CSC 4080: Artificial Intelligence in Medical Imaging and Health

Homework Set 1

Due date: Feb. 20, 2022

Short Answer questions:

1. Explain the image acquisition process for digital CT Imaging in math. (10%)

   For a 2D attenuation function $\mu(x,y)$, describing the X-ray attenuation per unit volume in some slice *z* in the human body, the Radon transform is given by all line integrals through this function:

$$R(\rho,\varphi)[\mu(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)\delta(\rho - xcos\varphi - ysin\varphi)dxdy$$

where δ is the Dirac-delta function and *s* is the line.

When light spreads in the homogeneous (均匀) medium, its intensity attenuation model is: $I = I_0 e^{-\int_L \mu(x)\,dx}$, where *u* is the attenuation coefficient and *x* is the length. The Radon Transform is $ln(\frac{I}{I_0}) =- \int_L \mu(x)\,dx = R_L\mu$.

Two variables (ρ and φ) in radon transform, assume that φ is fixed, only ρ changes. We apply Fourier transformation to $R_L\mu$:

$$\begin{aligned}
F_\rho R_\mu(\rho,\varphi) &= \int_{-\infty}^{\infty} e^{-2\pi i r\rho} R_\mu(\rho,\varphi)d\rho \\
&= \int_{-\infty}^{\infty} e^{-2\pi i r\rho}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x_1,x_2)\delta(\rho - x_1cos\varphi - x_2sin\varphi)dx_1dx_2d\rho \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x_1,x_2)\int_{-\infty}^{\infty} e^{-2\pi i r\rho}\,\delta(\rho - x_1cos\varphi - x_2sin\varphi)d\rho dx_1dx_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x_1,x_2)e^{-2\pi i r(x_1cos\varphi + x_2sin\varphi)} = F\mu(\xi_1,\xi_2)
\end{aligned}$$

Take the inverse of Fourier transform, μ can be calculated. Then the information of the thickness ot organs is found. The result is:

$$\begin{aligned}
\mu(\xi_1,\xi_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_\rho R_\mu(\rho,\varphi)e^{-2\pi i r(x_1cos\varphi + x_2sin\varphi)}\,dx_1dx_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ln(\frac{I(\rho,\varphi)}{I_0(\rho,\varphi)})e^{-2\pi i r(x_1cos\varphi + x_2sin\varphi)}\quad e^{-2\pi i r\rho}d\rho\,dx_1dx_2)
\end{aligned}$$

2. What is the purpose of filtering with a Hamming window filter when reconstructing CT images? (10%)

Combined with High pass filters, to reduce the amplification of high-frequencies. It is also a low pass filter, which presents a high degree of smoothing.

3. What is the advantage and disadvantages of X-ray, CT, ultrasound, and MRI? (10%)

Advantages of X-ray: It provides a lucid picture of humans' bones, and the thickness and the shape of organs. It's cheap.

Shortcomings for X-ray: It cannot view the 3D appearance of the human body. The radiation damages humans' health. Short at distinguishing organs with similar density and thickness and adjacent positions.

Advantages of CT: Form images of non-living objects. Create an image of one or more slices through the body.

Shortcomings of CT: Time consuming, expensive and a great amount of radiation.

Advantages of ultrasound: fast and timely, no pain and danger, non-invasive. It has been popularized in clinical applications and it is an important part of medical imaging. Because the equipment is not as expensive as CT or MRI equipment, US diagnosis can obtain arbitrary cross-sectional images of organs, and can also observe the activity of moving organs. It's cheap and convenient.

Shortcomings of ultrasound: the contrast resolution and spatial resolution of the image are not as high as that of CT or MRI.

Advantages of MRI: Examination of nervous system, head, cervical spine (颈椎), thoracic spine , lumbar spine , limbs and other parts has unique advantages. The biggest advantage is that it can not only display diseased tissues, but also reflect the physiological and biochemical information of living tissue functions and metabolic processes.

Shortcomings of MRI: the contrast resolution and spatial resolution of the image are not as high as that of CT or MRI.

