

10703 Deep Reinforcement Learning and Control

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Policy Gradient II

Used Materials

- **Disclaimer:** Much of the material and slides for this lecture were borrowed from Rich Sutton's RL class and David Silver's Deep RL tutorial

Policy-Based Reinforcement Learning

- ▶ So far we approximated the value or action-value function using parameters θ (e.g. neural networks)

$$V_\theta(s) \approx V^\pi(s)$$

$$Q_\theta(s, a) \approx Q^\pi(s, a)$$

- ▶ A policy was generated directly from the value function e.g. using ϵ -greedy
- ▶ In this lecture we will directly parameterize the policy

$$\pi_\theta(s, a) = \mathbb{P}[a \mid s, \theta]$$

- ▶ We will focus again on model-free reinforcement learning

Policy Gradient Theorem

- ▶ The policy gradient theorem generalizes the likelihood ratio approach to multi-step MDPs
 - ▶ Replaces instantaneous reward r with long-term value $Q^\pi(s, a)$
 - ▶ Policy gradient theorem applies to start state objective, average reward and average value objective
-

- ▶ For any differentiable policy $\pi_\theta(s, a)$, the policy gradient is

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

Monte-Carlo Policy Gradient (REINFORCE)

- ▶ Update parameters by stochastic gradient ascent
- ▶ Using policy gradient theorem
- ▶ Using return G_t as an unbiased sample of $Q^{\pi_\theta}(s_t, a_t)$

$$\Delta\theta_t = \alpha G_t \nabla_\theta \log \pi_\theta(s_t, a_t)$$

REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic)

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$, $\forall a \in \mathcal{A}, s \in \mathcal{S}, \boldsymbol{\theta} \in \mathbb{R}^n$

Initialize policy weights $\boldsymbol{\theta}$

Repeat forever:

 Generate an episode $S_0, A_0, R_1, \dots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$

 For each step of the episode $t = 0, \dots, T - 1$:

$G_t \leftarrow$ return from step t

$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G_t \nabla_{\boldsymbol{\theta}} \log \pi(A_t | S_t, \boldsymbol{\theta})$

Actor-Critic

- ▶ Monte-Carlo policy gradient still has **high variance**
- ▶ We can use a **critic** to estimate the action-value function:

$$Q_w(s, a) \approx Q^{\pi_\theta}(s, a)$$

- ▶ **Actor-critic algorithms** maintain two sets of parameters
 - **Critic Updates** action-value function parameters w
 - **Actor Updates** policy parameters θ , in direction suggested by critic
- ▶ Actor-critic algorithms follow an approximate policy gradient

$$\nabla_\theta J(\theta) \approx \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \ Q_w(s, a)]$$

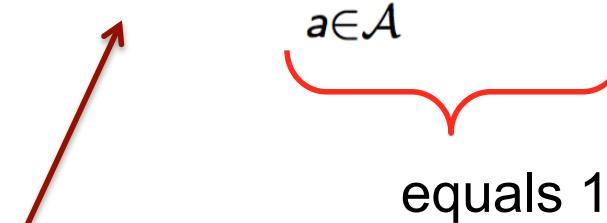
$$\Delta\theta = \alpha \nabla_\theta \log \pi_\theta(s, a) \ Q_w(s, a)$$

Reducing Variance Using a Baseline

- We can subtract a **baseline function** $B(s)$ from the policy gradient
- This can reduce variance, **without changing expectation!**

$$\begin{aligned}\mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) B(s)] &= \sum_{s \in \mathcal{S}} d^{\pi_\theta}(s) \sum_a \nabla_\theta \pi_\theta(s, a) B(s) \\ &= \sum_{s \in \mathcal{S}} d^{\pi_\theta} B(s) \nabla_\theta \sum_{a \in \mathcal{A}} \pi_\theta(s, a) \\ &= 0\end{aligned}$$

