A Cross-layer Design of Optimized Receiver for Wireless State Feedback Control

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***Abstract*—This paper considers a cross-layer design of communication and control layers for wireless state feedback control, and an optimized receiver is proposed. The proposed receiver is a maximum a posteriori probability (MAP)-based receiver that utilizes estimated state information of a Kalman filter-based state observer as a priori knowledge of transmitted state feedback. It takes advantages of the recursive structure of feedback control loop. Simulation results show that the proposed receiver can effectively suppress channel errors and improve the control performance.**

***Index Terms*—Cross-layer design, optimum receiver, maximum a posteriori probability, state observer, networked control systems.**

I. INTRODUCTION

Wireless networked control is an attractive research field in factory automation, smart grid, healthcare, disaster relief, and so on. Although wireless networked control has several advantages, such as the reconfigurability, mobility, and easy installation, there are some disadvantages, including data rate limitation, transmission delay, and channel errors, which cause the deterioration of the control performance.

In previous studies, several types of state observers and controllers that consider channel errors have been proposed to improve the control performance. For example, Kalman filter-based state observers [1]–[4] and linear quadratic optimal controllers [4]–[7] have been proposed for various control situations. These studies focus on suppressing the influence of channel errors from the viewpoint of a control layer without no modification to the communication layer.

From the viewpoint of a communication layer designed for networked control systems, a cooperative communication scheme [8], a hybrid automatic repeat request scheme [9], an adaptive modulation scheme [10], an adaptive power management scheme [11], and adaptive forward error correction schemes [12], [13] have been proposed to suppress channel errors. These studies are based on an attractive cross-layer concept that the communication layer utilizes the control layer's quality information and configures communication parameters to improve the performance of the control layer.

This paper presents a new approach to the cross-layer design of communication and control layers for wireless state feedback control. The proposal is a MAP-based receiver, which is a communication layer's technique; however, it utilizes control layer's information of a Kalman filter-based state observer as a priori knowledge of transmitted state feedback. It has a recursive structure between the communication layer's receiver and the control layer's state observer, which is well suited for feedback control loop. The improvement of the channel error probability of the receiver is supported by the control accuracy and vice versa; therefore, the proposed receiver can effectively suppress channel errors and improve the control performance.

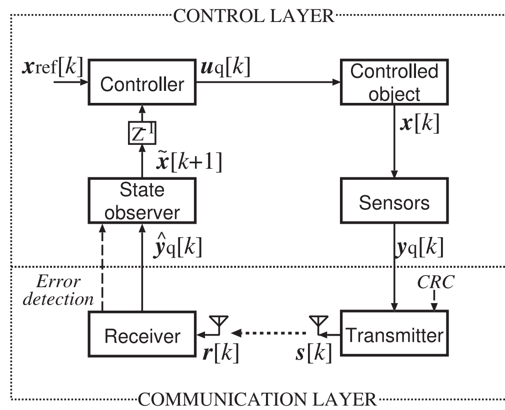

Fig. 1. Wireless state feedback control system.

II. SYSTEM MODEL

The considered wireless networked control system is shown in Figure 1. The system is a state feedback control system that wirelessly feed backs sensors' output. The control layer and the communication layer of the system is modeled as follows.

A. Control layer

1) Controller: Using a reference vector $\boldsymbol{x}_{\mathrm{ref}}[k]$ and an estimated state vector $\tilde{\boldsymbol{x}}[k]$ of the state observer, the controller calculates an input vector $\boldsymbol{u}[k]$ as

$$\boldsymbol{u}[k] = \boldsymbol{K}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{ref}}[k] - \tilde{\boldsymbol{x}}[k]), \tag{1}$$

where $k (= 0, 1, 2 \ldots)$ represents a time index of the discretized system with T_{s}. A linear quadratic Gaussian (LQG) gain $\boldsymbol{K}_{\mathrm{c}}$ [4] is employed, which minimizes $\lim_{K\to\infty} J$.

$$J = \frac{1}{K}\sum_{k=0}^{K}\left(\boldsymbol{x}[k]^{\mathrm{T}}\boldsymbol{Q}_{\mathrm{x}}\boldsymbol{x}[k] + \boldsymbol{u}[k]^{\mathrm{T}}\boldsymbol{Q}_{\mathrm{u}}\boldsymbol{u}[k]\right), \tag{2}$$

where $\boldsymbol{Q}_{\mathrm{x}}$ and $\boldsymbol{Q}_{\mathrm{u}}$ are arbitrary weight matrices.

