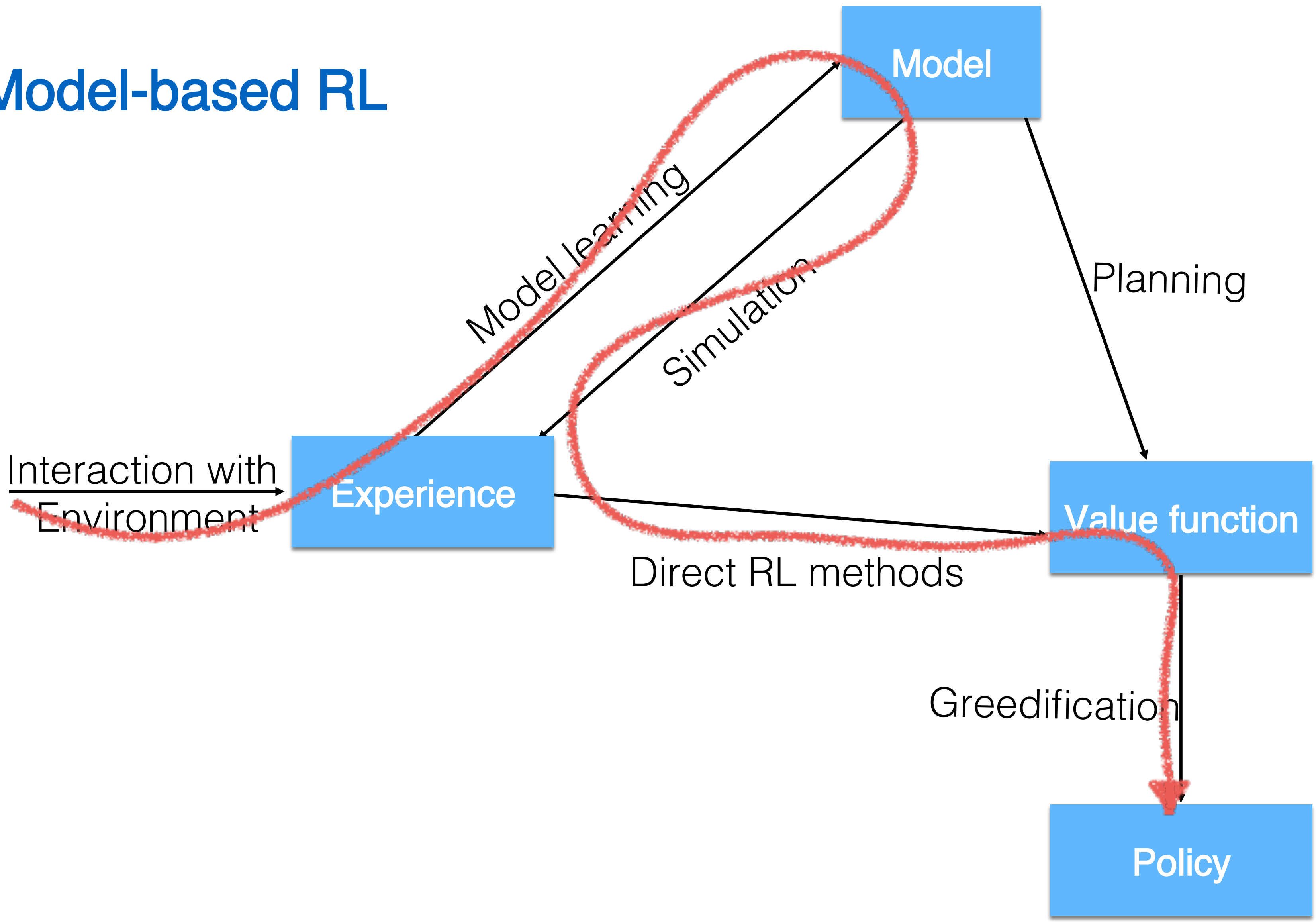


Model-based RL



Advantages of Model-Based RL

Advantages:

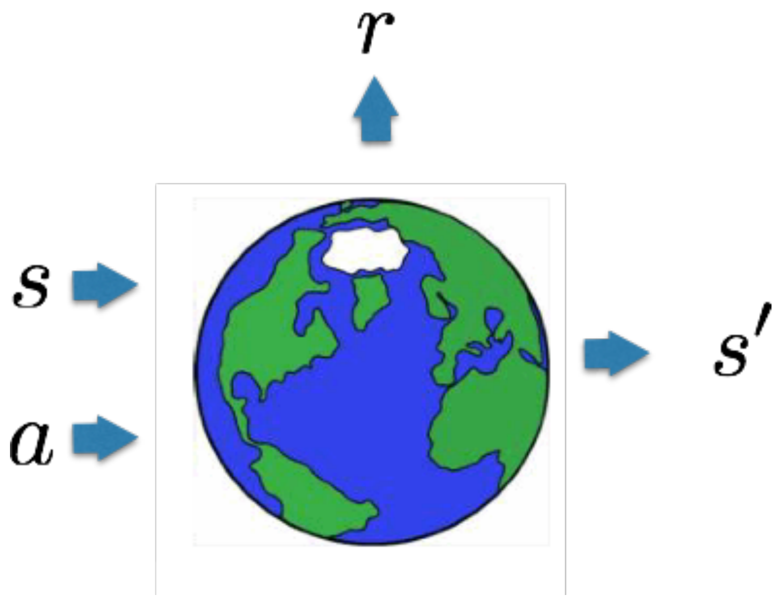
- Model learning transfers across tasks and environment configurations (learning physics)
- Better exploits experience in case of sparse rewards
- It is probably what the brain does (more later)
- Helps exploration: Can reason about model uncertainty

Disadvantages:

- First learn model, then construct a value function: Two sources of approximation error

What is a Model?

Model: anything the agent can use to predict how the environment will respond to its actions: -- specifically, the transition (dynamics) T(s’|s,a) and reward functions R(s,a).



this includes transitions of the state of the environment and the state of the agent.

What is a Model?

Model: anything the agent can use to predict how the environment will respond to its actions, specifically:

- the transition function (dynamics)
- reward function

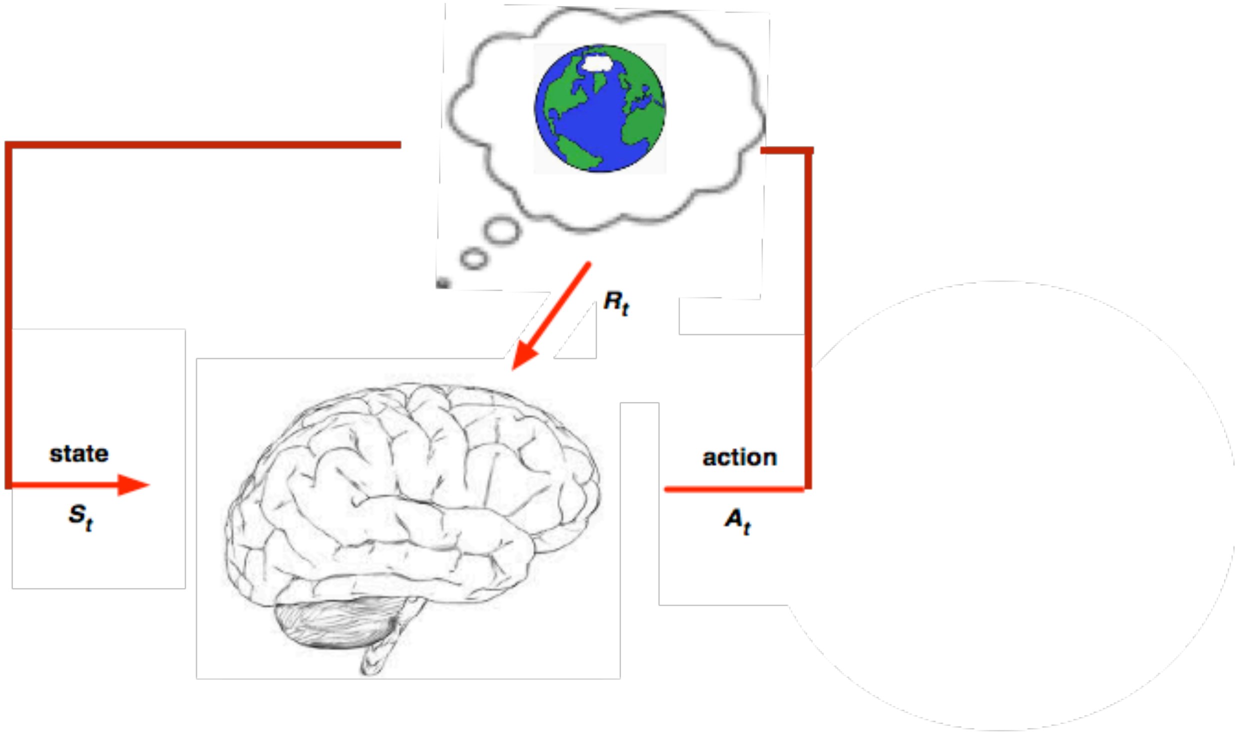
Distribution model: description of all possibilities and their probabilities, $T(s'|s,a)$ for all (s, a, s')

Sample model, a.k.a. a simulation model: produces sample experiences for a given ***s,*** often much easier to come by

Both types of models can be used to produce hypothetical experience (what if…)

Model Learning

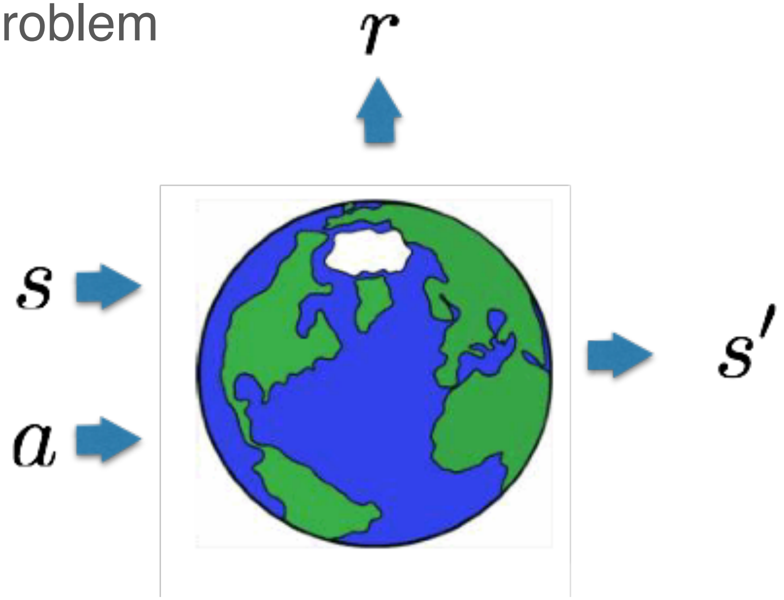
- If the model is unknown, we will learn the model.
- Learn value functions using both real and simulated experience
- Learn value functions online using model-based lookahead search



Model Learning

- Goal: estimate model $\mathcal{M}_\eta$ from experience $\{\mathcal{S}_1, \mathcal{A}_1, R_2, ..., \mathcal{S}_T\}$
- This can be thought as a supervised learning problem

