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## **Question 1**

From this problem, we can see that dynamic programming has a very important role in the question.

Subproblems: We can solve this problem by considering the subproblem  $\text{span}[i, j]$ : For every  $0 \leq i < k < j \leq n$ ,  $\text{span}[i, j] = \min(\text{span}[i, k] + \text{span}[k, j])$  while  $\text{span}[i, j] = (j - i)^2 + \max(A[i \dots j])$ .

Recurrence: For  $0 \leq i < j \leq n$ ,  $k = \text{argmin}(\text{span}[i, j])$ . We can then continue the algorithm for both spans  $[i, k]$  and  $[k, j]$  until  $k=i+1$  or  $j=k+1$ . And  $k$  is where we put pillars in the best solution.

Base case: no pillars, the total cost would be:  $\text{span}[0, n] = n^2 + \max(A[1 \dots n])$ .

Order of computation: when we get 2 subproblems spans  $[i, k]$  and  $[k, j]$ , we can solve the subproblems  $\text{span}[i, k]$  first.

Final answer: The maximum total cost is  $\text{span}[0, n]$  at last. And  $k$  which we get in every recurrence is where we decide to put pillars in the best solution.

Time complexity: As for each span  $[i, j]$  we only compute once using dynamic programming,  $0 \leq i < j \leq n$ . So, we need to compute  $n^2$  times.

Every time we need  $o(1)$  so the overall time complexity is  $o(n^2)$ .