Robust Beamforming for Wireless Information and Power Transmission

Zhengzheng Xiang and Meixia Tao, *Senior Member, IEEE*

Abstract—In this letter, we study the robust beamforming problem for the multi-antenna wireless broadcasting system with simultaneous information and power transmission, under the assumption of imperfect channel state information (CSI) at the transmitter. Following the worst-case deterministic model, our objective is to maximize the worst-case harvested energy for the energy receiver while guaranteeing that the rate for the information receiver is above a threshold for all possible channel realizations. Such problem is nonconvex with infinite number of constraints. Using certain transformation techniques, we convert this problem into a relaxed semidefinite programming problem (SDP) which can be solved efficiently. We further show that the solution of the relaxed SDP problem is always rank-one. This indicates that the relaxation is tight and we can get the optimal solution for the original problem. Simulation results are presented to validate the effectiveness of the proposed algorithm.

Index Terms—Energy harvesting, beamforming, worst-case robust design, semidefinite programming.

I. INTRODUCTION

ENERGY harvesting for wireless communication is able to extend the flying power of handheld devices and advocacy for green communication [1]- [3]. With the aid of this promising technique, the transmitter can transfer power to terminals who need to harvest energy to charge their devices, which is especially important for energy-constrained wireless networks. Beamforming is another promising technique which exploits channel state information (CSI) at the transmitter for information transmission [4]- [6]. In wireless networks with simultaneous transmission of power and information, beamforming is anticipated to play an important role as well.

The beamforming design with perfect knowledge of CSI at the transmitter was first considered in [7] to characterize the rate-energy region in a simplified three-node wireless broadcasting system. In practical scenarios, perfect knowledge of CSI may not be available due to many factors such as inaccurate channel estimation, quantization error, and time delay of the feedback.

The goal of this letter is to investigate the robust beamformer design with imperfect CSI for simultaneous information transmission and energy harvesting. In general, there are two classes of models to characterize imperfect CSI: the stochastic and deterministic (or worst-case) models. In the stochastic model, the CSI errors are often modeled as Gaussian random variables and the system design is then
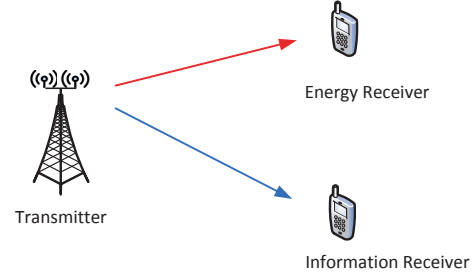
Fig. 1. A three-node wireless MISO system for simultaneous information and power transmission.

based on optimizing the average or outage performance [8], [9]. Alternatively, the deterministic model assumes that the CSI uncertainty, though not exactly known, is bounded by possible values [10], [11]. In this case, the system is optimized to achieve a given quality of service (QoS) for every possible CSI error if the problem is feasible, thereby, achieving absolute robustness. It was also shown in [12] that a bounded worst-case model is able to cope with quantization errors in CSI. In this letter, we shall employ the worst-case approach to address the robust beamforming design problem.

Consider the three-node system shown in Fig. 1, where we assume that the transmitter only has imperfect knowledge of the channels to both the information receiver and energy receiver. We formulate the worst-case robust beamforming problem for harvested energy maximization at the energy receiver while ensuring a minimum target rate at the information receiver. Since the original problem has infinite constraints due to the channel uncertainties, we first transform it into an easier problem which has finite constraints but is still nonconvex. Then we apply the semidefinite relaxation (SDR) and obtain a semidefinite programming (SDP) problem which can be solved efficiently. Finally we show that the optimal solution of the SDP problem is always rank-one, which means that the relaxation is tight and we can obtain the optimal solution of the original problem.

The rest of this letter is organized as follows. In Section II, the system model and the problem formulation are presented. Section III presents our proposed algorithm to find the solutions to the robust problems using convex optimization and rank relaxation, and show its optimality. Simulation results are given in Section IV. Finally, Section V concludes this letter.