Calculation problems:

1. Get the derivatives for the sigmoid function $\sigma(x) = \frac{1}{1+\exp(-x)}$ and softmax function $\mathrm{z_k} = \mathrm{softmax}(\mathrm{x_k}) = \frac{\exp(\mathrm{x_k})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})}$ (you should write down the explicit matrix form different from the slides). Talk about the similarities of these two functions and their applications, respectively. (10%)

$$\sigma'(x) = \begin{bmatrix} \frac{d(\frac{1}{1+\exp(-x_1)})}{dx_1} & \frac{d(\frac{1}{1+\exp(-x_1)})}{dx_2} & \cdots & \frac{d(\frac{1}{1+\exp(-x_1)})}{dx_n} \\ \frac{d(\frac{1}{1+\exp(-x_2)})}{dx_1} & \frac{d(\frac{1}{1+\exp(-x_2)})}{dx_2} & \cdots & \frac{d(\frac{1}{1+\exp(-x_2)})}{dx_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{d(\frac{1}{1+\exp(-x_n)})}{dx_1} & \frac{d(\frac{1}{1+\exp(-x_n)})}{dx_2} & \cdots & \frac{d(\frac{1}{1+\exp(-x_n)})}{dx_n} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\exp(-x_1)}{(1+\exp(-x_1))^2} & 0 & \cdots & 0 \\ 0 & \frac{\exp(-x_2)}{(1+\exp(-x_2))^2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \frac{\exp(-x_n)}{(1+\exp(-x_n))^2} \end{bmatrix}$$

$$z_k'(x) = \begin{bmatrix} \frac{d(\frac{\exp(\mathrm{x_1})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_1} & \frac{d(\frac{\exp(\mathrm{x_1})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_2} & \cdots & \frac{d(\frac{\exp(\mathrm{x_1})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_K} \\ \frac{d(\frac{\exp(\mathrm{x_2})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_1} & \frac{d(\frac{\exp(\mathrm{x_2})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_2} & \cdots & \frac{d(\frac{\exp(\mathrm{x_2})}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_K} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{d(\frac{\exp(\mathrm{x}_K)}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_1} & \frac{d(\frac{\exp(\mathrm{x}_K)}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_2} & \cdots & \frac{d(\frac{\exp(\mathrm{x}_K)}{\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i})})}{dx_K} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\sum_{\mathrm{i}=1,\mathrm{i}\neq 1}^{\mathrm{K}} \exp(\mathrm{x_i}+\mathrm{x_1})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \frac{-\exp(\mathrm{x_2}+\mathrm{x_1})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \cdots & \frac{-\exp(\mathrm{x}_K+\mathrm{x_1})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} \\ \frac{-\exp(\mathrm{x_1}+\mathrm{x_2})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \frac{\sum_{\mathrm{i}=1,\mathrm{i}\neq 2}^{\mathrm{K}} \exp(\mathrm{x_i}+\mathrm{x_2})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \cdots & \frac{-\exp(\mathrm{x}_K+\mathrm{x_2})}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} \\ \cdots & \cdots & \cdots & \\ \frac{-\exp(\mathrm{x_1}+\mathrm{x}_K)}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \frac{-\exp(\mathrm{x_2}+\mathrm{x}_K)}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \frac{-\exp(\mathrm{x_3}+\mathrm{x}_K)}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} & \frac{\sum_{\mathrm{i}=1,\mathrm{i}\neq \mathrm{K}}^{\mathrm{K}} \exp(\mathrm{x_i}+\mathrm{x}_K)}{(\sum_{\mathrm{i}=1}^{\mathrm{K}} \exp(\mathrm{x_i}))^2} \end{bmatrix}$$

Similarities:

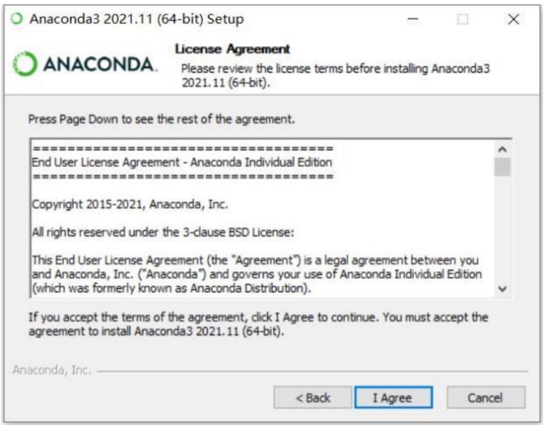
Both of functions have a range (0, 1). The outputs can represent probabilities in the output layer of CNN and NN in classification problems. However, sigmoid functions can only be used in the binary classification. Softmax functions is an extension of sigmoid functions at some extend.

