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Power Control and Resource Allocation for Multi-cell OFDM Networks

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***Abstract*—In this paper, we consider the problem of minimizing the total transmit power with power control and resource allocation in OFDM networks where mutual interference exists among cells. The signal-to-interference-and-noise-ratio (SINR) relation is interpreted by the load and power coupling model, where every resource is available for each user with a probability. These probability variables can be also named as load vector. To solve this problem, we develop one low-complexity distributed power control and resource allocation algorithm. Specially, each BS updates its load vector, power vector, rate vector information and broadcasts it to all other BSs. Having collected all the information, each BS calculates its own load vector, power vector and rate vector by solving a convex program and a linear program. Compared with the existing optimization algorithm, where each resource is allocated to at most one user in a period of time, our algorithm has lower computational complexity. Numerical results verify the effectiveness and convergence of our proposed algorithm.**

***Index Terms*—Energy minimization, OFDM networks, load coupling, power control, resource allocation.**

I. Introduction

Global mobile data traffic will increase nearly tenfold between 2014 and 2019 [1], fueled mainly by multimedia mobile applications. This increase will result in rapidly growing energy consumption [2]. In recent years, the energy consumed by information and communications technology (ICT) systems can take a portion of more than 3% of the world-wide electric power consumption [3]. The problem of energy minimization for communication system is very critical and there have been many works on green communication [4], [5], [6], [7]. In this paper, we are interested in the minimization of sum power consumption for multi-cell multi-user downlink OFDM systems with mutual interference.

For a multi-cell system where each subcarrier is taken by at most one user, [4] showed that Lagrange dual decomposition method can be used to find the optimal solution to sum power minimization problem with large number of subcarriers. For distributed OFDMA femtocell networks, [5] introduced a simple self-organization rule, based on minimizing cell transmit power. In [8], resource allocation problem in OFDM-based cognitive radio networks was formulated as mixed integer programming. This kind of resource allocation problems are also investigated in other aspects, such as relay networks [9] and wireless virtualization networks [10].

On other hand, [11] and [12] considered the load coupling systems, where each subcarrier can be used by a user with a probability. In the load and power coupling model, the load of a cell is defined as the average utilization level of RBs and the load coupling equation has been shown to give a good approximation for a multi-cell systems. Besides, the theoretical properties of load coupling equation model with fixed power were provided and the solution of load coupling equation can be obtained by solving an equivalent convex problem [13]. The load coupling model has been used in many applications, such as load balancing and data offloading in a heterogeneous wireless network [14], location planning of small cells.

The program problem in OFDM systems, where each subcarrier is taken by at most one user, is always a mixed integer program. Obtaining the optimal solution of this mixed integer program needs numerous computation. Thus, systems with load coupling model always has a low complexity. Previous work [15] using the load coupling model considers the minimization of sum power in cellular networks, where the channel gain between a user and a BS is the same for every subcarrier. The authors studied the property of load and power coupling model, and provided an iterative power allocation algorithm. However, the algorithm is not suitable for systems with frequency selective channel.

In this paper, we aim to minimize the total power consumption of all BSs with load and power coupling among cells, subject to user rate demand constraints. Comparing with existing works, the main contributions of this paper are summarized as follows:

- Differen from [15], where the channel gain between one user and a base station is the same for every subcarrier, the channel gain on different subcarrier in this paper is modeled as different value. We also extend the load and power coupling model for this situation.
- We provide a low-complexity distributed algorithm to solve the total power minimization problem, and prove the convergence to this distributed algorithm. Moreover, the implementation and complexity analysis of our proposed algorithm is also presented.
- The distributed power minimization problem is convex. It is proved that this convex problem can be solved by solving one convex problem with fewer variables and one linear program.

This paper is organized as follows. In Section II, we introduce the system model and provide the total power minimization problem formulation. Section III proposes a distributed power control and resource allocation algorithm.

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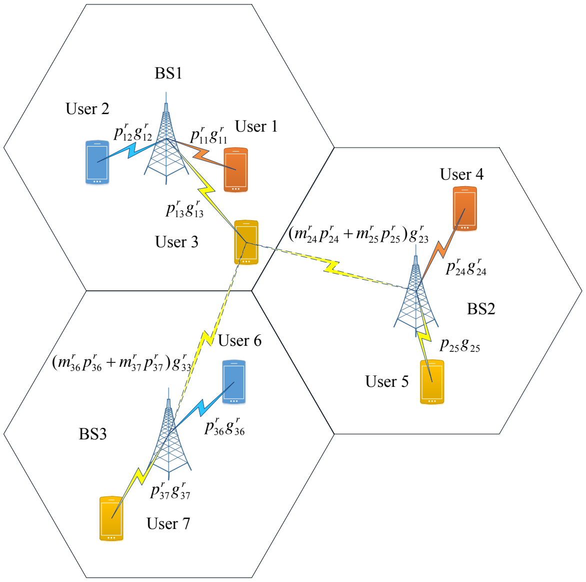

Fig. 1. System model.

Some numerical results are displayed in Section IV and conclusions are finally drawn in Section V.

