Part 1.

(1) concrete $= 0.85 \dfrac{f_{ck}}{1.5} \cdot 0.8x \cdot b$

$= 5440\, x_i$ N

steel: $F_s = \dfrac{f_{steel}}{1.15} \cdot A_s = 1186087$ N

$5440 x = 1186087$

$x = 218$ mm (the depth of neutral axis)

lever arm $: l = d - 0.4x = 557.8$ mm

$M_u = F_s \cdot l = 6.616 \times 10^8$ N·mm

Balanced moment of resistance:

$M_{bal} = 0.167 f_{ck} \cdot b \cdot d^2 = 833714100$ N·mm

The $M_u < M_{bal}$.

1b). concrete: $F_c = 0.85 \frac{f_{cm}}{\gamma_m} \cdot 300 \cdot 0.8x = 10200x$. N

Steel: $F_s = A_s \times f_s = 1445840$ N

$F_c = F_s \Rightarrow x = 141.75$ mm (neutral axis)

$z = \left(d - \frac{0.8x}{2}\right) = 588.3$ mm

$M_u = F_s \cdot z = 850587672$ N·mm

(c) $E_{eff} = \frac{E_c}{(1+\phi)} = \frac{30}{(1+2.4)} = 8.824$ kN/mm^2

the neutral axis depth, x

$b \cdot x \cdot \frac{x}{2} \cdot E_{eff} = E_s \cdot A_s \cdot (d-x)$

$1323.6x^2 = 545600(645 - x)$

$x^2 = 412.21(645 - x)$

$\Rightarrow 349.2$ mm

(ii):

Second moment of area of the cracked section.

$$I = \frac{bx^3}{3} + a_c A_s \cdot (d - x)^2$$

$$= \frac{300 \times 349.2^3}{3} + \frac{200}{8.824} \times 2728\,(645 - 349.2)^2$$

$$= 9668266014 \text{ mm}^4$$

$$f_{cc} = \frac{My}{I} = \frac{250 \times 10^6 \times 349.2}{9668266014} = 9.03 \text{ N/mm}^2$$

$$\varepsilon_{cc} = \frac{f_{cc}}{E_{eff}} = \frac{9.03}{8.824 \times 10^3} = 0.001 \text{ mm}$$

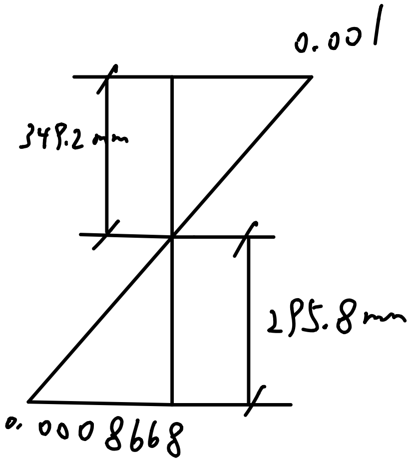
(iii) $$f_{st} = \frac{M \times (d-x)}{I} \times \frac{E_s}{E_{eff}}$$

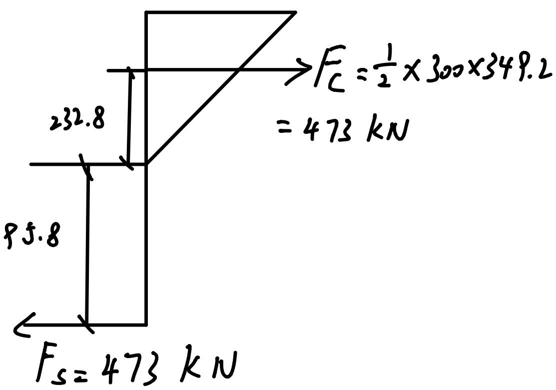
$$= \frac{250 \times 10^6 \times (645 - 349.2)}{9668266014} \times \frac{200}{8.824}$$

$$= 173.362 \text{ N/mm}^2$$

$$\varepsilon_{st} = \frac{f_{st}}{E_s} = \frac{173.362}{200000} = 0.0008668$$

(iv).





(v) $M = 473 \times (295.8 + 232.8) = 250027.8\ kN \cdot mm$

It is ok.