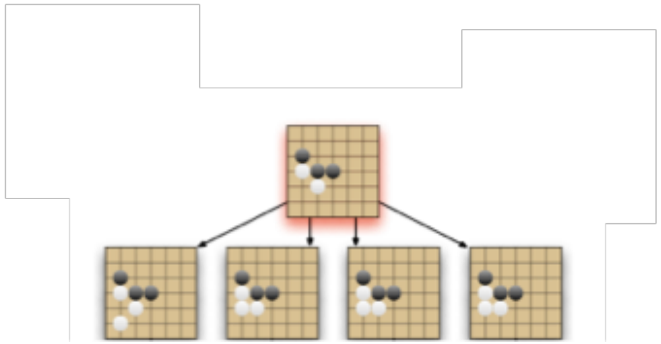
$$\mathcal{S}_1, \mathcal{A}_1 \rightarrow R_2, \mathcal{S}_2$$
$$\mathcal{S}_2, \mathcal{A}_2 \rightarrow R_3, \mathcal{S}_3$$
$$\vdots$$
$$\mathcal{S}_{T-1}, \mathcal{A}_{T-1} \rightarrow R_T, \mathcal{S}_T$$



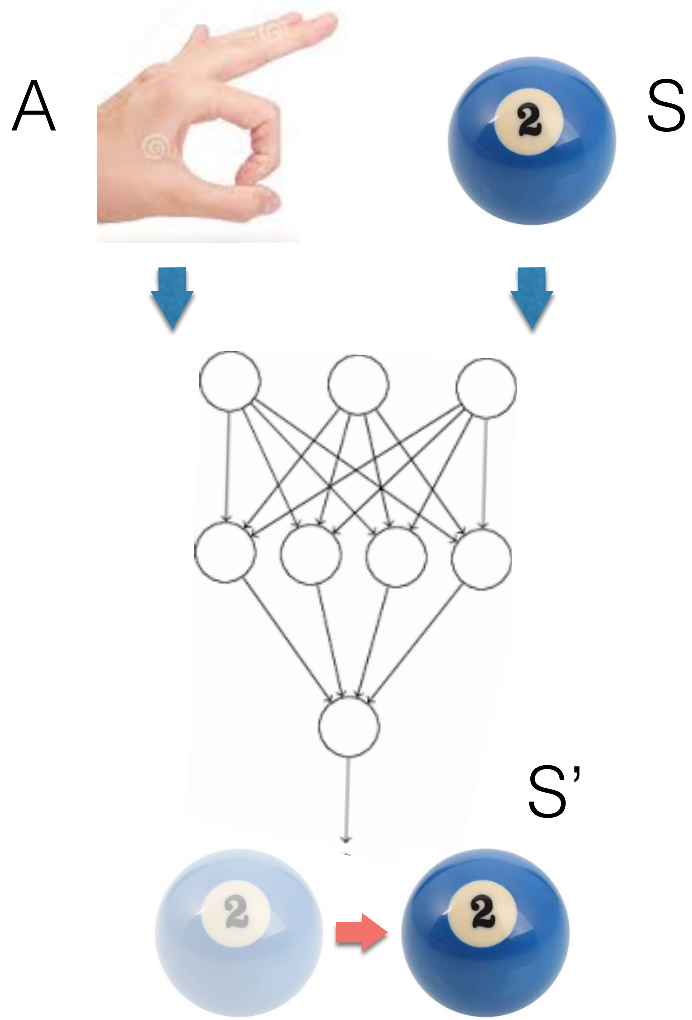
- Learning $s, a \rightarrow r$ is a *regression* problem
- Learning $s, a \rightarrow s'$ is a *density estimation* problem
- Pick loss function, e.g. mean-squared error, KL divergence
- Find parameters η that minimize empirical loss

Examples of Models for T(s'|s,a)

Table lookup model (tabular): book-keeping a probability of occurrence for each transition (s,a,s')

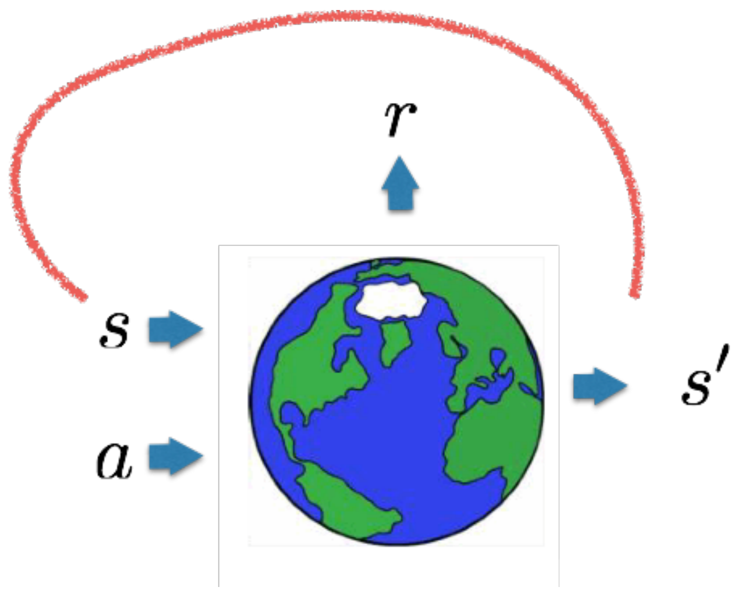


Transition function is approximated through some function approximator



A supervised learning problem?

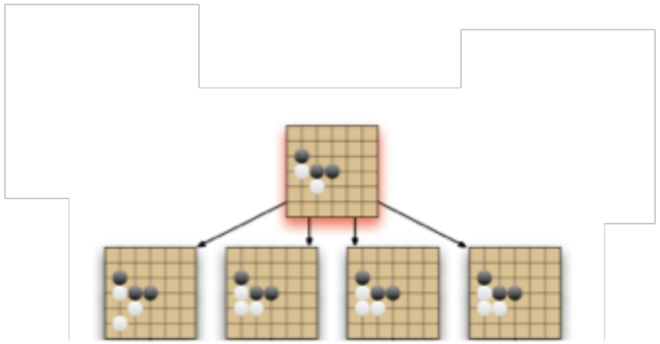
- To look ahead far in the future you need to chain your dynamic predictions
- Data is sequential
- i.i.d. assumptions break
- Errors accumulate in time



- Solutions:
  - Hierarchical dynamics models
  - Linear local approximations

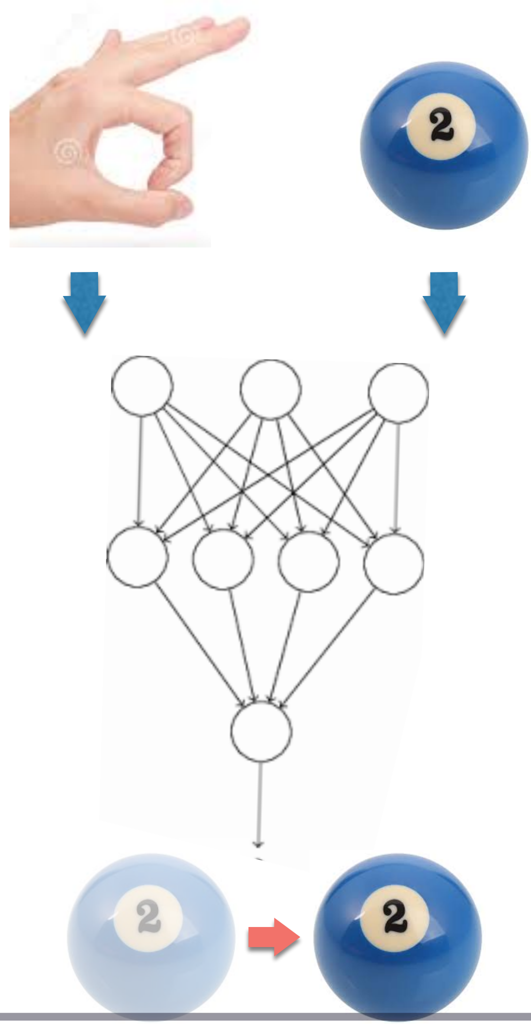
Examples of Models for T(s'|s,a)

Table lookup model (tabular): bookkeeping a probability of occurrence for each transition (s,a,s')



This Lecture

Transition function is approximated through some features



Later..

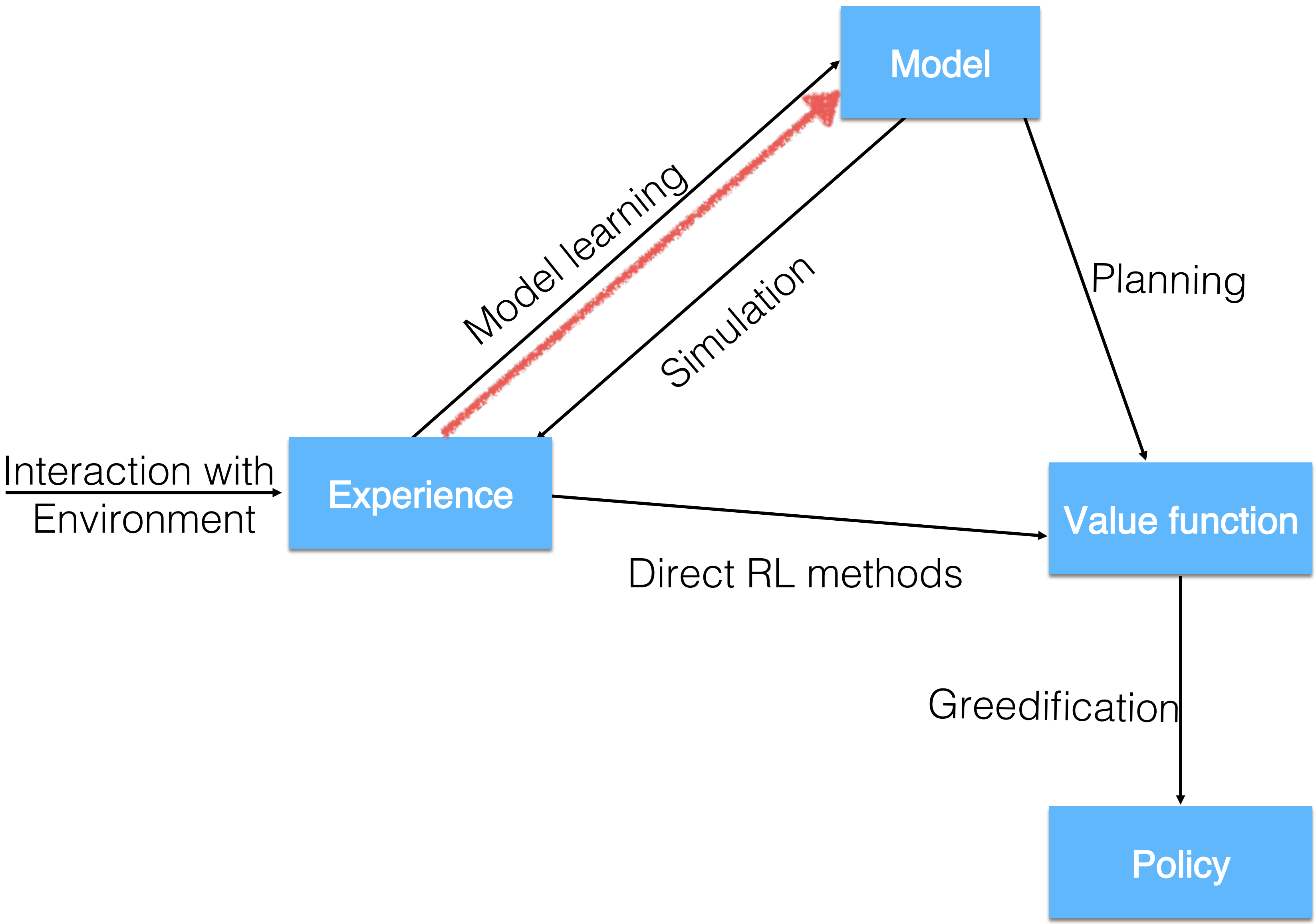

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Table Lookup Model

- Model is an explicit MDP, $\hat{\mathcal{T}}, \hat{\mathcal{R}}$
- Count visits $N(s,a)$ to each state action pair

$$\hat{T}(s'|s,a) = \frac{1}{N(s,a)} \sum_{t=1}^{\mathcal{T}} \mathbf{1}(S_t, A_t, S_{t+1} = s, a, s')$$