II. System Model

Consider a multi-cell OFDM network consisting of N base stations (BSs) denoted as the set $\mathcal{N} = \{1, 2, \cdots, N\}$, as shown in Fig.1. Each BS $i \in \mathcal{N}$ serves one unique group of users, denoted by the set $\mathcal{J}_i = \{J_{i-1}+1, J_{i-1}+2, \cdots, J_i\}$, where $J_0 = 0$, $J_i = \sum_{l=1}^{i} |\mathcal{J}_l|$, $|\cdot|$ is the cardinality of a set and $|\mathcal{J}_i| \geq 1$. We focus on the downlink scenarios where each BS transmits data to different users with different power and different number of resource blocks (RBs). Each BS is assumed to have R RBs, denoted by $\mathcal{R} = \{1, 2, \cdots, R\}$. On RB $r \in \mathcal{R}$, BS i transmits with power p_{ij}^r to user $j \in \mathcal{J}_i$. For notational convenience, we collect all transmit power of BS i as vector $\boldsymbol{p}_i = (p_{i(J_{i-1}+1)}^1, \cdots, p_{iJ_i}^1, \cdots, p_{i(J_{i-1}+1)}^R, \cdots, p_{iJ_i}^R)$, and denote $\boldsymbol{p} = (\boldsymbol{p}_1, \cdots, \boldsymbol{p}_N)^T$.

In our considered system each user can use all the R RBs, but users in the same cell can not use the same RBs at the same time. To show this, we introduce load variable $0 \leq m_{ij}^r \leq 1$, which is regarded as the fraction of RB r allocated to user $j \in \mathcal{J}_i$ in cell i by time division. Then the load of BS i on RB r can be calculated by the summation of load for serving every user $j \in \mathcal{J}_i$ on RB r, i.e. $\sum_{j \in \mathcal{J}_i} m_{ij}^r \leq 1$. Denote $\boldsymbol{m}_i = (m_{i(J_{i-1}+1)}^1, \cdots, m_{iJ_i}^1, \cdots, m_{i(J_{i-1}+1)}^R, \cdots, m_{iJ_i}^R)$ and $\boldsymbol{m} = (\boldsymbol{m}_1, \cdots, \boldsymbol{m}_N)^T$.

The load and power coupling is considered in our multi-cell OFDM network. The SINR model of user j in cell i on RB r can be formulated as [11], [12], [13],

$$\eta_{ij}^r = \frac{p_{ij}^r g_{ij}^r}{\sum_{k \in \mathcal{N} \setminus \{i\}} \sum_{l \in \mathcal{J}_k} m_{kl}^r p_{kl}^r g_{kj}^r + \sigma^2}, \tag{1}$$

where g_{ij}^r is the channel gain from BS i to user j on RB r and σ^2 represents the noise power. Intuitively, load proportion m_{kl}^r can be interpreted as the probability of receiving interference from BS k on RB r for meeting the rate demand of user $l \in \mathcal{J}_k$. Then, the achievable rate of user $j \in \mathcal{J}_i$ on RB r can be written as

$$t_{ij}^r = m_{ij}^r B \log_2(1 + \eta_{ij}^r), \forall i \in \mathcal{N}, j \in \mathcal{J}_i, r \in \mathcal{R}. \tag{2}$$

Denote vector $\boldsymbol{t}_i = (t_{i(J_{i-1}+1)}^1, \cdots, t_{iJ_i}^1, \cdots, t_{i(J_{i-1}+1)}^R, \cdots, t_{iJ_i}^R)$ and $\boldsymbol{t} = (t_1, \cdots, t_N)^T$.

Our aim is to minimize the total power minimization of all BSs on all RBs, subject to the constraints of minimal rate for every user, maximal transmit power of BS and load inequalities. The formulation is given below.

$$\min_{\boldsymbol{0} \leq \boldsymbol{m}, \boldsymbol{0} \leq \boldsymbol{p}, \boldsymbol{0} \leq \boldsymbol{t}} \quad \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{J}_i} m_{ij}^r p_{ij}^r \tag{3a}$$

$$\text{s.t.} \quad t_{ij}^r = m_{ij}^r B \log_2(1 + \eta_{ij}^r), \forall i, j, r \tag{3b}$$

$$\sum_{r \in \mathcal{R}} t_{ij}^r \geq d_{ij}, \forall i, j \tag{3c}$$

$$\sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{J}_i} m_{ij}^r p_{ij}^r \leq p_i^{\max}, \forall i \tag{3d}$$

$$\sum_{j \in \mathcal{J}_i} m_{ij}^r \leq 1, \forall i, r \tag{3e}$$

where d_{ij} is the minimal rate of user j in cell i, and $p_i^{\max}$ is the maximal transmit power of BS i.

Note that problem (3) is non-convex, it is difficult to obtain the globally optimal solution even by the centralized algorithm. In the following, we devise a novel distributed algorithm to deal with this problem with much lower computational complexity.

III. Distributed Algorithm

In this section, we investigate the optimality of load and rate for program (3). Then, we provide a distributed power control and resource allocation algorithm to solve program (3). Finally, the detailed implementation and complexity analysis of the distributed algorithm is presented.

