

# Energy-Efficient Optimization for Wireless Information and Power Transfer in Large-Scale MIMO Systems Employing Energy Beamforming

Xiaoming Chen, Xiumin Wang, and Xianfu Chen

**Abstract**—In this letter, we consider a large-scale multiple-input multiple-output (MIMO) system where the receiver should harvest energy from the transmitter by wireless power transfer to support its wireless information transmission. The energy beamforming in the large-scale MIMO system is utilized to address the challenging problem of long-distance wireless power transfer. Furthermore, considering the limitation of the power in such a system, this letter focuses on the maximization of the energy efficiency of information transmission (bit per Joule) while satisfying the quality-of-service (QoS) requirement, i.e. delay constraint, by jointly optimizing transfer duration and transmit power. By solving the optimization problem, we derive an energy-efficient resource allocation scheme. Numerical results validate the effectiveness of the proposed scheme.

**Index Terms**—Energy-efficient resource allocation, wireless information and power transfer, large-scale MIMO system, energy beamforming, QoS guarantee.

## I. INTRODUCTION

WIRELESS power transfer facilitates the battery charging to prolong the lifetime of wireless networks, especially under some extreme conditions, such as battlefield, underwater and body area networks [1]. Thus, wireless power transfer draws considerable research attentions from academics and industry [2]–[5].

In general, wireless power transfer from a power source to a receiver is implemented through electromagnetic propagation [6]. Similar to wireless information transmission, wireless power transfer also suffers from propagation loss, including path loss, shadowing and fast fading. Therefore, transfer efficiency is a critical and challenging issue for wireless power transfer. To solve this problem, multi-antenna techniques are introduced into wireless power transfer to improve the transfer efficiency by using energy beamforming. In [7], the design methods of the optimal transmit beam for power transfer were given in MIMO broadcast systems. Furthermore, considering the imperfect channel state information at the power source, a robust energy beamforming strategy was proposed in [8].

Manuscript received July 14, 2013. The associate editor coordinating the review of this letter and approving it for publication was K. Huang.

This work was supported by the Natural Science Foundation of China (No. 61301102, 61300212), the Natural Science Foundation of Jiangsu Province (No. BK20130820), the open research fund of National Mobile Communications Research Laboratory, Southeast University (No. 2012D16) and the Doctoral Fund of Ministry of Education of China (No. 20123218120022).

Xiaoming Chen is with the College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, and also with the National Mobile Communications Research Laboratory, Southeast University, China (e-mail: chenxiaoming@nuaa.edu.cn).

X. Wang is with the School of Computer and Information, Hefei University of Technology, Hefei, China (e-mail: wxiumin@hfut.edu.cn).

Xianfu Chen is with the VTT Technical Research Centre of Finland, Oulu, Finland (e-mail: xianfu.chen@vtt.fi).

Digital Object Identifier 10.1109/WCL.2013.092813.130514

However, due to the limited number of antennas at the power source, the energy efficiency based on the traditional multi-antenna systems is difficult to satisfy the practical requirement, especially when the transfer distance is not so short [9]. Recently, large-scale MIMO deploying a huge number of antenna at the transmitter is proposed to enormously improve the transmission performance by exploiting its large array gain [10] [11]. For example, in the 60GHz WiFi 802.11/ac, since small size antenna is possible, large-scale antenna beamforming is exactly what is being implemented. Similarly, large-scale MIMO can be used to effectively improve the performance of wireless power transfer.