$\boldsymbol{u}[k]$ is quantized to $\boldsymbol{u}_{\mathrm{q}}[k]$ and input to the controlled object. An L-bit mid-tread uniform quantizer is used for the quantization, and each vector element of $\boldsymbol{u}_{\mathrm{q}}[k]$ is quantized as an L-bit digital value; to simplify the notation, the same bit length is used for all vector elements.

2) Controlled object: The controlled object is assumed to be linear time invariant, and its state vector $\boldsymbol{x}[k]$ is related to the discrete-time state-space model of

$$\boldsymbol{x}[k+1] = \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}\boldsymbol{u}_{\mathrm{q}}[k] + \boldsymbol{w}[k], \tag{3}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are coefficient matrices; $\boldsymbol{w}[k]$ is a disturbance vector and assumed to be a white Gaussian random vector of which mean vector and covariance matrix are $\boldsymbol{0}$ and $\boldsymbol{W}$, respectively.

3) Sensors: The sensors observe N_{y} of N_{x} elements of $\boldsymbol{x}[k]$. Using the output matrix $\boldsymbol{C}$, which has only one "1" at each row and "0" for any other elements, the output vector is represented as $\boldsymbol{y}[k] = \boldsymbol{C}\boldsymbol{x}[k]$.

$\boldsymbol{y}[k]$ is quantized to $\boldsymbol{y}_{\mathrm{q}}[k]$ and input to the communication layer, i.e., the transmitter. An L-bit mid-tread uniform quantizer is used for the quantization, and each vector element of $\boldsymbol{y}_{\mathrm{q}}[k]$ is quantized as an L-bit digital value; to simplify the notation, the same bit length is used.

In this case, the relationship between $\boldsymbol{y}_{\mathrm{q}}[k]$ and $\boldsymbol{y}[k]$ can be represented as $\boldsymbol{y}_{\mathrm{q}}[k] = \boldsymbol{y}[k] + \boldsymbol{v}_{\mathrm{q}}[k]$, i.e.,

$$\boldsymbol{y}_{\mathrm{q}}[k] = \boldsymbol{C}\boldsymbol{x}[k] + \boldsymbol{v}_{\mathrm{q}}[k], \tag{4}$$

where $\boldsymbol{v}_{\mathrm{q}}[k]$ is a quantization noise vector that is modeled as a uniform random vector of which mean vector and covariance matrix are $\boldsymbol{0}$ and $\boldsymbol{V}_{\mathrm{q}}$, respectively. $\boldsymbol{V}_{\mathrm{q}}$ is given as a diagonal matrix whose i-th ($i = 0, 1, \ldots, N_{\mathrm{y}}-1$) diagonal element is $V_{\mathrm{q}\,i} = \Delta_i^2/12$, where Δ_i is the resolution of the uniform quantizer for $y_{\mathrm{q}\,i}$

4) State observer: The state observer receives $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$ and an error detection result from the communication layer, i.e., the receiver. Using them, the state observer estimates $\tilde{\boldsymbol{x}}[k+1]$ corresponding to the next state $\boldsymbol{x}[k+1]$ in accordance with the control system equations (3) and (4). A Kalman filter-based state observer [4] is employed and the calculation steps are described as follows:

⟨1. Gain calculation step⟩

$$\boldsymbol{K}_{\mathrm{o}}[k] = \boldsymbol{\Pi}[k]\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\Pi}[k]\boldsymbol{C}^{\mathrm{T}} + \boldsymbol{V}[k])^{-1} \tag{5}$$