A. Optimal Condition

We establish the optimal conditions for load vector $\boldsymbol{m}$ and rate vector $\boldsymbol{t}$.

Theorem 1: If program problem (3) is feasible, the optimal solution to minimize the total transmit power is such that the rate vector reaches the minimal rate constraints $\boldsymbol{t}^* = \boldsymbol{d}$ as well as load vector satisfies $\sum_{j \in \mathcal{J}_i} m_{ij}^{r*} = 1$, $\forall i \in \mathcal{N}, r \in \mathcal{R}$.

Theorem 1 can be proved by using the same method in [11, Lemma 2]. Thus, the proof of theorem 1 is omitted.

B. Distributed Power Control and Resource Allocation Algorithm

By the definition of load variable, the total power of BS i on RB r can be calculated as $q_i^r = \sum_{j \in \mathcal{J}_i} m_{ij}^r p_{ij}^r$. Denote power vector $\boldsymbol{q}_i = (q_i^1, \cdots, q_i^R)$ and $\boldsymbol{q} = (\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N)^T$.

With the definition of total power of BS, equation (2) can be reformulated as

$$t_{ij}^r = m_{ij}^r B \log_2\left(1 + \frac{p_{ij}^r g_{ij}^r}{\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2}\right). \quad (4)$$

Then, the following equation is obtained

$$\begin{aligned} p_{ij}^r &= \frac{\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2}{g_{ij}^r}(\mathrm{e}^{\ln(2)t_{ij}^r/(Bm_{ij}^r)} - 1) \\ &\triangleq f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}^r), \forall i\in\mathcal{N}, \forall j\in\mathcal{J}_i, \forall r\in\mathcal{R}. \quad (5) \end{aligned}$$

where $\boldsymbol{q}_{-i} = (\boldsymbol{q}_1, \cdots, \boldsymbol{q}_{i-1}, \boldsymbol{q}_{i+1}, \cdots, \boldsymbol{q}_N)^T$.

Equation (5) shows that power vector $\boldsymbol{p}$ can be replaced by the function of load vector $\boldsymbol{m}$, new power vector $\boldsymbol{q}$, and rate vector $\boldsymbol{t}$. Substituting equation (5) into program (3), the following equivalent program is obtained.

$$\min_{\boldsymbol{0}\le\boldsymbol{m},\boldsymbol{0}\le\boldsymbol{q},\boldsymbol{0}\le\boldsymbol{t}} \quad \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r \quad (6a)$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}^r) \le q_i^r, \forall i, r \quad (6b)$$

$$\sum_{r\in\mathcal{R}} t_{ij}^r \ge d_{ij}, \forall i, j \quad (6c)$$

$$\sum_{r\in\mathcal{R}} q_i^r \le p_i^{\max}, \forall i \quad (6d)$$

$$\sum_{j\in\mathcal{J}_i} m_{ij}^r \le 1, \forall i, r \quad (6e)$$

To show the equivalence, we note that if the pair $(\boldsymbol{m}, \boldsymbol{p}, \boldsymbol{t})$ is feasible in (3), then the pair $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$, where power $q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r$, $\forall i\in\mathcal{N}$, is feasible in (6), with the same objective value $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r = \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r$. It follows that the optimal value of (3) is greater than or equal to the optimal value of (6).

Conversely, if $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ is the optimal solution of (6), we can claim that

$$q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r f_{ij}^r(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}), \forall i\in\mathcal{N}, r\in\mathcal{R}.$$

If there exists at least one q_i^r which satisfies

$$q_i^r > \sum_{j\in\mathcal{J}_i} m_{ij} g_{ij}(m_{ij}, \boldsymbol{q}, t_{ij}).$$

With $\boldsymbol{m}$, $\boldsymbol{q}_{-i}$ and $\boldsymbol{t}$ fixed, let $\tilde{q}_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r g_{ij}(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}^r)$. Denote vector $\tilde{\boldsymbol{q}}_i = (q_i^1, \cdots, q_i^{r-1}, \tilde{q}_i^r, q_i^{r+1}, q_R)$, and $\tilde{\boldsymbol{q}} = (\boldsymbol{q}_1, \cdots, \boldsymbol{q}_{i-1}, \tilde{\boldsymbol{q}}_i, \boldsymbol{q}_{i+1}, \cdots, \boldsymbol{q}_N)$. Then, we can obtain $\sum_{l\in\mathcal{J}_k} m_{kl}^r\ g_{kl}(m_{kl}^r, \tilde{\boldsymbol{q}}_{-k}, t_{kl}^r) < \sum_{l\in\mathcal{J}_k} m_{kl}^r g_{kl}(m_{kl}^r, \boldsymbol{q}_{-k}, t_{kl}^r) \le q_k$, $\forall k \ne i$. Therefore, $(\boldsymbol{m}, \tilde{\boldsymbol{q}}, \boldsymbol{t})$ is feasible with $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} \tilde{q}_i^r < \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r$, which contradicts the fact that $(\boldsymbol{m}, \boldsymbol{q}, \boldsymbol{t})$ is the optimal solution. Thus, $q_i^r = \sum_{j\in\mathcal{J}_i} m_{ij}^r g_{ij}(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}^r)$, $\forall i\in\mathcal{N}, r\in\mathcal{R}$. The pair $(\boldsymbol{m}, \boldsymbol{p}, \boldsymbol{d})$, where $p_{ij}^r = g_{ij}(m_{ij}^r, \boldsymbol{q}_{-i}, t_{ij}^r)$, $\forall i\in\mathcal{N}, j\in\mathcal{J}_i, r\in\mathcal{R}$, is feasible in program problem (3) with the same objective value $\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{J}_i} m_{ij}^r p_{ij}^r = \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}} q_i^r$. As a result, we conclude that the optimal value of (3) is less than or equal to the optimal value of (6). Hence, program (3) is equivalent to program (6).