Intuitively, the ultimate purpose of wireless power transfer is to fulfill the need of the receiver for work. As a simple example, in medical field, the implanted equipment in body is powered through wireless power transfer, and then transmits the medical data to the outside receiver with the harvested energy. However, most of previous analogous works analyze and design the wireless power transfer without taking into consideration the utilization of the harvested energy. In this letter, we consider joint wireless information and power transfer in large-scale MIMO systems employing energy beamforming, where the power receiver transmits the information with the harvested energy, namely wireless powered communication. Since energy harvesting and information transmission are impossibly carried out simultaneously, time slot should be divided into the harvesting and the transmitting components. In order to optimize the performance, it is necessary to determine the optimal time switch point, namely time resource allocation. Moreover, transmit power at the power source, as another important resource, also affects the ultimate performance. Recently, energy-efficient communications, namely green communication, gain more and more prominence due to energy shortage and greenhouse effect [12]. This letter focuses on the maximization of energy efficiency, defined as the delivered information bits per Joule harvested energy, by jointly optimizing transfer duration and transmit power. The contributions of this letter lie in three folds. First, it solves the challenging problem of long-distance wireless information and power transfer. Second, it improves the energy efficiency enormously. Third, it provides high QoS guarantee.

The rest of this letter is organized as follows. We first give an overview of wireless information and power transfer in large-scale MIMO systems in Section II, and then derive an energy-efficient resource allocation scheme by maximizing the energy efficiency while satisfying the QoS requirement in Section III. In Section IV, we present some numerical results to validate the effectiveness of the proposed scheme. Finally, we conclude the whole paper in Section V.

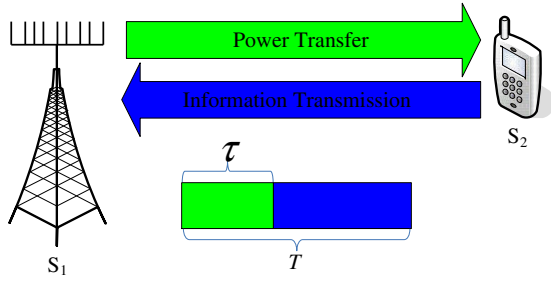


Fig. 1. An overview of the wireless information and power system.

## II. SYSTEM MODEL

We consider a time division duplex (TDD) large-scale MIMO system employing wireless information and power transfer, as shown in Fig.1, where the power source and the information receiver, namely  $S_1$ , is equipped with  $N_t$  antennas to enhance the power transfer efficiency by energy beamforming and improve the information transmission rate by receive combining, respectively. Note that the number of antennas  $N_t$  is quite large in the large-scale MIMO system. Similarly,  $S_2$  plays the roles as both power receiver and information transmitter. Due to space limitation,  $S_2$  deploys a single antenna for energy harvesting and information transmitting.  $S_2$  only has limited power to maintain the active state, so it needs to harvest enough energy from the outside for information transmission, such as  $S_1$ . Considering the limited storage,  $S_2$  should be charged from  $S_1$  slot by slot.

The whole system is operated in slotted time of length  $T$ . At the beginning of each time slot,  $S_1$  performs wireless power transfer by using energy beamforming with the duration  $\tau$ . With the harvesting energy,  $S_2$  transmits the information to  $S_1$  in the rest of the time slot  $T - \tau$ . We use  $\sqrt{\alpha}\mathbf{h}$  to denote the  $N_t$  dimensional channel vector from  $S_1$  to  $S_2$ , where  $\alpha$  represents the distance-dependent path loss and  $\mathbf{h}$  is the channel fast fading vector with independent and identically distributed (i.i.d.) zero mean and unit variance circular symmetric complex Gaussian random entries. According to the law of energy conservation, the harvested power at  $S_2$  from  $S_1$  can be expressed as [7]