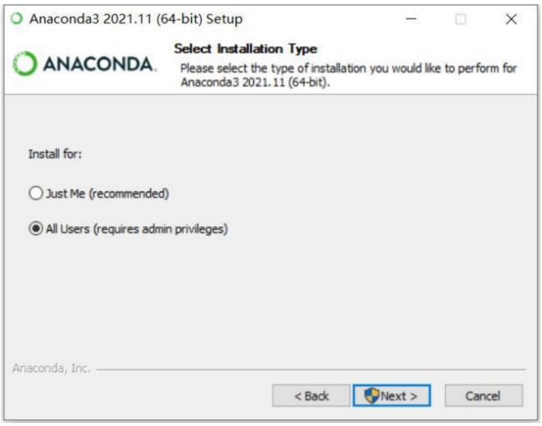
2. Write down the cross-entropy loss function, and its derivatives. Discuss how it works for classification tasks. Is there any disadvantage of using naïve cross-entropy for classification? (10%)

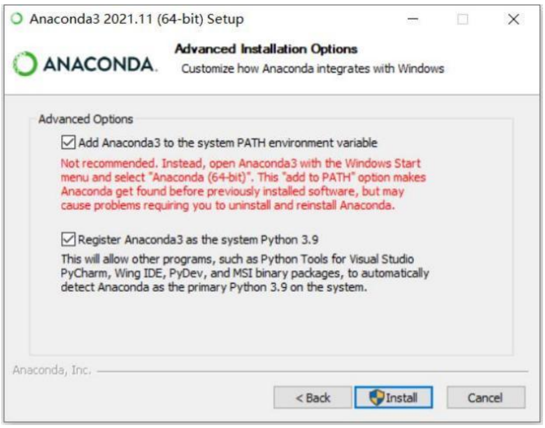
Assume **x** is the result vector of the last layer, and the probability generate by softmax function. Then **p**=softmax(**x**), Assume the target one is the i th vector, then the indicator function ***y*** **with** $y_i = 1, y_j = 0$ if $j \neq i$. $L = -\sum_i y_i \ log(p_i)$, $\frac{dL}{dx_i} = -\sum_i y_i \frac{dlog(p_i)}{dp_i}\frac{dp_i}{dx_i} = \sum_i y_i \frac{1}{p_i}\frac{dp_i}{dx_i}$. In the task, the softmax function transform the result into probability. Cross entropy function provides a judgement for the efficiency of the model. And the cross entropy function is differentiable, which means it can update the model by gradient descending. Yes, the disadvantage is that if p_i is very small, then the gradient will be very large(gradient explode) since it's in the denominator. And softmax function is similar to an exponential function. The repeating calculation of exponential and log functions.

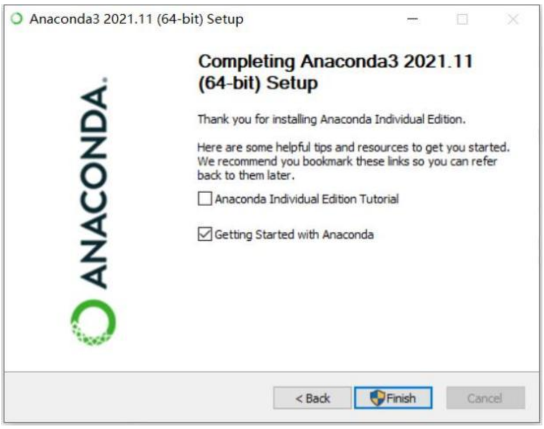
Programming problems:

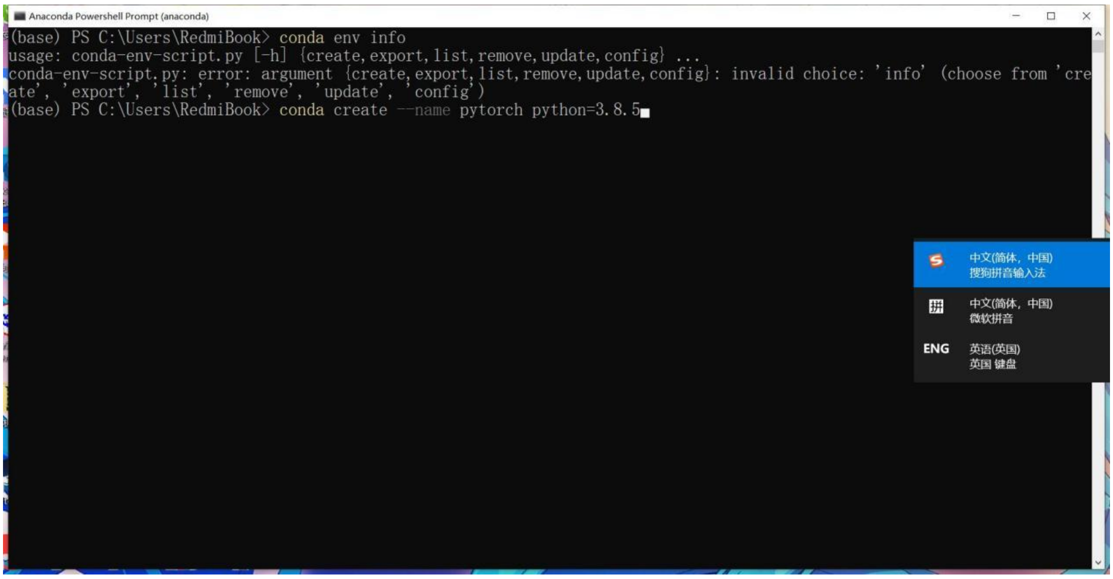
1. Install Anaconda (a very good Python version that can help you easily implement data science applications).
   Based on Anaconda, install some commonly used packages for Python, including Numpy, Pandas, Matplotlib, Seaborn, PyTorch (CPU version is OK if you don't have GPU on your device). Record your installation process in detail, screenshots for every step are needed. (10%)


Anaconda3 2021.11 (64-bit) Setup
ANACONDA.
License Agreement
Please review the license terms before installing Anaconda3 2021.11 (64-bit).
Press Page Down to see the rest of the agreement.
===================================
End User License Agreement - Anaconda Individual Edition
===================================
Copyright 2015-2021, Anaconda, Inc.
All rights reserved under the 3-clause BSD License:
This End User License Agreement (the "Agreement") is a legal agreement between you and Anaconda, Inc. ("Anaconda") and governs your use of Anaconda Individual Edition (which was formerly known as Anaconda Distribution).
If you accept the terms of the agreement, click I Agree to continue. You must accept the agreement to install Anaconda3 2021.11 (64-bit).
Anaconda, Inc.
< Back
I Agree
Cancel


Anaconda3 2021.11 (64-bit) Setup
ANACONDA.
Select Installation Type
Please select the type of installation you would like to perform for Anaconda3 2021.11 (64-bit).
Install for:
Just Me (recommended)
All Users (requires admin privileges)
Anaconda, Inc.
< Back
Next >
Cancel


Anaconda3 2021.11 (64-bit) Setup
ANACONDA.
Advanced Installation Options
Customize how Anaconda integrates with Windows
Advanced Options
Add Anaconda3 to the system PATH environment variable
Not recommended. Instead, open Anaconda3 with the Windows Start menu and select "Anaconda (64-bit)". This "add to PATH" option makes Anaconda get found before previously installed software, but may cause problems requiring you to uninstall and reinstall Anaconda.
Register Anaconda3 as the system Python 3.9
This will allow other programs, such as Python Tools for Visual Studio PyCharm, Wing IDE, PyDev, and MSI binary packages, to automatically detect Anaconda as the primary Python 3.9 on the system.
Anaconda, Inc.
< Back
Install
Cancel


Anaconda3 2021.11 (64-bit) Setup
ANACONDA.
Completing Anaconda3 2021.11 (64-bit) Setup
Thank you for installing Anaconda Individual Edition.
Here are some helpful tips and resources to get you started. We recommend you bookmark these links so you can refer back to them later.
Anaconda Individual Edition Tutorial
Getting Started with Anaconda
< Back
Finish
Cancel


