Low Complexity Cross-Layer Design with Packet Dependent Scheduling for Heterogeneous Traffic in Multiuser OFDM Systems

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***Abstract*—We propose an adaptive cross-layer design for the downlink multiuser orthogonal frequency division multiplexing (OFDM) systems, to maximize the weighted sum capacity of all users, where each user has multiple heterogeneous traffic queues simultaneously. A packet dependent (PD) scheduling scheme is employed at the medium access control (MAC) layer, which determines the packet transmission order by assigning different weights to different packets, and is shown by simulations more efficient than the previous methods where all packets in a queue have the same weight. The weight design in PD scheduling considers the delay, size and quality of service (QoS) priority level of packets. Each user weight employed in resource allocation at the physical (PHY) layer is obtained by summing up the weights of selected packets for the user. By properly choosing the number of packets used for weight calculation, the proposed cross-layer design requires a much lower overall complexity than the conventional queue based designs. It is also proven to achieve the maximum system stability region. Furthermore, simulation results show that the proposed suboptimal resource allocation algorithm provides a near-optimal performance while requiring a relatively low complexity.**

***Index Terms*—Cross-layer design, OFDM, scheduling, resource allocation, quality of service (QoS).**

I. Introduction

WITH the increasing demands for high capacity and multi-tasking services, wireless communication systems are expected to support multiple users, each of whom has multiple heterogeneous traffic queues simultaneously. For example, a user can use a mobile device to make a phone call, watch the Internet TV, and download files from a website at the same time. Adaptive resource management plays a crucial role in multiuser multi-tasking services. Orthogonal frequency division multiplexing (OFDM) [1][2][3], which divides a broadband channel into a set of orthogonal narrowband subcarriers, allows for a wide range of resource management approaches, and therefore is regarded as a promising technology for future wireless communications.

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Conventional communication networks, where each layer is designed to operate independently, do not utilize the spectrum and energy efficiently, especially in wireless networks which are confronted with time-varying fading channels, limited bandwidth, and competition of air resources among multiple users. Therefore, a cross-layer design of the wireless network is desirable.

Much research work has been carried out on utilizing the cross-layer information for resource allocation at the physical (PHY) layer or/and data scheduling at the medium access control (MAC) layer. In [4], adaptive subcarrier, bit and power allocation was investigated to minimize the total transmit power of an OFDM system with fixed data rate. An optimal joint subcarrier and power allocation algorithm was proposed in [5], to maximize the system capacity with fixed power and delay constraints. In [1], resource allocation is performed based on a utility function which reflects the quality of service (QoS). In [6], power allocation of a multimedia CDMA cellular network is determined by maximizing the weighted sum capacity. A modified largest weighted delay first (M-LWDF) scheduling scheme was proposed for the MAC layer of a single carrier system in [7], which assigns a higher priority to the queues that have a larger head-of-line (HOL) packet delay relative to the delay bound, a higher instantaneous data rate relative to the average data rate, and a higher requirement for the outage probability. In [8], an urgency and efficiency based packet scheduling scheme was employed for OFDM systems, based on a utility function of the HOL delay and channel quality. However, the work in [7] and [8] did not consider resource allocation at the PHY layer. A maximum delay utility (MDU) based cross-layer design was proposed in [9], which performs resource allocation and scheduling by maximizing a utility function of the delay. In [10], the genetic algorithm was employed for resource allocation and scheduling by maximizing the weighted sum capacity, where the weights are determined at the MAC layer according to the traffic delays.

Most previous work on scheduling was traffic queue based, where all packets contained in the selected queues are served until the data or the PHY resources are exhausted in each slot. This however leads to inefficiency if some packets in the unselected queues are more urgent than some packets in the selected queues. In [11], a packet based cross-layer scheduling approach was proposed for GSM/EDGE systems, which takes the packet delay and channel quality into account. However, it did not consider the system capacity. Furthermore, most previous work assumed that each user obtains a single traffic queue and did not consider a system with heterogeneous traffic for each user.

In this paper, we propose a novel adaptive cross-layer design to maximize the weighted sum capacity of the downlink multiuser multi-tasking OFDM systems. Our work is different in the following aspects. First, we consider an OFDM system where each user has multiple heterogeneous traffic queues at the same time. Second, we propose a packet dependent (PD) scheduling scheme, which determines the packet transmission order by assigning different weights to different packets, and therefore is more efficient than the conventional queue dependent scheduling [7][9] where all packets in a queue have the same weight. The packet weights in PD scheduling are determined based on the delay, packet size and QoS priority level of the packets. Third, our resource allocation is user based, by maximizing the weighted sum capacity of users rather than queues, and therefore requires a lower complexity than the conventional queue based resource allocation. The weight for each user is obtained by summing the weights of selected packets for that user. By properly choosing the number of selected packets for weight calculation, our proposed cross-layer design achieves a significant reduction of the overall complexity compared to the previous work [7][9]. Furthermore, we provide an intensive performance analysis of the proposed cross-layer design, which is proven to achieve the maximum system stability region. Simulation results show that, compared to the queue based M-LWDF [7] and MDU [9] scheduling schemes, the proposed PD scheduling scheme demonstrates significant performance advantages. Also, the proposed suboptimal resource allocation algorithm provides a performance close to that of the optimal algorithm, while requiring a lower complexity.

Section II presents the system model and problem formulation. The maximum weighted sum capacity (MWSC) based resource allocation and the PD data scheduling are proposed in Sections III and IV, respectively. Performance analysis and complexity analysis are respectively provided in Sections V and VI. Section VII shows simulation results, and Section VIII draws the conclusion.

II. System Model and Problem Formulation

A. System Model

We consider a general downlink time division duplexing (TDD) OFDM system, where the base station can acquire the channel state information (CSI) through the uplink dedicated pilots from all mobile stations at the beginning of each time slot, and use it for resource allocation and scheduling. The subcarrier and power controller at the PHY layer performs subcarrier and power allocation, and the traffic controller at the MAC layer performs data scheduling. With a cross-layer design, the QoS information obtained by the traffic controller is transferred to the subcarrier and power controller for resource allocation, and the resource allocation results are fed back to the traffic controller in the base station for scheduling of the data to be sent out in each slot.

We assume a total bandwidth of B which is divided into N independent subcarriers and shared by K users. The total transmit power from the base station is assumed to be p_T. The OFDM signaling is time-slotted and each time slot is of length T_{slot} [1]. We consider a quasi-static fading channel, where the channel gain is constant during each slot and is independent of the channel for other slots. We assume that each user has I_k traffic queues with heterogeneous delay constraints for the queues. Define Ω_k as the index set of subcarriers allocated to user k ($k \in \{1, ..., K\}$). Without loss of generality and for simplicity, we assume that each subcarrier is allocated to only one user [2][12]. Let $p_{k,n}$ denote the power allocated to user k on subcarrier n ($n \in \Omega_k$), $h_{k,n}$ the corresponding channel gain, and N_0 the single-sided power spectral density of additive white Gaussian noise (AWGN). Assuming perfect channel estimation, the achievable instantaneous data rate of user k on subcarrier n is expressed as:

$$R_{k,n} = \frac{B}{N} \log_2(1 + p_{k,n}\gamma_{k,n}) \tag{1}$$

where

$$\gamma_{k,n} = \frac{|h_{k,n}|^2}{N_0 B/N} \tag{2}$$

is the channel-to-noise power ratio for user k on subcarrier n. The total achievable instantaneous data rate of user k is given by:

$$R_k = \sum_{n \in \Omega_k} R_{k,n} \tag{3}$$

B. Problem Formulation

We employ a general maximum weighted sum capacity (MWSC) based cross-layer design, which is to maximize the weighted sum of all users' instantaneous capacities [6][10]. Letting W_k denote the weight for user k, which contains the QoS information and is determined by scheduling at the MAC layer, the cost function to be maximized is given by:

$$J = \sum_{k=1}^{K} W_k R_k \tag{4}$$

subject to (C1) $p_{k,n} \geq 0$, (C2) $\sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq p_T$, (C3) $R_k T_{\text{slot}} \leq Q_k$, (C4) $\Omega_1 \cup \ldots \cup \Omega_K \subseteq \{1, 2, \ldots, N\}$, and (C5) $\Omega_k \cap \Omega_j = \emptyset$ ($k \neq j$), where Q_k is the queue length of user k. The constraint in (C3) is to guarantee that no more resource is allocated to user k if the user already obtains sufficient resources to send all data out in current slot, to avoid waste of resources. It can be proved that (4) is a convex function by using a procedure similar to that in [12].

The MWSC based cross-layer design is general in that the weights W_k can be obtained by using a wide range of scheduling schemes. The resource allocation scheme based on maximizing the sum capacity [2] can be regarded as a special case of the MWSC based cross-layer design with equal weights. In this paper, we investigate a multi-objective optimization problem by taking into account not only the system capacity, but also the QoS traffic delay, QoS priority level and data amount. It is worth noting that there is generally not a single optimal solution to a multi-objective optimization problem. By projecting the information on delay, QoS priority level and data amount into the weights, our multi-objective optimization problem is formulated as a single cost function of MWSC as shown in (4), and we have been able to obtain a unique optimal solution.