$$P_{harv} = \eta\alpha P|\mathbf{h}^H \mathbf{w}|^2 \quad (1)$$

where  $P$  is the transmit power of  $S_1$ ,  $\eta$  is the efficiency ratio at  $S_2$  for converting the harvested energy to the electrical energy to be stored, which depends on the efficiency of the energy converter. Intuitively,  $\eta$  belong to the interval  $[0, 1]$ .  $\mathbf{w}$ , as the energy beamforming vector with unit norm, is used to adaptively adjust the energy transmit direction according to the instantaneous CSI  $\mathbf{h}$ , so as to maximize  $P_{harv}$ . We assume  $S_1$  has full CSI  $\mathbf{h}$  by estimating the uplink channel from  $S_2$  to  $S_1$  in the last time slot, due to channel reciprocity in TDD systems. Clearly,  $\mathbf{w} = \mathbf{h}/\|\mathbf{h}\|$ , namely maximum ratio transmission (MRT), can maximize  $P_{harv}$ . Thus, the total harvested energy at  $S_2$  during a time slot is  $Q_{harv} = \eta\alpha P\|\mathbf{h}\|^2\tau$ . After that,  $S_1$  and  $S_2$  toggle to the wireless information transfer mode. With the harvesting energy  $Q_{harv}$ ,  $S_2$  transmits the information to  $S_1$  in the rest of the time slot  $T - \tau$ , and the

receive signal can be expressed as

$$\mathbf{y} = \sqrt{\frac{Q_{harv}}{T - \tau}}\theta\mathbf{g}s + \mathbf{n} \quad (2)$$

where  $s$  is the normalized Gaussian distributed transmit signal,  $\mathbf{y}$  is the  $N_t$  dimensional receive signal vector, and  $\mathbf{n}$  is the additive Gaussian white noise with zero mean and variance matrix  $\sigma^2\mathbf{I}_{N_t}$ .  $\sqrt{\theta}\mathbf{g}$  denotes the unlink channel from  $S_2$  to  $S_1$ , where  $\theta$  is the distance-dependent path loss, which may be the same as  $\alpha$ , and  $\mathbf{g}$  is the channel fast fading vector distributed as  $\mathcal{CN}(0, \mathbf{I}_{N_t})$ . Note that  $\mathbf{g}$  may be the same as  $\mathbf{h}$  during a time slot due to channel reciprocity in TDD system.  $\sqrt{\frac{Q_{harv}}{T - \tau}}$  is the transmit power. Assuming perfect CSI at  $S_1$ , maximum ratio combining (MRC) is performed to maximize the information transmission rate. Assuming there is always data to be transmitted in each slot, then the information transmission rate is given by

$$\begin{aligned} R(P, \tau) &= W \log_2 \left( 1 + \frac{Q_{harv}\theta\|\mathbf{g}\|^2}{(T - \tau)\sigma^2} \right) \\ &= W \log_2 \left( 1 + \frac{\eta\alpha\theta P\tau\|\mathbf{h}\|^2\|\mathbf{g}\|^2}{(T - \tau)\sigma^2} \right) \end{aligned} \quad (3)$$

where  $W$  is the spectrum bandwidth. As mentioned earlier, in the large-scale MIMO system, the BS is equipped with a large number of antennas. According to the fact that  $\lim_{N_t \rightarrow \infty} \frac{\|\mathbf{g}\|^2}{N_t} = 1$  and  $\lim_{N_t \rightarrow \infty} \frac{\|\mathbf{h}\|^2}{N_t} = 1$ , namely channel hardening [13], we have

$$\begin{aligned} R(P, \tau) &= W \log_2 \left( 1 + \frac{\eta\alpha\theta N_t^2 P\tau}{(T - \tau)\sigma^2} \right) \\ &\approx W \log_2 \frac{\eta\alpha\theta N_t^2 P\tau}{(T - \tau)\sigma^2} \end{aligned} \quad (4)$$

where (4) holds true since the constant 1 becomes negligible when  $N_t \rightarrow \infty$ . Because only duration  $T - \tau$  is used for information transmission during a time slot, the average information transmission rate can be expressed as

$$\bar{R}(P, \tau) = \frac{T - \tau}{T} W \log_2 \frac{\eta\alpha\theta N_t^2 P\tau}{(T - \tau)\sigma^2} \quad (5)$$

*Remark:* The performance of wireless information and power transfer in traditional MIMO system is limited by the path loss  $\alpha$  and  $\theta$ , and thus it is only applicable for short-distance wireless information and power transfer. However, in the large-scale MIMO system, this challenging problem can be solved by adding antennas at the transmitter, so it can support relative long-distance transmission and high QoS guaranteed wireless services with a low transmit power.