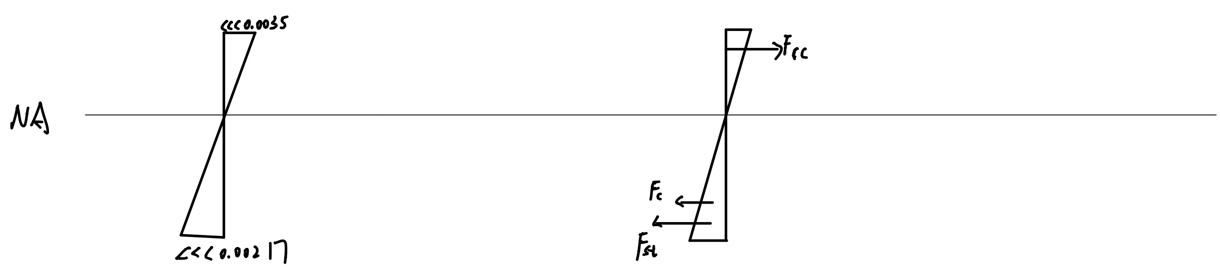
(d) $\phi = \frac{\text{Creep strain}}{\text{elastic strain}} \Rightarrow \phi \downarrow \quad E_{eff} = \frac{E_c}{(1+\phi)}, \quad E_{eff} \uparrow$

So the x will decrease, the neutral axial up, thus the second moment of area $\downarrow$, the EI will $\downarrow$

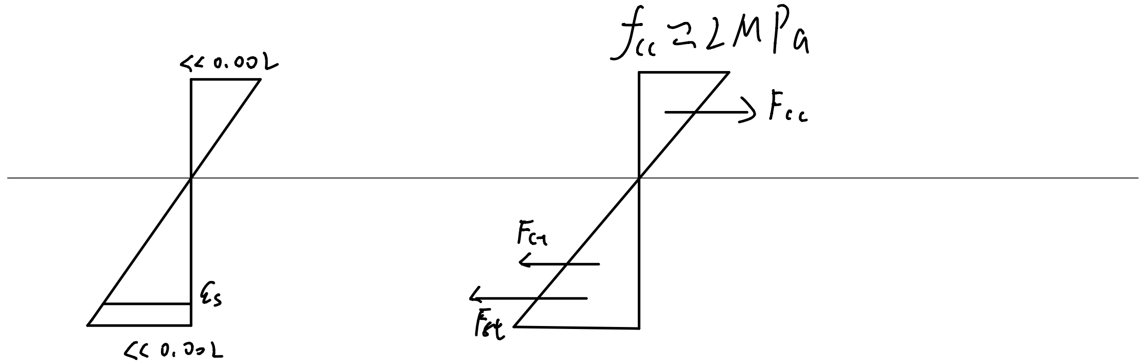
So $f_{st} \downarrow$; $f_{cc} \uparrow$

(e)