The above user based cross-layer design can be easily extended to the queue based cross-layer design, by replacing the user index k with the queue index i. Assuming that all users have ω queues each, $i.e.$, $I_k = \omega$ $(\forall k \in \{1, ..., K\})$, there are a total number of ωK queues in the system. The queue based MWSC cross-layer design [7][9] is to maximize the following cost function:

$$J = \sum_{i=1}^{\omega K} W_i R_i \tag{5}$$

subject to (C6) $p_{i,n} \geq 0$, (C7) $\sum_{i=1} \sum_{n \in \Omega_i} p_{i,n} \leq p_T$, (C8) $R_i T_{\text{slot}} \leq Q_i$, (C9) $\Omega_1 \cup \ldots \cup \Omega_{\omega K} \subseteq \{1, 2, \ldots, N\}$, and (C10) $\Omega_i \cap \Omega_j = \emptyset$ $(i \neq j)$, where W_i, R_i and Q_i denote the weight, instantaneous data rate and queue length of queue i, and Ω_i is the index set of subcarriers allocated to queue i.

Since the number of users is usually smaller than the number of queues (when $\omega > 1$), with given weights, the user based MWSC cross-layer design requires a lower complexity than the conventional queue based cross-layer design.

III. Maximum Weighted Sum Capacity based Resource Allocation

In this section, we present an optimal algorithm and a suboptimal algorithm for resource allocation based on the MWSC criterion.

A. Optimal MWSC based Resource Allocation

The optimal solution to (4) and (5) can be obtained by using the method in [5]. It can be derived that each subcarrier n $(\forall n \in \{1, ..., N\})$ is assigned to the user satisfying

$$k^* = \underset{k}{\operatorname{argmax}} \left\{ - \left[\frac{B(W_k - \mu_k T_{\text{slot}})}{N} - \frac{\zeta}{\gamma_{k,n}} \right]^+ + \frac{B(W_k - \mu_k T_{\text{slot}})}{N} \left[\log_2 \left(\frac{B \gamma_{k,n} (W_k - \mu_k T_{\text{slot}})}{N\zeta} \right) \right]^+ \right\} \tag{6}$$

and the corresponding power is given by

$$p_{k,n} = \begin{cases} \left[\frac{B(W_k - \mu_k T_{\text{slot}})}{N\zeta} - \frac{1}{\gamma_{k,n}} \right]^+ & k = k^* \\ 0 & k \neq k^* \end{cases} \tag{7}$$

where $[x]^+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$, μ_k and ζ are the Lagrange multipliers, which satisfy

$$p_T - \sum_{n=1}^{N} \sum_{k=1}^{K} \left[\frac{B(W_k - \mu_k T_{\text{slot}})}{N\zeta} - \frac{1}{\gamma_{k,n}} \right]^+ = 0 \tag{8}$$

and

$$Q_k - \frac{B T_{\text{slot}}}{N} \sum_{n=1}^{N} \left[\log_2 \left(\frac{B \gamma_{k,n} (W_k - \mu_k T_{\text{slot}})}{N\zeta} \right) \right]^+ = 0 \tag{9}$$

Since the search for solutions to (8) and (9) requires a high complexity [5], the optimal resource allocation algorithm is more suitable for the environment where the channels are constant over a large number of time slots. In this paper, we assume quasi-static fading channels, which vary in different time slots. Thus, a suboptimal resource allocation algorithm with a lower complexity is desirable.

B. Suboptimal Resource Allocation

To make a good tradeoff between complexity and performance, we propose a suboptimal resource allocation algorithm which performs subcarrier allocation and power allocation separately.

We first perform subcarrier allocation, assuming equal power across all subcarriers, and then perform power allocation. With equal power on each subcarrier, the subcarrier allocation should assign subcarrier n to user k rather than user j, if $W_k R_{k,n} > W_j R_{j,n}$. After subcarrier allocation, power allocation is performed by using Karush-Kuhn-Tucker (KKT) conditions [13]. Let Φ denote the user index set. The dynamic algorithm to implement the suboptimal MWSC based subcarrier and power allocation is described as follows:

1) Initialization: Set $\Phi = \{1, ..., K\}$, $R_k = 0$, $\Omega_k = \emptyset$ for $\forall k \in \Phi$ and $p_{k,n} = p_T/N$ $(\forall n \in \{1, ..., N\}, \quad \forall k \in \Phi)$.
2) For $n = 1$ to N
   If $\Phi \neq \emptyset$, find $k^*(n) = \underset{k \in \Phi}{\operatorname{argmax}} \{W_k R_k\}$, and assign subcarrier n to user $k^*(n)$; update $\Omega_{k^*(n)} = \Omega_{k^*(n)} \cup n$ and $R_{k^*(n)}$; if $R_{k^*(n)} T_{\text{slot}} \geq Q_{k^*(n)}$, *i.e.*, user $k^*(n)$ has obtained sufficient resources for sending out all data, remove $k^*(n)$ from set Φ.
3) Allocate power to subcarrier n $(\forall n \in \{1, ..., N\})$ by
$$p_{k,n} = \begin{cases} \frac{W_k \left(p_T + \sum_{m=1}^{K} \sum_{q \in \Omega_m} \frac{1}{\gamma_{m,q}} \right)}{\sum_{m=1}^{K} W_m |\Omega_m|} - \frac{1}{\gamma_{k,n}} & k = k^*(n) \\ 0 & k \neq k^*(n) \end{cases} \tag{10}$$
4) For $n = 1$ to N
   If $p_{k^*(n),n} \leq 0$, *i.e.*, there is no enough power for subcarrier n which has small $W_{k^*(n),n}$ and/or $\gamma_{k^*(n),n}$, subcarrier n will be "ignored" for power allocation and removed from set $\Omega_{k^*(n)}$.
5) Repeat Steps 3 and 4 until $p_{k,n} > 0$ $(\forall n \in \{1, ..., N\}, \forall k \in \{1, ..., K\})$.

Note that constraint (C3) is only applied to subcarrier allocation in the proposed suboptimal resource allocation algorithm, which has a dominant impact on the system capacity [2].

Consider users j and k $(j, k \in \{1, ..., K\}, j \neq k)$, and subcarriers m and n $(m \in \Omega_j$ and $n \in \Omega_k)$ which are two arbitrary subcarriers allocated to users j and k, respectively. Letting $\partial J / \partial p_{k,n} = \partial J / \partial p_{j,m} = 0$, we have

$$p_{j,m} - p_{k,n} \geq \frac{1}{\gamma_{k,n}} - \frac{1}{\gamma_{j,m}} \tag{11}$$

when $W_j \geq W_k$. According to (11), we have $p_{j,m} - p_{k,n} = \frac{1}{\gamma_{k,n}} - \frac{1}{\gamma_{j,m}}$ when $W_j = W_k$. This implies that when two subcarriers have equal weights, the subcarrier with a better channel gain is allocated more power, which is the same as the water-filling [14] result. When $W_j > W_k$, $p_{j,m} - p_{k,n} > \frac{1}{\gamma_{k,n}} - \frac{1}{\gamma_{j,m}}$, which means that the subcarrier corresponding to the user with a higher weight is allocated more power than the case of water-filling.

Subcarriers - queues?

IV. Packet Dependent Scheduling

The weights of the MWSC based cross-layer design described in Section II contain the QoS information, and can be obtained from scheduling at the MAC layer. In this section, we first review two queue based scheduling schemes: M-LWDF [7] and MDU [9], which can be combined with the queue based MWSC cross-layer design using the cost function given by (5). Then a packet based scheduling scheme is proposed in subsection IV-C. Notice that all scheduling schemes are performed in every time slot and the weights are updated in every slot as well.

A. Modified Largest Weighted Delay First Scheduling

The M-LWDF [7] scheduling scheme can keep the delays of most queues below a bound. We extend the work in [7] to the case of OFDM systems. Letting U_i and S_i respectively denote the delay tolerance and HOL packet delay for queue i, define δ_i as the maximum allowed probability that $S_i > U_i$. The weight of queue i is given by:

$$W_i = -\frac{S_i \log(\delta_i)}{U_i \overline{R}_i} \tag{12}$$

where $\overline{R}_i$ is the data rate of queue i averaged over slots.

Substituting (12) into (5), it can be deduced that with M-LWDF, the queue which has a higher HOL packet delay relative to the delay bound (S_i/U_i), a higher instantaneous data rate relative to the average data rate ($R_i/\overline{R}_i$), and a higher requirement for the outage probability (*i.e.*, a larger $-\log(\delta_i)$), is given a higher priority to be served.