## III. ENERGY-EFFICIENT POWER ALLOCATION

Considering the limitation of power resource and the requirement of green communications, energy efficiency becomes an important performance metric in wireless communications, especially for wireless power transfer systems. In this section, we attempt to derive a joint power and time resources allocation scheme to maximize the energy efficiency

while satisfying power, time and QoS constraints, which is equivalent to the following optimization problem:

$$J_1 : \max \quad \frac{\bar{R}(P, \tau)T}{P_0T + P\tau} \quad (6)$$

$$\text{s.t.} \quad P \leq P_{1,\max} \quad (7)$$

$$\frac{Q_{\text{harv}}}{T - \tau} \leq P_{2,\max} \quad (8)$$

$$\tau \leq T \quad (9)$$

$$\bar{R}(P, \tau) \geq r_{\min} \quad (10)$$

where  $P$  is the power consumption in the power amplifier at  $S_1$ , and  $P_0$  is the constant power consumption in the transmit filter, mixer, frequency synthesizer, and digital-to-analog converter which are independent of the actual transmit power. (6) is the so called energy efficiency, defined as information transmission bits per Joule. (7) is the transmit power constraint at  $S_1$ , (8) is the transmit power constraint at  $S_2$ , (9) is the time constraint at  $S_1$ , and (10) is the QoS constraint, where  $r_{\min}$  is the minimum transmission rate meeting a given QoS requirement, such as delay provisioning [14]. Due to channel hardening in large-scale MIMO systems, the constraint condition (8) is equivalent to  $P \leq \frac{P_{2,\max}(T-\tau)}{\eta\alpha N_t\tau}$ . According to (7),  $P$  must be less than or equal to  $P_{1,\max}$ , so the following condition should be satisfied  $\frac{P_{2,\max}(T-\tau)}{\eta\alpha N_t\tau} \geq P_{1,\max}$ . Thus, we have  $\tau \leq \frac{P_{2,\max}T}{\eta\alpha N_t P_{1,\max} + P_{2,\max}} = \tau_{\max}$ .

The objective function (6) in a fractional program is a ratio of two functions of the optimization variables  $P$  and  $\tau$ , resulting in  $J_1$  is a fractional programming problem, which is in general nonconvex. Following [15], the objective function is equivalent to  $\bar{R}(P, \tau)T - q^*(P_0T + P\tau)$  by exploiting the properties of fractional programming, where  $q^*$  is the energy efficiency when  $P$  and  $\tau$  are equal to the optimal value  $P^*$  and  $\tau^*$  of  $J_1$  respectively, namely  $q^* = \frac{\bar{R}(P^*, \tau^*)T}{P_0T + P^*\tau^*}$ . Thus,  $J_1$  is transformed as

$$J_2 : \max \quad \bar{R}(P, \tau)T - q^*(P_0T + P\tau) \quad (11)$$

$$\text{s.t.} \quad P \leq P_{1,\max} \quad (12)$$

$$\tau \leq \tau_{\max} \quad (13)$$

$$\tau \leq T \quad (14)$$

$$\bar{R}(P, \tau) \geq r_{\min} \quad (15)$$

It can be proved that  $\frac{\partial^2 \bar{R}(P, \tau)}{\partial^2 P} < 0$  and  $\frac{\partial^2 \bar{R}(P, \tau)}{\partial^2 \tau} < 0$ , so  $J_2$  is a convex optimization problem, which can be solved by the Lagrange multiplier method. By some arrangements, its Lagrange dual function can be written as

$$\begin{aligned} \mathcal{L}(\mu, \vartheta, \nu, v, P, \tau) &= \bar{R}(P, \tau)T - q^*(P_0T + P\tau) - \mu P \\ &+ \mu P_{1,\max} - \vartheta\tau + \vartheta\tau_{\max} - \nu\tau + \nu T \\ &+ v\bar{R}(P, \tau) - vr_{\min} \end{aligned} \quad (16)$$

where  $\mu \geq 0$ ,  $\vartheta \geq 0$ ,  $\nu \geq 0$  and  $v \geq 0$  are the Lagrange multipliers corresponding to the constraint (12), (13), (14) and (15), respectively. Therefore, the dual problem of  $J_2$  is given by

$$\min_{\mu, \vartheta, \nu, v} \max_{P, \tau} \mathcal{L}(\mu, \vartheta, \nu, v, P, \tau) \quad (17)$$