Notation: $(\cdot)^H$ and $\mathrm{Tr}\{\cdot\}$ stand for Hermitian transpose and the trace respectively. $|x|$ denotes the absolute value of the scalar x and $\|\mathbf{x}\|$ denotes the Euclidean norm of the vector $\mathbf{x}$. The function $\log(.)$ is taken to the base 2.

II. SYSTEM MODEL AND PROBLEM FORMULATION

With reference to Fig. 1, we consider a three-node multiple-input single-output (MISO) communication system, where the

transmitter has N antennas and each receiver has a single antenna. Let $\mathbf{h}^H$ and $\mathbf{g}^H$ denote the frequency-flat quasi-static $1 \times N$ complex channel vectors from the transmitter to the information receiver and the energy receiver respectively, and s denote the transmitted symbol. Then the received signals at the information receiver and the energy receiver are given by, respectively,

$$y_i = \mathbf{h}^H \mathbf{w} s + z_i \tag{1}$$
$$y_e = \mathbf{g}^H \mathbf{w} s + z_e \tag{2}$$

where $\mathbf{w}$ is the $N \times 1$ beamforming vector applied to the transmitter, and z_i and z_e are the additive white circularly symmetric Gaussian complex noise with variance $\sigma^2/2$ on each of their real and imaginary components.

For the energy receiver, it will harvest energy from its received signal. Thanks to the law of energy conservation, we can assume that the total harvested RF-band power, denoted by Q, is proportional to the power of the received baseband signal, i.e.,

$$Q = \eta|\mathbf{g}^H \mathbf{w}|^2 \tag{3}$$

where η is the efficiency ratio at the energy receiver for converting the harvested energy to electrical energy to be stored. Here we simply assume that $\eta = 1$ and the details for the converting process is beyond the scope of this letter.

Our objective is to maximize the harvested energy for the energy receiver while guaranteeing that the information rate for the information receiver is above a threshold. Mathematically, the problem is expressed as follows:

$$\mathbf{P}_0 : \max_{\mathbf{w}} \; |\mathbf{g}^H \mathbf{w}|^2 \tag{4}$$
$$\text{s.t.} \; \log\left(1 + \frac{|\mathbf{h}^H \mathbf{w}|^2}{\sigma^2}\right) \geq r \tag{5}$$
$$\|\mathbf{w}\|^2 \leq P \tag{6}$$

where r is the rate target for the information receiver and P is the power constraint at the transmitter. Similar problem has been considered in [7] with the objective of maximizing information rate subject to a minimum energy threshold.

Herein we consider that the transmitter has imperfect CSI of both receivers. In particular, the channels are modeled as

$$\mathbf{h} = \widehat{\mathbf{h}} + \Delta\mathbf{h} \tag{7}$$
$$\mathbf{g} = \widehat{\mathbf{g}} + \Delta\mathbf{g} \tag{8}$$

where $\widehat{\mathbf{h}}$ and $\widehat{\mathbf{g}}$ denote the estimated CSI known at the transmitter, $\Delta\mathbf{h}$ and $\Delta\mathbf{g}$ are the error vectors. We assume no statistical knowledge about the error vectors but that they are bounded by some possible values (also known to the transmitter) as

$$\|\Delta\mathbf{h}\| \leq \varepsilon \tag{9}$$
$$\|\Delta\mathbf{g}\| \leq \varepsilon \tag{10}$$

where ε is the radius of the uncertainty region. We assume that both the two receivers have perfect CSI knowledge.