⟨2. Update step⟩

If no bit error is detected,

$$\tilde{\boldsymbol{x}}_{+}[k] = \tilde{\boldsymbol{x}}[k] + \boldsymbol{K}_{\mathrm{o}}[k](\hat{\boldsymbol{y}}_{\mathrm{q}}[k] - \tilde{\boldsymbol{y}}[k]) \tag{6}$$

$$\boldsymbol{\Pi}_{+}[k] = (\boldsymbol{I} - \boldsymbol{K}_{\mathrm{o}}[k]\boldsymbol{C})\boldsymbol{\Pi}[k] \tag{7}$$

Otherwise,

$$\tilde{\boldsymbol{x}}_{+}[k] = \tilde{\boldsymbol{x}}[k] \tag{8}$$

$$\boldsymbol{\Pi}_{+}[k] = \boldsymbol{\Pi}[k] \tag{9}$$

⟨3. Prediction step⟩

$$\tilde{\boldsymbol{x}}[k+1] = \boldsymbol{A}\tilde{\boldsymbol{x}}_{+}[k] + \boldsymbol{B}\boldsymbol{u}_{\mathrm{q}}[k] \tag{10}$$

$$\boldsymbol{\Pi}[k+1] = \boldsymbol{A}\boldsymbol{\Pi}_{+}[k]\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{W} \tag{11}$$

where $\boldsymbol{I}$ is an identity matrix and $\tilde{\boldsymbol{y}}[k] = \boldsymbol{C}\tilde{\boldsymbol{x}}[k]$. The estimation error is $\boldsymbol{e}[k] = \boldsymbol{x}[k] - \tilde{\boldsymbol{x}}[k]$. For the above calculation, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{W}$, and $\boldsymbol{V}$ are assumed to be known at the state observer.

In this paper, in contrast to [4], $\boldsymbol{V}[k]$ is given as $\boldsymbol{V}[k] = \boldsymbol{V}_{\mathrm{q}} + \boldsymbol{V}_{\mathrm{ud}}[k]$, where $\boldsymbol{V}_{\mathrm{ud}}[k]$ is added to suppress the effect of undetected bit errors on $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$. This equation is based on the fact that the effect of quantization noise and that of channel bit errors can be separately described in the case of the uniform quantizer and natural binary index assignment (and also Gray code) [14]. The calculation of $\boldsymbol{V}_{\mathrm{ud}}[k]$ is explained in detail in Section III.

B. Communication layer

1) Transmitter: $\boldsymbol{y}_{\mathrm{q}}[k]$ is mapped to natural binary indices and interpreted as a data bit sequence of $N_{\mathrm{y}}L$-bit. Note that the natural binary index assignment is simple but one of good index assignments, which minimize the mean squared error distortion of uniformly quantized values resulted from channel bit errors [14]. Then, to detect bit errors at the receiver, the $N_{\mathrm{y}}L$-bit data sequence is encoded by an error detection code, namely, cyclic redundancy code (CRC) with M-bit parity; therefore, the total bit length of the transmitted data packet corresponding to $\boldsymbol{y}_{\mathrm{q}}[k]$ is $L_{\mathrm{p}} = N_{\mathrm{y}}L + M$. The encoded sequence is modulated to a digital signal and transmitted to the receiver. The transmitted signal is represented as $\boldsymbol{s}[k]$ in a signal space, and the transmitted signal energy per data bit is E_{b}.

2) Channel: The transmission of $\boldsymbol{s}[k]$ from the transmitter to the receiver is assumed to be completed within one control loop of T_{s}. The channel from the transmitter to the receiver is assumed to be a fading channel with additive white Gaussian noise (AWGN), where the received signal is represented as $\boldsymbol{r}[k] = \boldsymbol{H}[k]\boldsymbol{s}[k] + \boldsymbol{n}[k]$. $\boldsymbol{n}[k]$ is a white Gaussian random vector that the power spectrum density of each vector element is $N_0/2$. The channel matrix $\boldsymbol{H}[k]$ is assumed to be known at the receiver.

3) Receiver: In accordance with the optimal decision theory, the receiver first decides the most probable transmitted bit sequence from $\boldsymbol{r}[k]$, where the leading $N_{\mathrm{y}}L$-bit is demapped to $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$. The proposed receiver design is explained in detail in Section III. Next, the error detection is performed using the decided bit sequence. If no bit error is detected, $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$ is input to the control layer, i.e., the state observer; otherwise $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$ is discarded.