B. Maximum Delay Utility Scheduling

The so-called MDU scheduling scheme in [9] is to maximize the utility functions with respect to the delay. Define $f_i(\overline{S}_i)$ as a decreasing utility function of the average HOL packet delay $\overline{S}_i = \overline{Q}_i/\overline{\lambda}_i$ for queue i [15], where $\overline{S}_i$, $\overline{Q}_i$ and $\overline{\lambda}_i$ denote the short-term average HOL delay, the average queue length and the average arrival rate of queue i calculated in current slot, respectively. In practice, systems do not serve an empty queue. Thus, we have $\overline{\lambda}_i = \overline{R}_i$ [9] and $\overline{S}_i = \overline{Q}_i/\overline{R}_i$. The average length of queue i is updated during each slot. Letting Q_i denote the length of queue i in current slot, and $\overline{Q}_i'$ denote the average length of queue i calculated in previous slot, the average length of queue i calculated in current slot is given by $\overline{Q}_i = \alpha Q_i + (1-\alpha)\overline{Q}_i'$, where $0 < \alpha < 1$. Similar to M-LWDF [7], MDU is a queue based scheduling scheme, with the weight W_i in (5) expressed as:

$$W_i = \frac{-1}{\overline{\lambda}_i} \cdot \frac{\partial f_i(\overline{S}_i)}{\partial \overline{S}_i} \tag{13}$$

C. Packet Dependent Scheduling

The mechanism of the conventional queue based scheduling such as M-LWDF and MDU is to assign the same weight to all packets in a queue, and serve the selected queues until either the data or PHY layer resources are exhausted during each slot. This however leads to inefficiency if some packets in the unselected queues are more urgent than some packets in the currently served queues. We now propose a packet dependent (PD) scheduling scheme, which assigns different weights to different packets contained in the same queue. The packet weights are determined based on the delay, packet size and QoS priority level of the packets. Therefore, it is more flexible and efficient than the queue based scheduling.

Defining $U_{k,i}$ as the delay tolerance for queue i ($i \in \{1, ..., I_k\}$) of user k ($k \in \{1, ..., K\}$), a packet within this queue will be dropped if its waiting time is larger than $U_{k,i}$. In general, the delay tolerance of the voice, variable bit rate (VBR) video and BE traffic queues is set to be low, medium and high, respectively. A guard interval $G_{k,i}$ is introduced to reduce the packet drop rate, which means that the urgent packets belonging to queue i of user k should be given a high priority within the last $G_{k,i}$ duration before they timeout. Letting $t_c \in [0, \infty)$ denote the current time, for the lth packet that belongs to queue i of user k and arrives at time $t_l \in [0, t_c]$, the delay is given by $S_{k,i,l} = t_c - t_l$. Thus, the time left that the packet becomes urgent is denoted by $C_{k,i,l}$ (in msec), which is expressed as:

$$C_{k,i,l} = U_{k,i} - S_{k,i,l} - G_{k,i} \tag{14}$$

If $C_{k,i,l} < 0$, *i.e.*, $S_{k,i,l} > U_{k,i} - G_{k,i}$, it means that the packet is urgent, and will timeout within the guard interval of $G_{k,i}$. Hence, the packets with $C_{k,i,l} < 0$ should be given a high priority during this guard interval.

Further, let $D_{k,i,l}$ denote the packet size (in bits) of the packet that belongs to queue i of user k, and arrives at time t_l. Define $\beta_{k,i} \in [1, \infty)$ as the QoS priority level for queue i of user k. In general, the QoS priority levels for the voice traffic queues, video traffic queues and BE traffic queues are high, medium and low, respectively.

We design $W_{k,i,l}$, the weight of the to-be-served packets that belong to queue i of user k, and arrive at time t_l by using the following equation:

$$W_{k,i,l} = \begin{cases} \beta_{k,i} D_{k,i,l}/(C_{k,i,l}+1) & (C_{k,i,l} \geq 0) \\ \beta_{k,i} D_{k,i,l} & (C_{k,i,l} < 0) \end{cases} \tag{15}$$

According to (15), the proposed PD scheduling scheme assigns a higher weight to the packets in a queue which have a higher QoS priority level, a larger data amount and a fewer time left to become urgent. The packets with higher weights will be transmitted first, regardless if they are in the same queue or not. Each packet is assigned a unique sequence number, which ensures that it can be reassembled during transmission. Therefore, the proposed PD scheduling scheme is more flexible and efficient than conventional queue based scheduling schemes [7][9], where all packets in the same queue are assigned the same weight.

1) If $C_{k,i,l} \geq 0$, *i.e.*, $S_{k,i,l} \leq U_{k,i} - G_{k,i}$, the weight $W_{k,i,l}$ in (15) increases with the decrease of $C_{k,i,l}$. In particular, we have:

$$\partial W_{k,i,l}/\partial C_{k,i,l} = -\beta_{k,i} D_{k,i,l}/(C_{k,i,l}+1)^2 \tag{16}$$

This implies that with a large valued $C_{k,i,l}$, *i.e.*, a relatively small packet delay, the variation of the weight $W_{k,i,l}$ is less sensitive to the variation of $C_{k,i,l}$. In this case, the QoS priority level and data amount play a more important role in weight calculation. On the other hand, the fewer time left for the packet to become urgent, the larger the packet delay and the faster the weight increases.

2) If $C_{k,i,l} < 0$, *i.e.*, $S_{k,i,l} > U_{k,i} - G_{k,i}$, it means that the packets arriving at time t_l will timeout in less than $G_{k,i}$ msec, and should be given a high priority during the guard interval to avoid the drop of packets. In this case, $W_{k,i,l}$ in (15) maintains its maximum value, which is determined by the data amount and QoS priority level of the packets. For instance, if two voice packets (of the same QoS priority level) both fall in the guard interval, the packet with a larger data amount will be assigned a higher weight. On the other hand, if two packets are of the same data amount, the packet with a higher QoS priority level will obtain a higher weight.

To make a good tradeoff between performance and complexity, we select only the first $\Lambda_{k,i}$ to-be-served packets (most urgent packets) in queue i of user k for weight calculation of the MWSC based cross-layer design. The $\Lambda_{k,i}$ selected packets are divided into two sets: 1) the packets falling in the guard interval (*i.e.*, to time out within $G_{k,i}$), denoted by $\mathbb{L}^{\mathrm{U}}_{k,i}$; 2) the rest packets, denoted by $\overline{\mathbb{L}^{\mathrm{U}}_{k,i}}$. It can be deduced that $|\mathbb{L}^{\mathrm{U}}_{k,i}| + |\overline{\mathbb{L}^{\mathrm{U}}_{k,i}}| = \Lambda_{k,i}$. Using (15), the weight W_k in (4) for the current slot is given by:

$$\begin{aligned} W_k &= \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}^{\mathrm{U}}_{k,i} \cup \overline{\mathbb{L}^{\mathrm{U}}_{k,i}}} W_{k,i,l} \\ &= \sum_{i=1}^{I_k} \beta_{k,i} \left[\sum_{l \in \mathbb{L}^{\mathrm{U}}_{k,i}} D_{k,i,l} + \sum_{l \in \overline{\mathbb{L}^{\mathrm{U}}_{k,i}}} \frac{D_{k,i,l}}{C_{k,i,l}+1} \right] \end{aligned} \tag{17}$$

It is important to determine the value of $\Lambda_{k,i}$ in (17). With a small-valued $\Lambda_{k,i}$, the complexity decreases, while less QoS information of the to-be-served packets is obtained from the MAC layer for resource allocation. As a result, the performance of the cross-layer design is degraded. In this case, the urgent packets, represented by the first term in (17), play a more important role in weight calculation. On the other hand, with a large-valued $\Lambda_{k,i}$, the complexity increases, while more QoS information of the to-be-served packets is reflected in the weights, which leads to an improved system performance. In practice, a reasonable value of $\Lambda_{k,i}$ is below 100.

The proposed PD scheduling scheme can also be easily combined with the queue based MWSC resource allocation, whose complexity is higher than that of the user based MWSC with heterogeneous traffic, as can be shown in Section VI.

V. Performance Analysis

System stability plays a critical role in network design [16]. In this section, we analyze the stability of the proposed cross-layer PD scheduling scheme, assuming infinite buffer size, and no packet drop during transmission, as in [9] and [16].

Given optimization objective(s) and allocation constraints, define $\mathbf{R} = [R_1, ..., R_K]^T$ as an instantaneous transmission rate vector achieved by a resource allocation algorithm. Assuming that the channel state process is ergodic, let $\widetilde{C}$ denote the ergodic (long-term average) capacity region, which consists of the average transmission rate vectors $E\{\mathbf{R}\}$ achieved by all possible resource allocation algorithms and result in reliable communications [9]. Also define $int(\widetilde{C})$ as the interior of $\widetilde{C}$, which refers to the whole region of $\widetilde{C}$ except for its boundary [16]. Let $\widetilde{S}$ denote the stability region achieved by a scheduling algorithm, which is defined as the set of all possible average arrival rate vectors $E\{\boldsymbol{\lambda}\}$with which the system stability is guaranteed by the scheduling algorithm [16], where $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_K]^T$ denotes an instantaneous arrival rate vector. The maximum stability region that can be achieved by different scheduling algorithms is identical with $int(\widetilde{C})$ [1]. Therefore, if a system is stable, then $\widetilde{S} \subseteq int(\widetilde{C}) \subset \widetilde{C}$. It means that the average arrival rates locate in the interior of the ergodic capacity region, *i.e.*, $E\{\boldsymbol{\lambda}\} \in int(\widetilde{C})$. In other words, if a system is stable, there exists an average transmission rate vector $E\{\mathbf{R}\} \in \widetilde{C}$ so that $E\{\lambda_k\} < E\{R_k\}$ for all $k \in \{1, ..., K\}$. However, this is a necessary but insufficient condition for system stability.