To take the CSI errors into account, the problem $\mathbf{P}_0$ based on worst-case criterion can be formulated as

$$\mathbf{P}_1 : \max_{\mathbf{w}} \min_{\|\Delta\mathbf{g}\| \leq \varepsilon} |(\widehat{\mathbf{g}} + \Delta\mathbf{g})^H \mathbf{w}|^2 \tag{11}$$
$$\text{s.t.} \; \log\left(1 + \frac{|(\widehat{\mathbf{h}} + \Delta\mathbf{h})^H \mathbf{w}|^2}{\sigma^2}\right) \geq r, \forall \, \|\Delta\mathbf{h}\| \leq \varepsilon \tag{12}$$
$$\|\mathbf{w}\|^2 \leq P. \tag{13}$$

Since $\log(1 + x)$ is monotonically increasing for positive x, problem $\mathbf{P}_1$ can be reformulated as below

$$\mathbf{P}_1 : \max_{\mathbf{w}} \min_{\|\Delta\mathbf{g}\| \leq \varepsilon} |(\widehat{\mathbf{g}} + \Delta\mathbf{g})^H \mathbf{w}|^2 \tag{14}$$
$$\text{s.t.} \; |(\widehat{\mathbf{h}} + \Delta\mathbf{h})^H \mathbf{w}|^2 \geq \sigma^2(2^r - 1), \forall \|\Delta\mathbf{h}\| \leq \varepsilon \tag{15}$$
$$\|\mathbf{w}\|^2 \leq P. \tag{16}$$

It can be seen that the goal of the problem $\mathbf{P}_1$ is to maximize the harvested energy for the worst channel realization while guaranteeing that the information rate is above a threshold for all possible channel realizations.

III. SEMIDEFINITE PROGRAMMING SOLUTION

The key challenges in problem $\mathbf{P}_1$ are the channel uncertainties and the nonconvex constraints, which cause that $\mathbf{P}_1$ is a semi-infinite nonconvex quadratically constrained quadratic programming (QCQP) problem. It is well known that the general nonconvex QCQP problem is NP-hard and thus, intractable. However, as we will show in the following, due to the special structure of the objective function and the constraints, problem $\mathbf{P}_1$ can be reformulated as a convex SDP problem and solved optimally.

We first transform the above problem into a more tractable form. For the objective function of $\mathbf{P}_1$ in (14), we simplify it using an approach similar to the one developed in [10] and [13]. According to triangle inequality, we obtain

$$|\widehat{\mathbf{g}}^H \mathbf{w} + \Delta\mathbf{g}^H \mathbf{w}| \geq |\widehat{\mathbf{g}}^H \mathbf{w}| - |\Delta\mathbf{g}^H \mathbf{w}|. \tag{17}$$

Then applying the Cauchy-Schwarz inequality to the second term in the right-hand-side (RHS) of (17), we have

$$|\Delta\mathbf{g}^H \mathbf{w}| \leq \|\Delta\mathbf{g}\| \cdot \|\mathbf{w}\| \leq \varepsilon\|\mathbf{w}\|. \tag{18}$$

Plugging (18) into (17), we then have that

$$|\widehat{\mathbf{g}}^H \mathbf{w} + \Delta\mathbf{g}^H \mathbf{w}| \geq |\widehat{\mathbf{g}}^H \mathbf{w}| - |\Delta\mathbf{g}^H \mathbf{w}| \geq |\widehat{\mathbf{g}}^H \mathbf{w}| - \varepsilon\|\mathbf{w}\|. \tag{19}$$

An important observation about problem $\mathbf{P}_1$ is that its optimal solution is obtained only when the constraint in (16) is active, i.e., the transmitter should work with full power. Then we have

$$|\widehat{\mathbf{g}}^H \mathbf{w} + \Delta\mathbf{g}^H \mathbf{w}| \geq |\widehat{\mathbf{g}}^H \mathbf{w}| - \varepsilon\sqrt{P}. \tag{20}$$