To solve program (6), the distributed algorithm is adopted. In order to rewrite constraints (6b) in a convenient form, we let $u_{kn}^r = \sum_{l\in\mathcal{J}_k} g_{nl}^r(\mathrm{e}^{\ln(2)t_{kl}^r/(Bm_{kl}^r)} - 1)/g_{kl}^r$, $v_k^r = \sum_{l\in\mathcal{J}_k} \sigma^2/g_{kl}^r$, $\forall k, n\in\mathcal{N}, k\ne n, r\in\mathcal{R}$. Then, inequality (6b) can be reformulated as

$$q_k^r \ge \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn}^r q_n^r + u_{ki}^r q_i - v_k^r, \forall k\in\mathcal{N}\setminus\{i\}. \quad (7)$$

Thus, $q_i^r \le (q_k^r - \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn}^r q_n^r + v_k^r)/u_{ki}^r, \forall k\in\mathcal{N}\setminus\{i\}$. Denote $\boldsymbol{m}_{-i} = (\boldsymbol{m}_1, \cdots, \boldsymbol{m}_{i-1}, \boldsymbol{m}_{i+1}, \cdots, \boldsymbol{m}_N)^T$ and $\boldsymbol{t}_{-i} = (\boldsymbol{t}_1, \cdots, \boldsymbol{t}_{i-1}, \boldsymbol{t}_{i+1}, \cdots, \boldsymbol{t}_N)^T$. With power $\boldsymbol{q}_{-i}$, load $\boldsymbol{m}_{-i}$ and rate $\boldsymbol{t}_{-i}$ fixed, the power minimization problem of cell i can be formulated as follows.

$$\min_{\boldsymbol{0}\le\boldsymbol{m}_i,\boldsymbol{0}\le\boldsymbol{q}_i,\boldsymbol{0}\le\boldsymbol{t}_i} \quad \sum_{r\in\mathcal{R}} q_i^r \quad (8a)$$

$$\text{s.t.} \quad q_i^r \ge \sum_{j\in\mathcal{J}_i} a_{ij}^r m_{ij}^r(\mathrm{e}^{bt_{ij}^r/m_{ij}^r} - 1), \forall r \quad (8b)$$

$$q_i^r \le \bar{q}_i^r, \forall r \quad (8c)$$

$$\sum_{j\in\mathcal{J}_i} m_{ij}^r \le 1, \forall r \quad (8d)$$

$$\sum_{r\in\mathcal{R}} t_{ij}^r \ge d_{ij}, \forall j \quad (8e)$$

where $a_{ij}^r = (\sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2)/g_{ij}^r$, $b = \ln(2)/B$ and $\bar{q}_i^r = \min_{k\in\mathcal{N}\setminus\{i\}}(q_k^r - \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn} q_n^r + v_k^r)/u_{ki}^r, \forall i\in\mathcal{N}$, $j\in\mathcal{J}_i, r\in\mathcal{R}$. We omit the constraint (6d) here, because the optimal solution of program (8) always satisfies constraint (6d) if program (6) is feasible.

It can be verified that program (8) is convex. The Language function of (8) can be written as

$$\begin{aligned} \mathcal{L} = &\sum_{r\in\mathcal{R}} q_i^r + \sum_{r\in\mathcal{R}} \alpha_r(\sum_{j\in\mathcal{J}_i} a_{ij}^r m_{ij}^r(\mathrm{e}^{bt_{ij}^r/m_{ij}^r} - 1) - q_i^r) + \\ &\sum_{r\in\mathcal{R}} \beta_r(q_i^r - \bar{q}_i^r) + \sum_{r\in\mathcal{R}} \gamma_r(\sum_{j\in\mathcal{J}_i} m_{ij}^r - 1) + \\ &\sum_{j\in\mathcal{J}_i} \lambda_j(d_{ij} - \sum_{r\in\mathcal{R}} t_{ij}^r) \end{aligned} \quad (9)$$

where α_r, β_r, γ_r, λ_j are the non-negative dual variables associated with the corresponding constraints of problem (8), $\forall r\in\mathcal{R}, j\in\mathcal{J}_i$. According to [16], [17], the optimal solution should satisfy the KKT conditions of problem (8),

$$\begin{aligned} a_{ij}^r((1 - bt_{ij}^r/m_{ij}^r)\mathrm{e}^{bt_{ij}^r/m_{ij}^r} - 1)\alpha_r + \gamma_r &= 0, \forall j, r \quad (10a) \\ 1 - \alpha_r + \beta_r &= 0, \forall r \quad (10b) \\ ba_{ij}^r \mathrm{e}^{bt_{ij}^r/m_{ij}^r}\alpha_r - \lambda_j &= 0, \forall j, r \quad (10c) \end{aligned}$$