Let $Q_{k,Z}$ denote the queue length of user k up to current slot Z, which is given by

$$Q_{k,Z} = \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}^Z_{k,i}} D_{k,i,l} \tag{18}$$

where $\mathbb{L}^Z_{k,i}$ denotes the set of packets belonging to queue i, user k, up to current slot Z. It was shown in [17] that a system is regarded as stable if

$$\limsup_{P\to\infty} \frac{1}{P} \sum_{Z=1}^{P} \sum_{k=1}^{K} E\{Q_{k,Z}\} < \infty \tag{19}$$

which implies that the upper limit of the average queue length is finite. Substituting (19) into (18), we can deduce that a system is stable if

$$\limsup_{P\to\infty} \frac{1}{P} \sum_{Z=1}^{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}^Z_{k,i}} E\{D_{k,i,l}\} < \infty \tag{20}$$

Theorem 1: The proposed cross-layer PD scheduling scheme achieves the maximum stability region, *i.e.*, the PD scheduling scheme guarantees the system stability if the average data arrival rates locate in the interior of the ergodic capacity region.

The proof of *Theorem 1* is shown in Appendix. Given the optimization objective in (4) and allocation constraints in (C1)-(C5), let $\widetilde{C}$ denote the ergodic capacity region achieved by all possible resource allocation algorithms, including the algorithms presented in Section III. *Theorem 1* means that if $E\{\boldsymbol{\lambda}\} \in int(\widetilde{C})$, the proposed PD scheduling guarantees the system stability defined by (20). In other words, if there exists an average transmission rate vector $E\{\mathbf{R}\} \in \widetilde{C}$ so that $E\{\lambda_k\} < E\{R_k\}$ for all $k \in \{1, ..., K\}$, the system is stable. Therefore, with PD scheduling, $E\{\boldsymbol{\lambda}\} \in int(\widetilde{C})$, *i.e.*, $E\{\lambda_k\} < E\{R_k\}$ for all $k \in \{1, ..., K\}$, becomes a necessary and sufficient condition for system stability.

In the proof of *Theorem 1*, a Lyapunov drift [17] is employed, which is one of the most important methods to deal with the stability issues of queuing networks. Define a Lyapunov function [18] $Y(\mathbf{Q}_Z) = \sum_{k=1}^{K} y(Q_{k,Z})$ as a scalar measure of the queue length in the system, where

$\mathbf{Q}_Z = [Q_{1,Z}, ..., Q_{K,Z}]^T$, and $Y(.)$ is a nonnegative function. The Lyapunov drift is given by $E\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\}$, representing the expectation of the change of the Lyapunov function during two adjacent slots. According to (15), the packet weights determined by our proposed PD scheduling scheme contain the queue state information (packet size). From (27), which is shown again below for easy reference:

$$E\left\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\right\} \leq a - b\sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{l\in\mathbb{L}_{k,i}^{Z}} E\left\{W_{k,i,l}\right\} \quad (a, b \in (0, \infty))$$

it can be derived that the Lyapunov drift becomes negative when the average aggregate weight, which includes the information of the average queue length according to (15), increases to a certain bound. The negative Lyapunov drift means that the queue length decreases to reinstate the system stability. On the other hand, if the weights do not contain any queue information, for example, in the case of equal weight $W_k = 1$ ($k \in \{1, ..., K\}$), the increase of the queue length has no impact on the weights, and therefore has no impact on the Lyapunov drift. Thus, the system stability is not guaranteed by a scheduling approach which does not consider the queue state information.

VI. Complexity Analysis

In this section, we provide a complexity analysis of the proposed cross-layer MWSC based resource allocation and PD scheduling, in comparison to the existing scheduling schemes reviewed in Section IV. Without loss of generality and for simplicity, we assume that all users have ω queues each, *i.e.*, $I_k = \omega$ ($k \in \{1, ..., K\}$). Thus, there are a total number of ωK queues in the system.

The suboptimal user based MWSC resource allocation algorithm in described Subsection III-B requires K comparisons in each of N iterations for subcarrier allocation, which requires a complexity of $O(KN)$. Power allocation is performed subsequently, whose complexity is $O(N)$ and can be ignored compared to the complexity of subcarrier allocation. Hence, the complexity for one iteration in suboptimal resource allocation is $O(KN)$. To search for the Lagrange multiplier, the suboptimal resource allocation requires a small number of iterations (2-3 iterations on average), which is independent of the numbers of users and subchannels. The optimal resource allocation algorithm described in Subsection III-A requires the same complexity of $O(KN)$ as the suboptimal algorithm for one iteration, according to (6) and (7). However, it needs to search for two Lagrange multipliers at the same time, and therefore requires 10 times more iterations on average than the suboptimal resource allocation which requires searching for only one Lagrange multiplier. Due to its significant complexity reduction and little performance loss (shown by Fig. 1) over the optimal algorithm, only the suboptimal resource allocation algorithm is considered in the following of this section.

Compared to the user based MWSC resource allocation, the queue based MWSC resource allocation requires ωK comparisons in each of the N iterations for subcarrier allocation, and therefore its overall complexity is $O(\omega KN)$. This implies that the complexity of the user based MWSC resource allocation algorithm is lower than that of the queue based MWSC resource allocation algorithm with heterogeneous traffic, especially when ω, the number of queues per user, is large.

When the queue based MWSC resource allocation is combined with the M-LWDF scheduling, M-LWDF uses (12) once to calculate the weight for each of the ωK queues in each slot, and therefore its complexity is $O(\omega K)$. The complexity of resource allocation is $O(\omega KN)$, as discussed above. By considering only the higher order complexity of resource allocation and scheduling, the overall complexity of MWSC+M-LWDF is $O(\omega KN)$, which grows linearly with the numbers of subcarriers, users, and queues per user. Similarly, the overall complexity of MWSC+MDU is $O(\omega KN)$.

Incorporated with the PD scheduling, the complexity of the user based MWSC resource allocation is $O(KN)$, as discussed above. With the PD scheduling at the MAC layer, according to (17), $W_{k,i,l}$ is calculated for at most $\Lambda_{k,i}$ packets in each of the ωK queues. Therefore, the complexity of the PD scheduling is $O(\omega\Lambda_{k,i}K)$. With $\Lambda_{k,i} < N$ in general, we have the following discussion:

1) If $\omega = 1$, *i.e.*, each user has a single traffic queue, the overall complexity of MWSC+PD is $O(KN)$, which is the same as the complexity of MWSC+M-LWDF and MWSC+MDU.
2) If $1 < \omega \leq N/\Lambda_{k,i}$, which is the most likely scenario in practice, resource allocation plays a dominant role in the overall complexity of MWSC+PD, which is $O(KN)$. In this case, the overall complexity of the proposed cross-layer design is roughly independent of the number of queues per user, benefiting from the user based resource allocation. With given numbers of users and subcarriers, MWSC+PD achieves a complexity reduction of approximately ω times over MWSC+M-LWDF and MWSC+MDU, whose overall complexities are $O(\omega KN)$.
3) If $\omega > N/\Lambda_{k,i}$, the overall complexity of MWSC+PD is $O(\omega\Lambda_{k,i}K)$. Therefore, the overall complexity of MWSC+PD is around $N/\Lambda_{k,i}$ times lower than that of MWSC+M-LWDF and MWSC+MDU.

The above analysis is summarized in TABLE I, where the overall complexity of each cross-layer design takes account of only the higher order complexity of resource allocation and scheduling, as it is dominant with the increase of the parameters values [19]. It can be concluded that by properly choosing the number of packets used for weight calculation, MWSC+PD requires a lower overall complexity than MWSC+M-LWDF and MWSC+MDU when there are multiple heterogeneous traffic queues for each user simultaneously. For instance, with $K = 10$ users, $N = 512$ subcarriers, $\omega = 3$ queues per user and $\Lambda_{k,i} = 100$, 75 and 50 for voice, video and BE traffic, MWSC+PD achieves a complexity reduction of around 3 times.