The inequality becomes equality when $\Delta\mathbf{g} = -\frac{\mathbf{w}}{\|\mathbf{w}\|}\varepsilon e^{-j\theta}$, where θ is the angle between $\widehat{\mathbf{g}}^H$ and $\mathbf{w}$. Note that it has been assumed that $|\widehat{\mathbf{g}}^H \mathbf{w}| \geq \varepsilon\|\mathbf{w}\|$ in (19), and $|\widehat{\mathbf{g}}^H \mathbf{w}| \geq \varepsilon\sqrt{P}$ in (20). This assumption essentially means that the errors $\Delta\mathbf{g}$ is sufficiently small or equivalently ε is sufficiently small. It is a practical assumption since large channel estimation errors can cause large beamforming errors and no robustness can

be guaranteed in such case. Then combining (17)-(20), we conclude that

$$\min_{\|\Delta\mathbf{g}\|\leq\varepsilon} |(\widehat{\mathbf{g}}+\Delta\mathbf{g})^H\mathbf{w}|^2 = \left||\widehat{\mathbf{g}}^H\mathbf{w}| - \varepsilon\sqrt{P}\right|^2. \tag{21}$$

For the infinite number of constraints in (15), we can similarly have that

$$|\widehat{\mathbf{h}}^H\mathbf{w} + \Delta\mathbf{h}^H\mathbf{w}| \geq |\widehat{\mathbf{h}}^H\mathbf{w}| - \varepsilon\sqrt{P}. \tag{22}$$

Here, the equality holds when $\Delta\mathbf{h} = -\frac{\mathbf{w}}{\|\mathbf{w}\|}\varepsilon e^{-j\varphi}$ with φ being the angle between $\widehat{\mathbf{h}}^H$ and $\mathbf{w}$. Then in order to meet the constraints for all possible $\Delta\mathbf{h}$, we just need to satisfy the following

$$|\widehat{\mathbf{h}}^H\mathbf{w}| - \varepsilon\sqrt{P} \geq \sigma\sqrt{2^r - 1}. \tag{23}$$

Then the robust beamforming problem $\mathbf{P_1}$ can be rewritten as follows

$$\mathbf{P_1}: \quad \max_{\mathbf{w}} \ |\widehat{\mathbf{g}}^H\mathbf{w}|^2 \tag{24}$$

$$\text{s.t.} \ |\widehat{\mathbf{h}}^H\mathbf{w}|^2 \geq \left(\varepsilon\sqrt{P} + \sigma\sqrt{2^r - 1}\right)^2 \tag{25}$$

$$\|\mathbf{w}\|^2 \leq P. \tag{26}$$

Although the problem $\mathbf{P_1}$ is much easier now, it is still a nonconvex QCQP problem. We then apply the semidefinite relaxation and obtain the following relaxed problem:

$$\mathbf{P_2}: \quad \max_{\mathbf{W}\succeq\mathbf{0}} \ \text{Tr}\{\widehat{\mathbf{G}}\mathbf{W}\} \tag{27}$$

$$\text{s.t.} \quad \text{Tr}\{\widehat{\mathbf{H}}\mathbf{W}\} \geq \left(\varepsilon\sqrt{P} + \sigma\sqrt{2^r - 1}\right)^2 \tag{28}$$

$$\text{Tr}\{\mathbf{W}\} \leq P \tag{29}$$

where $\widehat{\mathbf{G}} = \widehat{\mathbf{g}}\widehat{\mathbf{g}}^H$ and $\widehat{\mathbf{H}} = \widehat{\mathbf{h}}\widehat{\mathbf{h}}^H$. Notice that the rank-one constraint has been dropped and $\mathbf{P_2}$ is a relaxed version of $\mathbf{P_1}$. The problem $\mathbf{P_2}$ is a standard SDP problem which is convex and can be solved efficiently using the software package [14].

At this point, an important question is that whether the optimal solution of $\mathbf{P_2}$ is rank-one. If $\mathbf{W}$ is rank-one, then the optimal beamformer for the original problem $\mathbf{P_1}$ can be extracted by eigenvalue decomposition. Otherwise, the solution of $\mathbf{P_2}$ is only an upper bound of $\mathbf{P_1}$ and the beamformer extracted from $\mathbf{W}$ is not guaranteed to be globally optimal. Generally there is no guarantee that an algorithm for solving SDP problems will give the desired rank-one solution. However, in some special cases such as [15]-[17], the relaxation is proven to be exact and thus there always exists a rank-one solution. Whether the relaxation is tight for our proposed algorithm will be addressed in the following theorem.

