Answer Set Programming[1]

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COMP4418

[1] Slides designed by Christoph Schwering

Non-Monotonic Reasoning

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Non-Monotonic Reasoning

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ASP at a Glance

- ASP $=$ Answer Set Programming
  - ASP $\neq$ Microsoft's Active Server Pages

- ASP belongs to logic programming
  - Like Prolog: $Head \leftarrow Body$ or $Head$:– $Body$.
  - Like Prolog: negation as failure
  - Unlike Prolog: $Head$ may be empty $\Rightarrow$ constraints

- Declarative programming
  - Unlike Prolog: no procedural control
  - Order has no impact on semantics

- ASP programs compute *models*
  - Unlike Prolog: not query-oriented, no resolution
  - Unlike Prolog: not Turing-complete
  - Tool for problems in NP and NP^{NP}

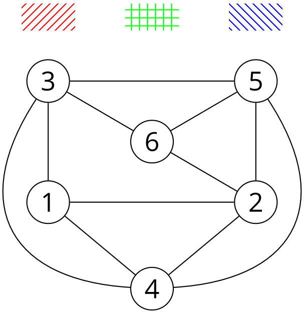
Motivation for ASP and this Lecture

- Very useful in practice!
  - Declarative problem solving
  - Very fast to write
  - Very fast to run
  - Few experts
- Interesting case study
  - Small, simple core language
  - Great expressivity by reduction to core language
- Knowing the theory is essential

Example: Graph Colouring

Definition: graph colouring problem

Input: graph with vertices V and edges $E \subseteq V \times V$, set of colors C.
Is there a mapping $m : V \rightarrow C$ with $m(x) \neq m(y)$ for all $(x, y) \in E$?



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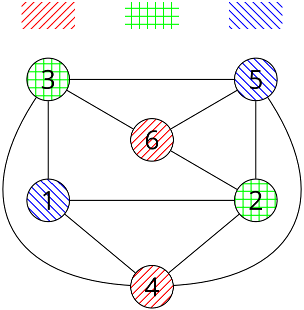

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- Graph Coulouring is NP-complete
  - NP: guess solution, verify in polynomial time
  - NP-complete: among hardest in NP
- Many applications:
  - Mapping (neighbouring countries to different colors)
  - Compilers (register allocation)
  - Scheduling (e.g., conflicting jobs to different time slots)
  - Allocation problems, Sudoku, ...

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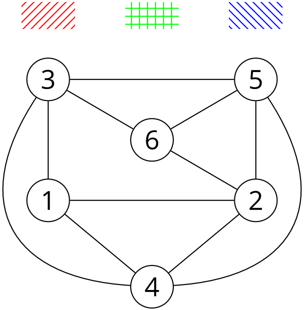

$c(r)\,.\ c(g)\,.\ c(b)\,.$
$v(1)\,.\ v(2)\,.\ v(3)\,.\ v(4)\,.\ v(5)\,.\ v(6)\,.$
$e(1,2)\,.\ e(1,3)\,.\ e(1,4)\,.$
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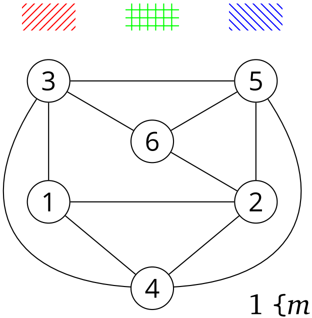


```
c(r). c(g). c(b).
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e(1,2). e(1,3). e(1,4).
e(2,4). e(2,5). e(2,6).
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`1 {m(X,C) : c(C)} 1 :- v(X).` guess mapping m

`:- e(X,Y), m(X,C), m(Y,C).` verify $m(X) \neq m(Y)$

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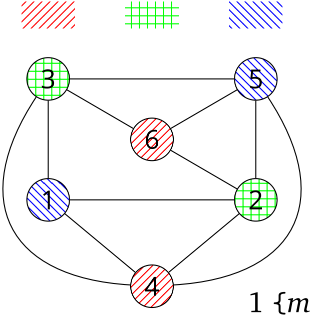

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Applications of ASP

- Automated product configuration
- Linux package manager
- Decision-support system for space shuttle
- Bioinformatics (diagnosis, inconsistency detection)
- General game playing

- Several implementations are available
- For this lecture: **Clingo**
  - `https://potassco.org/`
  - `https://github.com/potassco/clingo/releases/`
  - `https://potassco.org/clingo/run/`