Low moment

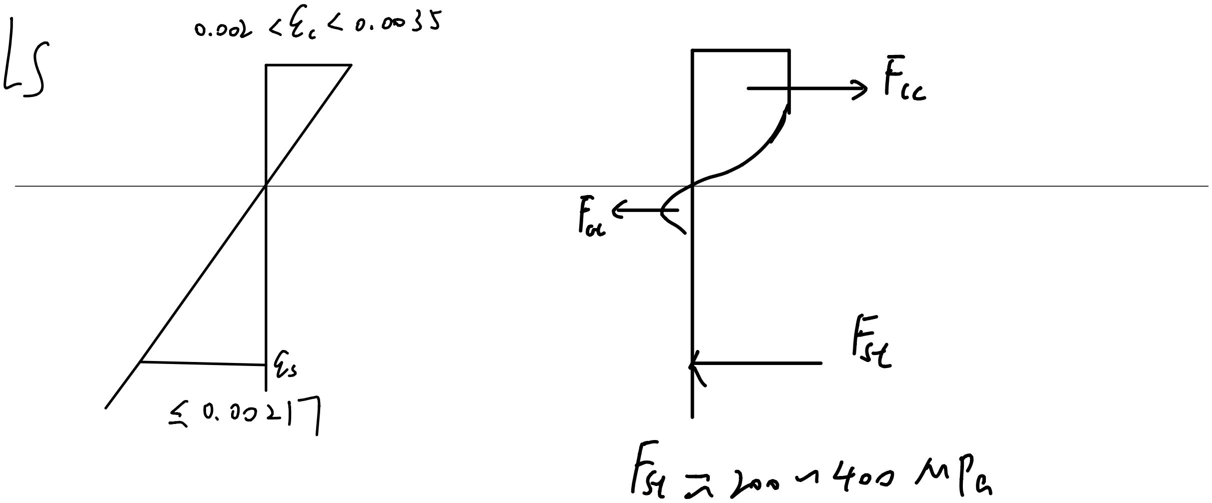


NA



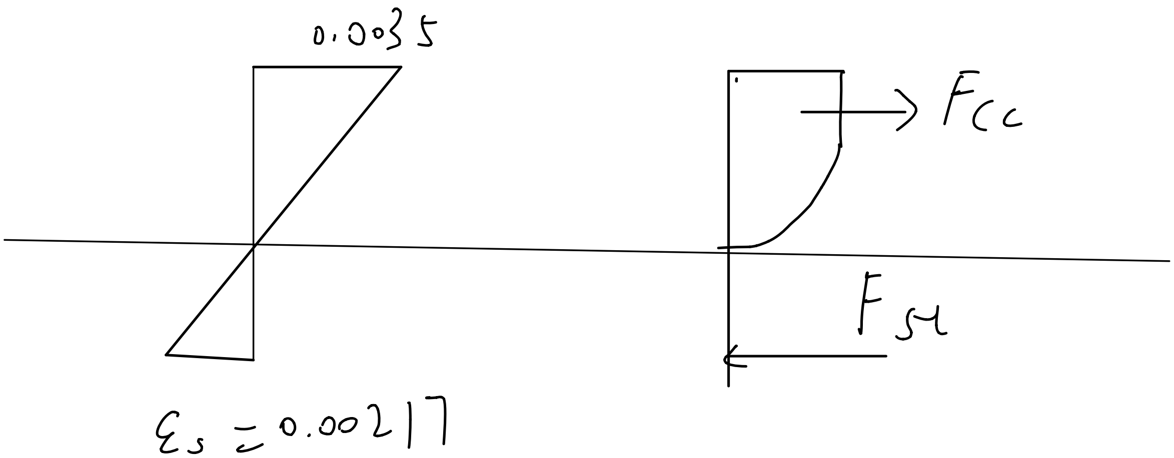
<< 0.002
ε_s
<< 0.002
$f_{cc} \simeq 2$ MPa
F_{cc}
F_{ct}
F_{st}

SLS



$0.002 < \varepsilon_c < 0.0035$
ε_s
≤ 0.00217
F_{cc}
F_{ct}
F_{st}
$F_{st} \simeq 200 \sim 400$ MPa

ULS



0.0035
F_{cc}
F_{st}
$\varepsilon_s = 0.00217$

Part B.

1. ULS: $(1.35 \times 3.7) + (1.5 + 3.3) = 9.945 \ kN/m^2$

SLS: $3.3 \ kN/m^2$

For ULS:

UDL: $9.945 \times 3 = 29.835 \ kN/m$

Maximum moment: $\frac{wL^2}{8} = 238.68 \ kN \cdot m$

Shear: $V = \frac{wL}{2} = 119.34 \ kN$

Try $406 \times 140 \times 46$ S355 UB

$h = 403.2 \ mm$ $b = 142.2 \ mm$ $t_w = 6.8 \ mm$

$t_f = 11.2 \ mm$ $r = 10.2 \ mm$ $I_{yy} = 15685 \ cm^4$ $I_{zz} = 538 \ cm^4$

$W_{el,yy} = 778 \ cm^3$ $W_{pl,yy} = 888 \ cm^3$ $A = 58.6 \ cm^2$

$I_w = 0.207 \times 10^{12} \ mm^6$ $I_t = 19 \times 10^4 \ mm^4$

$\varepsilon = \sqrt{\frac{235}{355}} = 0.8136$

Outstand flange: $\frac{c_f}{t_f} = 5.13 < 9\varepsilon \Rightarrow$ class 1

Internal web: $\frac{c_w}{t_w} = 53 \leq 72\varepsilon = 58.58 \Rightarrow$ class 1.