Theorem 1: The optimal solution $\mathbf{W}$ for problem $\mathbf{P_2}$ is rank-one.

Proof: Please refer to Appendix A. ∎

According to Theorem 1, we can see that problem $\mathbf{P_2}$ is indeed equivalent to the original problem $\mathbf{P_1}$, which means that the relaxation is tight. So in order to solve the problem $\mathbf{P_1}$, we first solve the SDP problem $\mathbf{P_2}$ and obtain the resulting rank-one matrix $\mathbf{W}^\star$. Apply eigenvalue decomposition on $\mathbf{W}^\star$ as

$$\mathbf{W}^\star = \alpha^\star\mathbf{w}^\star\mathbf{w}^{\star H}. \tag{30}$$

The optimal solution of $\mathbf{P_1}$ is then obtained as $\mathbf{w} = \sqrt{\alpha^\star}\mathbf{w}^\star$.
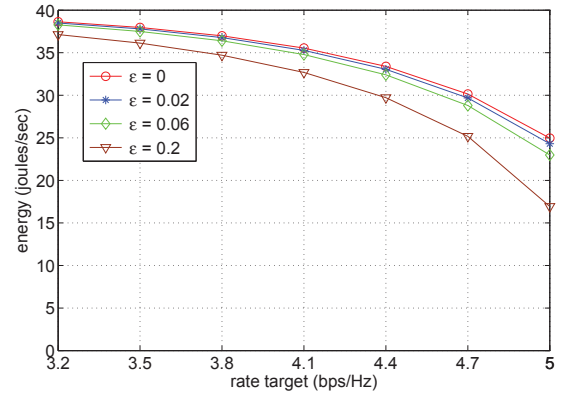

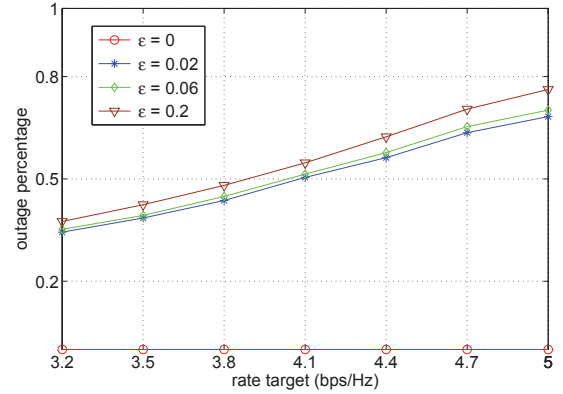Fig. 2.   Average harvested energy for the robust beamforming design.

Fig. 3.   Outage percentage for the nonrobust beamforming design.

IV. SIMULATION RESULTS

In this section, we present numerical results to evaluate the performance of the proposed robust beamforming algorithm. We consider the three-node MISO system in which the transmitter has four antennas ($N = 4$). We set the power $P = 10$ and noise covariance $\sigma^2 = 1$. The channel from the transmitter to each receiver is assumed as the normalized Rayleigh fading channel. The rate target for the information receiver is set to be smaller than $\log(1 + 10\|\mathbf{h}\|^2)$ in order for the problem to be feasible. For simplicity, we normalize these channel vectors with respect to the number of transmit antennas as $\|\mathbf{h}\|^2 = \|\mathbf{g}\|^2 = 4$. Thus the feasible region for the rate target is $0 \leq r \leq \log(1 + 40)$. A total of 100 independent normalized channel realizations are simulated. For each channel realization, 100 channel uncertainty samples are generated.

In Fig. 2, we plot the average harvested energy versus different targets of information rate for different levels of bounded channel uncertainty. The special case with the CSI (i.e., $\varepsilon = 0$) is also simulated. It can be seen that the performance loss is small when the CSI error is not big. Also the performance gap increases when the rate target becomes larger.