Anaconda Powershell Prompt (anaconda)
(base) PS C:\Users\RedmiBook> conda env info
usage: conda-env-script.py [-h] {create, export, list, remove, update, config} ...
conda-env-script.py: error: argument {create, export, list, remove, update, config}: invalid choice: 'info' (choose from 'create', 'export', 'list', 'remove', 'update', 'config')
(base) PS C:\Users\RedmiBook> conda create --name pytorch python=3.8.5
中文(简体，中国)
搜狗拼音输入法
中文(简体，中国)
微软拼音
ENG
英语(英国)
英国 键盘

Anaconda Powershell Prompt (anaconda)

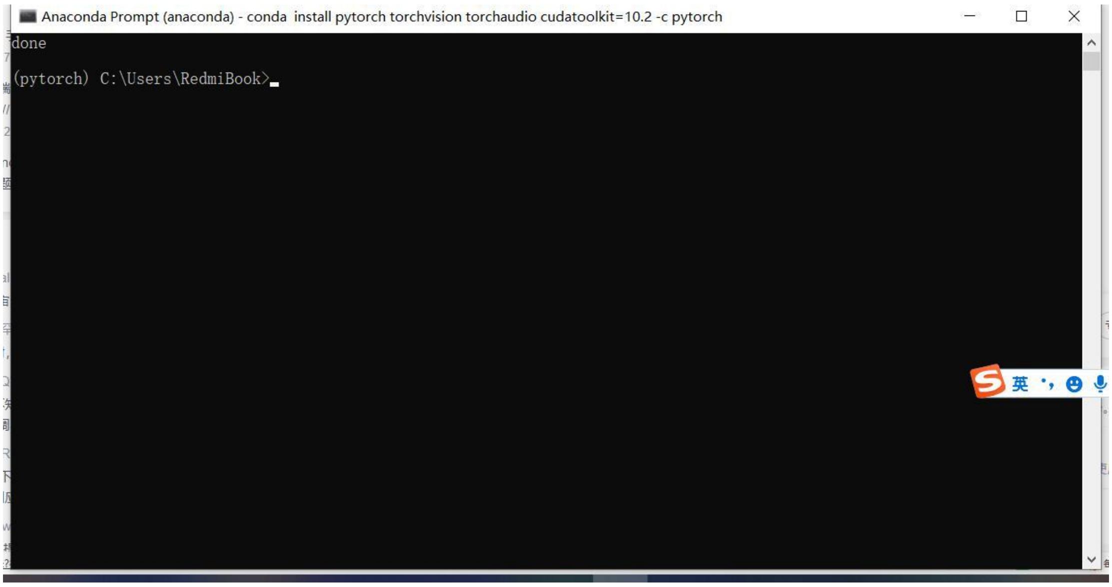
```
    package                    |            build
    ---------------------------|-----------------
    ca-certificates-2021.10.26 |       haa95532_4         116 KB
    certifi-2021.10.8          |   py38haa95532_2         152 KB
    openssl-1.1.1m             |       h2bbff1b_0         4.8 MB
    pip-21.2.2                 |   py38haa95532_0         1.9 MB
    python-3.8.5               |       h5fd99cc_1        15.7 MB
    setuptools-58.0.4          |   py38haa95532_0         779 KB
    sqlite-3.37.2              |       h2bbff1b_0         799 KB
    wheel-0.37.1               |     pyhd3eb1b0_0          33 KB
    wincertstore-0.2           |   py38haa95532_2          15 KB
    ------------------------------------------------------------
                                           Total:        24.2 MB

The following NEW packages will be INSTALLED:

  ca-certificates    pkgs/main/win-64::ca-certificates-2021.10.26-haa95532_4
  certifi            pkgs/main/win-64::certifi-2021.10.8-py38haa95532_2
  openssl            pkgs/main/win-64::openssl-1.1.1m-h2bbff1b_0
  pip                pkgs/main/win-64::pip-21.2.2-py38haa95532_0
  python             pkgs/main/win-64::python-3.8.5-h5fd99cc_1
  setuptools         pkgs/main/win-64::setuptools-58.0.4-py38haa95532_0
  sqlite             pkgs/main/win-64::sqlite-3.37.2-h2bbff1b_0
  vc                 pkgs/main/win-64::vc-14.2-h21ff451_1
  vs2015_runtime     pkgs/main/win-64::vs2015_runtime-14.27.29016-h5e58377_2
  wheel              pkgs/main/noarch::wheel-0.37.1-pyhd3eb1b0_0
  wincertstore       pkgs/main/win-64::wincertstore-0.2-py38haa95532_2


Proceed ([y]/n)?
```