Function of state s , but
not action a



- A good baseline is the state value function $B(s) = V^{\pi_\theta}(s)$

Reducing Variance Using a Baseline

- We can subtract a **baseline function** $B(s)$ from the policy gradient
- This can reduce variance, **without changing expectation!**

$$\mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) B(s)] = 0$$

- A good baseline is the state value function $B(s) = V^{\pi_\theta}(s)$
- So we can rewrite the policy gradient using the advantage function:

$$A^{\pi_\theta}(s, a) = Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) A^{\pi_\theta}(s, a)]$$

- Note that it is the exact same policy gradient:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

Estimating the Advantage Function

- ▶ The advantage function can significantly reduce variance of policy gradient
- ▶ So the critic should really estimate the advantage function
- ▶ For example, by estimating both $V^{\pi_\theta}(s)$ and $Q^{\pi_\theta}(s, a)$
- ▶ Using two function approximators and two parameter vectors:

$$V_v(s) \approx V^{\pi_\theta}(s)$$

$$Q_w(s, a) \approx Q^{\pi_\theta}(s, a)$$

$$A(s, a) = Q_w(s, a) - V_v(s)$$

- ▶ And updating both value functions by e.g. TD learning

Dueling Networks

- ▶ Split Q-network into two channels
- ▶ Action-independent value function $V(s, v)$
- ▶ Action-dependent advantage function $A(s, a, w)$

$$Q(s, a) = V(s, v) + A(s, a, w)$$

- ▶ Advantage function is defined as:

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s).$$

Estimating the Advantage Function

- For the true value function $V^{\pi_\theta}(s)$ the TD error:

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

is an unbiased estimate of the advantage function:

$$\begin{aligned}\mathbb{E}_{\pi_\theta} [\delta^{\pi_\theta} | s, a] &= \mathbb{E}_{\pi_\theta} [r + \gamma V^{\pi_\theta}(s') | s, a] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s) \\ &= A^{\pi_\theta}(s, a)\end{aligned}$$

- So we can use the TD error to compute the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \delta^{\pi_\theta}]$$

- Remember the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) A^{\pi_\theta}(s, a)]$$

Estimating the Advantage Function

- For the true value function $V^{\pi_\theta}(s)$ the TD error:

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

is an unbiased estimate of the advantage function

- So we can use the TD error to compute the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \delta^{\pi_\theta}]$$

- In practice we can use an approximate TD error

$$\delta_v = r + \gamma V_v(s') - V_v(s)$$

- This approach only requires one set of critic parameters v

Critic

- ▶ Critic can estimate **value function** $V_v(s)$ from various targets. For example, from previous lectures:
- ▶ For MC, the target is the return G_t

$$\Delta v = \alpha(G_t - V_v(s_t)) \nabla_v V_v(s)$$

- ▶ V_v can be a **deep neural network** with parameters v .
- ▶ For TD(0), the target is the TD target $r + \gamma V(s')$

$$\Delta v = \alpha(r_{t+1} + \gamma V_v(s_{t+1}) - V_v(s_t)) \nabla_v V_v(s)$$

- ▶ So critic is updated to minimize MSE w.r.t. target, given by MC or TD(0)

$$(V^{\pi_\theta}(s_t) - V_v(s_t))^2$$

Actor

- ▶ The policy gradient can also be estimated as follows:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) \ A^{\pi_{\theta}}(s, a)]$$

- ▶ Monte-Carlo policy gradient uses error from complete return

$$\Delta \theta = \alpha(G_t - V_v(s_t)) \nabla_{\theta} \log \pi_{\theta}(s_t, a_t)$$

- ▶ Actor-critic policy gradient uses the one-step TD error

$$\Delta \theta = \alpha(r + \gamma V_v(s_{t+1}) - V_v(s_t)) \nabla_{\theta} \log \pi_{\theta}(s_t, a_t)$$

Advantage Actor-Critic Algorithm

One-step Actor-Critic (episodic)