VII. Simulation Results

We use simulation results to demonstrate performance of the proposed cross-layer design for a system with a total transmit power $p_T = 1$ W, a slot duration of $T_{\text{slot}} = 2$

TABLE I
COMPLEXITY OF CROSS-LAYER OPTIMIZATION SCHEMES

Complexity	MWSC + M-LWDF [7]	MWSC + MDU [9]	MWSC + PD
Resource Allocation	$O(\omega KN)$	$O(\omega KN)$	$O(KN)$
Scheduling	$O(\omega K)$	$O(\omega K)$	$O(\omega\Lambda_{k,i}K)$
Overall	$O(\omega KN)$	$O(\omega KN)$	$O(KN)$ $(\omega \leq N/\Lambda_{k,i})$ $O(\omega\Lambda_{k,i}K)$ $(\omega > N/\Lambda_{k,i})$

msec, and a total bandwidth of $B = 5$ MHz which is divided into $N = 512$ subcarriers. We assume that each user has $I_k = 3$ $(\forall k \in \{1, ..., K\})$ queues: voice, VBR video and BE traffic queues. The delay tolerance for voice, video and BE traffic is set to be $U^{\text{voice}}_{k,i} = 100$ msec, $U^{\text{video}}_{k,i} = 400$ msec and $U^{\text{BE}}_{k,i} = 1000$ msec $(\forall k \in \{1, ..., K\}, \forall i \in \{1, ..., I_k\})$, respectively. The guard interval $G_{k,i}$ is set to be 10 msec for all types of traffic. The packets arrive at an interval of $T^{k,i}_{\text{a}} = 1$ msec $(\forall k \in \{1, ..., K\}, \forall i \in \{1, ..., I_k\})$. The packet arrival rates of voice and BE traffic are constantly $\lambda^{\text{voice}}_{k,i} = 64$ Kbps and $\lambda^{\text{BE}}_{k,i} = 500$ Kbps $(\forall k \in \{1, ..., K\}, \forall i \in \{1, ..., I_k\})$, respectively. The packet arrival rate of the variable bit rate (VBR) video traffic, $\lambda^{\text{video}}_{k,i}$ $(\forall k \in \{1, ..., K\}, \forall i \in \{1, ..., I_k\})$, follows a truncated exponential distribution in each state, with the maximum, minimum and mean of 420 Kbps, 120 Kbps and 239 Kbps, respectively, where the duration of each state follows an exponential distribution with mean 160 msec. It can be calculated that the average packet sizes for voice, video and BE traffic are 64 bits, 239 bits and 500 bits, respectively. The QoS priority levels for the voice, video and BE traffic are respectively set to be $\beta^{\text{voice}}_{k,i} = 1024$, $\beta^{\text{video}}_{k,i} = 512$ and $\beta^{\text{BE}}_{k,i} = 1$ as a result of testing, to provide the best performance. Based on different requirements of different traffic types on performance and complexity, the number of voice, video and BE packets used for weight calculation in (17) are set to be $\Lambda^{\text{voice}}_{k,i} = 100$, $\Lambda^{\text{video}}_{k,i} = 75$ and $\Lambda^{\text{BE}}_{k,i} = 50$ $(\forall k \in \{1, ..., K\}, \forall i \in \{1, ..., I_K\})$, respectively. The channel is modeled to be of six independent Rayleigh fading paths with an exponentially delay profile and a root-mean-square (RMS) delay spread of 0.5 μsec. The signal-to-noise ratio (SNR) is defined as the average ratio of the received signal power to noise power for each user.

Fig. 1 demonstrates the average delays of the voice and video traffic achieved by using both the optimal and suboptimal resource allocation algorithms proposed in Section III, and the PD scheduling scheme proposed in Sebsection IV-C. The optimal resource allocation employs the bisection method [20] to search for the Lagrange multipliers. Compared to the optimal algorithm, the suboptimal resource allocation algorithm provides a similar performance, at a lower complexity, as discussed in Section VI.

In the following, we present a performance comparison of the proposed PD scheduling scheme with M-LWDF and MDU. For MDU, the utility functions defined in [9] are used. For the purpose of consistency and complexity, we employ the MWSC based suboptimal resource allocation discussed in subsection III-B at the PHY layer for all the cross-layer schemes.

Figs. 2 to 5 demonstrate the impact of the number of users on performance of different cross-layer deigns, with SNR = 20 dB. The system and BE traffic throughputs are

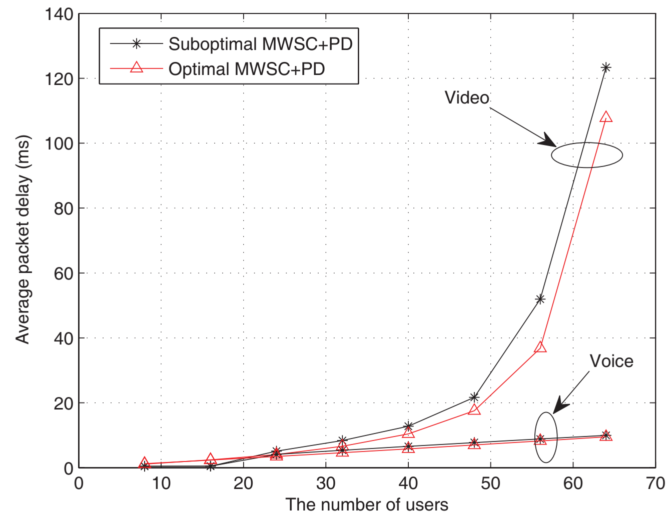


Fig. 1. Impact of the number of users on average voice and video packet delays of optimal and suboptimal resource allocation schemes (SNR = 20 dB).

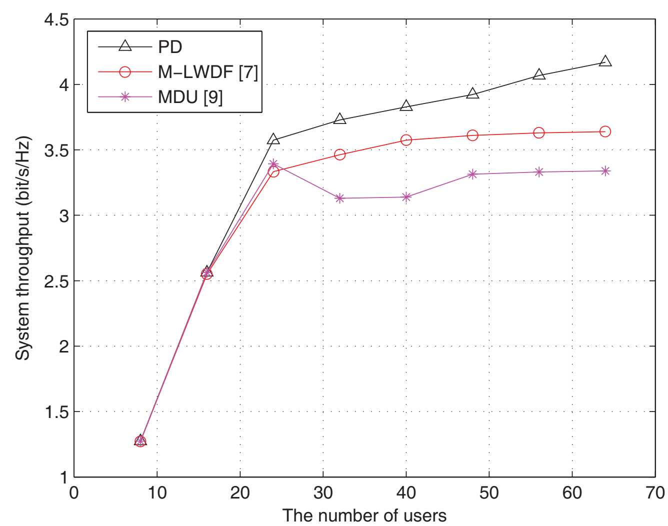


Fig. 2. Impact of the number of users on system throughput of PD, M-LWDF and MDU scheduling schemes (SNR = 20 dB).

shown in Fig. 2 and Fig. 3, respectively. PD demonstrates significant performance advantages over MDU and M-LWDF with a wide range of the number of users ($K = 24 \sim 64$ users), while achieving a complexity reduction of about three times, as discussed in Section VI. In particular, the system throughput achieved by PD increases with the increase of the number of users, due to enhanced multiuser diversity. With given resources, when the number of users increases, there is a higher degree of freedom for resource allocation, resulting in

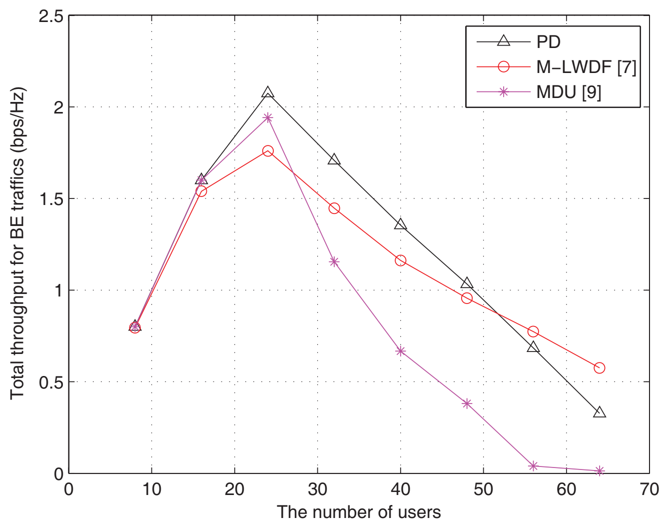


Fig. 3. Impact of the number of users on BE traffic throughput of PD, M-LWDF and MDU scheduling schemes (SNR = 20 dB).

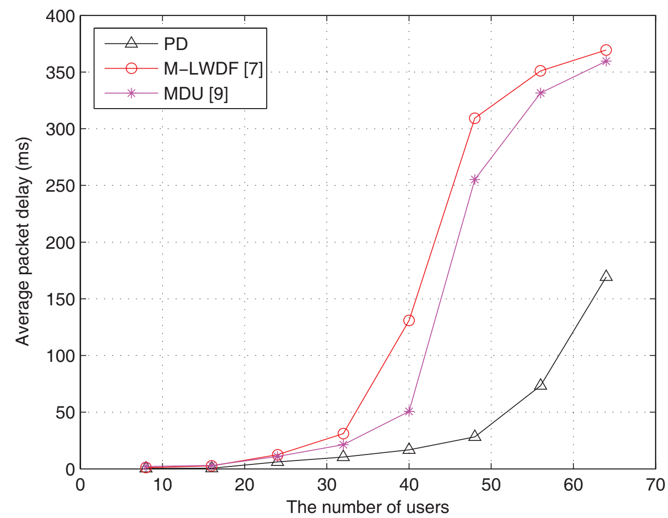


Fig. 5. Impact of the number of users on average video packet delays of PD, M-LWDF and MDU scheduling schemes (SNR = 20 dB).