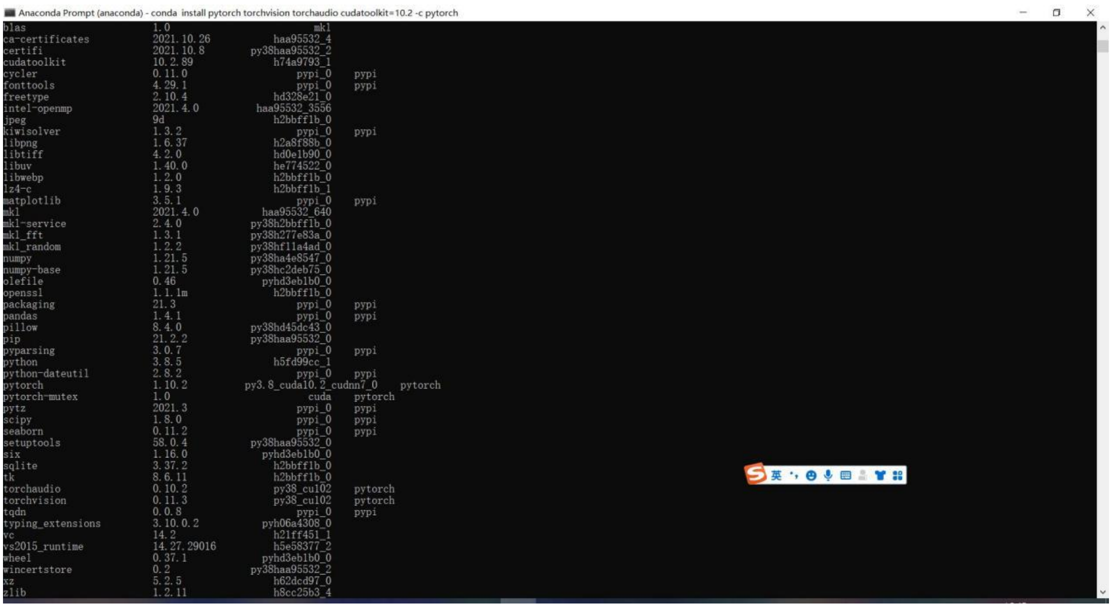
Anaconda Powershell Prompt (anaconda)

```
done
#
# To activate this environment, use
#
#     $ conda activate pytorch
#
# To deactivate an active environment, use
#
#     $ conda deactivate

(base) PS C:\Users\RedmiBook>
```

Anaconda Powershell Prompt (anaconda)

```
done
#
# To activate this environment, use
#
#     $ conda activate pytorch
#
# To deactivate an active environment, use
#
#     $ conda deactivate

(base) PS C:\Users\RedmiBook> conda activate pytorch
(pytorch) PS C:\Users\RedmiBook> conda list
# packages in environment at D:\anaconda\envs\pytorch:
#
# Name                    Version                   Build  Channel
ca-certificates           2021.10.26           haa95532_4
certifi                   2021.10.8        py38haa95532_2
openssl                   1.1.1m               h2bbff1b_0
pip                       21.2.2           py38haa95532_0
python                    3.8.5                h5fd99cc_1
setuptools                58.0.4           py38haa95532_0
sqlite                    3.37.2               h2bbff1b_0
vc                        14.2                 h21ff451_1
vs2015_runtime            14.27.29016          h5e58377_2
wheel                     0.37.1             pyhd3eb1b0_0
wincertstore              0.2              py38haa95532_2
(pytorch) PS C:\Users\RedmiBook> conda install pytorch torchvision torchaudio cudatoolkit=10.2 -c pytorch
```