Input: a differentiable policy parameterization $\pi(a|s, \theta)$, $\forall a \in \mathcal{A}, s \in \mathcal{S}, \theta \in \mathbb{R}^n$

Input: a differentiable state-value parameterization $\hat{v}(s, \mathbf{w})$, $\forall s \in \mathcal{S}, \mathbf{w} \in \mathbb{R}^m$

Parameters: step sizes $\alpha > 0$, $\beta > 0$

Initialize policy weights θ and state-value weights w

Repeat forever:

Initialize S (first state of episode)

$$I \leftarrow 1$$

While S is not terminal:

$$A \sim \pi(\cdot | S, \theta)$$

Take action A , observe S', R

$$\delta \leftarrow R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w}) \quad \text{(if } S' \text{ is terminal, then } \hat{v}(S', \mathbf{w}) \doteq 0\text{)}$$

$$\mathbf{w} \leftarrow \mathbf{w} + \beta \delta \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

$$\theta \leftarrow \theta + \alpha I \delta \nabla_{\theta} \log \pi(A|S, \theta)$$

$$I \leftarrow \gamma I$$

$$S \leftarrow S'$$

So Far: Summary of PG Algorithms

- ▶ The policy gradient has many equivalent forms

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) G_t] && \text{REINFORCE} \\ &= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) Q^w(s, a)] && \text{Q Actor-Critic} \\ &= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) A^w(s, a)] && \text{Advantage Actor-Critic} \\ &= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) \delta] && \text{TD Actor-Critic}\end{aligned}$$

- ▶ Each leads a stochastic gradient ascent algorithm
- ▶ Critic uses **policy evaluation** (e.g. MC or TD learning) to estimate $Q^{\pi}(s, a)$, $A^{\pi}(s, a)$ or $V^{\pi}(s)$

Bias in Actor-Critic Algorithms

- ▶ Approximating the policy gradient introduces **bias**

$$Q_w(s, a) \approx Q^{\pi_\theta}(s, a)$$

- ▶ A biased policy gradient may not find the right solution
- ▶ Luckily, if we choose value function approximation carefully
- ▶ Then we can avoid introducing any bias
- ▶ i.e. we can still follow the exact policy gradient

Compatible Function Approximation

- ▶ If the following two conditions are satisfied:
 1. Value function approximator is compatible to the policy

$$\nabla_w Q_w(s, a) = \nabla_\theta \log \pi_\theta(s, a)$$

- 2 Value function parameters w minimize the mean-squared error

$$\varepsilon = \mathbb{E}_{\pi_\theta} [(Q^{\pi_\theta}(s, a) - Q_w(s, a))^2]$$

- ▶ Then the policy gradient is exact,

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \ Q_w(s, a)]$$

- ▶ Remember:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \ Q^{\pi_\theta}(s, a)]$$

Proof

- ▶ If w is chosen to minimize mean-squared error, gradient of ε w.r.t. w must be zero,

$$\nabla_w \varepsilon = 0$$

$$\mathbb{E}_{\pi_\theta} [(Q^\theta(s, a) - Q_w(s, a)) \nabla_w Q_w(s, a)] = 0$$

$$\mathbb{E}_{\pi_\theta} [(Q^\theta(s, a) - Q_w(s, a)) \nabla_\theta \log \pi_\theta(s, a)] = 0$$

$$\mathbb{E}_{\pi_\theta} [Q^\theta(s, a) \nabla_\theta \log \pi_\theta(s, a)] = \mathbb{E}_{\pi_\theta} [Q_w(s, a) \nabla_\theta \log \pi_\theta(s, a)]$$

- ▶ So $Q_w(s, a)$ can be substituted directly into the policy gradient,

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q_w(s, a)]$$

Alternative Policy Gradient Directions

- Gradient ascent algorithms can follow any **ascent direction**
- A good ascent direction can significantly speed convergence
- Also, a policy can often be **reparametrized** without changing action probabilities
- For example, increasing score of all actions in a softmax policy
- The vanilla gradient is sensitive to these reparametrizations