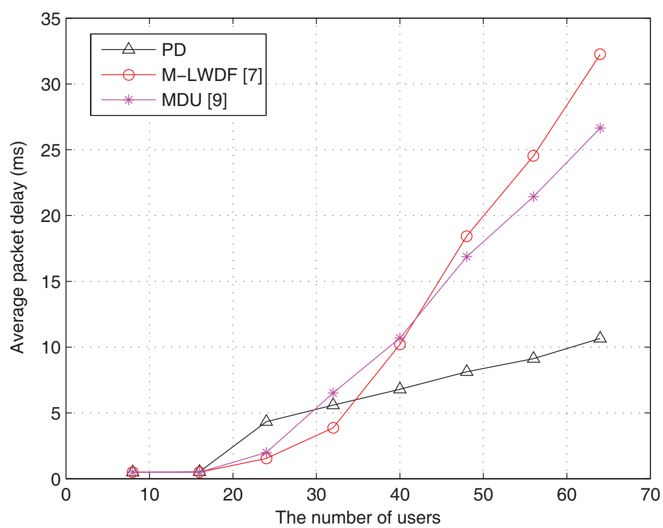


Fig. 4. Impact of the number of users on average voice packet delays of PD, M-LWDF and MDU scheduling schemes (SNR = 20 dB).

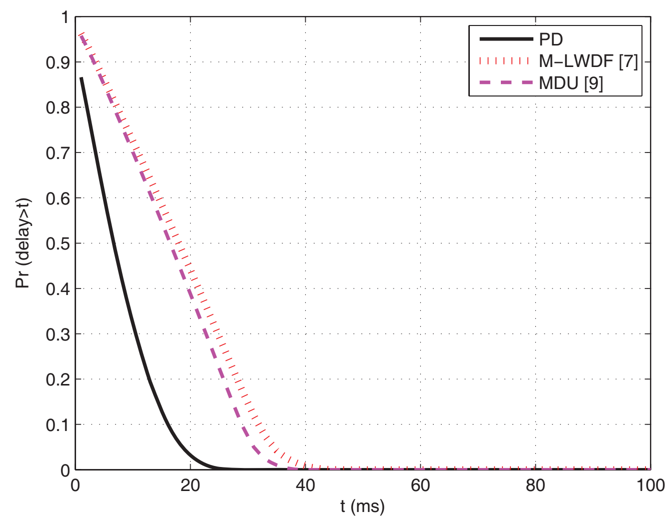


Fig. 6. Voice traffic delay probability of PD, M-LWDF and MDU scheduling schemes with $K = 48$ users (SNR = 20 dB).

enhanced multiuser diversity. On the other hand, the increase in the number of users would cause the increase in the weight difference of the HOL packets of various queues, and thus the decrease in the system throughput. This impact on the system throughput is more significant with MDU/M-LWDF than with PD. With PD, the packets with larger weights are served first, whichever queues they belong to. Whereas with MDU/M-LWDF, all packets in a queue which has a larger weight are served first. Therefore, the system throughputs of MDU and M-LWDF benefit less from multiuser diversity with the increase of the number of users in this case. With a large number of users, the system throughput achieved by PD is much higher than that achieved by MDU/M-LWDF. Within the range of small to moderate number of users ($K = 16 \sim 48$), as shown in Fig. 3, PD outperforms MDU and M-LWDF in terms of the BE throughput. With a large number of users ($K = 56 \sim 64$), the BE throughput achieved by PD is the second highest in the three. Meanwhile, since PD allocates more resources to QoS traffic in this case, it achieves up to two times lower delays for QoS traffic than M-LWDF and MDU, as can be seen in Figs. 4 and 5.

Fig. 4 shows the impact of the number of users on the average voice traffic delay of different cross-layer designs. PD achieves a much lower delay than M-LWDF and MDU with a wide range of the number of users ($K = 40 \sim 64$ users). For instance, the average voice delays for both MDU and M-LWDF with $K = 64$ users are about 30 msec, while it is only 10 msec for PD. A similar trend can be observed in Fig. 5 for the average video traffic delay. With $K = 64$ users, the video delay of PD is around 2 times lower than the delays of MDU and M-LWDF.

The voice and video delay outage probabilities are demonstrated in Figs. 6 and 7, respectively, where $Pr\ (delay > t)$ denotes the probability that a packet has a delay of greater than t. As can be seen in Fig. 6, given the delay tolerance of 100 msec, all the voice packets are guaranteed to be transmitted after 30 msec using PD, which is 10 msec less than the case

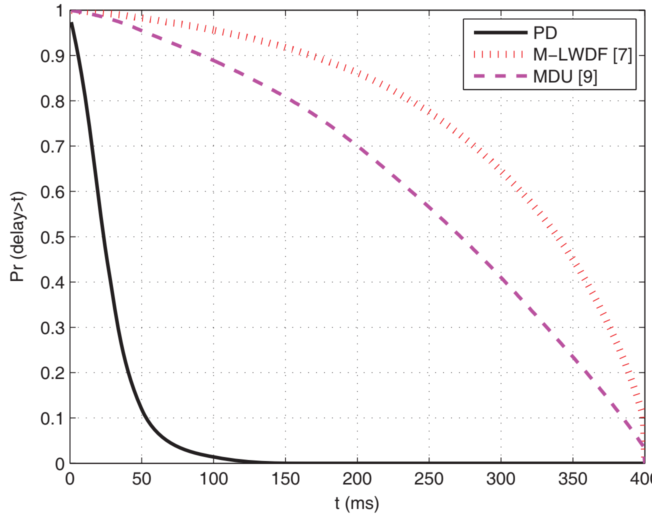


Fig. 7. Video traffic delay probability of PD, M-LWDF and MDU scheduling schemes with $K = 48$ users (SNR $= 20$ dB).

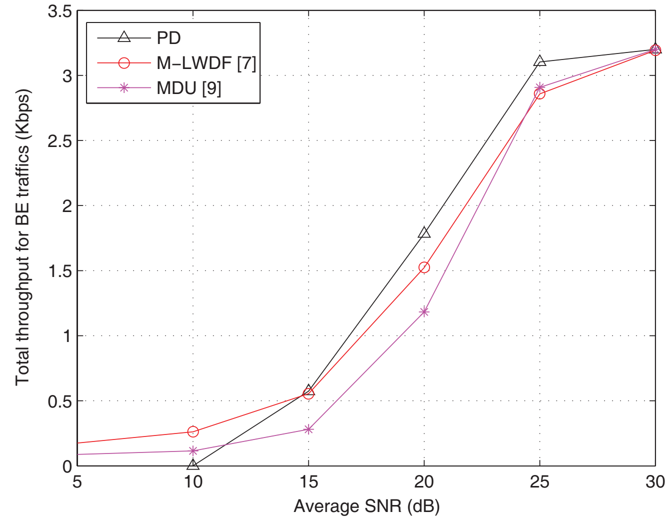


Fig. 9. Impact of SNR on BE traffic throughput of PD, M-LWDF and MDU scheduling schemes with $K = 32$ users.

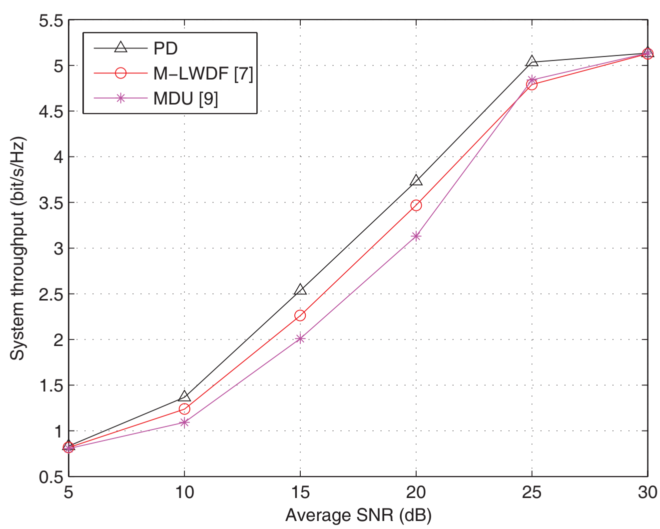


Fig. 8. Impact of SNR on system throughput of PD, M-LWDF and MDU scheduling schemes with $K = 32$ users.

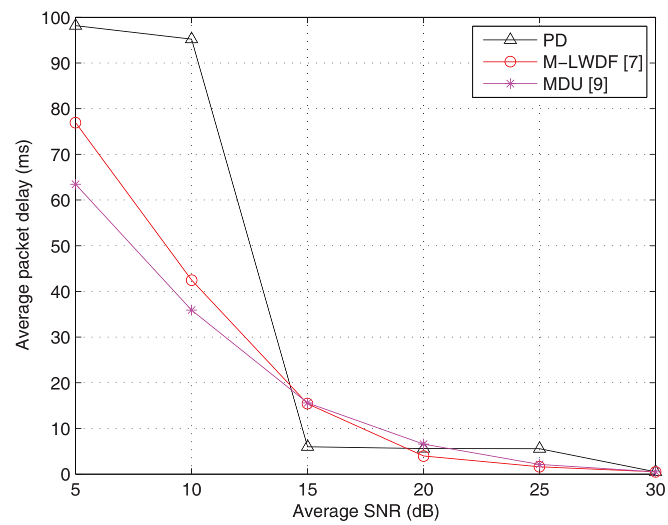


Fig. 10. Impact of SNR on average voice packet delays of PD, M-LWDF and MDU scheduling schemes with $K = 32$ users.

using MDU and M-LWDF. It is shown in Fig. 7 that at a delay time of $t = 160$ msec, PD will have had all video packets transmitted, while M-LWDF and MDU still have more than 90% and 80% outstanding packets to be served, respectively. With a delay tolerance of $t = 400$ msec, the video packet drop rates of M-LWDF and MDU are 3.7% and 9%, respectively, while PD results in a zero packet drop rate.