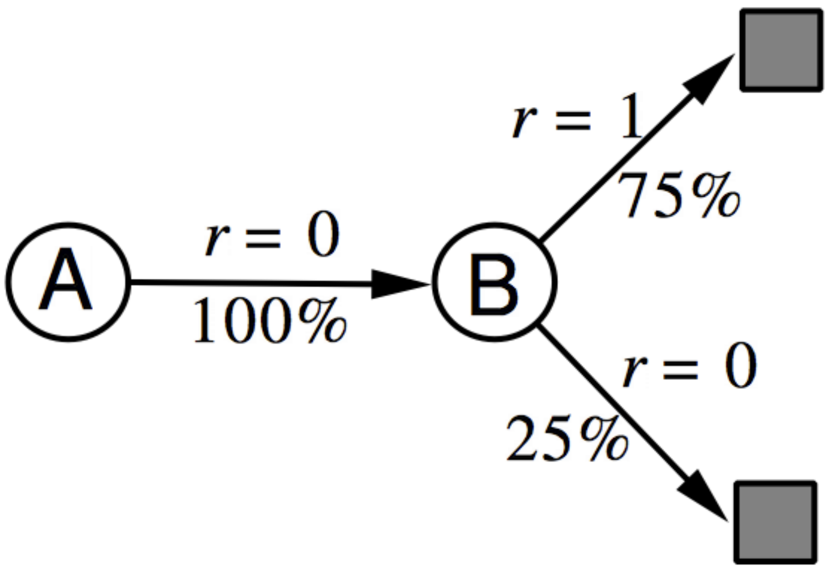
$$\hat{R}(s,a) = \frac{1}{N(s,a)} \sum_{t=1}^{\mathcal{T}} \mathbf{1}(S_t, A_t = s, a) R_t$$

- Alternatively
  - At each time-step t, record experience tuple $\langle S_t, A_t, R_{t+1}, S_{t+1} \rangle$
  - To sample model, randomly pick tuple matching $\langle s, a, \cdot, \cdot \rangle$

A simple Example

Two states *A,B*; no discounting; 8 episodes of experience

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0



We have constructed a table lookup model from the experience

Planning with a Model

Given a model $\mathcal{M}_\eta = \langle T_\eta, \mathcal{R}_\eta \rangle$

Solve the MDP $\langle \mathcal{S}, \mathcal{A}, T_\eta \mathcal{R}_\eta \rangle$

Using favorite planning algorithm

- Value iteration
- Policy iteration
- Tree search

curse of dimensionality!

Planning with a Model

Given a model $\mathcal{M}_\eta = \langle T_\eta, \mathcal{R}_\eta \rangle$

Solve the MDP $\langle \mathcal{S}, \mathcal{A}, T_\eta \mathcal{R}_\eta \rangle$

Using favorite planning algorithm

- Value iteration
- Policy iteration
- Tree search
- Sample-based planning

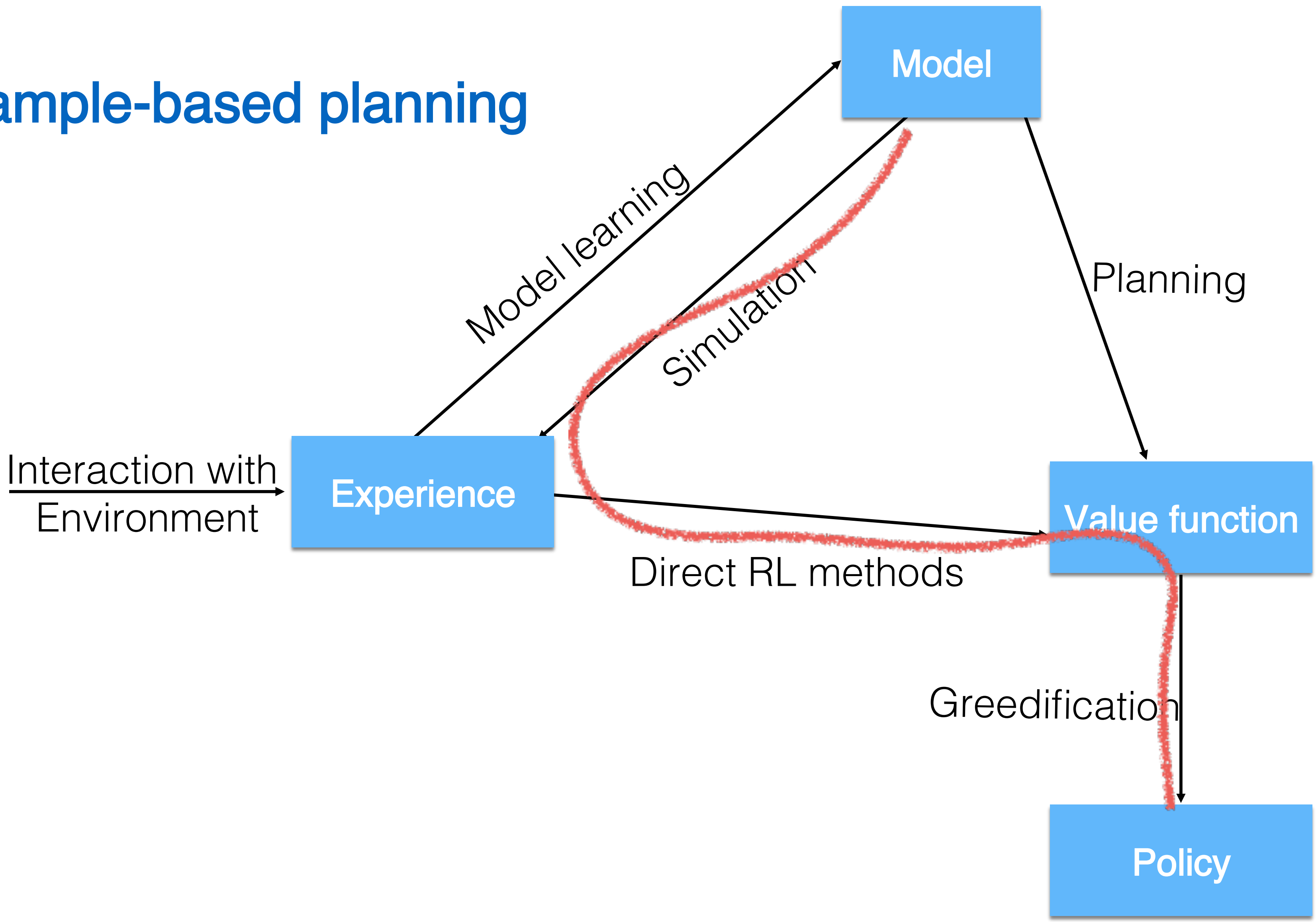
Sample-based Planning

- Use the model only to generate samples, not using its transition probabilities and expected immediate rewards
- Sample experience from model

$$S_{t+1} \sim T_\eta(S_{t+1}|S_t, A_t)$$
$$R_{t+1} = \mathcal{R}_\eta(R_{t+1}|S_t, A_t)$$

- Apply model-free RL to samples, e.g.:
  - Monte-Carlo control
  - Sarsa
  - Q-learning
- Sample-based planning methods are often more efficient: rather than exhaustive state sweeps: we focus on what is likely to happen

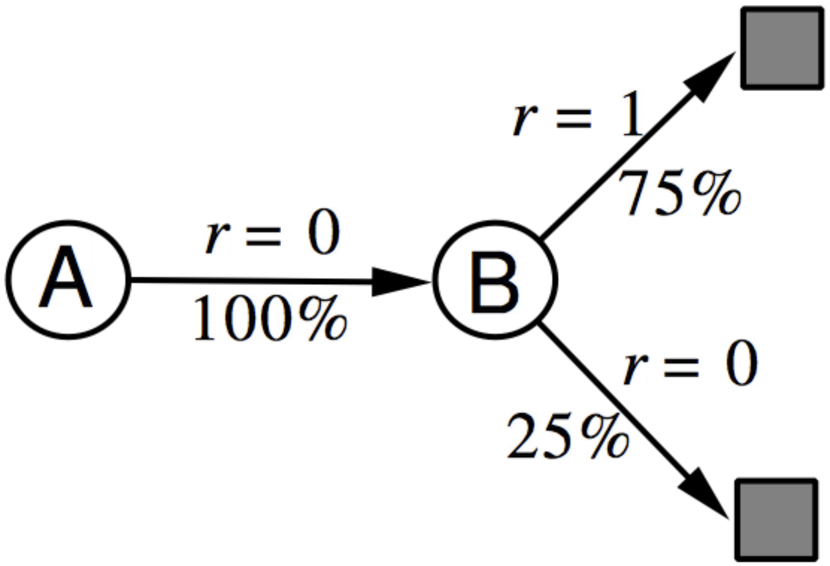
Sample-based planning



A Simple Example

- Construct a table-lookup model from real experience
- Apply model-free RL to sampled experience

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0



e.g. Monte-Carlo learning: $\mathrm{v}(A) = 1, \mathrm{v}(B) = 0.75$

Planning with an Inaccurate Model

Given an imperfect model $< \mathcal{T}_\eta, \mathcal{R}_\eta > \neq < \mathcal{T}, \mathcal{R} >$

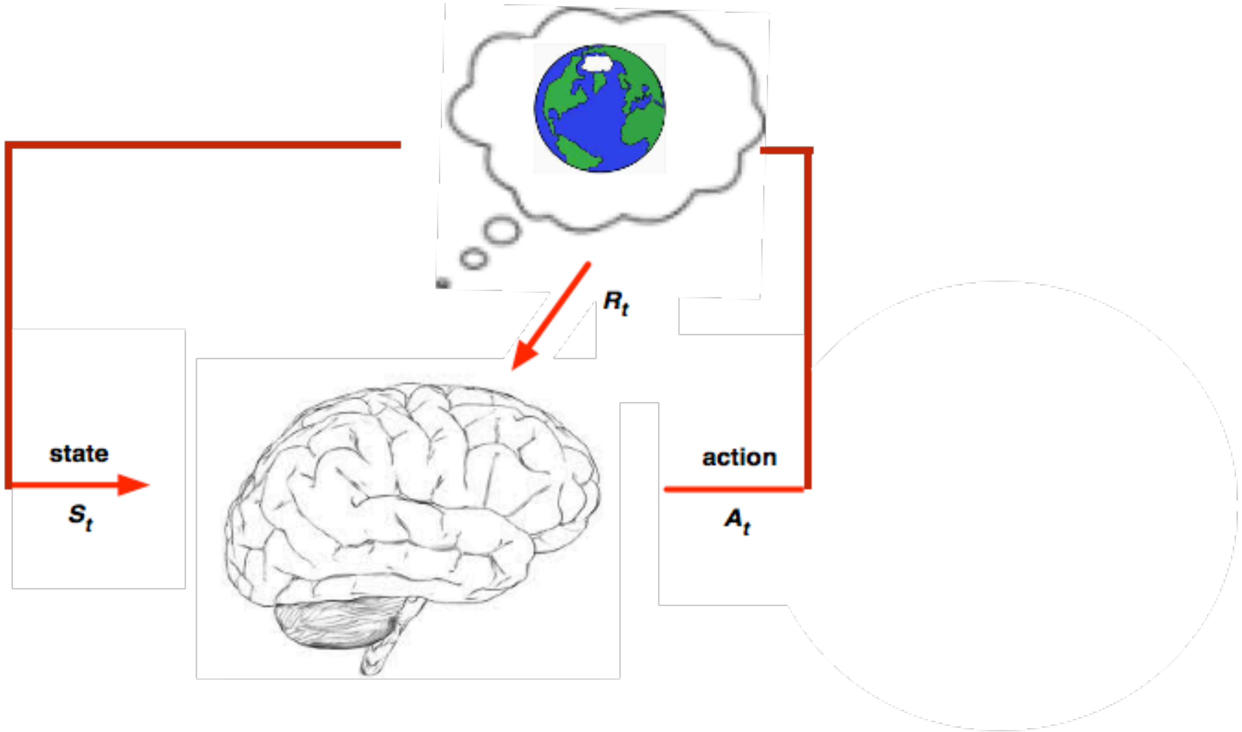
- Performance of model-based RL is limited to optimal policy for approximate MDP $< \mathcal{S}, \mathcal{A}, \mathcal{T}_\eta, \mathcal{R}_\eta >$
- i.e. Model-based RL is only as good as the estimated model