From (10b), it is obtained that $\alpha_r = 1 + \beta_r \ge 1$. Then from (10c), it follows that $bt_{ij}^r/m_{ij}^r = \ln(\lambda_j/(ba_{ij}^r\alpha_r))$. By applying this fact to (10a), α_r, γ_r and λ_j should satisfy the equality $(1 - \ln(\lambda_j/ba_{ij}^r\alpha_r))\lambda_j/b - a_{ij}^r\alpha_r + \gamma_r = 0, \forall j, r$. To solve this equality, the following minimization problem is constructed,

$$\min_{\mathbf{1}\le\boldsymbol{\alpha},\mathbf{0}\le\boldsymbol{\lambda},\mathbf{0}\le\boldsymbol{\zeta}} \sum_{r\in\mathcal{R}}\sum_{j\in\mathcal{J}_i}\left(\zeta_j^r+\sum_{k\in\mathcal{J}_i,k\neq j}T(\zeta_j^r-\zeta_k^r)^2\right) \tag{11a}$$

$$\zeta_j^r \ge a_{ij}^r\alpha_r-\frac{\lambda_j}{b}+\frac{\lambda_j}{b}\ln\frac{\lambda_j}{ba_{ij}^r\alpha_r},\forall j,r \tag{11b}$$

where $\boldsymbol{\alpha}=(\alpha_1,\cdots,\alpha_R)^T$, $\boldsymbol{\lambda}=(\lambda_1,\cdots,\lambda_{\mathcal{J}_i})^T$, $\boldsymbol{\zeta}=(\zeta_1^1,\ \cdots,\ \beta_{\mathcal{J}_i}^1,\cdots,\zeta_1^R,\cdots,\beta_{\mathcal{J}_i}^R)^T$, and penalty factor T is a large positive constant. The minimization problem (11) can be proved convex. Denote $U_{ij}^r(\alpha_r,\lambda_j)=a_{ij}^r\alpha_r-\frac{\lambda_j}{b}+\frac{\lambda_j}{b}\ln\frac{\lambda_j}{ba_{ij}^r\alpha_r}$. Then, the Hessian matrix can be obtained by

$$\nabla^2U_{ij}^r(\alpha_r,\lambda_j)=\begin{bmatrix}\lambda_j/(b\alpha_r^2) & -1/(b\alpha_r)\\ -1/(b\alpha_r) & 1/(b\lambda_j)\end{bmatrix}\succeq\mathbf{0}. \tag{12}$$

Convex program (11) can also be effectively solved by using KKT conditions. It can be observed that the dimension of all variables in program (11) is less than in program (8). As a result, optimal $\boldsymbol{\alpha}^*$, $\boldsymbol{\lambda}^*$, $\boldsymbol{\zeta}^*$can be obtained by solving convex problem (11) with interior-point method. Denote $\boldsymbol{\gamma}^*=(\zeta_1^{1*},\cdots,\zeta_1^{R*})^T$, $\boldsymbol{\beta}^*=\boldsymbol{\alpha}^*-\mathbf{1}$. By inserting $bt_{ij}^r/m_{ij}^r=\ln(\lambda_j^*/(ba_{ij}^r\alpha_r^*))$ into (8), the optimal $\boldsymbol{m}_i$, $\boldsymbol{q}$ and $\boldsymbol{t}_i$ can be obtained by solving the following linear program,

$$\min_{\mathbf{0}\le\boldsymbol{m}_i,\mathbf{0}\le\boldsymbol{q}_i,\mathbf{0}\le\boldsymbol{t}_i}\ \sum_{r\in\mathcal{R}}q_i^r \tag{13a}$$

$$\text{s.t.}\quad q_i^r=\sum_{j\in\mathcal{J}_i}a_{ij}^rm_{ij}^r\left(\frac{\lambda_j^*}{ba_{ij}^r\alpha_r}-1\right),\forall r \tag{13b}$$

$$q_i^r\le\bar{q}_i^r,\forall r \tag{13c}$$

$$\beta_r^*(q_i^r-\bar{q}_i^r)=0,\forall r \tag{13d}$$

$$\sum_{j\in\mathcal{J}_i}m_{ij}^r\le 1,\forall r \tag{13e}$$

$$\gamma_r^*(\sum_{j\in\mathcal{J}_i}m_{ij}^r-1)=0,\forall r \tag{13f}$$

$$\sum_{r\in\mathcal{R}}t_{ij}^r\ge d_{ij},\forall j \tag{13g}$$

$$\lambda_j^*(\sum_{r\in\mathcal{R}}t_{ij}^r-d_{ij})=0,\forall j \tag{13h}$$

$$t_{ij}^r=m_{ij}^r\ln\frac{\lambda_j^*}{ba_{ij}^r\alpha_r},\forall j,r \tag{13i}$$

To solve problem (8), we have the following theorem.