2. Here's two basic Python programming problems: (10%)

1) Write a program to sum all the numbers in a list;

Q2(1)

```
list= [i for i in range(100)]
s=0
for i in list:
    s+=i
s
```

[3]

```
4950
```

2) Write a program to find all prime numbers in a list.

```python
def isprime(M):
    for i in range(2,int(np.sqrt(M))+1):
        if M%i==0:
            return True
    return False
list= [i for i in range(100)]

ans=[]
for i in list:
    if isprime(i)==False:
        ans.append(i)
ans
```

3. Here's two basic Numpy programming problems: (10%)

   1) Given an array, calculate its softmax result;

```python
def softmax(x):
    return np.exp(x)/np.sum(np.exp(x))
x=np.array([1,2,3])
softmax(x)
```

```
array([0.09003057, 0.24472847, 0.66524096])
```

   2) use the softmax result to calculate the cross-entropy loss (you are free to construct the input x, and ground truth label y by yourself).

```python
x=np.array([1,2,3])
y=softmax(x)
-np.log(y[0])
```

```
2.40760596444438
```

4. Medical data exploration. You are going to work with a real medical image dataset, including exploring data labels, visualizing and observing the data, along with processing the data. Please fill in the blanks in "data_exploration.ipynb" and successfully run all the cells. (20%)

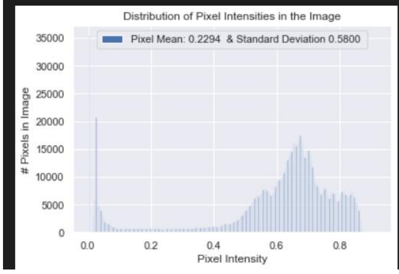
```python
# Plot a histogram of the distribution of the pixels
### Your code here ###
raw_image=raw_image.reshape(1,1048576)
sns.distplot(raw_image,bins=len(np.unique(raw_image)),kde=False,color='b')
import matplotlib.patches as mp
info = mp.Patch(label='Pixel Mean: %.4f  & Standard Deviation %.4f '%(raw_image.std(),raw_image.mean()))
### End your code ###
plt.legend(handles=[info],loc='upper center')
plt.title('Distribution of Pixel Intensities in the Image')
plt.xlabel('Pixel Intensity')
plt.ylabel('# Pixels in Image')
```

✓ 0.6s

Python

```
D:\anaconda\envs\pytorch\lib\site-packages\seaborn\distributions.py:2619: FutureWarning: `distplot` is a deprecated function and will be removed in a future version. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).
  warnings.warn(msg, FutureWarning)
```

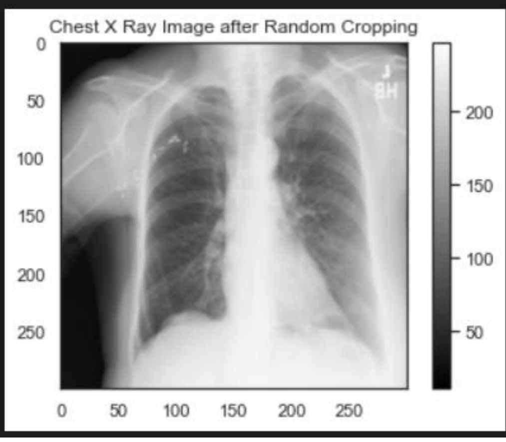
```
Text(0, 0.5, '# Pixels in Image')
```



```
# Create transformation for data standardization

norm_mean = [0.6, 0.6, 0.6]
norm_std = [0.225, 0.225, 0.225]
data_transform = transforms.Compose([
    transforms.Resize((320, 320)),
    transforms.ToTensor(),
    ### Your code here, use API like transforms.XXX() ###
    transforms.Normalize(norm_mean, norm_std)
    ### End your code ###
])
dataset = XRayDataset('nih/images-small', data_transform)
```
✓ 0.8s

```
# Random cropping

data (module) transforms ns.Compose([
    transforms.Resize((320, 320)),
    ### Your code here, use API like transforms.XXX() ###
    transforms.RandomCrop(300),
    ### End your code ###
    transforms.ToTensor()
])
dataset = XRayDataset('nih/images-small', data_transform)

sns.set_style("white")
generated_image = dataset.__getitem__(0)
inverted_image = transform_invert(copy.deepcopy(generated_image), data_transform)
plt.imshow(inverted_image, cmap='gray')
plt.colorbar()
plt.title('Chest X Ray Image after Random Cropping')
```
✓ 0.3s