When the model is inaccurate, planning process will compute a suboptimal policy

- Solution 1: when model is wrong, use model-free RL
- Solution 2: reason explicitly about model uncertainty

Combine real and simulated experience

- If the model is unknown, we will learn the model.
- Learn value functions using both real and simulated experience
- Learn value functions online using model-based lookahead search



Real and Simulated Experience

We consider two sources of experience

- Real experience - Sampled from environment (true MDP)

$$S' \sim T(s'|s, a)$$

$$R = r(s, a)$$

- Simulated experience - Sampled from model (approximate MD)

$$S' \sim T_\eta(S'|S, A)$$
$$R = \mathcal{R}_\eta(\mathcal{R}|\mathcal{S}, \mathcal{A})$$

Integrating Learning and Planning

Model-Free RL

- No model
- Learn value function (and/or policy) from real experience

Integrating Learning and Planning

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- No model
- Learn value function (and/or policy) from real experience

Model-Based RL (using Sample-Based Planning)

- Learn a model from real experience
- Plan value function (and/or policy) from simulated experience

Integrating Learning and Planning

Model-Free RL

- No model
- Learn value function (and/or policy) from real experience

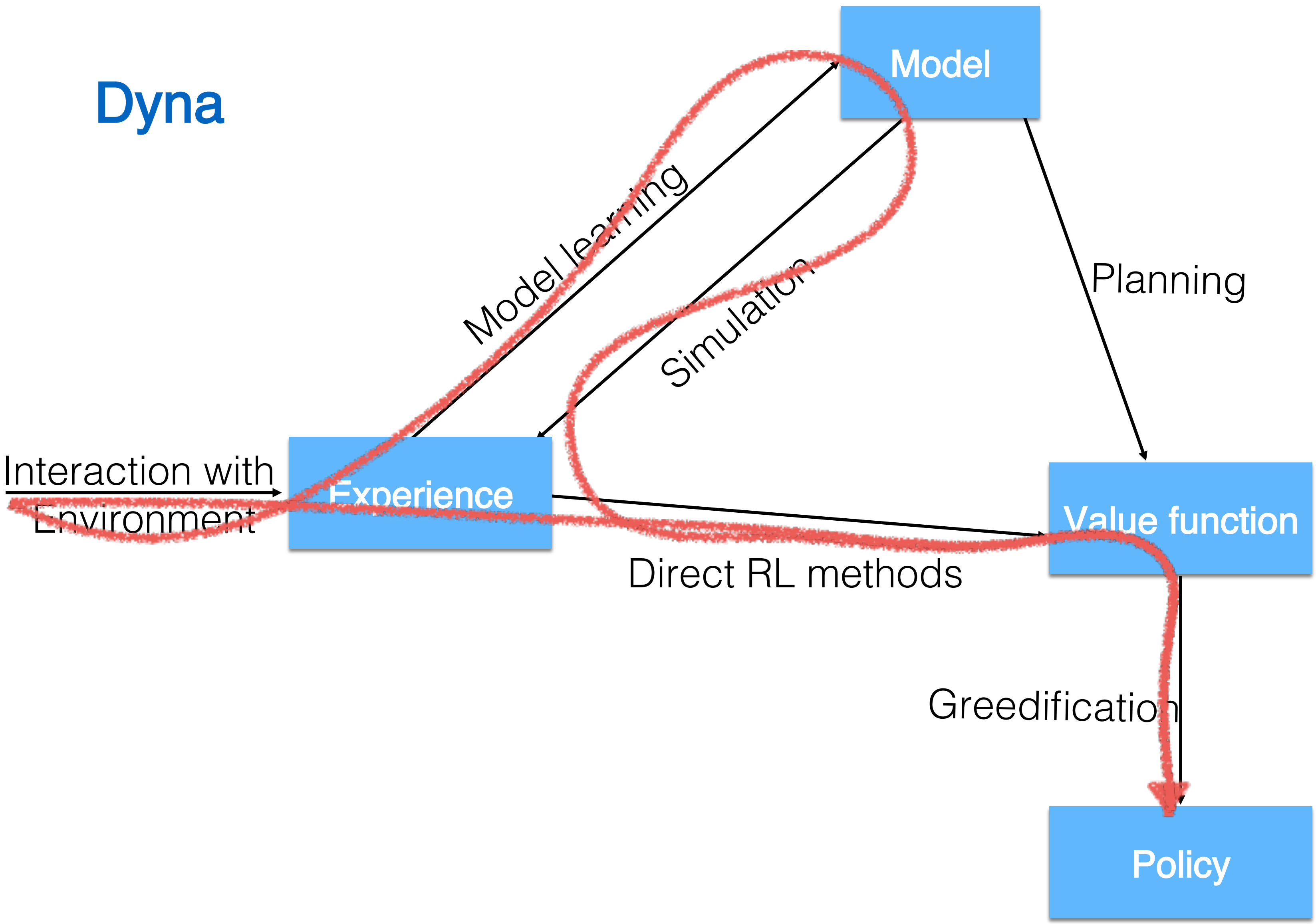
Model-Based RL (using Sample-Based Planning)

- Learn a model from real experience
- Plan value function (and/or policy) from simulated experience

Dyna

- Learn a model from real experience
- Learn and plan value function (and/or policy) from real and simulated experience

Dyna



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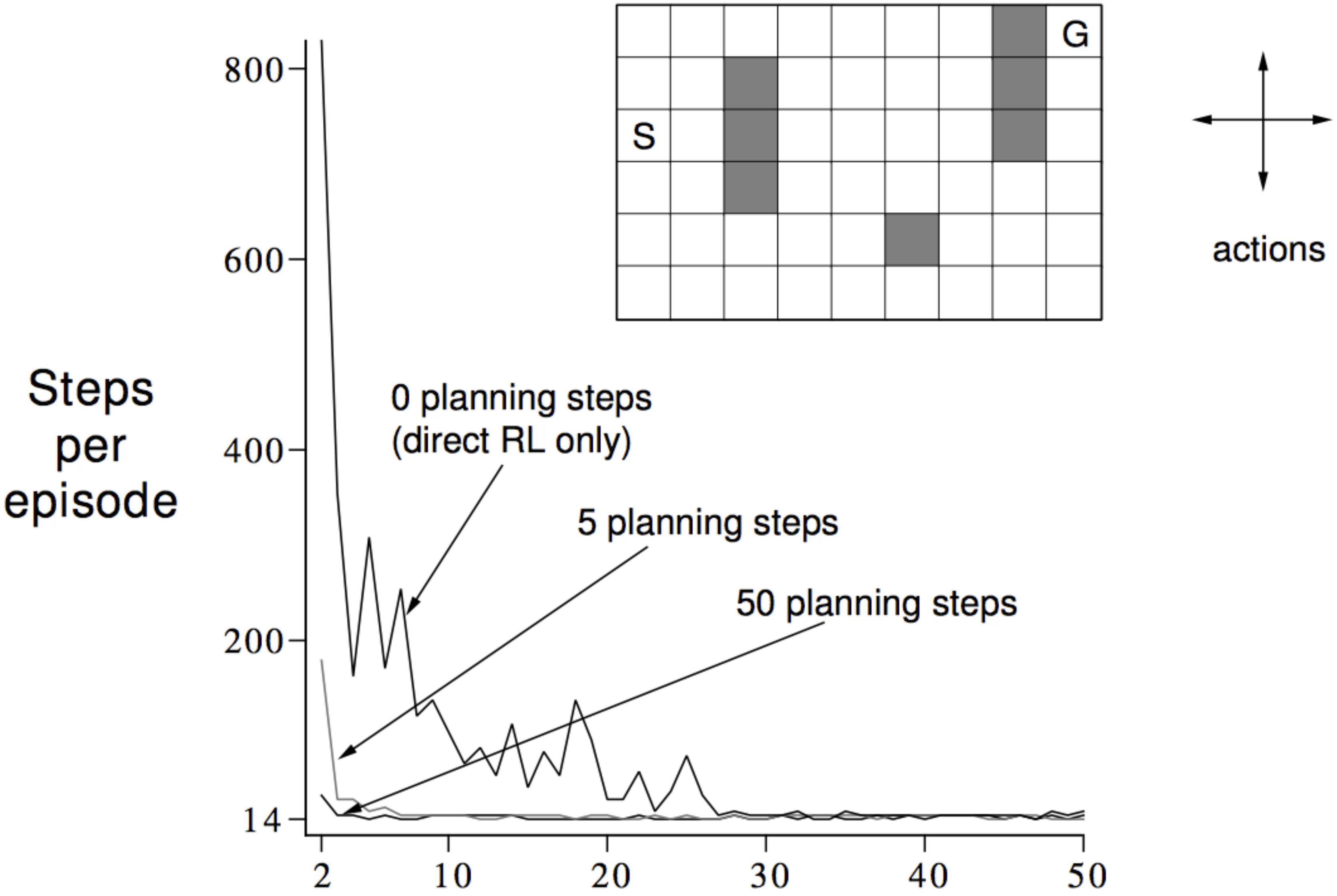
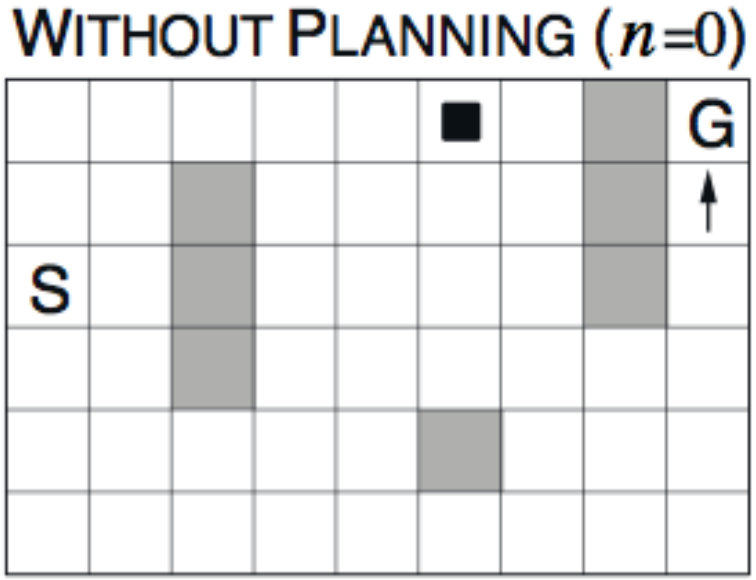
Dyna-Q Algorithm

Initialize $Q(s, a)$ and $Model(s, a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Do forever:
  (a) $S \leftarrow$ current (nonterminal) state
  (b) $A \leftarrow \varepsilon$-greedy(S, Q)
  (c) Execute action A; observe resultant reward, R, and state, S'
  (d) $Q(S, A) \leftarrow Q(S, A) + \alpha\left[R + \gamma \max_a Q(S', a) - Q(S, A)\right]$
  (e) $Model(S, A) \leftarrow R, S'$ (assuming deterministic environment)
  (f) Repeat n times:
    $S \leftarrow$ random previously observed state
    $A \leftarrow$ random action previously taken in S
    $R, S' \leftarrow Model(S, A)$
    $Q(S, A) \leftarrow Q(S, A) + \alpha\left[R + \gamma \max_a Q(S', a) - Q(S, A)\right]$

