Social Choice

- Consider the following profile with 100 voters and 5 candidates. E.g. there are 33 voters with preference $A \succ B \succ C \succ D \succ E$.

33	16	3	8	18	22
A	B	C	C	D	E
B	D	D	E	E	C
C	C	B	B	C	B
D	E	A	D	B	D
E	A	E	A	A	A

  1. Which candidate is elected under the plurality rule? **A**
  2. Which candidate is elected under the Borda rule? **B**
  3. Who is the Condorcet winner (if one exists)? **C**
  4. Which candidate is elected under Plurality with runoff? **E**

- The Gibbard-Sattherthwaite Theorem is a famous impossibility result concerning the strategyproofness of voting rules. Explain why the voting rule that elects the candidate whose name comes first alphabetically is both strategyproof and non-dictatorial, even when there are three or more alternatives. **not surjective**

- For each of the following axioms, describe a voting rule which does **not** satisfy it, and explain why your proposed rule does not satisfy the axiom.

  – Anonymity **Select first preference of first alphabetically ordered voter**
  – Neutrality **First alphabetically ordered candidate**
  – Monotonicity **Scoring vector (1,2,3,..)**
  – Pareto Optimality **Select outcome with second highest Borda score**
  – Independence of Irrelevant Alternatives **First alphabetically ordered candidate**
  – Non-dictatorial **Random dictator**
  – Condorcet-extension **Borda**
  – Strategyproofness **Almost any non-dictatorial rule**

- Prove or disprove the following: **ALL FALSE**

  1. The Borda rule is strategyproof.
  2. The Condorcet rule is strategyproof.
  3. The Condorcet winner always exists.

- Suppose you are holding an election to choose a single candidate and need to decide on a voting rule. For each of the scenarios below, what voting rule would you use and why?

  1. There are two candidates to choose from. **Plurality, May's theorem**

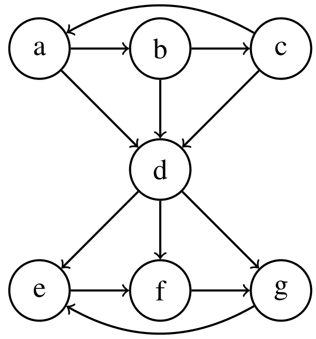

Figure 1: Tournament

2. There are an odd number of voters, each with single-peaked preferences. **Black's Median voter theorem**
3. No voter can achieve a better outcome by lying about their true preferences on their ballot. **Random dictator, strategyproof**

- In the tournament in Figure 1, all the arcs missing from the figure are downward arcs. For the tournament in Figure 1, find the

  (a) the uncovered set **a,b,c**

  (b) the top cycle **a,b,c**

  (c) the set of Copeland winners **a,b,c**

  (d) the set of Banks winners **a,b,c**

  (e) the set of Condorcet winners **None**

  and give arguments for your answers.

- Consider the following preference profile of voters.

$$1 : b \succ a \succ c \succ d$$
$$2 : c \succ a \succ b \succ d$$
$$3 : d \succ b \succ a \succ c$$

  Prove or disprove that the preference profile is single-peaked.

  Prove or disprove that a Condorcet winner exists for the preference profile.

  TRUE and TRUE

- Consider an approval-based committee voting problem in which each voter approves of a set of alternatives and target k number of alternatives are to be selected. Design a program to check if there exists a committee of k alternatives such that each voter has at least one approved alternative in the committee.