Investigating the Radial Stability of Boson Stars in $D \geq 4$ Dimensions

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Boson stars (BS) are a simple model of compact objects arising from the minimal coupling of a complex scalar field to gravity [1]. They are of interest in a wide variety of areas, including as dark matter candidates and as useful models in numerical relativity, allowing us to explore the physics of compact objects and large-mass-ratio binary systems in a computationally tractable way. A certain class of BS, which we call *solitonic* models, can additionally serve as a true black hole (BH) mimicker, possessing stable photon orbits and casting shadows very similar to that of an equal-mass BH [2]. This raises the intriguing possibility that a population of mergers involving horizonless ultracompact objects (UCOs) could be among the ostensible binary BH mergers detected by LIGO and other gravitational wave experiments.

A theoretical issue with this proposal is that UCOs may be unstable on astrophysical time scales, inevitably collapsing into ordinary BHs. Past numerical results suggested that this might be the case for our solitonic models [3], but we have recently found after a careful numerical investigation evidence that they might be stable after all. This involved a semi-analytical perturbative analysis, computing the fundamental frequencies of radial oscillations using methods similar to Ref. [4], as well as dynamical evolutions in spherical symmetry, axisymmetry, and full 3+1 evolution.

There have also been past investigations into higher-dimensional BSs. A recent investigation [5] found that the most basic class of BSs are always radially unstable in $D > 4$ dimensions. However, early numerical results in $D = 5$ dimensions suggest that our solitonic models may be radially stable in higher dimensions. If true, this would result in the first radially-stable class of higher-dimensional BSs in the literature, and potentially the first such class of higher-dimensional UCOs. This project would involve:

1. Extending the perturbative analysis already done in $D = 4$ to higher dimensions. There is an analytical component here, and experience with Maple/Mathematica may be helpful, though

not required. There is also a numerical component, potentially involving modifications to codes written in C++ and FORTRAN.

2. Dynamical evolutions of higher-dimensional BS models in spherical symmetry. The evolution code is written in C++ but works almost as-is in higher dimensions, so it's likely that few modifications would be required. This will be a laptop/desktop-scale computing job.

While you will likely learn a little about C++ in particular as you go, note that deep knowledge of C++/FORTRAN is neither expected nor required: most modifications will be of fairly straightforward kind like changing algebraic expressions to their higher-dimensional analogues, and you will mostly be changing parameters and running these codes. Experience with Python or a similar scripting language will be helpful for data analysis and visualization.

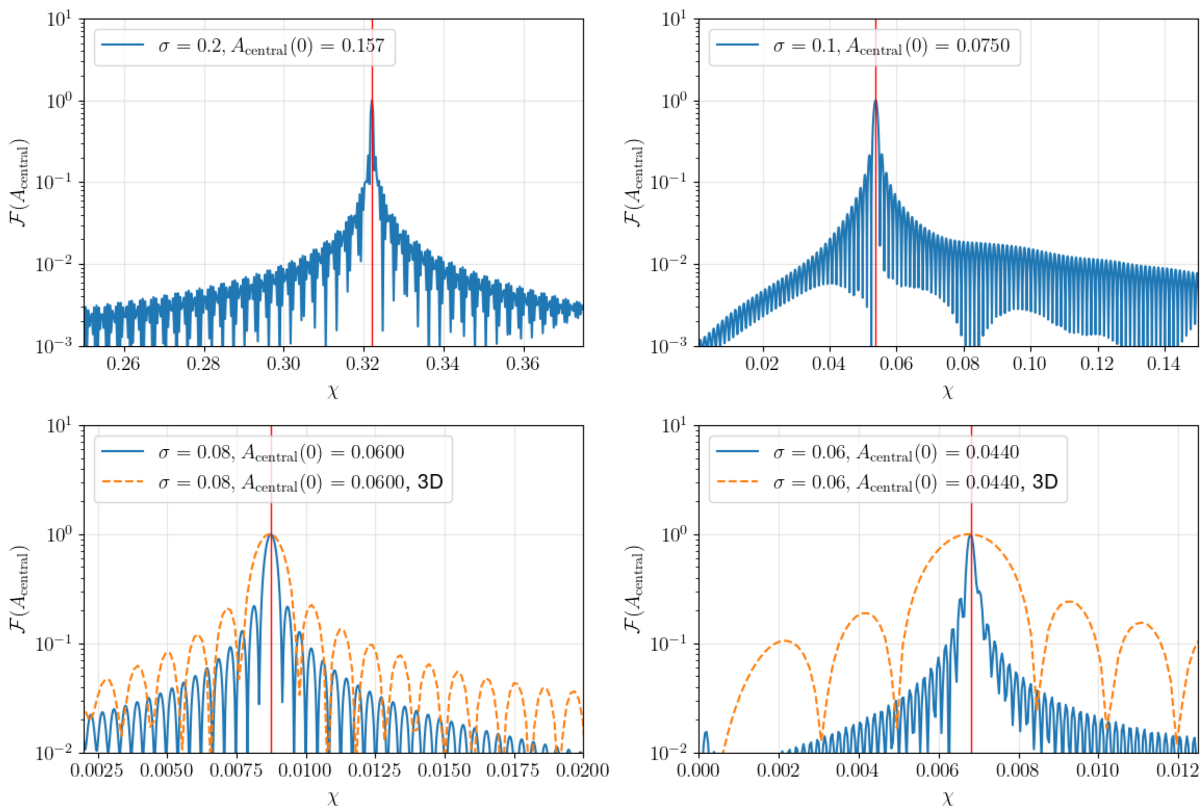

Figure 1: Power spectrum of central amplitude over time for a selection of $D = 4$ solitonic models, with the red line in each case corresponding to the χ_0 value computed in the previous section. Units on the y-axis are arbitrary, rescaled so the maximum is at 1. The two plots on the bottom correspond to the models with light rings considered in subsequent sections.

References

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[4] M. Gleiser and R. Watkins, "Gravitational stability of scalar matter," *Nuclear Physics B*, vol. 319, no. 3, pp. 733–746, 1989.

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