COMP9414: Artificial Intelligence

Lecture 2b: Informed Search

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Informed (Heuristic) Search

- Uninformed methods of search are capable of systematically exploring the state space in finding a goal state
- However, uninformed search methods are very inefficient
- With the aid of problem-specific knowledge, informed methods of search are more efficient
- All implemented using a priority queue to store frontier nodes

This Lecture

- Heuristics
- Informed Search Methods
  - Best-First Search
  - Greedy Search
  - A^* Search
  - Iterative Deepening A^* Search

Heuristics

- Heuristics are "rules of thumb"
- Heuristics are criteria, methods or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal. "Heuristics" (Pearl 1984)
- Can make use of heuristics in deciding which is the most "promising" path to take during search
- In search, heuristic must be an underestimate of actual cost to get from current node to any goal – an admissible heuristic
- Denoted $h(n)$; $h(n) = 0$ whenever n is a goal node

Heuristics – Example

- 8-Puzzle – number of tiles out of place

2	8	3		1	2	3
1	6	4	$\longrightarrow$	8		4
	7	5		7	6	5

- Therefore $h(n) = 5$

Heuristics – Example

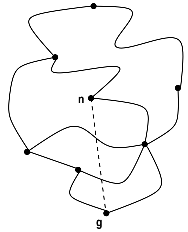
- 8-Puzzle – Manhattan distance (distance tile is out of place)

2	8	3		1	2	3
1	6	4	$\longrightarrow$	8		4
	7	5		7	6	5

- Therefore $h(n) = 1 + 1 + 0 + 0 + 0 + 1 + 1 + 2 = 6$

Heuristics – Example

- Another common heuristic is the straight-line distance ("as the crow flies") from node to goal

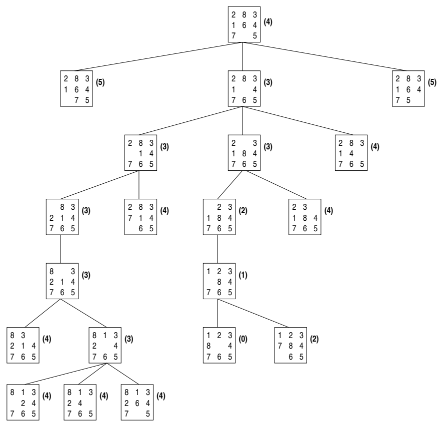


- Therefore $h(n) =$ distance from n to g

Greedy Search

- **Idea:** Expand node with the smallest estimated cost to reach the goal
- Use heuristic function $h(n)$ to order nodes on frontier, i.e. choose node for expansion with lowest $h(n)$
- Analysis
  - Similar to depth-first search; tends to follow single path to goal
  - Not optimal, incomplete
  - Time $O(b^m)$; Space $O(b^m)$
  - However, good heuristic can reduce time and space complexity significantly

Greedy Search



A* Search

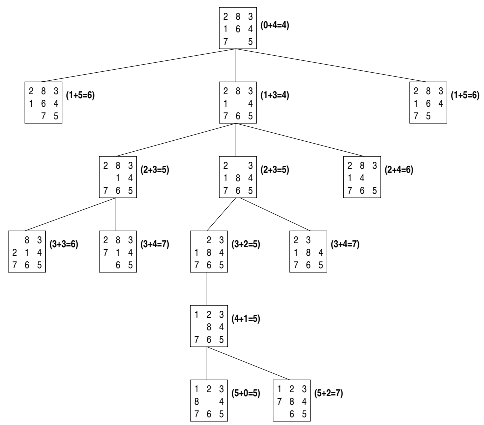
- **Idea:** Use both cost of path generated and estimate to goal to order nodes on the frontier
- $g(n)$ = cost of path from start to n; $h(n)$ = estimate from n to goal
- Order priority queue using function $f(n) = g(n) + h(n)$
- $f(n)$ is the estimated cost of the cheapest solution extending this path
- Expand node from frontier with smallest f-value
- Essentially combines uniform-cost search and greedy search

A* Algorithm

OPEN – nodes on frontier; CLOSED – expanded nodes

OPEN = $\{\langle s_0, nil\rangle\}$ where s_0 is the initial state
while OPEN is not empty
  remove from OPEN a node $n = \langle s, p\rangle$ with minimal $f(n)$
  place n on CLOSED
  if s is a goal state **return** success (with path p)
  for each edge e connecting s and a successor state s' with cost c
    if $\langle s', p'\rangle$ is on CLOSED **then if** $cost(p \oplus e) = cost(p) + c < cost(p')$
      then remove $\langle s', p'\rangle$ from CLOSED and put $\langle s', p \oplus e\rangle$ on OPEN
    else if $\langle s', p'\rangle$ is on OPEN **then if** $cost(p \oplus e) = cost(p) + c < cost(p')$
      then replace $\langle s', p'\rangle$ by $\langle s', p \oplus e\rangle$ on OPEN
    else if $\langle s', p''\rangle$ is not on OPEN **then** put $\langle s', p \oplus e\rangle$ on OPEN
return failure