Given  $\mu$ ,  $\vartheta$ ,  $\nu$  and  $v$ , the optimal transmit power  $P^*$  and transfer duration  $\tau^*$  can be derived by jointly solving the

following KKT conditions

$$\begin{aligned} \frac{\partial \mathcal{L}(\mu, \vartheta, \nu, v, P, \tau)}{\partial P} &= (T + v) \frac{\partial \bar{R}(P, \tau)}{\partial P} - q^*\tau - \mu \\ &= \frac{W(T + v)(T - \tau)}{\ln 2TP} - q^*\tau - \mu \\ &= 0 \end{aligned} \quad (18)$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}(\mu, \vartheta, \nu, v, P, \tau)}{\partial \tau} &= (T + v) \frac{\partial \bar{R}(P, \tau)}{\partial \tau} - q^*P - \vartheta - \nu \\ &= W(T + v) \left( \frac{1}{(T - \tau) \ln 2} \right. \\ &\quad \left. - \frac{1}{T} \log_2 \frac{\eta\alpha\theta N_t^2 P\tau}{(T - \tau)\sigma^2} \right) - q^*P - \vartheta - \nu \\ &= 0 \end{aligned} \quad (19)$$

Moreover,  $\mu$ ,  $\vartheta$ ,  $\nu$  and  $v$  can be updated by the gradient method, which are given by

$$\mu(n + 1) = [\mu(n) - \Delta_\mu(P_{1,\max} - P)]^+ \quad (20)$$

$$\vartheta(n + 1) = [\vartheta(n) - \Delta_\vartheta(\tau_{\max} - \tau)]^+ \quad (21)$$

$$\nu(n + 1) = [\nu(n) - \Delta_\nu(T - \tau)]^+ \quad (22)$$

and

$$v(n + 1) = [v(n) - \Delta_v(\bar{R}(P, \tau) - r_{\min})]^+ \quad (23)$$

where  $n$  is the iteration index, and  $\Delta_\mu$ ,  $\Delta_\nu$  and  $\Delta_v$  are the positive iteration steps. Inspired by the Dinkelbach method [16], we propose an iterative algorithm as follows

---

#### Algorithm 1: Energy-Efficient Resource Allocation

---

- 1) Initialization: Given  $N_t$ ,  $W$ ,  $T$ ,  $\eta$ ,  $\alpha$ ,  $\theta$ ,  $P_0$ ,  $P_{1,\max}$ ,  $P_{2,\max}$ ,  $r_{\min}$ ,  $\Delta_\mu$ ,  $\Delta_\nu$  and  $\Delta_v$ . Let  $\mu = 0$ ,  $\vartheta = 0$ ,  $\nu = 0$ ,  $v = 0$ ,  $P = 0$  and  $q^* = \frac{\bar{R}(P, \tau)T}{P_0T + P\tau}$ .  $\epsilon$  is a sufficiently small positive real number.
  - 2) Update  $\mu$ ,  $\vartheta$ ,  $\nu$  and  $v$  according to (20), (21), (22) and (23), respectively.
  - 3) Computing the optimal  $P^*$  and  $\tau^*$  by jointly solving the equations (18) and (19).
  - 4) If  $\bar{R}(P^*, \tau^*)T - q^*(P_0T + P^*\tau^*) > \epsilon$ , then set  $q^* = \frac{\bar{R}(P^*, \tau^*)T}{P_0T + P^*\tau^*}$ , and go to 2). Otherwise,  $P^*$  is the optimal transmit power and  $\tau^*$  is the optimal transfer duration.
- 

