1(a)

(i) $T=\sum_{i=1}^{n} a_i X_i$

$ET = E(\sum_{i=1}^{n} a_i X_i) = \sum_{i=1}^{n} a_i E(X_i)$

$E(X_i)=\mu \Rightarrow ET = \sum_{i=1}^{n} a_i \cdot \mu$

to satisfy $ET=\mu \Rightarrow \sum_{i=1}^{n} a_i = 1$

(ii) $var(T) = \sum_{i=1}^{n} a_i^2 var(X_i) = \sum_{i=1}^{n} a_i^2 \sigma^2$

create $F = \sigma^2 \sum_{i=1}^{n} a_i^2 + \lambda(\sum_{i=1}^{n} a_i - 1)$

$\frac{\partial F}{\partial a_i} = 2\sigma^2 a_i + \lambda = 0 \Rightarrow a_i = -\frac{\lambda}{2\sigma^2}$

$\Rightarrow n a_i = 1 \quad \therefore a_i = \frac{1}{n}$

This makes the estimator T behave as a mean which consistent with my guess.

(iii) $var(T) = \sum_{i=1}^{n} a_i^2 var(X_i) = \sum_{i=1}^{n} (\frac{1+b}{n})^2 \sigma^2$

$= \frac{\sigma^2}{n}(1+b)^2$

$\frac{\partial var(T)}{\partial b} = 0 \Rightarrow b = -1$

$\Rightarrow T = 0$

has less MSE but high bias

using this is unreasonable

(iv) $MSE(T) = 0$

$MSE(X) = \frac{\mu^2}{n}$

Can't use this choice in practice

(b)

(i) $[bias(\hat{m}(x_0))]^2 = (f(x_0) - y_{x_0})^2$

$= (f(x_0) - \frac{1}{K}\sum_{i=1}^{K} y_i)^2$

$= (f(x_0) - \frac{1}{K}\sum_{i=1}^{K}(f(z_i) + \varepsilon))^2$

$= (f(x_0) - \frac{1}{K}\sum_{i=1}^{K} f(z_i) - \frac{1}{K}\sum_{i=1}^{K}\varepsilon)^2 \Leftarrow mean(\varepsilon)=0$

$= (f(x_0) - \frac{1}{K}\sum_{i=1}^{K} f(z_i))^2$

(ii) $var(\hat{m}(x_0)) = E((\hat{m}(x_0) - f(x_0))^2)$

$= E((\frac{1}{K}\sum_{i=1}^{K} y_i - f(x_0))^2)$

$= E((\frac{1}{K}\sum_{i=1}^{K} f(z_i) - f(x_0) + \frac{1}{K}\sum_{i=1}^{K}\varepsilon)^2) \Leftarrow mean(\varepsilon)=0$

$= E((\frac{1}{K}\sum_{i=1}^{K} f(z_i) - f(x_0))^2)$

(iii) $MSE = bias^2(\hat{m}(x_0)) + var(\hat{m}(x_0)) + \varepsilon^2$

$= (f(x_0) - \frac{1}{K}\sum_{i=1}^{K} f(z_i))^2 + E((\frac{1}{K}\sum_{i=1}^{K} f(z_i) - f(x_0))^2) + \varepsilon^2$

when $k=1$ it is low bias and high variance

when $k=\infty$ it is high bias and low variance

The larger the k, the ~~lower~~ higher the bias

There must be an k that makes the bias and variance the lowest and when greater or less than this optimal k, it cause the bias and variance to become larger.