III. Proposed Receiver

The structure of our cross-layer optimized receiver is shown in Figure 2. The proposed receiver is a MAP-based receiver, which is a communication layer's technique; however, it utilizes control layer's information of the Kalman filter-based state observer as a priori knowledge of the transmitted output vector. It has a recursive structure between the communication layer's receiver and the control layer's state observer. On the basis of MAP criterion, the communication layer's receiver minimizes the channel error probability on $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$. In addition, it gives the covariance matrix of the effect of undetected bit errors on $\hat{\boldsymbol{y}}_{\mathrm{q}}[k]$ to the control layer's state observer.

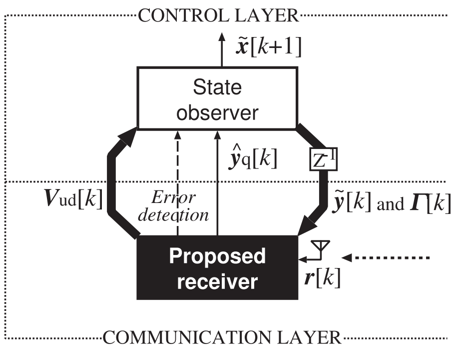

Fig. 2. Structure of the proposed receiver.

A. MAP receiver with estimated state information of state observer

First, we give a brief description of the maximum likelihood (ML) receiver, which is an optimal receiver with no a priori knowledge of the transmitted output vector. It decides each element $\hat{y}_{{\rm q}\,i}[k]$ of $\hat{\boldsymbol{y}}_{\rm q}[k]$ to minimize the channel error probability $\mathrm{P}(\hat{y}_{{\rm q}\,i}[k] \neq y_{{\rm q}\,i}[k])$ in accordance with the following criterion.

$$\begin{aligned}\hat{y}_{{\rm q}\,i}[k] &= \underset{y_{{\rm q}\,i}[k]}{\arg\max}\, \mathrm{P}(\boldsymbol{r}_i[k] \mid y_{{\rm q}\,i}[k]) \\ &= \underset{y_{{\rm q}\,i}[k](\equiv \boldsymbol{s}_i[k])}{\arg\max} \left\{ -\frac{1}{N_0}\|\boldsymbol{r}_i[k] - \boldsymbol{H}_i[k]\boldsymbol{s}_i[k]\|^2 \right\}, \qquad (12)\end{aligned}$$

where "≡" denotes equivalent information resulted from the one-to-one mapping. $\boldsymbol{r}_i[k]$, $\boldsymbol{s}_i[k]$, and $\boldsymbol{H}_i[k]$ denote the subvector of $\boldsymbol{r}[k]$, that of $\boldsymbol{s}[k]$, and the submatrix of $\boldsymbol{H}[k]$, respectively, and correspond to each element $y_{{\rm q}\,i}[k]$ of $\boldsymbol{y}_{\rm q}[k]$.

In contrast to the ML receiver, the proposed MAP receiver utilizes the previously calculated $\tilde{\boldsymbol{y}}[k]$ of the control layer's state observer as a priori knowledge of the transmitted output vector. It is formulated as a MAP decision with conditional a priori probability $\mathrm{P}(y_{{\rm q}\,i}[k] \mid \tilde{y}_i[k])$ corresponding to each element $\tilde{y}_i[k]$ of $\tilde{\boldsymbol{y}}[k]$. To minimize $\mathrm{P}(\hat{y}_{{\rm q}\,i}[k] \neq y_{{\rm q}\,i}[k])$, the proposed receiver decides $\hat{y}_{{\rm q}\,i}[k]$ in accordance with the following criterion.

$$\hat{y}_{{\rm q}\,i}[k] = \underset{y_{{\rm q}\,i}[k]}{\arg\max}\, \mathrm{P}(\boldsymbol{r}_i[k] \mid y_{{\rm q}\,i}[k])\ \mathrm{P}(y_{{\rm q}\,i}[k] \mid \tilde{y}_i[k]) \qquad (13)$$