Theorem 2: If convex program (8) is feasible, then (8) has the same optimal solution with linear program (13), where $\boldsymbol{\alpha}^*$ and $\boldsymbol{\lambda}^*$ are obtained by solving program (11).

Proof: We first show that the optimal solution $(\boldsymbol{\alpha}^*,\boldsymbol{\lambda}^*,\boldsymbol{\zeta}^*)$ of (11) should satisfy $\zeta_j^{r*}=a_{ij}^r\alpha_r^*-\frac{\lambda_j^*}{b}+\frac{\lambda_j^*}{b}\ln\frac{\lambda_j^*}{ba_{ij}^r\alpha_r^*}$. We prove it by contradiction. Since program (8) is feasible, there exists a solution $(\boldsymbol{\alpha}',\boldsymbol{\beta}',\boldsymbol{\gamma}',\boldsymbol{\lambda}',\boldsymbol{m}',\boldsymbol{t}')$ to KKT conditions (10). If there exists $\zeta_j^{r1*}\neq\zeta_j^{r2*}$ in $\boldsymbol{\zeta}^*$, then $(\boldsymbol{\alpha}',\boldsymbol{\lambda}',\boldsymbol{\zeta}')$, where $\zeta_j^{r\prime}=\gamma_r',\forall j\in\mathcal{J}_i,r\in\mathcal{R}$, is feasible for program (11) with a smaller objective value. If there exists $\zeta_j^r>a_{ij}^r\alpha_r^*-\frac{\lambda_j^*}{b}+\frac{\lambda_j^*}{b}\ln\frac{\lambda_j^*}{ba_{ij}^r\alpha_r^*}$, then $(\boldsymbol{\alpha}^*,\boldsymbol{\lambda}^*,\boldsymbol{\zeta}')$, where $\zeta_j^{r\prime}=a_{ij}^r\alpha_r^*-\frac{\lambda_j^*}{b}+\frac{\lambda_j^*}{b}\ln\frac{\lambda_j^*}{ba_{ij}^r\alpha_r^*},\zeta_k^{s\prime}=\zeta_k^{s*},\forall(k,s)\neq(j,r)$, is feasible for program (11) with a smaller objective value. Let $\boldsymbol{\gamma}^*=(\zeta_1^{1*},\cdots,\zeta_1^{R*})^T$, $\boldsymbol{\beta}^*=\boldsymbol{\alpha}^*-\mathbf{1}$, and denote $(\boldsymbol{m}^*,\boldsymbol{q}^*,\boldsymbol{t}^*)$ as the solution of (13). From (13b-13i), it follows that $(\boldsymbol{\alpha}^*,\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\lambda}^*,\boldsymbol{m}^*,\ \boldsymbol{t}^*)$ is a solution to KKT conditions (10). From the theory of convex optimization, the theorem is proofed.

As a result, we present our distributed power control and resource allocation (DPCRA) algorithm in algorithm 1. The convergence of algorithm 1 is given by theorem 3.

Algorithm 1 Distributed Power Control and Resource Allocation (DPCRA) Algorithm

1: Initialize any feasible $\boldsymbol{m}^{(0)}=(\boldsymbol{m}_1^{(0)},\cdots,\boldsymbol{m}_N^{(0)})^T$, $\boldsymbol{q}^{(0)}=(\boldsymbol{q}_1^{(0)},\cdots,\boldsymbol{q}_N^{(0)})^T$, $\boldsymbol{t}^{(0)}=(\boldsymbol{t}_1^{(0)},\cdots,\boldsymbol{t}_N^{(0)})^T$. Set the accuracy ϵ, the iteration number $n=1$, and the maximal iteration number $N_{\max}$.
2: **for** $i=1,2,\cdots,N$ **do**
3: Let $\boldsymbol{q}_{-i}^{(n-1)}=(\boldsymbol{q}_1^{(n)},\cdots,\boldsymbol{q}_{i-1}^{(n)},\boldsymbol{q}_{i+1}^{(n-1)},\cdots,\boldsymbol{q}_N^{(n-1)})^T$, BS i solves problem (11) to obtain the dual variables $\boldsymbol{\alpha}^*$, $\boldsymbol{\beta}^*$, $\boldsymbol{\gamma}^*$, $\boldsymbol{\lambda}^*$ of problem (8);
4: The optimal $\boldsymbol{m}_i^{(n)},\boldsymbol{q}_i^{(n)},\boldsymbol{t}_i^{(n)}$ are obtained by solving linear program (13)
5: **end for**
6: If $n>N_{\max}$ or $|\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}(q_i^{r(n)}-q_i^{r(n-1)})|/\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{N}}q_i^{r(n-1)}<\epsilon$, terminate. Otherwise, set $n=n+1$ and go to step 2.

Theorem 3: Assuming $N_{\max}\to\infty$, the sequence of load, power and rate vectors generated by the sequential updating DPCRA algorithm will converge.

