Cooperative Games

- Consider the coalitional game (N, v) such that $N = \{1,2,3\}$ and v is defined as follows:

S	$\{1,2\}$	$\{1,3\}$	$\{1,2,3\}$	$\emptyset$	$\{2\}$	$\{3\}$	$\{1\}$	$\{2,3\}$
$v(S)$	1	1	1	0	0	0	0	0

  For this game, compute the nucleolus, Shapley value and Banzhaf indices.

  Nucleolus: 1,0,0

  Shapley value: 4/6, 1/6, 1/6

  Banzhaf value: 3/5, 1/5, 1/5

- Consider the coalitional game (N, v) such that $N = \{1,2,3\}$ and v is defined as follows:

S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	0	1	0	1	4	3	5	8

  Compute the Shapley value and explain how you computed it. **14/6, 17/6, 17/6**

- Consider the coalitional game (N, v) such that $N = \{1,2,3\}$ and v is defined as follows:

S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	0	0	0	0	500	750	500	1000

  For this game, compute the nucleolus and explain how you computed it. **1250/3 500/3 1250/3**

  Also compute the Shapley value and explain how you computed it. **375 250 375**

- Consider the coalitional game (N, v) such that $N = \{1,2,3\}$ and v is defined as follows:

S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	0	4	3	2	4	3	2	12

  For this game, compute the nucleolus and explain how you computed it. **5 4 3**

  Also compute the Shapley value and explain how you computed it. **5 4 3**

- Consider a weighted voting game in which the quota is 12 and the countries have the following weights:
  - France: 4
  - Germany: 4
  - Italy: 4
  - Belgium: 2
  - Netherlands: 2
  - Luxembourg: 1

  What is the Banzhaf and Shapley value of Luxembourg? **0, 0**