Dyna-Q on a Simple Maze



Midway in 2nd Episode



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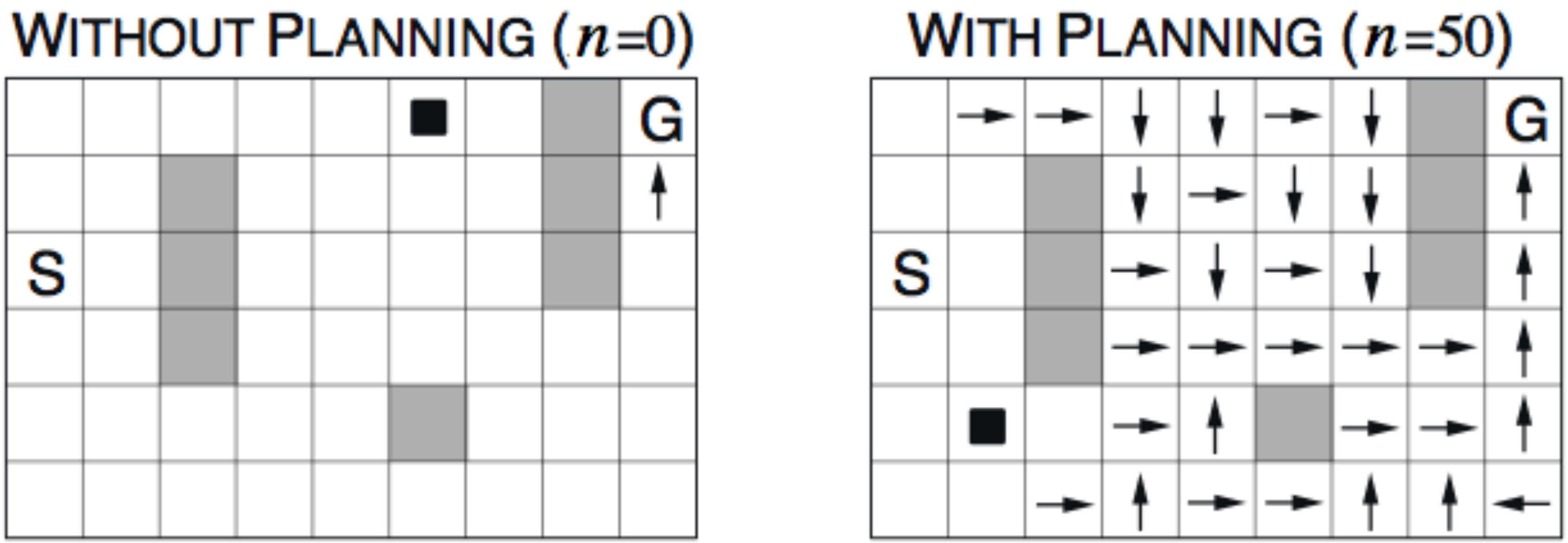
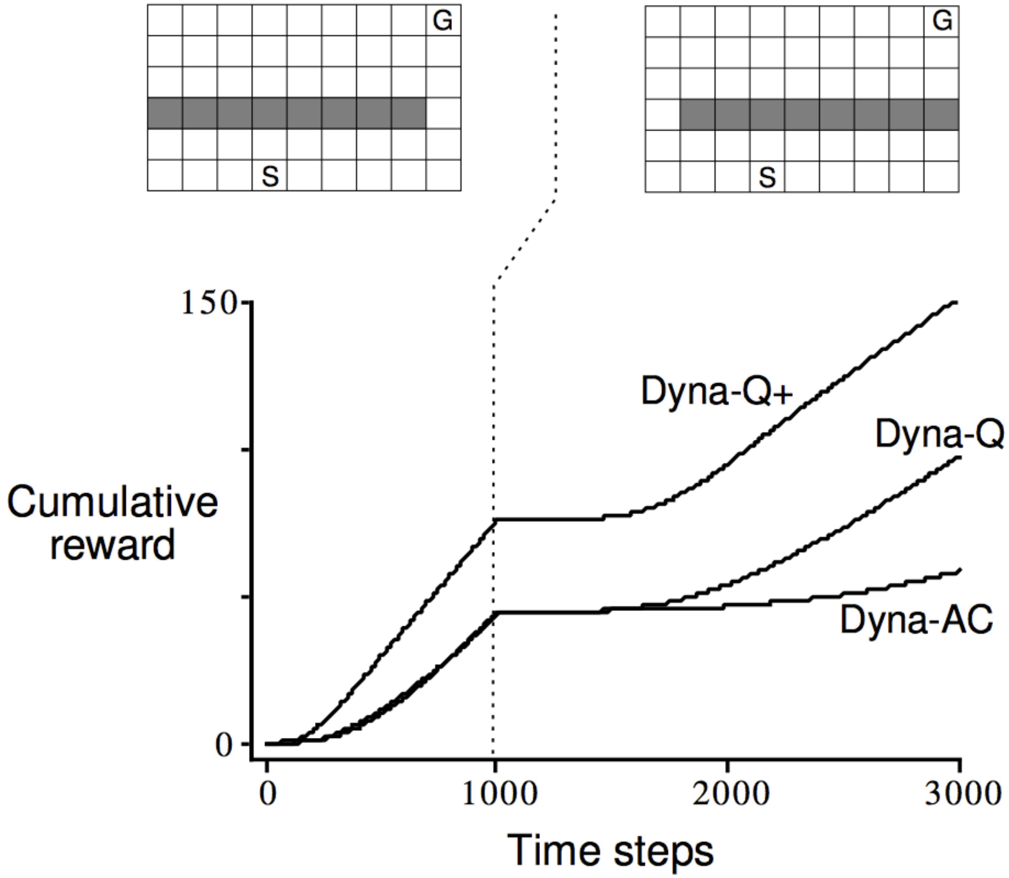


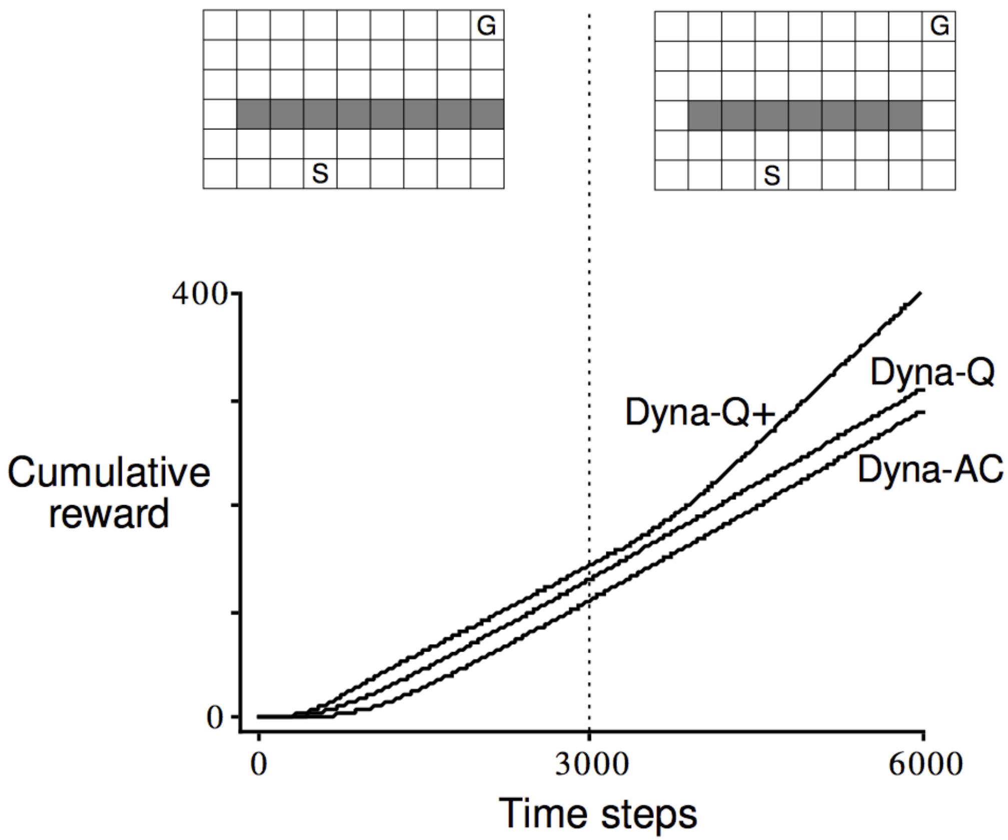
Dyna-Q with an Inaccurate Model

- The changed environment is harder



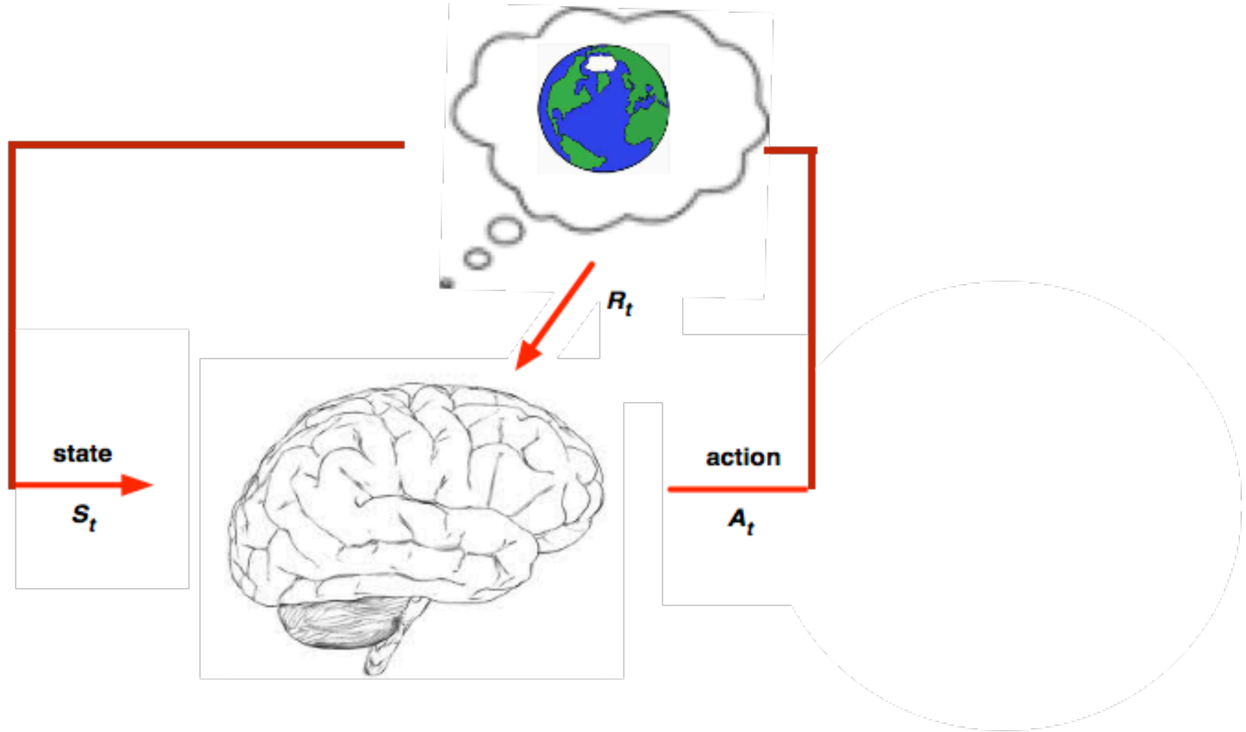
Dyna-Q with an Inaccurate model Cont.