Natural Policy Gradient

- ▶ The **natural policy gradient** is parameterization independent
- ▶ it finds ascent direction that is closest to vanilla gradient, when changing policy by a small, fixed amount

$$\nabla_{\theta}^{nat} \pi_{\theta}(s, a) = G_{\theta}^{-1} \nabla_{\theta} \pi_{\theta}(s, a)$$

- ▶ where G_{θ} is the **Fisher information matrix**

$$G_{\theta} = \mathbb{E}_{\pi_{\theta}} \left[\nabla_{\theta} \log \pi_{\theta}(s, a) \nabla_{\theta} \log \pi_{\theta}(s, a)^T \right]$$

Natural Actor-Critic

- ▶ Using compatible function approximation,

$$\nabla_w A_w(s, a) = \nabla_\theta \log \pi_\theta(s, a)$$

- ▶ The **natural policy gradient** simplifies,

$$\begin{aligned}\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) A^{\pi_\theta}(s, a)] \\ &= \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \nabla_\theta \log \pi_\theta(s, a)^T w] \\ &= G_\theta w\end{aligned}$$

$$\nabla_\theta^{nat} J(\theta) = w$$

Under linear model:

$$A^{\pi_\theta}(s, a) = \phi(s)^\top w$$

- ▶ i.e. update actor parameters in direction of critic parameters

Summary of Policy Gradient Algorithms

- ▶ The policy gradient has many equivalent forms

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) G_t] \quad \text{REINFORCE}$$

$$= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) Q^w(s, a)] \quad \text{Q Actor-Critic}$$

$$= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) A^w(s, a)] \quad \text{Advantage Actor-Critic}$$

$$= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) \delta] \quad \text{TD Actor-Critic}$$

$$= \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) \delta e] \quad \text{TD}(\lambda) \text{ Actor-Critic}$$

$$G_{\theta}^{-1} \nabla_{\theta} J(\theta) = w \quad \text{Natural Actor-Critic}$$

- ▶ Each leads a stochastic gradient ascent algorithm
- ▶ Critic uses **policy evaluation** (e.g. MC or TD learning) to estimate $Q^{\pi}(s, a)$, $A^{\pi}(s, a)$ or $V^{\pi}(s)$

Caption Generation with Visual Attention



A man riding a horse
in a field.

Caption Generation with Visual Attention

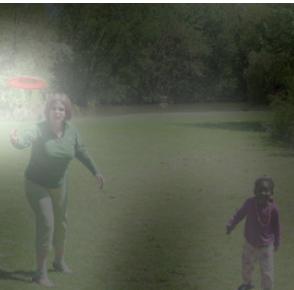


A man riding a horse
in a field.

Caption Generation with Visual Attention



A woman is throwing a frisbee in a park.



A dog is standing on a hardwood floor.



A stop sign is on a road with a mountain in the background.



A little girl sitting on a bed with a teddy bear.



A group of people sitting on a boat in the water.