Figs. 8 to 11 show the impact of SNR on performance of different cross-layer schemes with $K = 32$ users. The system throughput is demonstrated in Fig. 8, where PD outperforms M-LWDF and MDU when the SNR is below 30 dB. At SNR $=$ 15 dB, the bandwidth efficiency achieved by PD is around 12% or 25% higher than those achieved by M-LWDF or MDU, respectively. The throughput of the BE traffic is shown in Fig. 9. PD outperforms M-LWDF and MDU from a moderate to high SNR range. MDU and M-LWDF achieve a performance similar to that of PD only at a high SNR, when there are sufficient resources for them to transmit all the QoS and BE traffic queues (PD, MDU and M-LWDF reach the saturation of the BE traffic throughput at SNR $= 30$ dB).

Fig. 10 demonstrates the average delay of the voice traffic. Within a moderate to high SNR range, the voice delay of PD is almost the same as those of M-LWDF and MDU. At a low SNR, the voice delay of PD is not as good as those of M-LWDF and MDU. This is because M-LWDF and MDU concentrate limited resources on selected voice queues in each time slot and serve all packets in those queues, resulting in a lower average voice delay than PD, which serves the packets having higher delays in all queues first.

The average delay of the video traffic versus SNR is shown in Fig. 11, where PD outperforms M-LWDF and MDU when SNR is at $10 \sim 25$ dB. It can also be observed that the higher the SNR, the more advantages of PD over M-LWDF and MDU. At a low SNR, PD achieves a similar video delay compared to M-LWDF and MDU, while providing the lowest packet drop rate, as verified by simulations.

It can be deduced from Figs. 2 to 11 that with a wide range of the number of users ($K = 28 \sim 64$ users), and

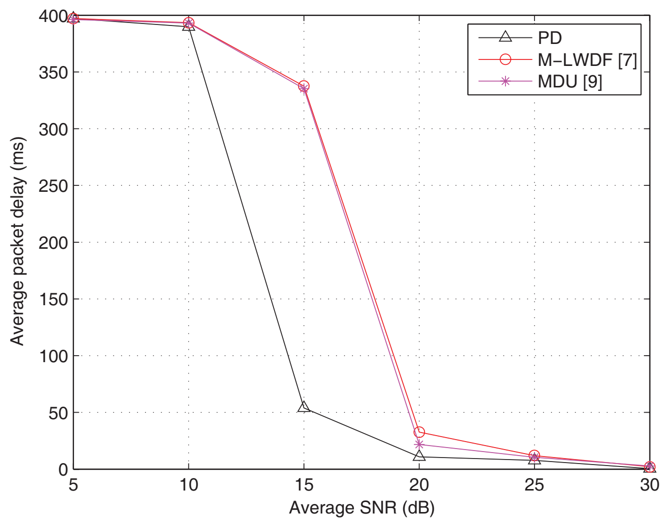


Fig. 11. Impact of SNR on average video packet delays of PD, M-LWDF and MDU scheduling schemes with $K = 32$ users.

with a moderate to high SNR ($15 \sim 25$ dB), PD outperforms the queue based M-LWDF and MDU in terms of the system throughput, BE traffic throughput, and QoS traffic delays, while achieving a complexity reduction of around 3 times over M-LWDF and MDU, as discussed in Section VI.

VIII. Conclusion

We have proposed an adaptive cross-layer design with the PD scheduling at the MAC layer, for downlink multiuser multitasking OFDM systems with heterogeneous traffic. The proposed PD scheduling scheme provides significant performance advantages over the queue based M-LWDF [7] and MDU [9] scheduling schemes in terms of the system throughput, BE traffic throughput, QoS traffic delays and QoS traffic outage probabilities, with a wide range of the number of users, and a moderate to high SNR range. It is also proven to achieve the maximum system stability region. By properly choosing the number of packets used for weight calculation, our user based cross-layer design requires a much lower overall complexity than conventional queue based designs. The proposed suboptimal resource allocation algorithm provides a performance close to that of the optimal algorithm, at a much lower complexity.

Appendix: Proof of Theorem 1

Define $\delta_k = \lambda_k - R_k$ ($\forall k \in \{1, ..., K\}$) as the difference between the data arrival rate λ_k and the data transmission rate R_k for user k in current slot L, which is a finite value since both δ_k and R_k are finite. When the average data arrival rates locate in the interior of the capacity region $\widetilde{C}$, there exists at least one average transmission rate vector $E\{\mathbf{R}\} \in \widetilde{C}$ so that $E\{\lambda_k\} < E\{R_k\}$ for all $k \in \{1, ..., K\}$, *i.e.*, $E\{\delta_k\} < 0$.

We define a Lyapunov function [18] $Y(\mathbf{Q}_Z) = \sum_{k=1}^{K} y(Q_{k,Z})$, where $y(Q_{k,Z})$ is a nonnegative function and its derivative is given by

$$g(Q_{k,Z}) = \frac{\partial y(Q_{k,Z})}{\partial Q_{k,Z}} = \beta_{\max} Q_{k,Z} \tag{21}$$

where $\beta_{\max} = \underset{k,i}{\operatorname{argmax}} \beta_{k,i}$. It is obvious that $g(x) \in [0, \infty)$ ($\forall x < \infty$).

Define $\Delta_{k,Z} = Q_{k,Z+1} - Q_{k,Z}$ as the difference between the queue lengths in two consecutive slots $(Z+1)$ and Z for user k. Using the mean value theorem [21], the Lyapunov drift can be written as:

$$\begin{aligned} &E\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\} \\ &= \sum_{k=1}^{K} E\{y(Q_{k,Z+1}) - y(Q_{k,Z})\} \\ &= \sum_{k=1}^{K} E\{\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z})\} \end{aligned} \tag{22}$$

where $0 < v_{k,Z} < 1$ ($\forall k \in \{1, ..., K\}$). According to (21), we have

$$\begin{aligned} &\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z}) \\ &= \Delta_{k,Z}\beta_{\max} Q_{k,Z} + \Delta_{k,Z}^2 \beta_{\max} v_{k,Z} \end{aligned} \tag{23}$$

We now discuss two scenarios for (23) in the following. Note that the data arriving in slot Z will be served in subsequent slot(s).

Scenario 1: $Q_{k,Z} \le R_k T_{\text{slot}}$, *i.e.*, all the to-be-served data of user k can be sent out during slot Z. Meanwhile, the amount of data that arrives in slot Z and is to be served in subsequent slot(s) is $\lambda_k T_{\text{slot}}$. Hence, the queue length for user k at the beginning of slot $(Z+1)$ is $Q_{k,Z+1} = \lambda_k T_{\text{slot}}$, and the difference between the queue lengths of slots $(Z+1)$ and Z for user k is $\Delta_{k,Z} = Q_{k,Z+1} - Q_{k,Z} = \lambda_k T_{\text{slot}} - Q_{k,Z}$. Since $0 \le Q_{k,Z} \le R_k T_{\text{slot}}$, we have $\delta_k T_{\text{slot}} \le \Delta_{k,Z} \le \lambda_k T_{\text{slot}}$. In this case, it is easy to derive that there exists an upper bound $\sigma_1 \in (0, \infty)$ for (23), *i.e.*, $\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z}) \le \sigma_1$.

Scenario 2: $Q_{k,Z} > R_k T_{\text{slot}}$, *i.e.*, not all the to-be-served data of user k can be sent out during slot Z. Hence, the data of size $(Q_{k,Z} - R_k T_{\text{slot}})$ is left unserved at the end of slot Z. Meanwhile, the data of size $\lambda_k T_{\text{slot}}$ arrives in slot Z, waiting to be served in subsequent slot(s). Thus, the to-be-served queue length for user k at the beginning of slot $(Z+1)$ is $Q_{k,Z+1} = Q_{k,Z} - R_k T_{\text{slot}} + \lambda_k T_{\text{slot}} = Q_{k,Z} + \delta_k T_{\text{slot}}$. Thus, the difference between queue lengths of slots $(Z+1)$ and Z for user k is $\Delta_{k,Z} = Q_{k,Z+1} - Q_{k,Z} = \delta_k T_{\text{slot}}$. In this case, it is easy to derive that there exists an upper bound $\sigma_2 \in (0, \infty)$ for $\Delta_{k,Z}^2 \beta_{\max} v_{k,Z}$ in (23), *i.e.*, $\Delta_{k,Z}^2 \beta_{\max} v_{k,Z} \le \sigma_2$. Therefore, we have $\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z}) \le \sigma_2 + \delta_k T_{\text{slot}} \beta_{\max} Q_{k,Z}$.