- The changed environment is easier

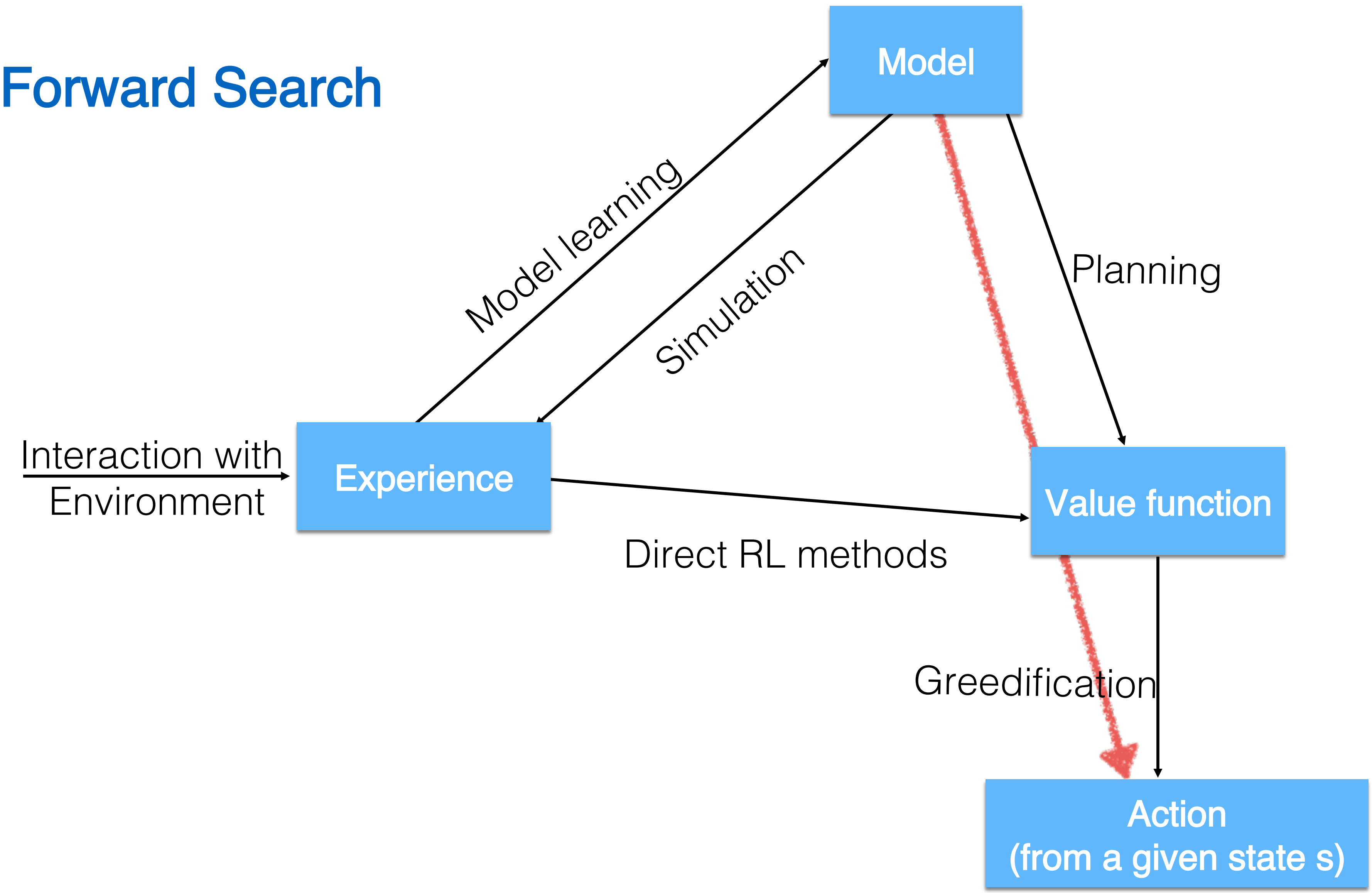


Sampling-based look-ahead search

- If the model is unknown, we will learn the model.
- Learn value functions using both real and simulated experience
- Learn value functions online using model-based lookahead search

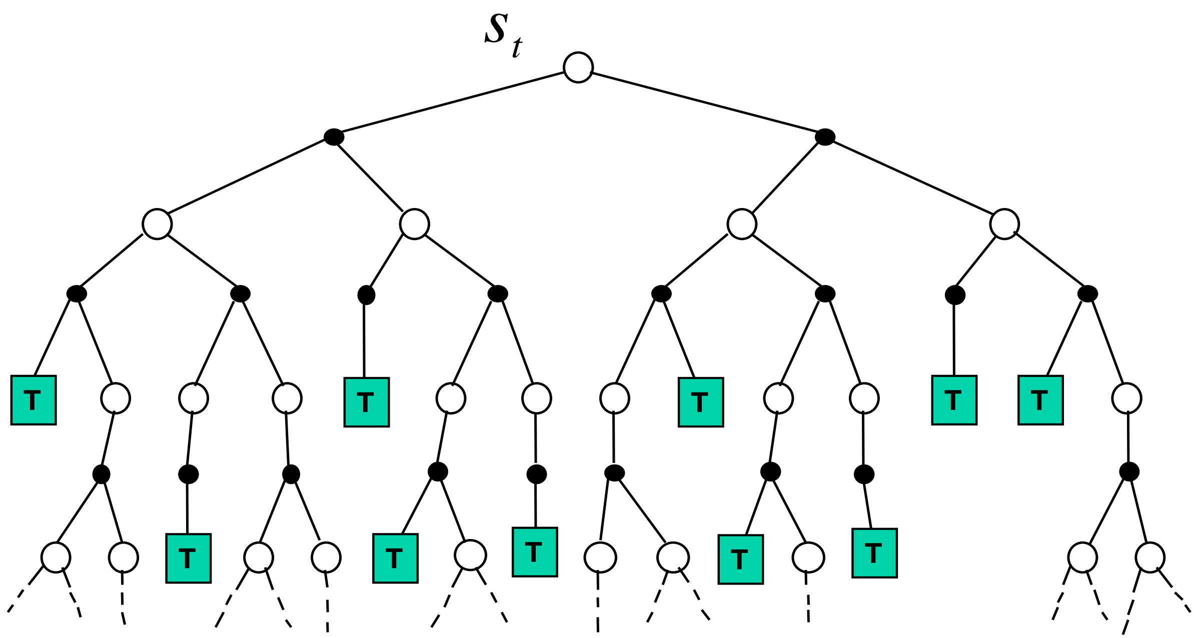


Forward Search



Forward Search

- Prioritizes the state the agent is currently in
- Using a model of the MDP to look ahead (exhaustively)
- Builds a search tree with the current state at the root
- Focus on sub-MDP starting from now, often dramatically easier than solving the whole MDP



Why Forward search?

Why don't we learn a value function directly for every state offline, so that we do not waste time online?

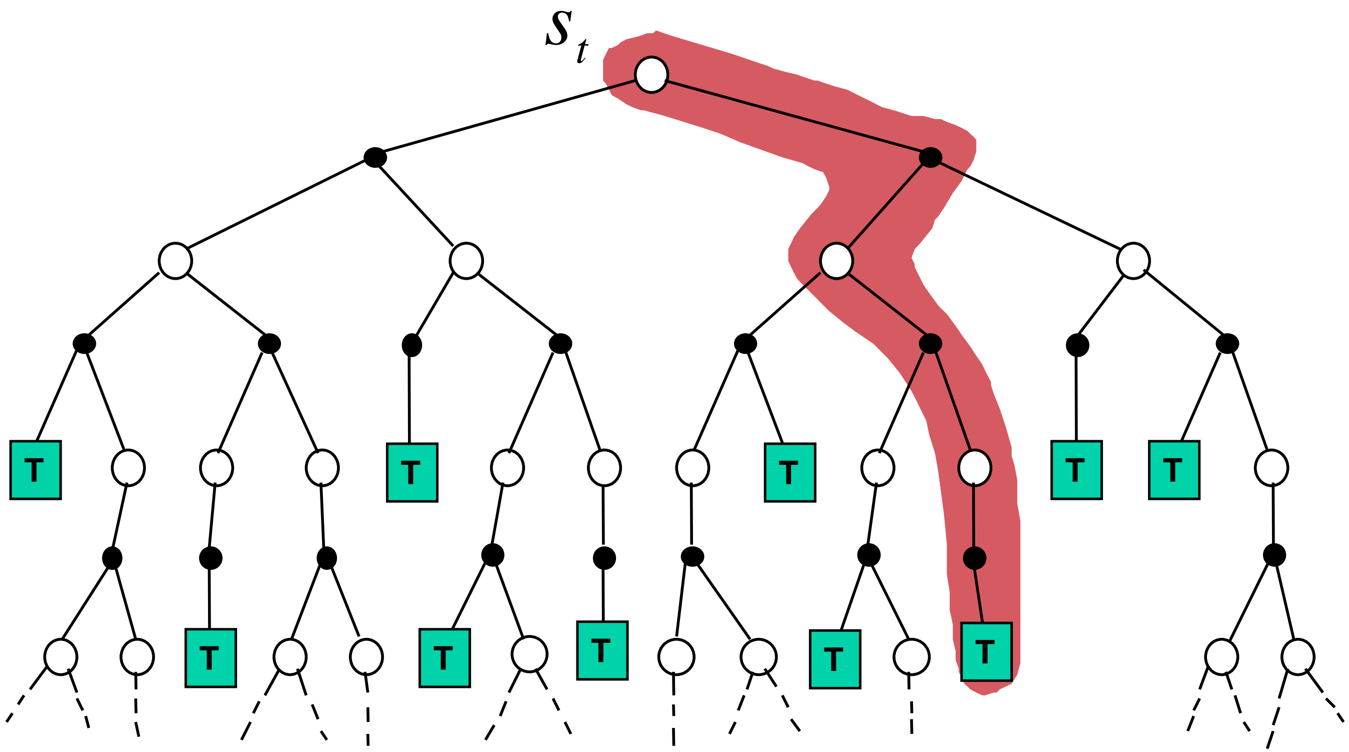
- Because the environment has many many states (consider Go 10^170, Chess 10^48, real world ….)
- Very hard to compute a good value function for each one, most you will never even visit
- Thus, it makes sense, condition on the current state you are in, to try to estimate the value function of the relevant part of the state space online
- Use the the online forward search to pick the best action

Disadvantages:

- Nothing is learnt from episode to episode

Simulation-based Search I

- Forward search paradigm using sample-based planning
- Simulate episodes of experience starting from now with the model
- Apply model-free RL to simulated episodes



Simulation-Based Search II

- Simulate episodes of experience from now with the model

$$\{s_t^k, \mathcal{A}_t^k, R_{t+1}^k, ..., S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu$$

- Apply model-free RL to simulated episodes
  - Monte-Carlo control $\quad \rightarrow \quad$ Monte-Carlo search

Simple Monte-Carlo Search

- Given a model $\mathcal{M}_\nu$ and a simulation policy π
- For each action $a \in \mathcal{A}$
  - Simulate K episodes from current (real) state s:

$$\{s_t, a, R_{t+1}^k, S_{t+1}^k, A_{t+1}^k, ..., S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu, \pi$$

  - Evaluate action value function of the root by mean return (Monte-Carlo evaluation)

$$Q(s_t, a) = \frac{1}{K} \sum_{k=1}^{K} G_t \xrightarrow{P} q_\pi(s_t, a)$$

- Select current (real) action with maximum value

$$a_t = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \; Q(s_t, a)$$

Monte-Carlo Tree Search (Evaluation)

