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Question 1

1.1:

First, according to the question, for each positive integer x, the volume of the fridge that can be bought can be found as 3x *(2x+1) *(2^x), and the time complexity of each such calculation is O (1). It is worth noting that all three parts of the expression for volume with respect to x are increasing functions, so the expression for volume with respect to x is also increasing, i.e., the volume increases as x increases.

Next, the dichotomy can be used to try to find the smallest x. The specific steps are as follows. Clearly, x takes values in the range [1, V]. For each iteration of the domain of values [a, b], the volume is calculated by taking [b-a)/2] as x.If the resulting volume is less than V, the next interval is [[b-a)/2], b], if the resulting volume is equal to V, the minimum x value is [b-a)/2], if the resulting volume is greater than V, the next interval is [a, [b-a)/2]].

Thus, after at most log(V-1) iterations we can find the minimum x value, and each iteration requires a time complexity of O (1), so the time complexity of the whole algorithm is O (log(V-1)) = O (logV).

1.2:

According to the analysis in solution of 1.1, for each positive integer x, the volume of the fridge that can be bought can be found as 3x *(2x+1) *(2^x), and the time complexity of each such calculation is O (1). It is worth noting that all three parts of the expression for volume with respect to x are increasing functions, so the expression for volume with respect to x is also increasing, i.e., the volume increases as x increases.

The time complexity of O(logV) can only be obtained using the dichotomy method in 1.1. To reduce the time complexity to O (log (log V)), the dichotomy method needs to be further extended.

Similar to the dichotomous method, this time we also use a gradual reduction of the domain of values taken, starting from the initial domain of values [1, V] of x. For each iteration of the domain of values [a, b], the volume is calculated by taking [a+√(b-a)] as the checkpoint. If the resulting volume is less than V, the next interval is [[a+√(b-a)], b], if the resulting volume is equal to V, the minimum x value is [a+√(b-a)], if the resulting volume is greater than V, the next interval is [a, [a+√(b-a)]].

Therefore, according to such an algorithm, at each iteration, the range of values of the original n numbers will be reduced to the square root of n and no longer to n/2 as in the dichotomy method. Thus, the total number of iterations required in this problem is log(log(V-1)), each time requiring a volume calculation of time complexity O(1), so the overall time complexity of the algorithm is O(log(log(V-1)))=O(log(logV)).