$\mathrm{P}(y_{{\rm q}\,i}[k] \mid \tilde{y}_i[k])$ is obtained from the state observer. First, the relationship between $\boldsymbol{y}_{\rm q}[k]$ and $\tilde{\boldsymbol{y}}[k]$ is formulated as

$$\begin{aligned}\boldsymbol{y}_{\rm q}[k] &= \tilde{\boldsymbol{y}}[k] + \boldsymbol{C}\boldsymbol{e}[k] + \boldsymbol{v}_{\rm q}[k] \quad (\because\ \boldsymbol{e}[k] = \boldsymbol{x}[k] - \tilde{\boldsymbol{x}}[k]) \\ &= \tilde{\boldsymbol{y}}[k] + \boldsymbol{\gamma}[k], \qquad (14)\end{aligned}$$

where $\boldsymbol{\gamma}[k] = \boldsymbol{C}\boldsymbol{e}[k] + \boldsymbol{v}_{\rm q}[k]$. The mean vector of $\boldsymbol{\gamma}[k]$ is $\boldsymbol{0}$. For ease of computation, approximating $\boldsymbol{C}\boldsymbol{e}[k]$ and $\boldsymbol{v}_{\rm q}[k]$ are independent, the covariance matrix of $\boldsymbol{\gamma}[k]$ is given as $\boldsymbol{\Gamma}[k] = \boldsymbol{C}\boldsymbol{\Pi}[k]\boldsymbol{C}^{\rm T} + \boldsymbol{V}_{\rm q}$; then, roughly approximating $\boldsymbol{\gamma}[k]$ as uncorrelated Gaussian, $\mathrm{P}(y_{{\rm q}\,i}[k] \mid \tilde{y}_i[k])$ is given as

$$\mathrm{P}(y_{{\rm q}\,i}[k] \mid \tilde{y}_i[k]) \approx \frac{1}{\sqrt{2\pi\Gamma_i[k]}} \exp\left(-\frac{1}{2\Gamma_i[k]}(y_{{\rm q}\,i}[k] - \tilde{y}_i[k])^2\right), \qquad (15)$$

where $\Gamma_i[k]$ denotes the i-th diagonal element of $\boldsymbol{\Gamma}[k]$. Note that the same calculation as $\boldsymbol{\Gamma}[k]$ is available from (5) in the state observer; therefore, no additional calculation is required to obtain $\boldsymbol{\Gamma}[k]$.

Finally, substituting (15) to (13), the MAP criterion is simplified as

$$\begin{aligned}\hat{y}_{{\rm q}\,i}[k] = \underset{y_{{\rm q}\,i}[k](\equiv \boldsymbol{s}_i[k])}{\arg\max} \Bigg\{ &-\frac{1}{N_0}\|\boldsymbol{r}_i[k] - \boldsymbol{H}_i[k]\boldsymbol{s}_i[k]\|^2 \\ &- \frac{1}{2\Gamma_i}(y_{{\rm q}\,i}[k] - \tilde{y}_i[k])^2 \Bigg\}. \qquad (16)\end{aligned}$$

Thus, the proposed receiver decides $\hat{\boldsymbol{y}}_{\rm q}[k]$ by utilizing $\tilde{\boldsymbol{y}}[k]$ and $\boldsymbol{\Gamma}[k]$ that were previously calculated in the state observer. Note that the received parity bits are decided in accordance with the ML decision because a priori knowledge of the parity bits is not available from the state observer.

B. Covariance matrix of the effect of undetected bit errors for state observer

Even if no bit error is detected by the error detection after the MAP decision, undetected bit errors may be included in $\hat{\boldsymbol{y}}_{\rm q}[k]$ with a probability $P_{\rm ud}[k]$. This is not negligible in low SNR and leads to the deterioration of the control performance; therefore, to suppress the effect of undetected bit errors on $\hat{\boldsymbol{y}}_{\rm q}[k]$, the receiver should give an appropriate $\boldsymbol{V}_{\rm ud}[k]$ to the state observer.