Proof: The proof is established by showing that when one BS updates its power vector by solving problem (8), the sum power of all BSs is non-increasing. Let $\boldsymbol{q}=(\boldsymbol{q}_1,\cdots,\boldsymbol{q}_N)^T$ denote the power vector of all BSs before BS i starts to update its power vector and $\boldsymbol{q}$ is the feasible solution of program (8). Let $\tilde{\boldsymbol{q}}_i$ denote the updated power-vector of BS i with given $\boldsymbol{q}_{-i}$ and $\tilde{\boldsymbol{q}}=(\boldsymbol{q}_1,\cdots,\boldsymbol{q}_{i-1},\tilde{\boldsymbol{q}}_i,\boldsymbol{q}_{i+1},\cdots,\boldsymbol{q}_N)^T$. From (8c), it can be obtained that $\tilde{\boldsymbol{q}}$ is also a feasible solution of program (8). Then, we have

$$\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}}q_k^r=\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}\setminus\{i\}}q_k^r+\sum_{r\in\mathcal{R}}q_i^r \tag{14a}$$

$$\ge\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}\setminus\{i\}}q_k^r+\sum_{r\in\mathcal{R}}\tilde{q}_i^r=\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{N}}\tilde{q}_k^r \tag{14b}$$

where the inequality follows from the fact that $\tilde{\boldsymbol{q}}_i$ is the optimal power vector of BS i by solving (6) with given $\boldsymbol{q}_{-i}$. Hence, the DCPRA algorithm must converge.

C. Implementation Method for the DPCRA Algorithm

To successfully implement the DPCRA algorithm, BS i needs to compute $(\boldsymbol{m}_i,\boldsymbol{q}_i,\boldsymbol{t}_i)$, which includes two aspects according to (8): 1) coefficients $a_{ij}^r,\forall j\in\mathcal{J}_i,r\in\mathcal{R}$, 2) maximal transmit power $\bar{q}_i^r,\forall r\in\mathcal{R}$. In order to obtain coefficients a_{ij}^r, user j should transmit the power message of total received interference $I_j^r = \sum_{k\in\mathcal{N}\setminus\{i\}} q_k^r g_{kj}^r + \sigma^2$ to BS i. Then a_{ij}^r can be obtained as $a_{ij}^r = I_j^r / g_{ij}^r$, where channel gain g_{ij}^r can be estimated at BS k through the pilot sequence. For maximal transmit power $\bar{q}_i^r$, each BS k transmit the message of power $q_{ki}^r \triangleq (q_k^r - \sum_{n\in\mathcal{N}\setminus\{k,i\}} u_{kn}^r q_n^r + v_k^r)/u_{ki}^r$ to BS i, $\forall k \in \mathcal{N}\setminus\{i\}, r \in \mathcal{R}$. To obtain q_{ki}^r, BS k needs four quantities according to (7): power q_n^r, power gain g_{nl}^r, g_{kl}^r and noise power σ^2, $\forall n \in \mathcal{N} \setminus \{k\}, r \in \mathcal{R}$. Since every BS broadcasts its power message to other BSs after updating transmit power, power q_n^r is always known by BS k. Power gain g_{nl}^r between user $l \in \mathcal{J}_k$ and BS n on RB r is approximately estimated at BS k according to the location messages. Power gain g_{kl}^r can be estimated at BS k by the pilot sequence. It is assumed the noise power σ^2 is known at each BS. Based on these equalities, BS i can update its load vector $\boldsymbol{m}_i$, power vector $\boldsymbol{q}_i$ and load vector $\boldsymbol{t}_i$.

D. Complexity analysis

For our proposed DPCRA algorithm, in each iteration the complexity lies in solving the convex optimization (8), which almost involves a complexity of $O(R^3M^3)$ [18]. For the simplicity of analysis, it is assumed that the number of users in each cell is M. Hence, the total complexity of the DPCRA algorithm is $O(K_{\text{DPCRA}}NR^3M^3)$, where K_{DPCRA} denotes the total number of iterations of the DPCRA algorithm. For the search method of resource allocation problem in multiuser OFDM systems with proportional rate constraints [19], the complexity of obtaining the optimal solution is $O(N^RM^R)$. As for QA-EBR algorithm in [20], the complexity is $O(RN^3(M+1)^3 \log 2(1/\epsilon))$.

IV. Numerical Results

In this section, we provide numerical results to illustrate the theoretical findings. The three-site 3GPP LTE network layout is illustrated in Fig.2, where there are 15 cells in total. A total number of 450 users (blue dots and red dots in Fig.2) are generated for these 15 cells. One half users are generated with one hotspot of 70 m radius per macro cell area, while the other half users are uniformed distributed in the whole area. The total number of RBs is 100 for each cell, and each RB follows the LTE standard of 180 KHz bandwidth. The 3-sector antenna pattern is used for each site and the gain for the 3-sector, of which 3dB beamwidth in degrees is 70 degrees, is 14dBi.

The path channel is modeled as a frequency-selective channel consisting of six independent Rayleigh multipaths. Each multipath component is modeled by Clarkes flat fading model [21]. It is supposed that the antenna gain of each user equipment is 0 dBi. Each user is assumed to be served by the cell with the best channel gain. The red dots in Fig.2 stand for users belonging to cell 9. The traffic demand is 1.2 Mbps for all users in a duration of one second.