In order to show how important it is to take the channel uncertainty into account when designing the beamformers, let us assume that the beamforming design takes place under the assumption of perfect CSI at the transmitter while in fact there is some uncertainty associated with the CSI used in the design problem, which we call the "nonrobust beamforming

design". Fig. 3 shows percentage of outage [1] at different rate targets for the nonrobust design. We observe that the channel uncertainty, when not considered in the design process, leads to frequent violations of the rate target at the information receiver. However, for our proposed worst-case robust beam-forming algorithm, the rate target is always satisfied and no outage happens.

V. CONCLUSION

In this letter, we consider the worst-case robust beamforming design for the wireless communication system with both information and energy receivers when the CSI is imperfect. By means of semidefinite relaxation, we transform the original robust design problem into a SDP problem. Then we prove that such relaxation is tight and we can always obtain the optimal solution of the original problem. The performance of the proposed beamforming algorithm has been demonstrated by simulations. Future research directions may include the robust beamforming design for the more general broadcasting systems with multiple information receivers and multiple energy receivers.

APPENDIX A
PROOF OF THEOREM 1

Denote $\beta \triangleq \left(\varepsilon \sqrt{P} + \sigma \sqrt{2^r - 1} \right)^2$, the Lagrangian of $\mathbf{P}_2$ is given by

$$L(\mathbf{W}, \lambda, \mu) = \text{Tr}\{\widehat{\mathbf{G}}\mathbf{W}\} + \lambda \left(\text{Tr}\{\widehat{\mathbf{H}}\mathbf{W}\} - \beta \right) \quad (31)$$
$$- \mu (\text{Tr}\{\mathbf{W}\} - P),$$

where λ and μ are the dual variables. The Lagrange dual function is then defined as

$$g(\lambda, \mu) = \max_{\mathbf{W} \succeq \mathbf{0}} L(\mathbf{W}, \lambda, \mu). \quad (32)$$

Since $\mathbf{P}_2$ is convex with strong duality, we can solve it by solving its dual problem

$$\mathbf{D}_2 : \min_{\lambda \geq 0, \mu \geq 0} g(\lambda, \mu). \quad (33)$$

Denote the optimal solution of $\mathbf{D}_2$ as $(\lambda^\star, \mu^\star)$, then the matrix $\mathbf{W}^\star$ that maximizes $L(\mathbf{W}, \lambda^\star, \mu^\star)$ is the optimal solution of $\mathbf{P}_2$, which means that we can find $\mathbf{W}^\star$ through the following problem

$$\max_{\mathbf{W} \succeq \mathbf{0}} \text{Tr}\{\widehat{\mathbf{G}}\mathbf{W}\} - \text{Tr}\left\{ \left(\mu^\star \mathbf{I} - \lambda^\star \widehat{\mathbf{H}} \right) \mathbf{W} \right\} \quad (34)$$

where the constant terms has been discarded. In order for problem (34) to have a bounded value, it is shown as follows that the matrix $\mu^\star \mathbf{I} - \lambda^\star \widehat{\mathbf{H}}$ should be positive definite. Suppose $\mu^\star \mathbf{I} - \lambda^\star \widehat{\mathbf{H}}$ is not positive definite, then we can choose $\mathbf{W} = t\mathbf{w}\mathbf{w}^H$, where $t > 0$ and $\text{Tr}\{t(\mu^\star \mathbf{I} - \lambda^\star \widehat{\mathbf{H}})\mathbf{w}\mathbf{w}^H\} \leq 0$. Due to the independence of $\widehat{\mathbf{g}}$ and $\widehat{\mathbf{h}}$, it follows that $\text{Tr}\{t\widehat{\mathbf{G}}\mathbf{w}\mathbf{w}^H\} > 0$ with probability one. Let $t \to +\infty$, the optimal value in (34) will be unbounded, which is a contradiction of the optimality of $(\lambda^\star, \mu^\star)$.

---

[1] We call the outage happens when the rate target is not satisfied at the information receiver.

