

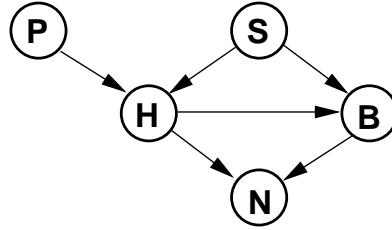
COMP9414: Artificial Intelligence

Solutions 5: Reasoning with Uncertainty

1. $P(A \wedge B) = P(A|B).P(B)$
 $P(B \wedge A) = P(B|A).P(A)$
 Now $P(A \wedge B) = P(B \wedge A)$ [why exactly?]
 Therefore $P(A|B).P(B) = P(B|A).P(A)$
 Rearranging gives Bayes' Rule $P(A|B) = \frac{P(B|A).P(A)}{P(B)}$ if $P(B) > 0$

2. $P(Mumps|\neg Fever) = \frac{P(\neg Fever|Mumps).P(Mumps)}{P(\neg Fever)}$
 $= \frac{(1-P(Fever|Mumps)).P(Mumps)}{1-P(Fever)}$
 $= \frac{(1-\frac{3}{4}).\frac{1}{15000}}{1-\frac{1}{1000}}$
 $= 0.0000167$

3. (i)



$$P(H \wedge B \wedge S \wedge P \wedge N) = P(H|P \wedge S).P(B|S \wedge H).P(S).P(P).P(N|H \wedge B)$$

- (ii) $P(H \wedge B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(B|H \wedge S).P(S).P(P).P(\neg N|H \wedge B)$
 $= 0.8 \times 0.4 \times 0.1 \times 0.2 \times 0.1$
 $= 0.00064$
 $P(H \wedge \neg B \wedge S \wedge P \wedge \neg N) = P(H|S \wedge P).P(\neg B|H \wedge S).P(S).P(P).P(\neg N|H \wedge \neg B)$
 $= 0.8 \times 0.6 \times 0.1 \times 0.2 \times 0.5$
 $= 0.00480$
 $P(H \wedge B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge B)$
 $= 0.4 \times 0.3 \times 0.9 \times 0.2 \times 0.1$
 $= 0.00216$
 $P(H \wedge \neg B \wedge \neg S \wedge P \wedge \neg N) = P(H|\neg S \wedge P).P(\neg B|H \wedge \neg S).P(\neg S).P(P).P(\neg N|H \wedge \neg B)$
 $= 0.4 \times 0.7 \times 0.9 \times 0.2 \times 0.5$
 $= 0.02520$
 $P(H \wedge B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge B)$
 $= 0.6 \times 0.4 \times 0.1 \times 0.8 \times 0.1$
 $= 0.00192$
 $P(H \wedge \neg B \wedge S \wedge \neg P \wedge \neg N) = P(H|S \wedge \neg P).P(\neg B|H \wedge S).P(S).P(\neg P).P(\neg N|H \wedge \neg B)$
 $= 0.6 \times 0.6 \times 0.1 \times 0.8 \times 0.5$
 $= 0.0144$
 $P(H \wedge B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge B)$
 $= 0.02 \times 0.3 \times 0.9 \times 0.8 \times 0.1$
 $= 0.000432$
 $P(H \wedge \neg B \wedge \neg S \wedge \neg P \wedge \neg N) = P(H|\neg S \wedge \neg P).P(\neg B|H \wedge \neg S).P(\neg S).P(\neg P).P(\neg N|H \wedge \neg B)$
 $= 0.02 \times 0.7 \times 0.9 \times 0.8 \times 0.5$
 $= 0.00504$

$$(iii) P(P|H \wedge \neg N) = \frac{P(P \wedge H \wedge \neg N)}{P(H \wedge \neg N)} = \frac{0.0328}{0.054592} = 0.60082$$

Note:

$$P(P \wedge H \wedge \neg N) = \sum_{b,s} P(H \wedge b \wedge s \wedge P \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520$$

$$P(H \wedge \neg N) = \sum_{b,s,p} P(H \wedge b \wedge s \wedge p \wedge \neg N) = 0.00064 + 0.00480 + 0.00216 + 0.02520 + 0.00192 + 0.0144 + 0.000432 + 0.00504 = 0.05452$$