In Fig.3, we illustrate the power solutions of each BS with equal load for every user and our proposed DCPRA algorithm. In the equal load method, the fractional number of RBs is the same for every user in each cell. It is obvious that the proposed

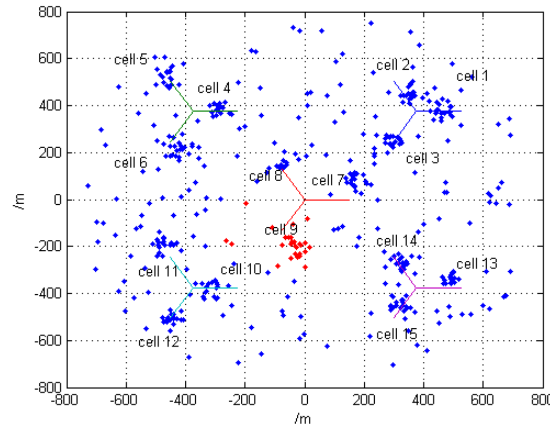

Fig. 2. Network configuration and user distribution.

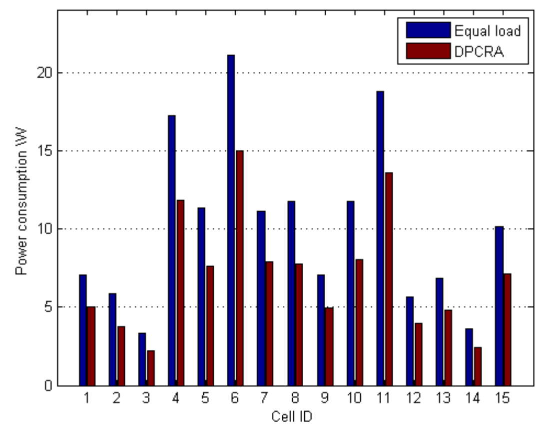

Fig. 3. Optimal energy consumption in each cell.

DCPRA algorithm significantly outperforms the equal load method in sense of requiring lower power for every BS. The total power of DCPRA algorithm is almost 32% smaller than the simple equal load method. This is due to the fact that the channel gain and interference level is always different for different user even in the same cell. The total power consumption of BS can be reduced by properly scheduling the fractional number of RBs for different users.

Fig.4 shows the convergence behavior of DCPRA algorithm. It is obvious that the DCPRA algorithm converges quit quickly and we can see from this figure at least 10 iterations are required for the convergence of the iterative procedure, i.e. $N_{\text{DPCRA}} = 10$. According to the complexity analysis, the DPCRA algorithm has much lower complexity than the search mehod in [19].

From Fig.5, it is apparent that the solution values by DCPRA algorithm grows rapidly in the high-region demand. We observe that the power consumption of cell 6 is much higher than cell 3 in high rate demand region. From the user distribution in Fig.2, the channel gain between user in cell 6

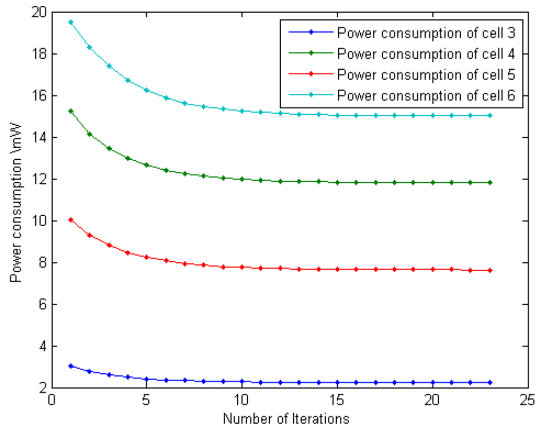

Fig. 4. Convergence behavior of DCPRA.

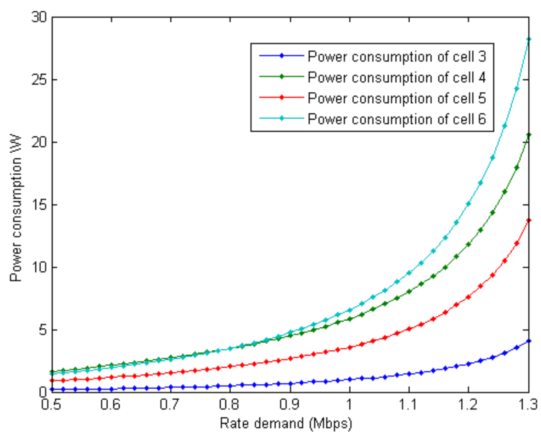

Fig. 5. Transmission energy with respect to user's rate demand.

and the corresponding BS is always low. Besides, there exist many users in other cells close to cell 6, which causes high interference. For the above two reasons, the total power of cell 6 is very high , especially in high user rate demand situation.

V. Conclusion

In this paper, we studied the sum power minimization problem with load and power coupling for OFDM networks. To solve this problem, we proposed a low-complexity distributed power control and resource allocation algorithm. This algorithm is strictly proved to converge. The implementation and complexity analysis are also analyzed. Numerical results show that our distributed algorithm converges to the solution in a few iterations.

Acknowledgments

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