- Given a model $\mathcal{M}_\nu$
- Simulate K episodes from current state s_t using current simulation policy π

$$\{s_t, A_t^k, R_{t+1}^k, S_{t+1}^k, ..., S_T^k\}_{k=1}^K \sim \mathcal{M}_\nu, \pi$$

- Build a search tree containing visited states and actions
- Evaluate states $Q(s, a)$ by mean return of episodes from s, a for all states and actions in the tree

$$Q(s_t, a) = \frac{1}{N(s,a)} \sum_{k=1}^{K} \sum_{u=t}^{T} \mathbf{1}(S_u, A_u = s, a) G_u \xrightarrow{P} q_\pi(s, a)$$

- After search is finished, select current (real) action with maximum value in search tree

$$a_t = \underset{a \in \mathcal{A}}{\operatorname{argmax}} \; Q(s_t, a)$$

Monte-Carlo Tree Search (Simulation)

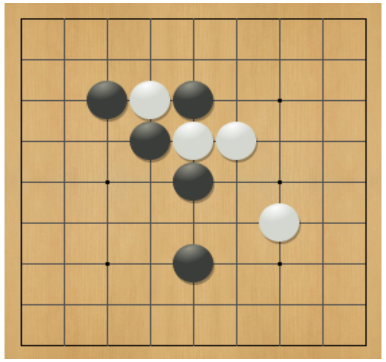
- In MCTS, the simulation policy π improves
- Each simulation consists of two phases (in-tree, out-of-tree)
  - Tree policy (improves): pick actions to maximize $Q(s, a)$
  - Default policy (fixed): pick actions randomly
- Repeat (each simulation)
  - Evaluate states $Q(s, a)$ by Monte-Carlo evaluation
  - Improve there policy, e.g. by $\epsilon - \mathrm{greedy}(Q)$
- Monte-Carlo control applied to simulated experience
- Converges on the optimal search tree, $Q(S, A) \rightarrow q * (S, A)$

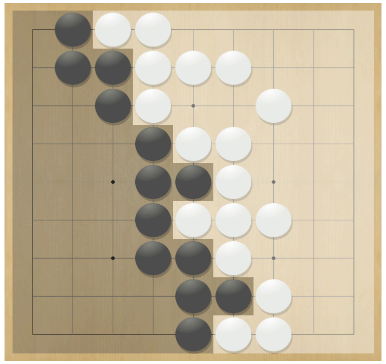
Case Study: the Game of Go

- The ancient oriental game of Go is 2500 years old
- Considered to be the hardest classic board game
- Considered a grand challenge task for AI (*John McCarthy*)
- Traditional game-tree search has failed in Go

Rules of Go

- Usually played on 19x19, also 13x13 or 9x9 board
- Simple rules, complex strategy
- Black and white place down stones alternately
- Surrounded stones are captured and removed
- The player with more territory wins the game





Position Evaluation in Go

- How good is a position s ?
- Reward function (undiscounted):

$R_t = 0$ for all non-terminal steps $t < \mathcal{T}$

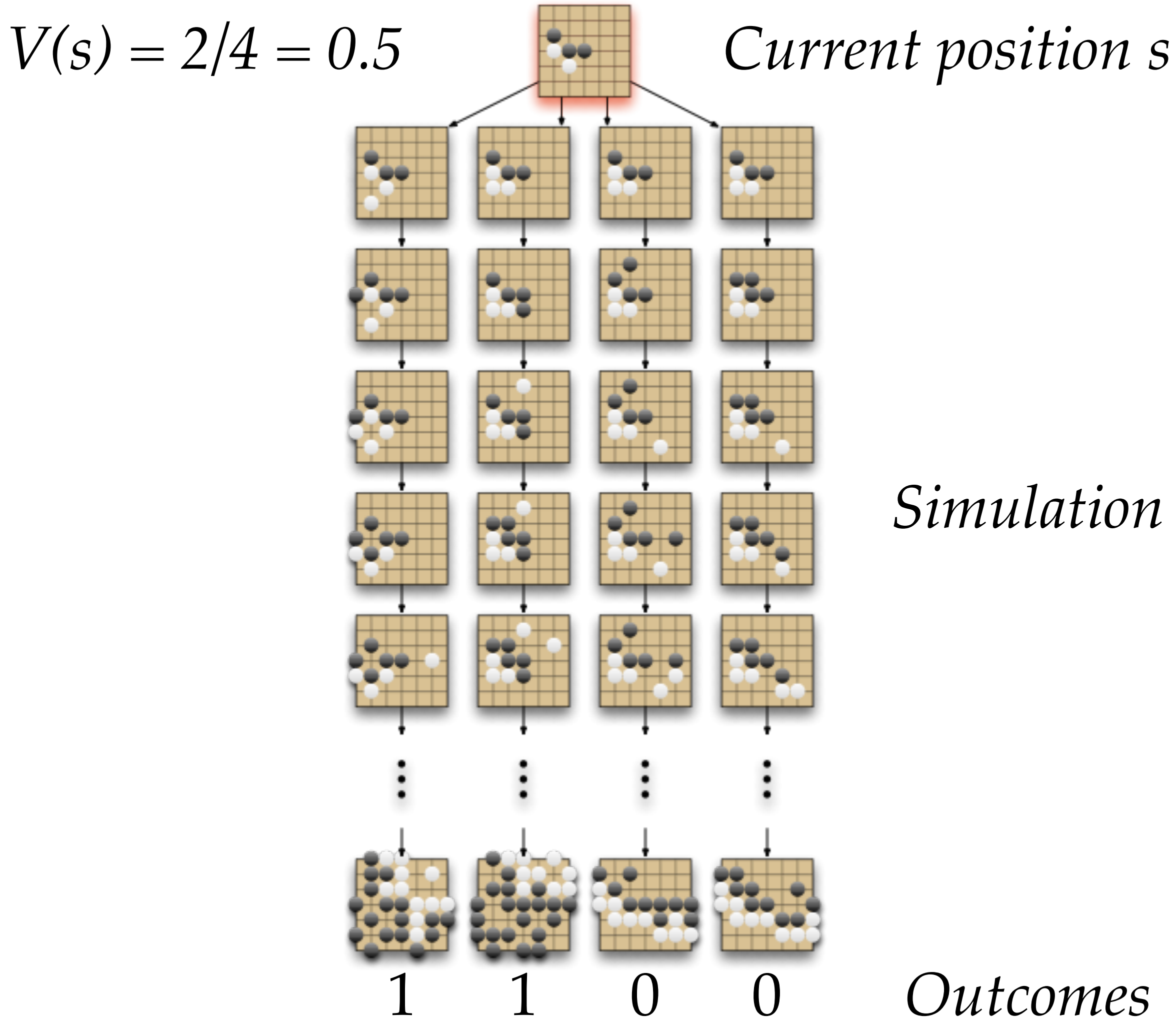
$$R_{\mathcal{T}} = \begin{cases} 1, & \text{if Black wins.} \\ 0, & \text{if White wins.} \end{cases}$$

- Policy $\pi = \langle \pi_B, \pi_W \rangle$ selects moves for both players
- Value function (how good is position s):

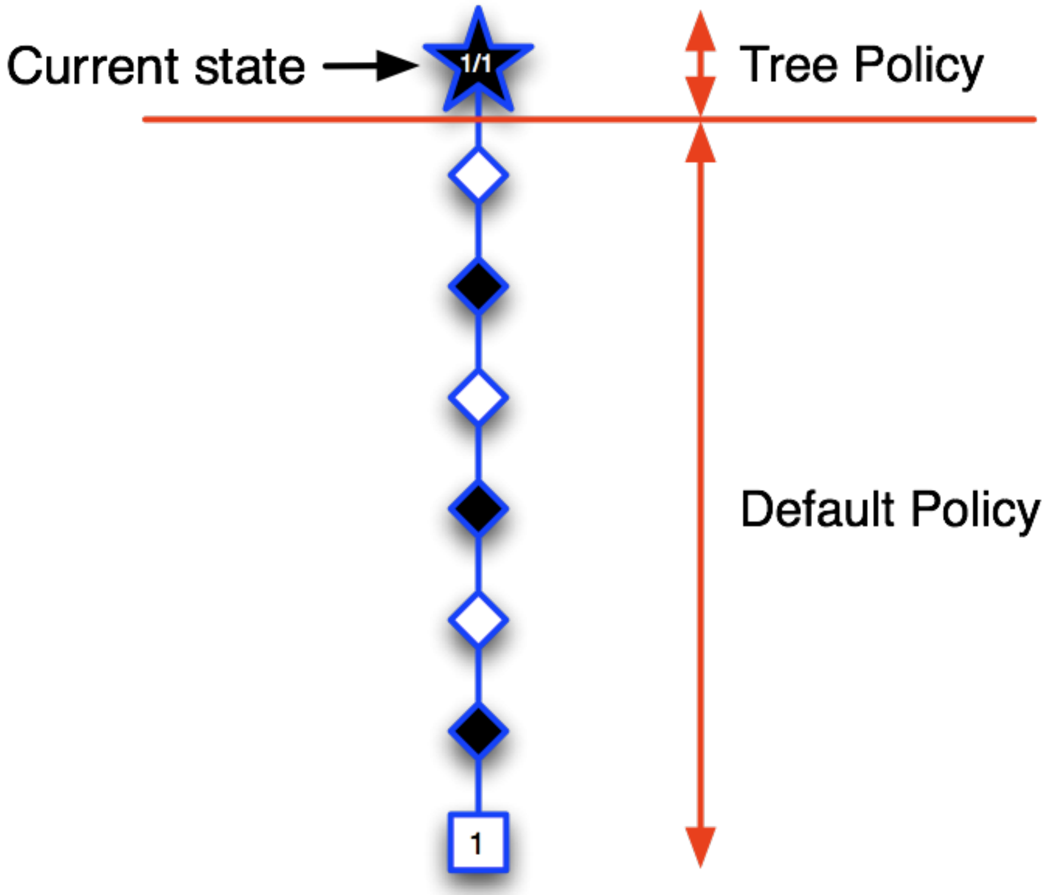
$$v_\pi(s) = \mathbb{E}_\pi[R_{\mathcal{T}} | S = s] = \mathbb{P}[Black \;\; wins | S = s]$$