We could also use direct inference, though this provides no advantage in this example. To do this, we need a conditional version of Bayes' Rule: $P(B|A, C) = \frac{P(A|B, C)P(B|C)}{P(A|C)}$

$$\text{Then } P(P|H, \neg N) = \frac{P(\neg N|H, P).P(P|H)}{P(\neg N|H)}$$

$$\begin{aligned} \text{Now } P(\neg N|H, P) &= P(\neg N|H, B, P).P(B|H, P) + P(\neg N|H, \neg B, P).P(\neg B|H, P) \\ &= P(\neg N|H, B).P(B|H, P) + P(\neg N|H, \neg B).P(\neg B|H, P) \\ &= P(\neg N|H, B).(P(B|H, S, P).P(S|H, P) + P(B|H, \neg S, P).P(\neg S|H, P)) + \\ &\quad P(\neg N|H, \neg B).(P(\neg B|H, S, P).P(S|H, P) + P(\neg B|H, \neg S, P).P(\neg S|H, P)) \\ &= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S|P) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S|P)) + \\ &\quad P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S|P) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S|P))] / P(H|P) \\ &= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S)) + \\ &\quad P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S))] / P(H|P) \end{aligned}$$

Also $P(P|H) = P(H|P).P(P)/P(H)$, so cancelling $P(H|P)$

$$\begin{aligned} P(\neg N|H, P).P(P|H) &= [P(\neg N|H, B).(P(B|H, S).P(H|S, P).P(S) + P(B|H, \neg S).P(H|\neg S, P).P(\neg S)) + \\ &\quad P(\neg N|H, \neg B).(P(\neg B|H, S).P(H|S, P).P(S) + P(\neg B|H, \neg S).P(H|\neg S, P).P(\neg S))] . P(P) / P(H) \end{aligned}$$

This generates four terms exactly as above, and similarly $P(\neg N|H)$ generates eight terms as above. The extra $P(H)$ cancel each other out.

4. Let A stand for "Alarm", B for "Burglary" and E for "Earthquake".

Then by Bayes' Rule

$$P(B|A) = P(A|B).P(B)/P(A) = (P(A|B \wedge E).P(E).P(B) + P(A|B \wedge \neg E).P(\neg E).P(B)) / P(A),$$

and as in lectures

$$\begin{aligned} P(A) &= P(A|B \wedge E).P(E).P(B) + P(A|B \wedge \neg E).P(\neg E).P(B) + P(A|\neg B \wedge E).P(E).P(\neg B) + \\ &\quad P(A|\neg B \wedge \neg E).P(\neg E).P(\neg B) \end{aligned}$$

So $P(B|A) = (0.95 \times 0.002 \times 0.001 + 0.94 \times 0.998 \times 0.001) / P(A)$ and

$$\begin{aligned} P(A) &= 0.95 \times 0.002 \times 0.001 + 0.94 \times 0.998 \times 0.001 + 0.29 \times 0.002 \times 0.999 + \\ &\quad 0.001 \times 0.998 \times 0.999 \end{aligned}$$

$$\text{Thus } P(B|A) = 0.00094002 / 0.002516442 = 0.3735512$$

Intuitively, the "true positives" (when there really is a burglary) account for roughly only 10/26 of the cases when the alarm is ringing (around 0.001 of the time), while the "false positives" account for 16/26 cases (6/26 when the alarm is ringing because of an earthquake, due to a false positive rate around 0.3 and prior of 0.002, so around 0.0006 of the time, and 10/26 when there is neither a burglary nor an earthquake, due to a false positive rate of 0.001 and a prior close to 1, so around 0.001 of the time). The rough calculation is $10/26 = 0.001 / (0.001 + 0.0006 + 0.001)$. That is, the false positives significantly outweigh the true positives in this scenario.

5. By the Chain Rule, $P(A \wedge B \wedge C) = P(B|A, C).P(A|C).P(C) = P(A|B, C).P(B|C).P(C)$. So, provided $P(C) \neq 0$ and $P(A|C) \neq 0$, the conditional version of Bayes' Rule follows.