Since it is difficult to directly derive $\boldsymbol{V}_{\rm ud}[k]$ of the proposed receiver, we give a simple approximation of

$$\boldsymbol{V}_{\rm ud}[k] \approx \frac{P_{\rm ud}[k]}{(1 - P_{\rm e}[k]) + P_{\rm ud}[k]}\boldsymbol{V}_{\rm c}[k], \qquad (17)$$

where $P_{\rm e}[k]$ is the packet error rate (PER). The denominator, $(1 - P_{\rm e}[k]) + P_{\rm ud}[k]$, is the probability that no bit error is detected by the error detection. $\boldsymbol{V}_{\rm c}[k]$ is the mean squared error distortion resulted from channel bit errors, where the error detection code is not employed, and given as a diagonal matrix whose i-th diagonal element is given as follows [14]:

$$V_{{\rm c}\,i}[k] = \sum_{\ell=0}^{L-1} (2^{\ell}\Delta_i)^2 p_{\ell+iL}[k], \qquad (18)$$

where $p_n[k]$ is a bit error rate (BER) at the n-th bit of the decided bit sequence, i.e., $p_{\ell+iL}[k]$ is the BER at the ℓ-th bit of $\hat{y}_{{\rm q}\,i}[k]$. The PER is given as

$$P_{\rm e}[k] = 1 - \sum_{n=0}^{L_{\rm p}-1}(1 - p_n[k]). \qquad (19)$$

The undetected error probability is upper-bounded by a function of the worst BER $p_{n'}[k](= \max_{n=0}^{L_{\rm p}-1} p_n[k])$ as follows:

$$P_{\rm ud}[k] \leq \sum_{d=d_{\min}}^{L_{\rm p}} \alpha_d\, p_{n'}[k]^d (1 - p_{n'}[k])^{L_{\rm p}-d}, \qquad (20)$$

where $d_{\min}$ and α_d are the minimum distance and weight distribution of the CRC.

To summarize (17)-(20), $\boldsymbol{V}_{\rm ud}[k]$ is calculated as a function of $\{p_n[k] \,|\, n = 0, 1, \ldots, L_{\rm p} - 1\}$. At the proposed receiver,

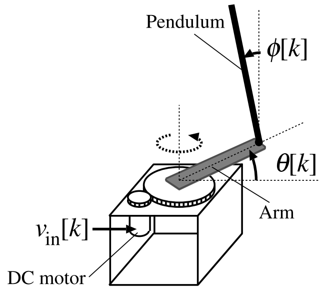

Fig. 3. Rotary inverted pendulum.

TABLE I
ROTARY INVERTED PENDULUM PARAMETERS.

Mass of the pendulum	m_{p}	0.016 (kg)
Length of the pendulum	l_{p}	0.20 (m)
Length of the arm	r_{a}	0.20 (m)
Central Moment of inertia of the arm	J_{a}	0.0048 (kgm^2)
DC motor's resistance	R_{m}	8.3 (Ω)
DC motor's torque constant	K_{m}	0.023 (Nm/A,Vs/rad)
Gear ratio (Arm/DC motor)	K_{g}	120/16
Gravitational constant	g	9.81 (m/s^2)

TABLE II
COMMUNICATION PARAMETERS.

Modulation	Binary phase shift keying
Error detection code	10-bit CRC
Fading channel model	Uncorrelated Rayleigh fading (static within one packet transmission)
Channel noise model	AWGN

$p_n[k]$ is estimated by known approaches [15] and [16] of MAP decoding. Note that the above calculation is applicable to not only the proposed receiver but also the ML receiver.

IV. NUMERICAL RESULTS

A. Simulation parameters

Computer simulations were performed to evaluate the performance of the proposed scheme. A rotary inverted pendulum was employed as the controlled object. Figure 3 shows the basic structure of the rotary inverted pendulum, and Table I shows the parameters based on RealTEC RTC05 [17]. In this controlled object, $\boldsymbol{u}[k]$ is a one-dimensional vector $\boldsymbol{u}[k] = [u_{\mathrm{in}}[k]]$, where $u_{\mathrm{in}}[k]$ denotes the input voltage to the direct current (DC) motor. $\boldsymbol{x}[k]$ is a four-dimensional vector $\boldsymbol{x}[k] = [\phi[k] \;\; \theta[k] \;\; \dot{\phi}[k] \;\; \dot{\theta}[k]]^{\mathrm{T}}$, where $\phi[k]$ and $\dot{\phi}[k]$ are the angle and angular velocities of the pendulum, respectively. $\theta[k]$ and $\dot{\theta}[k]$ are the angle and angular velocities of the arm, respectively. The coefficient matrices of (3) are given as follows:

$$\boldsymbol{A} = e^{\boldsymbol{A}_{\mathrm{c}} T_{\mathrm{s}}}, \quad \boldsymbol{B} = \int_0^{T_{\mathrm{s}}} e^{\boldsymbol{A}_{\mathrm{c}}\tau} \boldsymbol{B}_{\mathrm{c}} \, \mathrm{d}\tau \,, \tag{21}$$

where

$$\boldsymbol{A}_{\mathrm{c}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2g\frac{J_{\mathrm{a}}+m_{\mathrm{p}}r_{\mathrm{a}}^2}{l_{\mathrm{p}}J_{\mathrm{a}}} & 0 & 0 & \frac{2r_{\mathrm{a}}K_{\mathrm{g}}^2K_{\mathrm{m}}^2}{R_{\mathrm{m}}l_{\mathrm{p}}J_{\mathrm{a}}} \\ -\frac{m_{\mathrm{p}}r_{\mathrm{a}}g}{J_{\mathrm{a}}} & 0 & 0 & -\frac{K_{\mathrm{g}}^2K_{\mathrm{m}}^2}{R_{\mathrm{m}}J_{\mathrm{a}}} \end{bmatrix}, \boldsymbol{B}_{\mathrm{c}} = \begin{bmatrix} 0 \\ 0 \\ -\frac{2r_{\mathrm{a}}K_{\mathrm{g}}K_{\mathrm{m}}}{R_{\mathrm{m}}l_{\mathrm{p}}J_{\mathrm{a}}} \\ \frac{K_{\mathrm{g}}K_{\mathrm{m}}}{R_{\mathrm{m}}J_{\mathrm{a}}} \end{bmatrix}. \tag{22}$$

The angles are observed by sensors, but the angular velocities are not, i.e., the output matrix is written as

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{23}$$

The quantization bit length is set to $L = 10$ and the resolution of the mid-tread uniform quantizer is set to $8.0/2^{L-1}$(V) for $\boldsymbol{u}_{\mathrm{q}}[k]$ and $\Delta_1 = \Delta_2 = \pi/2^{L-1}$(rad) for $\boldsymbol{y}_{\mathrm{q}}[k]$.

The control objective of the rotary inverted pendulum is to enable the arm angle $\theta[k]$ to follow the target value $\Theta[k]$ while maintaining the pendulum's upright position, i.e., $\boldsymbol{x}_{\mathrm{ref}}[k] = [0 \;\; \Theta[k] \;\; 0 \;\; 0]^{\mathrm{T}}$. Here, $\Theta[k]$ is set to a rectangular signal alternately switching between $\pi/2$(rad) and $-\pi/2$(rad) every 10(s). To limit $|u_{\mathrm{in}}[k]| \leq 8.0$, the weight matrices for the LQG gain are adequately set to $\boldsymbol{Q}_{\mathrm{x}} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}$ and $\boldsymbol{Q}_{\mathrm{u}} = [0.24]$. $\boldsymbol{x}[0] = \tilde{\boldsymbol{x}}[0] = \boldsymbol{0}$ and $\boldsymbol{\Pi}[0] = \boldsymbol{0}$ are set as initial values. For simplicity, $\boldsymbol{W}$ is set to a diagonal matrix of which all diagonal elements are σ_{w}^2, where σ_{w} is used as a variable for performance evaluation.

Table II shows parameters about the communication from the transmitter to the receiver. For the error detection, the best 10-bit CRC, which has the polynomial 0x2b9 in implicit +1 octal notation, is selected according to [18].