## IV. NUMERICAL RESULTS

To examine the effectiveness of the proposed energy-efficient resource allocation scheme, we present several numerical results in the following scenarios: we set  $W = 10$  KHz,  $T = 5$  ms,  $\eta = 0.8$ ,  $\sigma^2 = 1$ ,  $r_{\min} = 12$  Kb/s,  $P_0 = 45$ Watt and  $P_{1,\max} = P_{2,\max} = 15$ Watt. In addition, we set  $\alpha = \theta$  for convention. It is found that the proposed energy-efficient resource allocation scheme converges after no more than 20 times iterative computation in the all simulation scenarios.

Fig.2 compares the energy efficiencies of the proposed power and duration joint optimization scheme and duration optimization scheme with  $N_t = 100$ . Intuitively, it is optimal

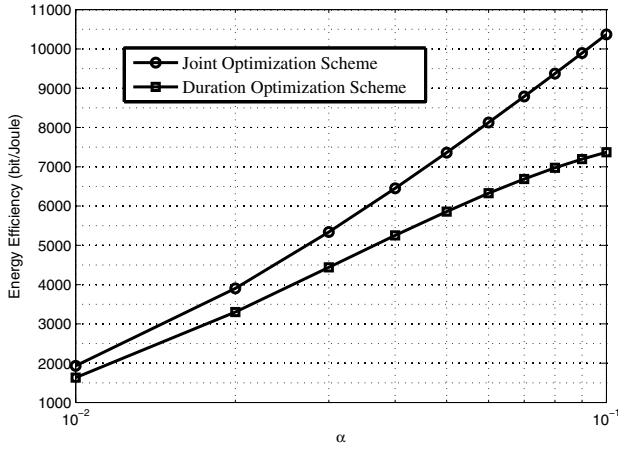


Fig. 2. Performance comparison of the proposed and the fixed resource allocation schemes.

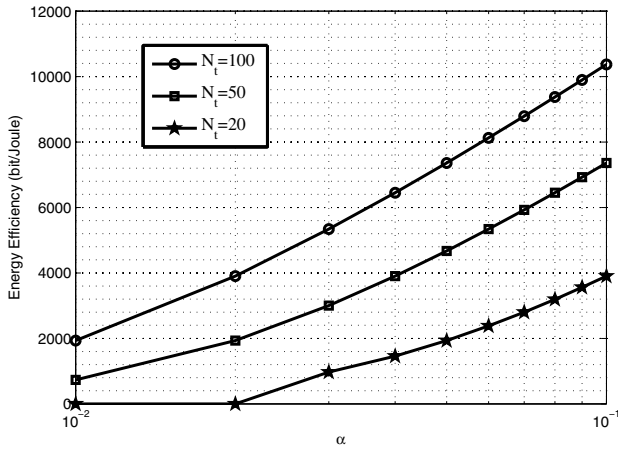


Fig. 3. Performance comparison of the proposed resource allocation scheme with different numbers of BS antennas.

to use  $P_{1,\max}$  as the transmit power at  $S_1$  in the sense of maximizing the harvesting energy, so we set  $P = P_{1,\max}$  and optimize  $\tau$  only for the duration optimization scheme based on Algorithm 1. As seen in Fig.2, the joint optimization scheme performs better than the duration optimization one obviously, as the latter fixes the transmit power, which also has a greater impact on energy efficiency. For example, when  $\alpha = 0.05$ , there is about 1.5Kb/J gain. As  $\alpha$  increases, the performance gain becomes larger. Therefore, the proposed scheme can effectively increase the energy efficiency of wireless information and power transfer.