A* Search



A* Search – Analysis

Subject to conditions on next slide:

- Optimal (and optimally efficient)
- Complete
- Number of nodes searched (and stored) still exponential in worst case
  - Unless the error in the heuristic grows no faster than the log of the actual path cost $h^*(n)$ of reaching a goal from n:
    $$|h(n) - h^*(n)| \leq O(\log h^*(n))$$
  - Which almost never happens: for many heuristics, this error is at least proportional to the path cost

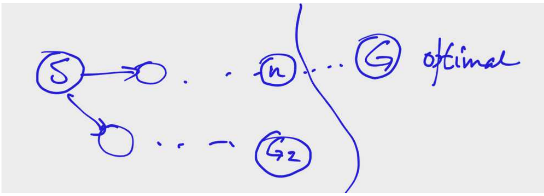
A* Search – Optimality

- Conditions on state space graph
  - Each node has a finite number of successors
  - Every arc in the graph has cost greater than some $\varepsilon > 0$
- Condition on heuristic function $h(n)$: admissibility
  - For every node n, the heuristic never overestimates the actual cost $h^*(n)$ of reaching a goal from n, i.e. $h(n) \leq h^*(n)$

A* Search – Optimal Efficiency

- A* is optimally efficient for a given heuristic: of the optimal search algorithms that expand search paths from the root node, there is no other optimal algorithm that expands fewer nodes in finding a solution
- Monotonic heuristic – along any path, the f-cost never decreases
  - Follows from triangle inequality
    $$h(n) \leq cost(n, n') + h(n')$$
- Common property of admissible heuristics
  - If this holds, don't need CLOSED set – big saving
  - If not, the path cost for n' connected to n can be set to:
    (Pathmax Equation) $\quad f(n') = \max(f(n),\ g(n') + h(n'))$

Proof of the Optimality of the A* Algorithm



G: optimal goal node; G_2: another goal node selected by A*
n: node on frontier on optimal path to G; $h^*(n)$: true cost to goal from n
Suppose A* chose G_2 rather than n
Then: $g(G_2) = f(G_2) \leq f(n)$ since G_2 is a goal node and A* chose G_2
$= g(n) + h(n)$ by definition
$\leq g(n) + h^*(n)$ by admissibility
$\leq g(G)$ since G is a goal node on a path from n
and that path is an optimal path to G

This means G_2 is also optimal, and hence, so is any node returned by A*

Heuristics – Properties

- h_2 dominates h_1 iff $h_2(n) \geq h_1(n)$ for any node n
- A* expands fewer nodes on average using h_2 than h_1
  - Every node for which $f(n) < f^*$ is expanded
    So n is expanded whenever $h(n) < f^* - g(n)$
    So any node expanded using h_2 is expanded using h_1
  - Always better to use an (admissible) heuristic with higher values
- Suppose there are a number of admissible heuristics for a problem $h_1(n),\ h_2(n),\ \ldots,\ h_k(n)$
  - Then $max_{i \leq k} h_i(n)$ is a more powerful admissible heuristic
  - Therefore can design a range of heuristics for special cases

Generating Heuristics

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then #tiles-out-of-place gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then Manhattan distance gives the shortest solution
- For TSP: let path be any structure that connects all cities
  $\Longrightarrow$ minimum spanning tree heuristic

Iterative Deepening A* Search

- IDA* performs repeated depth-bounded depth-first searches as in Iterative Deepening, however the bound is based on $f(n)$
- Start by using f-value of initial state
- If search ends without finding a solution, repeat with new bound of minimum f-value exceeding previous bound
- IDA* is optimal and complete with the same provisos as A*
- Due to depth-first search, space complexity = $O(\frac{bf^*}{\delta})$ (where δ = smallest operator cost and f^* = optimal solution cost) – often $O(bd)$ is a reasonable approximation
- Another variant – SMA* (Simplified Memory-Bounded A*) – makes full use of memory to avoid expanding previously expanded nodes

Conclusion

- Informed search makes use of problem-specific knowledge to guide progress of search
- This can lead to a significant improvement in performance
- Much research has gone into admissible heuristics
- Even on the automatic generation of admissible heuristics