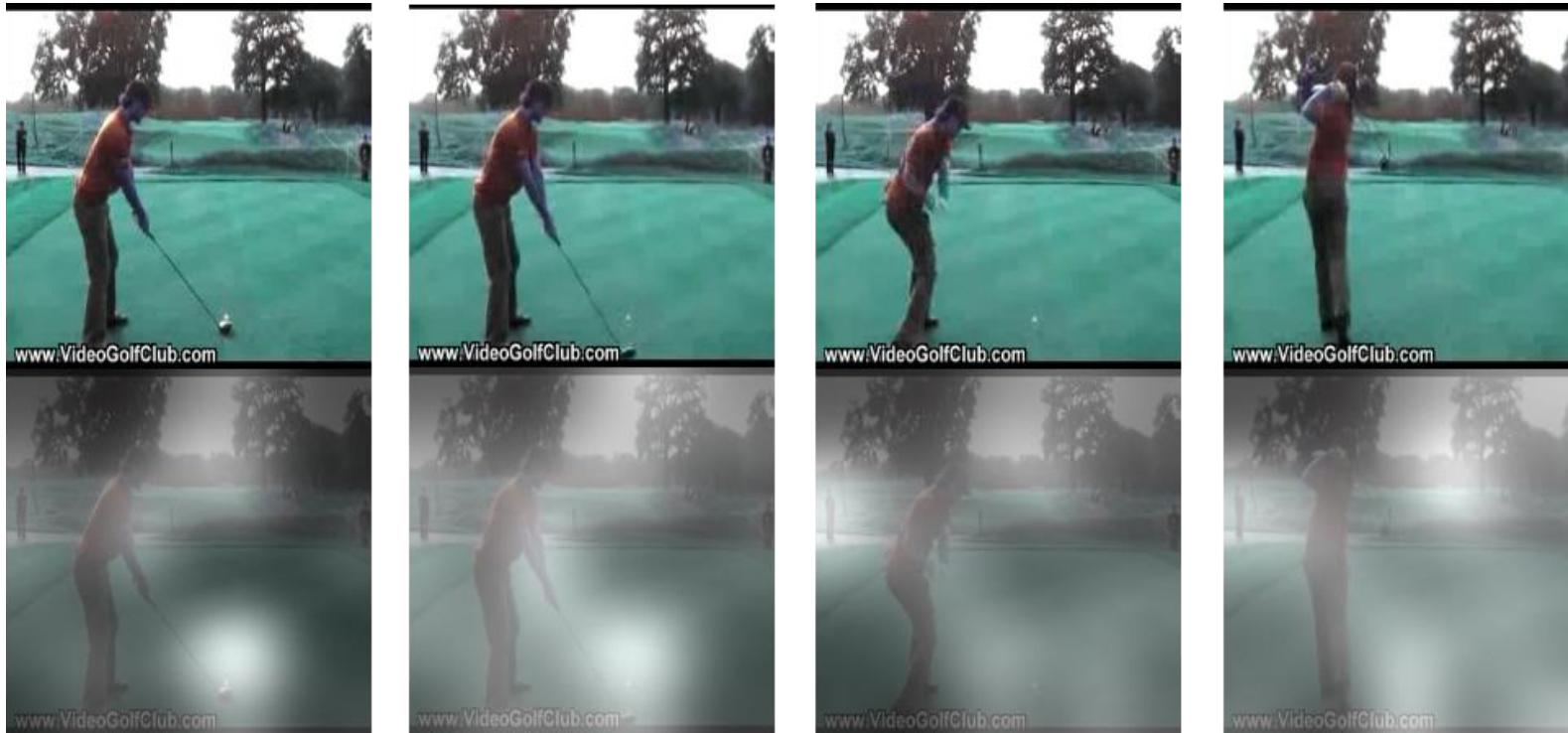


A giraffe standing in a forest with trees in the background.



Improving Action Recognition

- Consider performing action recognition in a video:



- Instead of processing each frame, we can process only a small piece of each frame.