Using the above discussion for two scenarios, we can obtain:

$$\begin{aligned} &E\{\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z})\} \\ &\le \sigma_1 + \sigma_2 + E\{\delta_k T_{\text{slot}} \beta_{\max} Q_{k,Z}\} \\ &\le \sigma_1 + \sigma_2 + \overline{\delta}_{\max} T_{\text{slot}} E\{\beta_{\max} Q_{k,Z}\} \end{aligned} \tag{24}$$

where $\overline{\delta}_{\max} = \underset{k}{\operatorname{argmax}} \left(E\left\{\delta_k\right\}\right) < 0$. Since $\beta_{\max} Q_{k,Z} \ge \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}_{k,i}^Z} W_{k,i,l} \ge 0$ can be easily derived by using (15) and (18), (24) becomes

$$\begin{aligned} &E\{\Delta_{k,Z} g(Q_{k,Z} + v_{k,Z}\Delta_{k,Z})\} \\ &\le \sigma_1 + \sigma_2 + \overline{\delta}_{\max} T_{\text{slot}} \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}_{k,i}^Z} E\left\{W_{k,i,l}\right\} \end{aligned} \tag{25}$$

Substituting (25) into (22) leads to

$$E\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\} \le K(\sigma_1 + \sigma_2) + \overline{\delta}_{\max} T_{\text{slot}} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \tag{26}$$

Letting $a = K(\sigma_1 + \sigma_2)$ and $b = -\overline{\delta}_{\max} T_{\text{slot}}$, (26) reduces to

$$E\left\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\right\} \le a - b \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \quad \forall Z \in [1, \infty) \tag{27}$$

Using (27), the Lyapunov drift $E\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\}$ averaged over slots $Z = 1$ to P is given by

$$\begin{aligned} &\frac{1}{P} \sum_{Z=1}^{P} E\left\{Y(\mathbf{Q}_{Z+1}) - Y(\mathbf{Q}_Z)\right\} \\ &= \frac{1}{P}\left[E\left\{Y(\mathbf{Q}_{P+1}) - Y(\mathbf{Q}_1)\right\}\right] \\ &\le a - \frac{b}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \end{aligned} \tag{28}$$

Since $Y(\mathbf{Q}_Z) \ge 0$ ($\forall Z \in [1, \infty)$), it can be derived from (28) that

$$\begin{aligned} &\frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \\ &\le \frac{a}{b} + \frac{1}{bP}\left[E\left\{Y(\mathbf{Q}_1) - Y(\mathbf{Q}_{P+1})\right\}\right] \\ &\le \frac{a}{b} + \frac{E\left\{Y(\mathbf{Q}_1)\right\}}{bP} \end{aligned} \tag{29}$$

Letting $P \to \infty$ and taking $\limsup$ of (29) yield

$$\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \le \frac{a}{b} \tag{30}$$

According to (15), we can obtain

$$\begin{aligned} &\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{D_{k,i,l}\} \\ &= \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z(U)}} E\left\{\frac{W_{k,i,l}}{\beta_{k,i}}\right\} \\ &+ \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \overline{\mathbb{L}_{k,i}^{Z(U)}}} E\left\{\frac{(C_{k,i,l}+1) W_{k,i,l}}{\beta_{k,i}}\right\} \\ &= \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\left\{\frac{W_{k,i,l}}{\beta_{k,i}}\right\} \\ &+ \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \overline{\mathbb{L}_{k,i}^{Z(U)}}} E\left\{\frac{C_{k,i,l} W_{k,i,l}}{\beta_{k,i}}\right\} \end{aligned} \tag{31}$$

where $\mathbb{L}_{k,i}^{Z(U)}$ denotes the set of packets belonging to queue i of user k and falling in the guard interval at slot Z, and $\overline{\mathbb{L}_{k,i}^{Z(U)}}$ denotes the set consisting of the rest packets. Since $W_{k,i,l} \in [0, \infty)$ and $\beta_{k,i} \in [1, \infty)$, the first term of (31) satisfies

$$\begin{aligned} &\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\left\{\frac{W_{k,i,t}}{\beta_{k,i}}\right\} \\ &< \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} \end{aligned} \tag{32}$$

In the second term of (31), $C_{k,i,l}$ is in the range of $0 < C_{k,i,l} \le U_{k,i} - G_{k,i}$. Thus, there exists $M \in [0, \infty)$ so that $C_{k,i,l} W_{k,i,l} / \beta_{k,i} < M$. Letting $U_{\max} = \underset{k,i}{\operatorname{argmax}} U_{k,i}$ and $I_{\max} = \underset{k}{\operatorname{argmax}} I_k$, the second term of (31) satisfies

$$\begin{aligned} &\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \overline{\mathbb{L}_{k,i}^{Z(U)}}} E\left\{\frac{C_{k,i,l} W_{k,i,l}}{\beta_{k,i}}\right\} \\ &< K I_{\max} M (U_{k,i} - G_{k,i}) < K I_{\max} M U_{\max} \end{aligned} \tag{33}$$

Using (32) and (33), (31) can be written as

$$\begin{aligned} &\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{D_{k,i,l}\} \\ &< \limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{W_{k,i,l}\} + K I_{\max} M U_{\max} \end{aligned} \tag{34}$$

Substituting (30) into (34), and using $a, b \in (0, \infty)$ yield

$$\begin{aligned} &\limsup_{P\to\infty} \frac{1}{P} \sum_{k=1}^{K} \sum_{i=1}^{I_k} \sum_{Z=1}^{P} \sum_{l \in \mathbb{L}_{k,i}^{Z}} E\{D_{k,i,l}\} \\ &< \frac{a}{b} + K I_{\max} M U_{\max} < \infty \end{aligned} \tag{35}$$

Therefore, the system is stable as defined by (20).

References

[1] G. C. Song and Y. Li, "Utility-based resource allocation and scheduling in OFDM-based wireless broadband networks," *IEEE Commun. Mag.*, vol. 43, pp. 127–134, Dec. 2005.

[2] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 21, pp. 171–178, Feb. 2003.

[3] C. Anton-Haro, P. Svedman, M. Bengtsson, A. Alexiou, and A. Gameiro, "Cross-layer scheduling for multi-user MIMO systems," *IEEE Commun. Mag.*, vol. 44, pp. 39–45, Sept. 2006.

[4] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit and power allocation," *IEEE JSAC*, vol. 17, pp. 1747–1758, Oct. 1999.

[5] D. S. W. Hui, V. K. N. Lau, and W. H. Lam, "Cross-layer design for OFDMA wireless systems with heterogeneous delay requirements," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2872–2880, Aug. 2007.

[6] D. Veronesi, S. Tomasin, and N. Benvenuto, "Cross-layer optimization for multimedia traffic in CDMA cellular networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 1379–1388, Apr. 2008.

[7] M. Andrews, K. Kumaran, K. Ramanan, A. Stolyar, P. Whiting, and R. Vijayakumar, "Providing quality of service over a shared wireless link," *IEEE Commun. Mag.*, vol. 2, pp. 150–154, Feb. 2001.

[8] S. Ryu, B. Ryu, H. Seo, and M. Shin, "Urgency and efficiency based packet scheduling algorithm for OFDMA wireless system," in *Proc. IEEE ICC'2005*, Seoul, South Korea, 2005.

[9] G. C. Song, "Cross-layer resource allocation and scheduling in wireless multicarrier networks," Ph.D. dissertation, Georgia Institute of Technology, 2005.

[10] Y. Yu and W. Zhou, "Resource allocation for OFDMA system based on genetic algorithm," in *Proc. IEEE IWCLD'2007*, vol. 10, Tai'an, China, Sep. 2007, pp. 65–69.

[11] K. B. Johnsson and D. C. Cox, “An adaptive cross-layer scheduler for improved QoS support of multiclass data services on wireless systems,” *IEEE J. Sel. Areas Commun.*, vol. 23, pp. 334–343, Feb. 2005.

[12] Z. K. Shen and J. G. Andrews, “Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints,” *IEEE Trans. Wireless Commun.*, vol. 4, pp. 2726–2737, Nov. 2005.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[14] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 1991.

[15] J. D. C. Little, “A proof of the queueing formula $l = \lambda w$,” *Operations Research*, vol. 9, pp. pp. 383–387, May 1961.

[16] V. K. N. Lau and Y. K. R. Kwok, *Channel-Adaptive Technologies and Cross-Layer Designs for Wireless Systems with Multiple Antennas: Theory and Applications*. Wiley, 2006.

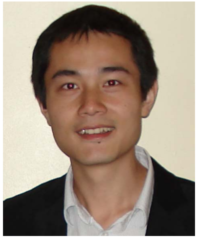
[17] L. Georgiadis, M. J. Neely, and L. Tassiulas, *Resource Allocation and Cross-Layer Control in Wireless Networks*. Now Publishers Inc, 2006.

[18] A. Erylimaz, R. Srikant, and J. R. Perkins, “Stable scheduling policies for fading wireless channels,” *IEEE/ACM Trans. Networking*, vol. 13, pp. 411–424, Apr. 2005.

[19] T. H. Corman, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*. MIT Press, 2001.

[20] J. Bentley, *Programming Pearls*. Addison-Wesley, 1999.

[21] W. Rudin, *Principles of Mathematical Analysis*. McGraw-Hill, 1964.



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