Define $\mathbf{Q} \triangleq (\mu^\star \mathbf{I} - \lambda^\star \widehat{\mathbf{H}}) \succ \mathbf{0}$ and let $\overline{\mathbf{W}} = \mathbf{Q}^{1/2}\mathbf{W}\mathbf{Q}^{1/2}$, the problem in (34) is then rewritten as

$$\max_{\overline{\mathbf{W}} \succeq 0} (\mathbf{Q}^{-1/2}\widehat{\mathbf{g}})^H \overline{\mathbf{W}}(\mathbf{Q}^{-1/2}\widehat{\mathbf{g}}) - \text{Tr}\{\overline{\mathbf{W}}\}. \quad (35)$$

Then we claim that the optimal solution of (35) is always rank-one. Suppose the optimal solution $\overline{\mathbf{W}}^\star$ is not rank-one, without loss of generality, we can assume its rank is k $(2 \leq k \leq N)$ and decompose it as $\overline{\mathbf{W}}^\star = \sum_{j=1}^k \alpha_j \overline{\mathbf{w}}_j \overline{\mathbf{w}}_j^H$. Then we choose another $\overline{\mathbf{W}}^{\star\prime} = (\sum_{j=1}^k \alpha_j) \overline{\mathbf{w}}_i \overline{\mathbf{w}}_i^H$, where $i = \arg \max_{j \in \{1, \dots, k\}} |(\mathbf{Q}^{-1/2}\widehat{\mathbf{g}})^H \overline{\mathbf{w}}_j|$. Then $\overline{\mathbf{W}}^{\star\prime}$ can achieve a larger value than $\overline{\mathbf{W}}^\star$, which is a contradiction.

From the above discussions, it is known that $\overline{\mathbf{W}}^\star$ is always rank-one. Since $\mathbf{W}^\star = \mathbf{Q}^{-1/2}\overline{\mathbf{W}}^\star\mathbf{Q}^{-1/2}$, we must have that $\mathbf{W}^\star$ is rank-one, which completes the proof of Theorem 1.

REFERENCES

[1] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. 2008 IEEE Int. Symp. Inf. Theory*, pp. 1612–1616.

[2] P. Grover and A. Sahai, "Shannon meets Tesla: wireless information and power transfer," in *Proc. 2010 IEEE Int. Symp. Inf. Theory*, pp. 2363–2367.

[3] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, "Optimal energy management policies for energy harvesting sensor nodes," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, Apr. 2010.

[4] J. Qiu, R. Zhang, Z.-Q. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," in *IEEE Trans. Signal Process.*, vol. 59, no. 11, Nov. 2011.

[5] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, May 2010.

[6] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, Apr. 2010.

[7] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," in *Proc. 2011 IEEE Global Communications Conference*.

[8] A. M. Tulino, A. Lozano, and S. Verdu, "Capacity-achieving input covariance for single-user multi-antenna channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 2, Mar. 2006.

[9] X. Zhang, D. P. Palomar, and B. Ottersten, "Statistically robust design of linear MIMO transceivers," *IEEE Trans. Signal Process.*, vol. 56, no. 8, Aug. 2008.

[10] S. Vorobyov, A. Gershman, and Z. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, Feb. 2003.

[11] J. Wang and D. P. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Process.*, vol. 57, no. 8, Aug. 2009.

[12] M. Botros and T. N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE J. Sel. Topics Signal Process.*, vol. 1, no. 4, pp. Dec. 2007.

[13] K. Phan, S. Vorobyov, N. Sidiropoulos, and C. Tellambura, "Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming," *IEEE Trans. Signal Process.*, vol. 57, no. 6, June 2009.

[14] M. Grant and S. Boyd, *CVX: Matlab software for disciplined convex programming*. Available: http://stanford.edu.boyd/cvx.

[15] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Ant. in Wireless Commun*, L. C. Godara, editor. CRC Press, 2001.

[16] G. Zheng, K. Wong, A. Paulraj, and B. Ottersten, "Robust collaborative-relay beamforming," *IEEE Trans. Signal Process.*, vol. 57, no. 8, Aug. 2009

[17] Y. Huang and S. Zhang, "Complex matrix decomposition and quadratic programming," *Math. Oper. Res.*, vol. 32, no. 3, 2007.