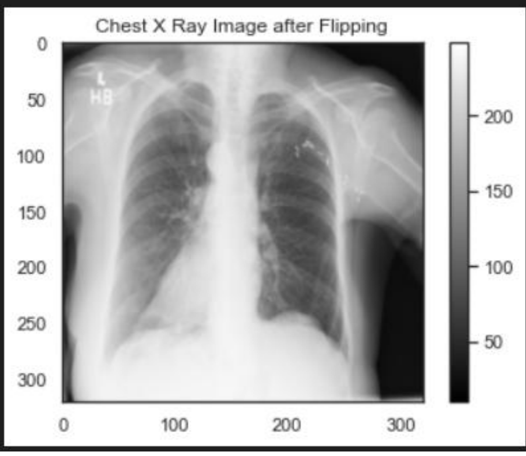
```
Text(0.5, 1.0, 'Chest X Ray Image after Random Cropping')
```



```python
# Horizontal flip

data_transform = transforms.Compose([
    transforms.Resize((320, 320)),
    ### Your code here, use API like transforms.XXX() ###
    transforms.RandomHorizontalFlip(p=1),
    ### End your code ###
    transforms.ToTensor()
])
dataset = XRayDataset('nih/images-small', data_transform)

sns.set_style("white")
generated_image = dataset.__getitem__(0)
inverted_image = transform_invert(copy.deepcopy(generated_image), data_transform)
plt.imshow(inverted_image, cmap='gray')
plt.colorbar()
plt.title('Chest X Ray Image after Flipping')
```

✓ 0.2s

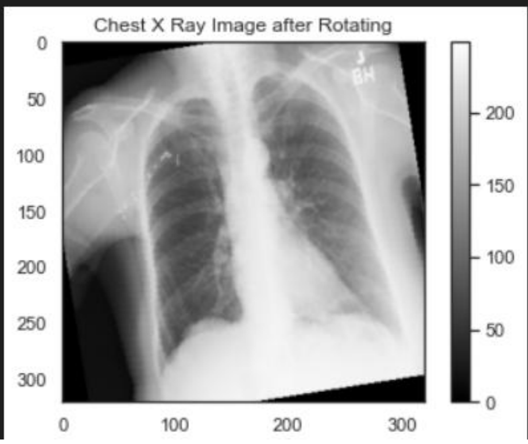
```
Text(0.5, 1.0, 'Chest X Ray Image after Flipping')
```



```python
# Random rotation

data_transform = transforms.Compose([
    transforms.Resize((320, 320)),
    ### Your code here, use API like transforms.XXX() ###
    transforms.RandomRotation(20),
    ### End your code ###
    transforms.ToTensor()
])
dataset = XRayDataset('nih/images-small', data_transform)

sns.set_style("white")
generated_image = dataset.__getitem__(0)
inverted_image = transform_invert(copy.deepcopy(generated_image), data_transform)
plt.imshow(inverted_image, cmap='gray')
plt.colorbar()
plt.title('Chest X Ray Image after Rotating')
```

✓ 0.2s

```
Text(0.5, 1.0, 'Chest X Ray Image after Rotating')
```



```python
# Color jittering

data_transform = transforms.Compose([
    transforms.Resize((320, 320)),
    ### Your code here, use API like transforms.XXX() ###
    transforms.ColorJitter(brightness=0.1, contrast=0, saturation=1, hue=0.1),
    ### End your code ###
    transforms.ToTensor()
])
dataset = XRayDataset('nih/images-small', data_transform)

sns.set_style("white")
generated_image = dataset.__getitem__(0)
inverted_image = transform_invert(copy.deepcopy(generated_image), data_transform)
plt.imshow(inverted_image, cmap='gray')
plt.colorbar()
plt.title('Chest X Ray Image after Color Jittering')
```

✓ 0.4s

```
Text(0.5, 1.0, 'Chest X Ray Image after Color Jittering')
```