The sampling interval is set to $T_{\mathrm{s}} = 0.01$(s). The simulation time is 1000(s) and the number of simulation runs is 10^4. Here, once $|\phi[k]| > \pi/2$, it is assumed that the pendulum has fallen. Once the pendulum falls, the simulation run is terminated.

B. Performance evaluation

The performance of the communication layer is evaluated by the BER at the ML/MAP receiver. The performance of the control layer is evaluated by the root mean squared error (RMSE) of the arm angle against the ideal control case without system disturbance, quantization, and channel errors. The performance is evaluated under a smaller system disturbance, $\sigma_{\mathrm{w}} = 10^{-5}$, and a larger system disturbance, $\sigma_{\mathrm{w}} = 10^{-3}$. Figures 4 and 5 show the BER and RMSE performance versus E_{b}/N_0, respectively. In Figures 4 and 5, the performance of the proposed MAP receiver, denoted as "Proposed," is compared with the ML receiver, denoted as "ML". Note that "ML" also has the covariance matrix of Section III-B. This is because without it, the system does not work well at all and the pendulum always falls in the given SNR range.

From Figure 4, we can see that the proposed receiver outperforms the ML receiver and results in smaller BER. The ML receiver does not depend on the control layer's information; therefore, the BER performance of the ML receiver for $\sigma_{\mathrm{w}} = 10^{-5}$ and that for $\sigma_{\mathrm{w}} = 10^{-3}$ are the same. On the other hand, the BER performance of the proposed receiver

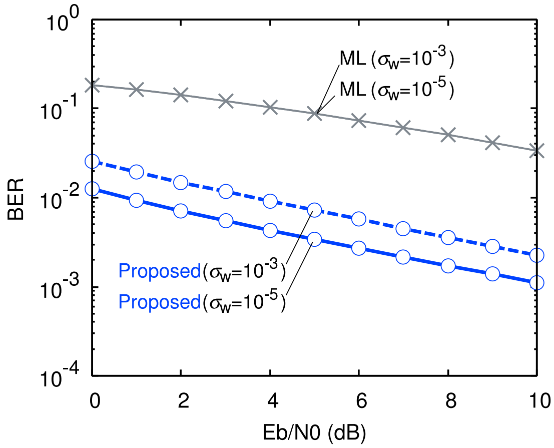

Fig. 4. BER performance versus E_b/N_0 under different system disturbances.

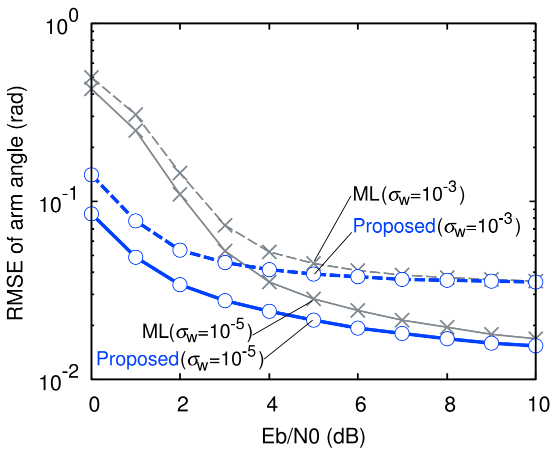

Fig. 5. RMSE performance versus E_b/N_0 under different system disturbances.

for $\sigma_w = 10^{-5}$ is better than that for $\sigma_w = 10^{-3}$ because the proposed receiver depends on the accuracy of the control layer, which is better for smaller system disturbance.

Figure 5 shows that as the system disturbance increases, the RMSE performance of both schemes deteriorates. We can also see that the proposed receiver outperforms the ML receiver and results in larger RMSE improvement, particularly in lower SNR. As the SNR increases, the influence of channel errors on the control performance decreases significantly; therefore, the performance of the proposed receiver and that of the ML receiver converge in high SNR.

V. Conclusion

We considered a cross-layer design of optimized receiver for wireless state feedback control. The proposed receiver is a MAP-based receiver connected with a Kalman filter-based state observer in the control layer by a recursive structure, and utilizes estimated state information as a priori knowledge of transmitted state feedback. We showed that the proposed receiver can effectively suppress channel errors on received state feedback and improve the control performance.

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