Qiyao Zhou

Z5379852

Question 4

4.1：

Since k can be both positive and negative, the expression for the probability of Alice arriving k minutes before Bob is also discussed categorically：

When k is positive, we have P = $\sum_{1}^{n-k} a_t\, b_{t+k}$

When k is negative, we have P = $\sum_{1}^{n+k} a_{t-k}\, b_t$

4.2：

According to the question, the probability expression for Alice arriving earlier than Bob is P = $\sum_{1}^{n-1}(\sum_{t}^{n} b_x)\, a_t$

= $a_1(b_2+b_3+\ldots+b_n) + a_2(b_3+b_4+\ldots+b_n)+\ldots+a_{n-1}b_n$

= $b_n(a_1 + a_2 + \cdots + a_{n-1}) + b_{n-1}(a_1 + a_2 + \cdots + a_{n-2}) + b_2a_1$

= $b_n(1 - a_n) + b_{n-1}(1 - a_n - a_{n-1}) + \cdots + b_2(1 - a_n - a_{n-1} - \cdots - a_2)$

So, the algorithm for solving the probability required by the question can be obtained from this equation as follows:

1.Temporarily create an X to store the probability of Alice arriving earlier than Bob's Alice side and set the initial value to 1, with the initial result value first set to 0.

2.Iterate through the sequence of probabilities of Bob's arrival time from bn to b2, setting the probability of reaching bk, subtracting ak from X for each iteration, then multiplying the processed X with bk and adding it to the resulting value.

3.The result of the traversal is the desired probability.

In terms of time complexity, the algorithm only needs to traverse the probability sequence once in total, with only one subtraction and one multiplication operation each time, so the total time complexity is O(n), which clearly meets the O(nlogn) requirement of the question.

4.3:

According to the question, the probability expression for Alice and Bob arrive an even number of minutes apart is P = $\sum_{1}^{n}(\sum_{1\le t+2k\le n \ and \ k\neq 0} b_{t+2k})a_t$, Noting that t+2k and t always maintain the same parity, you can first iterate through the Bob arrival time probability sequence to find the odd time probability sum and the even time probability sum first, as follows:

1. Iterate through the sequence of Bob's arrival time probabilities and add the probabilities to P_{odd} if odd, otherwise add them to P_{even} to find the odd time probability and even time probability sum of Bob's arrival time.
2. Iterate through the sequence of Alice's arrival time probabilities again, noting that a_k is multiplied by (P_{odd}-b_k) if it is the probability of an odd time and added to the result, otherwise it is multiplied by (P_{even}-b_k) and added to the result.

3. The result of the traversal is the desired probability.

   In terms of time complexity, each of the two iterations takes O(n), and the two iterations are independent of each other, so the total time complexity is also O(n).