Improving Action Recognition

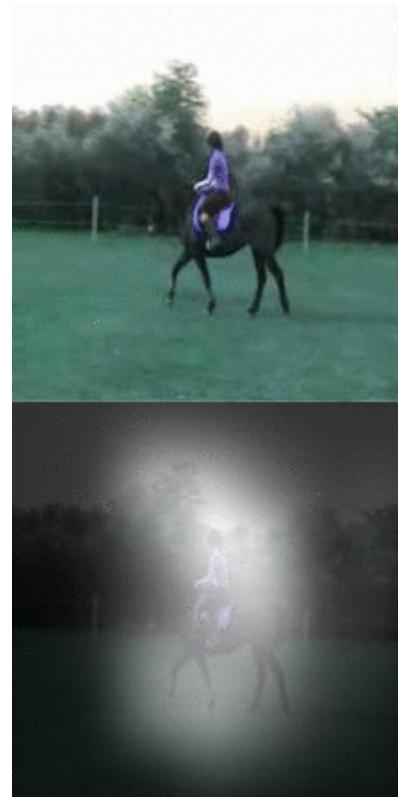
Cycling



Soccer juggling



Horse back riding



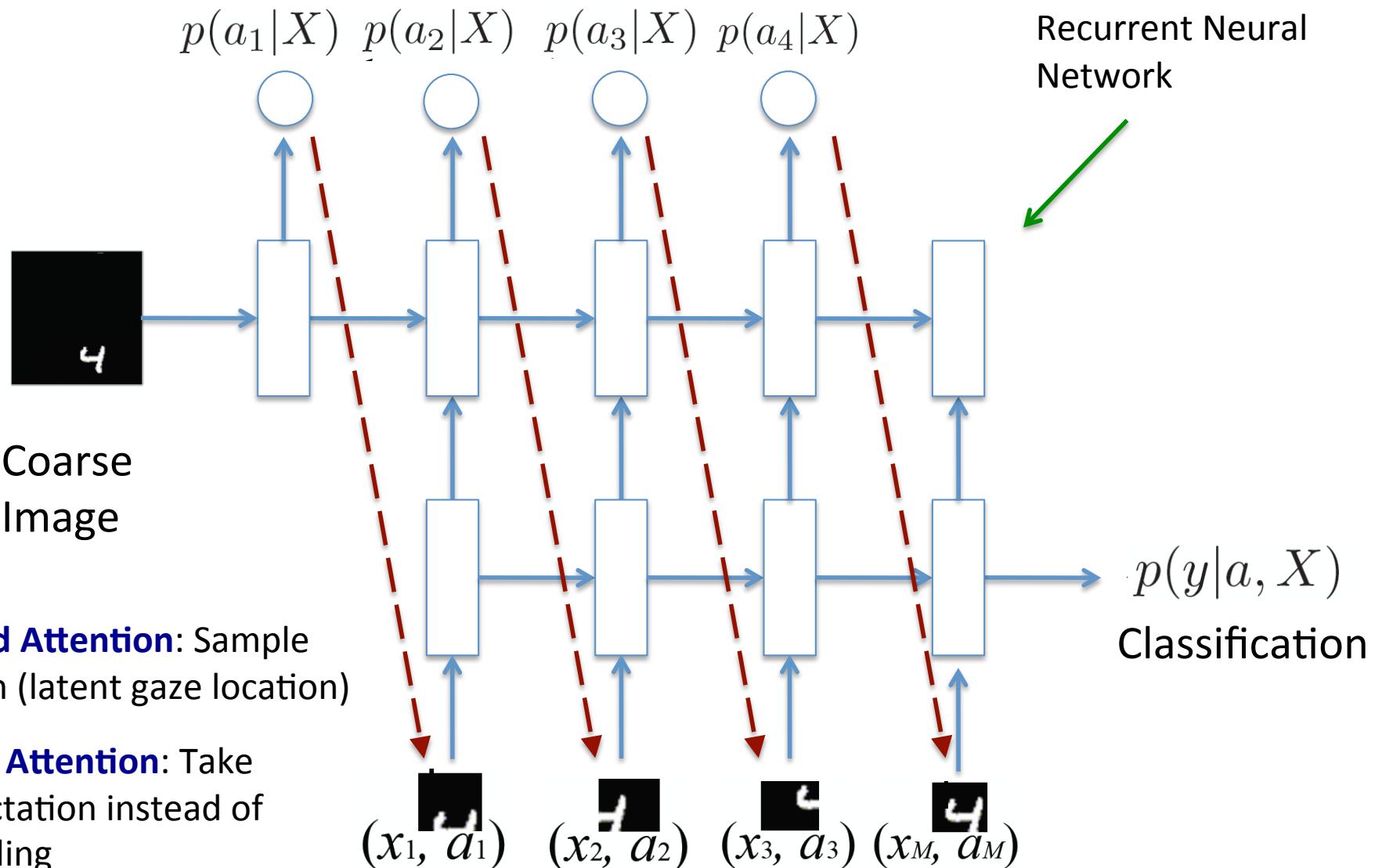
Basketball Shooting



Recurrent Attention Model

Sample action:

$$\tilde{a}_1 \sim p(a_1|X) \quad \tilde{a}_2 \sim p(a_2|X)$$



Model Setup

- We assume that we have a dataset with labels y for the supervised prediction task (e.g. object category).
- **Goal:** Learn an attention policy: The best locations to attend to are the ones which lead the model to predict the correct class.



Model Definition

- We aim to maximize the probability of correct class by marginalizing over the actions (or latent gaze locations):

$$\mathcal{LL} = \log p(y|X, W) = \log \sum_a p(a|X, W)p(y|a, X, W).$$

where

- W is the set of parameters of the recurrent network.
- a is a set of actions (latent gaze locations, scale).
- X : is the input (e.g. image, video frame).

For clarity of presentation, I will sometimes omit conditioning on W or X . It should be obvious from the context.

Variational Learning

- Previous approaches used variational lower bound:

$$\begin{aligned}\mathcal{LL} &= \log \sum_a p(a|X, W) p(y|a, X, W) \geq \\ &\sum_a q(a|y, X) \log p(y, a|X, W) + \mathcal{H}[q] = \mathcal{F}.\end{aligned}$$

- Here $q(a|y, X)$ is some approximation to posterior over the gaze locations.
- In the case where q is the prior, $q(a|y, X) = p(a|X, W)$, the variational bound becomes:

$$\mathcal{F} = \sum_a p(a|X, W) \log p(y|a, X, W).$$

Ba et.al., ICLR 2015
Mnih et.al., NIPS 2014

REINFORCE

$$\mathcal{F} = \sum_a p(a|X, W) \log p(y|a, X, W).$$

- Derivatives w.r.t model parameters:

$$\frac{\partial \mathcal{F}}{\partial W} = \sum_a p(a|X, W) \left[\frac{\partial \log p(y|a, X, W)}{\partial W} + \underbrace{\log p(y|a, X, W) \frac{\partial \log p(a|X, W)}{\partial W}}_{\text{Very bad term as it is unbounded.}} \right].$$

Very bad term as it is unbounded.
Introduces high variance in the estimator.

- Need to introduce heuristics (e.g. replacing this term with a 0/1 discrete indicator function, which leads to REINFORCE algorithm of Williams, 1992).

Ba et.al., ICLR 2015
Mnih et.al., NIPS 2014

REINFORCE

$$\mathcal{F} = \sum_a p(a|X, W) \log p(y|a, X, W).$$

- Derivatives w.r.t model parameters:

$$\frac{\partial \mathcal{F}}{\partial W} = \sum_a p(a|X, W) \left[\frac{\partial \log p(y|a, X, W)}{\partial W} + \log p(y|a, X, W) \frac{\partial \log p(a|X, W)}{\partial W} \right].$$

- The stochastic estimator of the gradient is given by:

$$\frac{\partial \mathcal{F}}{\partial W} \approx \frac{1}{M} \sum_{m=1}^M \left[\frac{\partial \log p(y|\tilde{a}^m, X, W)}{\partial W} + \log p(y|\tilde{a}^m, X, W) \frac{\partial \log p(\tilde{a}^m|X, W)}{\partial W} \right].$$

where we draw M actions from the prior: $\tilde{a}^m \sim p(a|X, W)$.

MNIST Attention Demo

- Actions contain:
 - Location: 2-d Gaussian latent variable
 - Scale: 3-way softmax over 3 different scales