$$v_*(s) = \max_{\pi_B} \min_{\pi_W} \mathrm{v}_\pi(s)$$

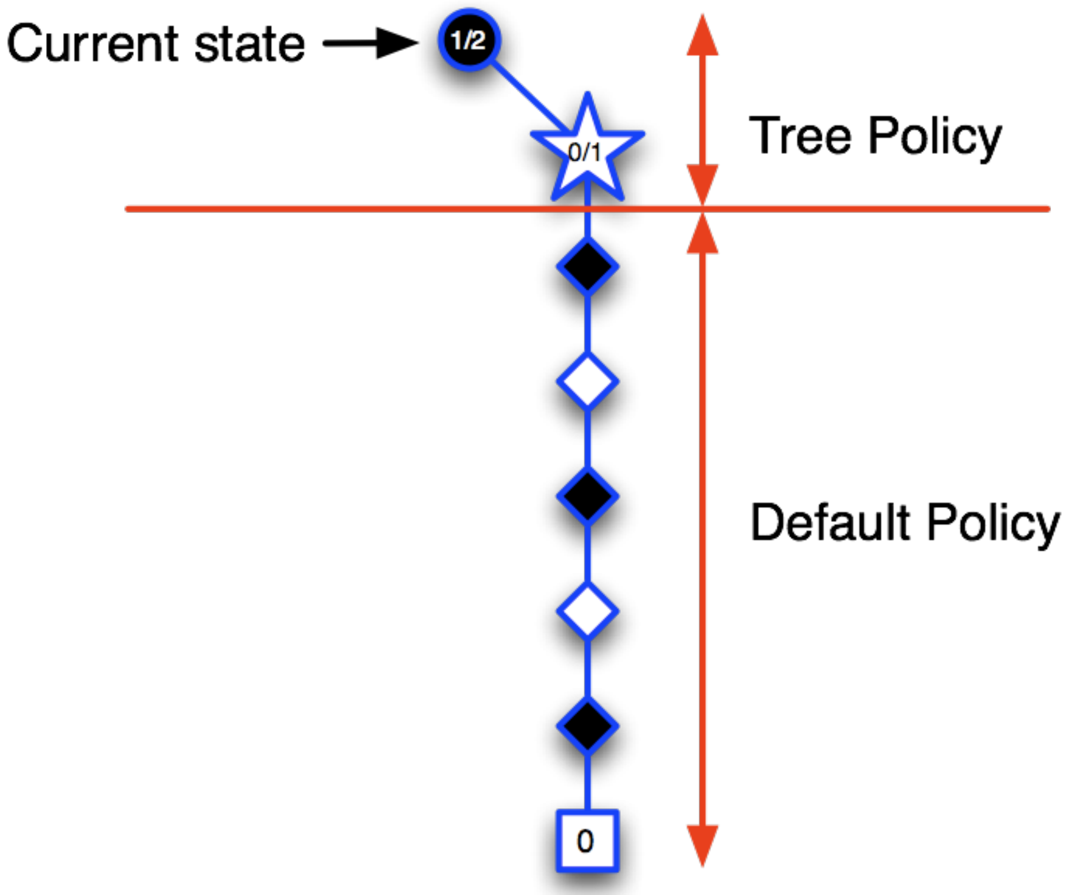
Monte-Carlo Evaluation in Go



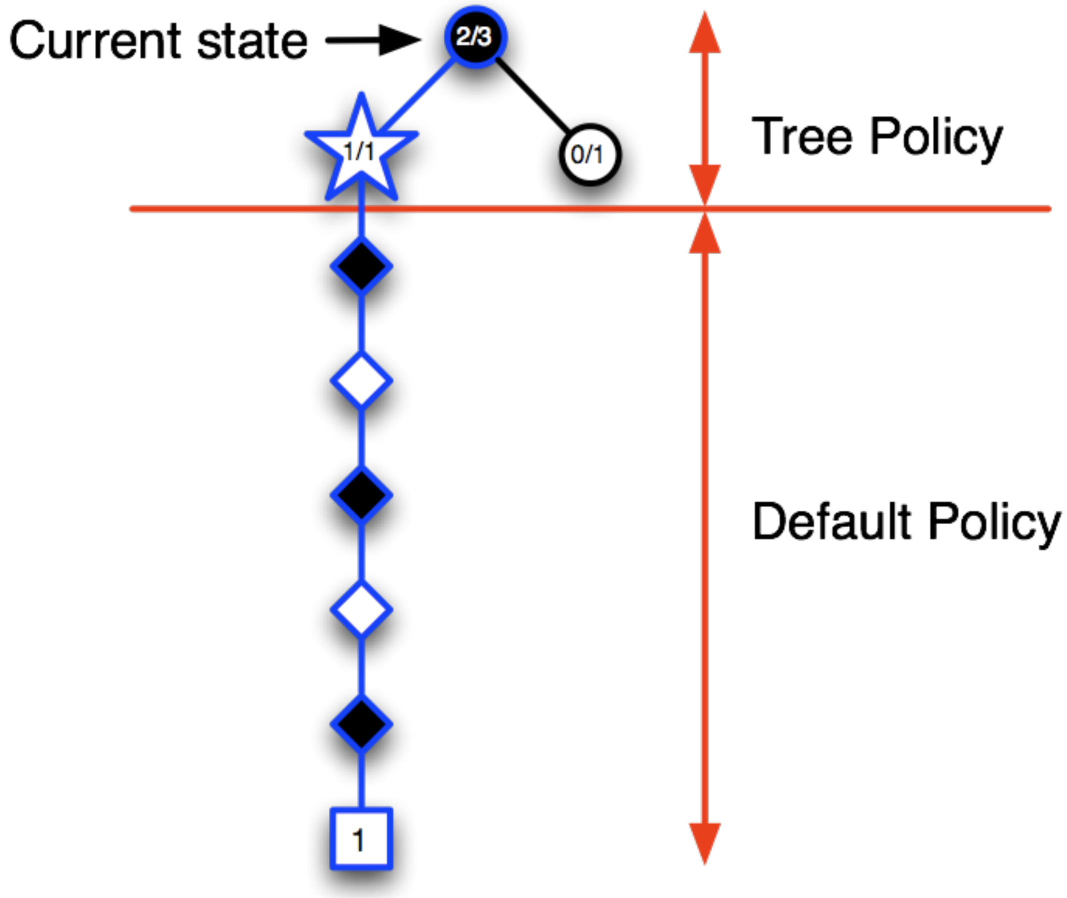
Applying Monte-Carlo Tree Search



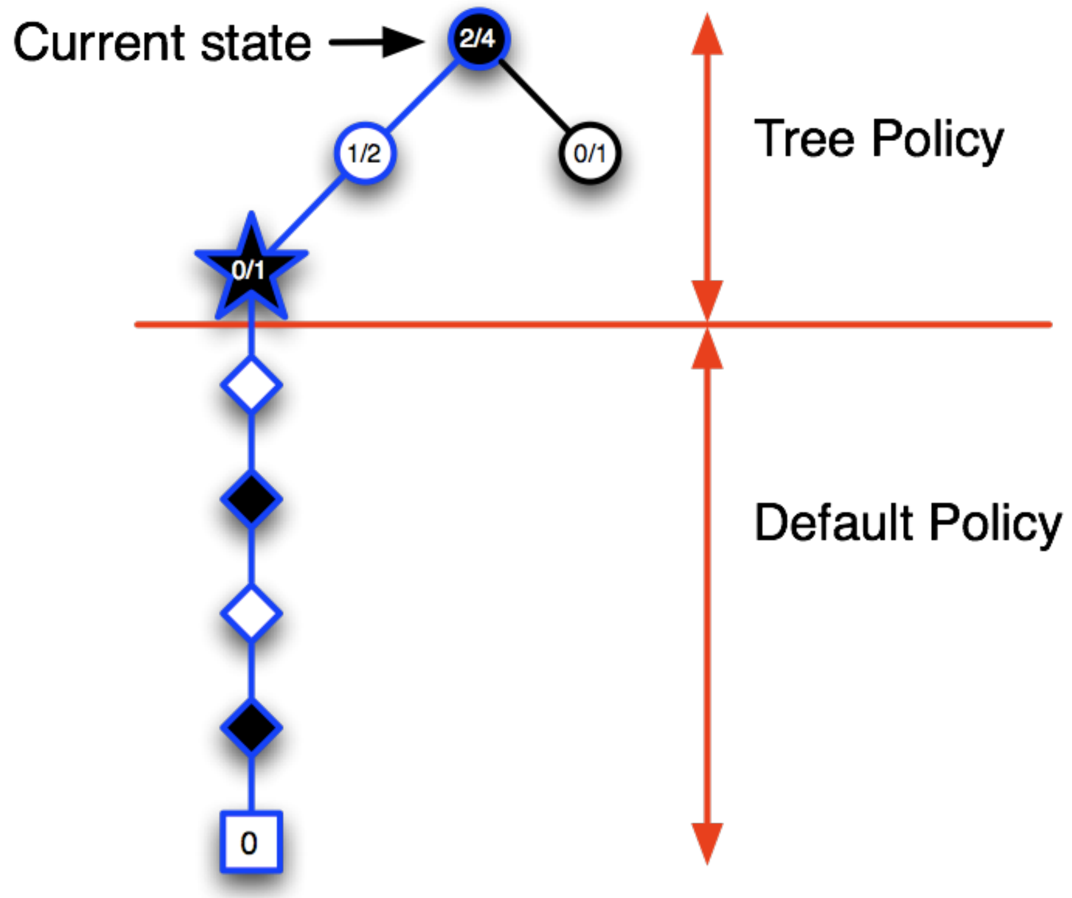
Applying Monte-Carlo Tree Search



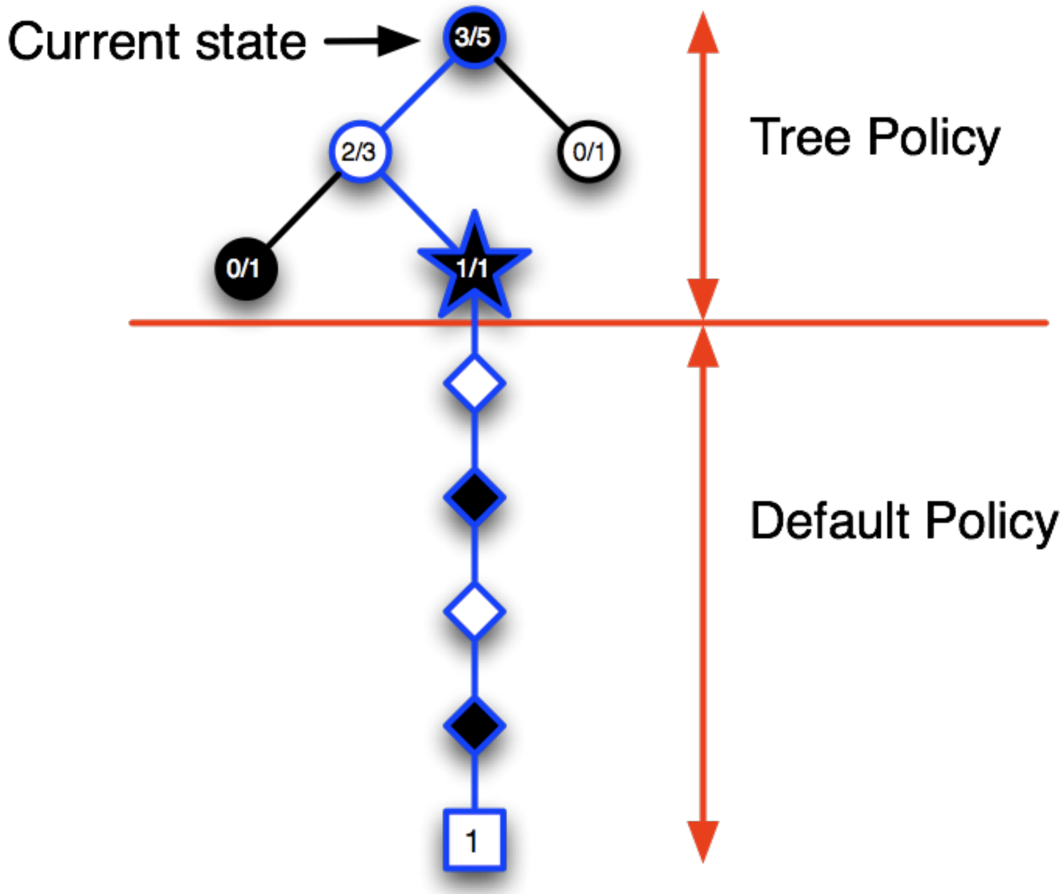
Applying Monte-Carlo Tree Search



Applying Monte-Carlo Tree Search



Applying Monte-Carlo Tree Search



Advantages of MC Tree Search

- Highly selective: best-first search
- Evaluate states dynamically (unlike e.g. DP)
- Uses sampling to break curse of dimensionality
- Computationally efficient, anytime, parallelizable

Combining offline and online value function estimation

- Use policy networks to have priors on Q(s,a):

$$a_t = \operatorname{argmax}_a (Q(s_t, a) + u(s_t, a))$$

$$u(s,a) \propto \frac{P(s,a)}{1 + N(s,a)} \qquad P(s,a) = \pi_\sigma(a|s)$$

- Use fast and light policy networks for rollouts (instead of random policy)