Fig.3 investigates the effect of the number of antennas at  $S_1$  on the energy efficiency of the proposed scheme. Clearly, as  $N_t$  increases, there is significant performance gain in terms of energy efficiency, especially for the large  $\alpha$ . For example, when  $\alpha = 0.05$ , there is about 2.5Kb/J gain by increasing  $N_t$  from 50 to 100. It is found that when  $\alpha \leq 0.02$ , the energy efficiency in the case of  $N_t = 20$  reduces to zero. This is because the harvested power can not support the given

QoS requirement under such an antenna setup. However, with the increase of antenna number, i.e  $N_t = 50$ , the wireless information and power transfer system can support the QoS guaranteed service even with  $\alpha = 0.01$ . Hence, the proposed scheme can realize long-distance and high QoS provisioning wireless information and power transfer.

## V. CONCLUSION

A major contribution of this letter is the introduction of the large-scale MIMO technique into wireless information and power transfer. By exploiting the advantage of the large-scale MIMO, this letter realizes long-distance and QoS guaranteed wireless information and power transfer. Considering the demand for green communications, an energy-efficient resource allocation scheme is proposed by jointly optimizing transmit power and transfer duration. Numerical results confirm the effectiveness of the proposed scheme.

## REFERENCES

- [1] F. Zhang, S. A. Hackworth, X. Liu, H. Chen, R. J. Sclabassi, and M. Sun, "Wireless energy transfer platform for medical sensor and implantable devices," in *Proc. 2009 IEEE EMBS Int. Conf.*, pp. 1045–1048.
- [2] K. Huang and E. G. Larsson, "Simultaneous information and power transfer for broadband wireless systems." Available: <http://arxiv.org/pdf/1211.6868v2.pdf>
- [3] M. K. Watfa, H. Al-Hassanieh, and S. Selman, "Multi-hop wireless energy transfer in WSNs," *IEEE Commun. Lett.*, vol. 15, no. 12, pp. 1275–1277, Sep. 2011.
- [4] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. 2008 IEEE Int. Symp. Inf. Theory*, pp. 1612–1616.
- [5] J. Gozalvez, "Witricity-the wireless power transfer," *IEEE Veh. Technol. Mag.*, vol. 2, no. 2, pp. 38–44, June 2007.
- [6] P. Grover and A. Sahai, "Shannon meets Tesla: wireless information and power transfer," in *Proc. 2010 IEEE Int. Symp. Inf. Theory*, pp. 2363–2367.
- [7] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," in *Proc. 2011 IEEE Globecom*, pp. 1–5.
- [8] Z. Xiang and M. Tao, "Robust beamforming for wireless information and power transmission," *IEEE Wireless Commun. Lett.*, vol. 1, no. 4, pp. 372–375, Aug. 2012.
- [9] X. Chen, C. Yuen, and Z. Zhang, "Wireless energy and information transfer tradeoff for limited feedback multi-antenna systems with energy beamforming," *IEEE Trans. Veh. Technol.*, 2013.
- [10] T. L. Marzetta, "Noncooperative cellular wireless with unlimited number of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- [11] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling to MIMO: opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 20, no. 1, pp. 40–60, Jan. 2013.
- [12] F. Chu, K. Chen, and G. Fettweis, "Green resource allocation to minimize receiving energy in OFDMA cellular systems," *IEEE Commun. Lett.*, vol. 16, no. 3, pp. 372–374, Jan. 2012.
- [13] B. M. Hochwald, T. L. Marzetta, and V. Tarokh, "Multiple-antenna channel hardening and its implications for rate-feedback and scheduling," *IEEE Tran. Inf. Theory*, vol. 50, no. 9, pp. 1893–1909, Sept. 2004.
- [14] X. Chen, Z. Zhang, S. Chen, and C. Wang, "Adaptive mode selection for multiuser MIMO downlink employing rateless codes with QoS provisioning," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 790–799, Feb. 2012.
- [15] D. W. K. Ng, E. S. Lo, and R. Schober, "Energy-efficient resource allocation for secure OFDMA systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2572–2585, Jul. 2012.
- [16] W. Dinkelbach, "On nonlinear fractional programming," *Manage. Sci.*, vol. 13, no. 7, pp. 492–498, Mar. 1967. Available: <http://www.jstor.org/stable/2627691>.