Bending: $M_{c,Rd} = \frac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = \frac{888000 \times 335}{1} = 297,480,000$ N·mm

$\frac{M_{Ed}}{M_{c,Rd}} = \frac{238680000}{297480000} < 1$ ∴ It is ok for Bending

Shear: $A_v = A - 2 \times b \times t_f + (t_w + 2 \cdot r) \cdot t_f$

$= 5860 - 2 \times 142.2 \times 11.2 + (6.8 + 2 \times 10.2) \times 11.2 = 2979.36 mm^2$

$V_{c,Rd} = \frac{A_v \cdot (f_y/\sqrt{3})}{\gamma_{M0}} = \frac{2979.36 \times (335/\sqrt{3})}{1} = 576245$ N

$\frac{V_{Ed}}{V_{c,Rd}} = \frac{119340}{576245} < 1$ ∴ It is ok for shear

LTB: $k = k_w = 1$

uniform load $C_1 = 1.132$ $C_2 = 0.454$. $z_g = \frac{d}{2} = 201.6$ mm.

$E = 210000$ N/mm² $G = 81000$ N/mm²

$$M_{cr} = \frac{C_1 \pi^2 \cdot E \cdot I_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(kL)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 z_g)^2} - (C_2 z_g) \right\}$$

$$= \frac{1.132 \cdot \pi^2 \cdot 210000 \times 538 \times 10^4}{(8000)^2} \cdot \left\{ \sqrt{\frac{0.207 \times 10^{12}}{538 \times 10^4} + \frac{(8000)^2 \times 81000 \times 19 \times 10^4}{\pi^2 \times 210000 \times 538 \times 10^4} + (0.454 \times 201.6)^2} - (0.454 \times 201.6) \right\}$$

$= 5.45 \times 10^7$ N·mm

$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = 2.4$

$h/b = 2.8 > 2$ Curve C. $\alpha_{LT} = 0.49$

$$\Phi_{LT} = 0.5[1 + 0.49(2.4 - 0.4) + 0.75 \times 2.4^2]$$
$$= 3.15$$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - 0.75 \times \lambda_{LT}^2}} = 0.18 \quad \begin{cases} \leq 1 \\ \leq \frac{1}{\lambda_{LT}^2} = 0.1736 \ (\times) \end{cases}$$

$$\therefore \chi_{LT} = 0.18$$

$$f = 1 - 0.5(1 - k_c)[1 - 2(\bar{\lambda}_{LT} - 0.8)^2] \qquad k_c = 0.94$$
$$= 1.1236 \quad \text{but } f \leq 1$$

$$\therefore f = 1 \Rightarrow \chi_{LT,mod} = \frac{\chi_{LT}}{1} = 0.18$$

$$M_{b,Rd} = \chi_{LT} \cdot W_{pl,y} \cdot \frac{f_y}{\gamma_{M1}} = 0.18 \times 888 \times 10^3 \times \frac{355}{1}$$
$$= 56743200 \ N \cdot mm$$
$$= 56.74 \ kN \cdot m < M_{Ed}$$

$\therefore$ Cannot bear it.

$$SLS: = 33 \times 3 = 9.9 \ kN/m$$

$$\delta = \frac{5 w L^4}{384 E I} = \frac{5 \times 9.9 \times 8000^4}{384 \times 210000 \times 15685 \times 10^4} = 16.03 \ mm$$

$$\frac{L}{360} = 22.22 \qquad \delta < 22.22 \qquad \therefore ok$$

$\therefore$ So we choose another section to calculate.

Estimated the cross-section:

$$W_y = \frac{M_{Ed} \cdot \gamma_{M1}}{f_y \cdot 0.1736} = 3872815 \ mm^3$$

Try 610 x 229 x 140.

$h = 617.2$ mm $\quad b = 230.2$ mm $\quad t_w = 13.1$ mm $\quad t_f = 22.1$

$r = 12.7$ mm $\quad I_{yy} = 111777 \ cm^4 \quad I_{zz} = 4505 \ cm^4 \quad W_{pl,y} = 4152 \ cm^3$

$I_w = 3.99 \ dm^6 \quad I_T = 216 \ cm^4 \quad A = 178 \ cm^2$

$\varepsilon = \sqrt{\frac{235}{335}} = 0.81 \qquad Z_g = \frac{h}{2} = 308.6$ mm

Internal parts: $\frac{C_t}{t_t} = 41.8 < 72\varepsilon \quad \Rightarrow$ Class 1

Outstand parts: $\frac{C_w}{t_w} = 4.34 < 9\varepsilon = 7.29 \quad \Rightarrow$ Class 1

Bending:

$$M_{c,Rd} = \frac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = \frac{4152000 \times 355}{1} = 1473960000 \ N \cdot mm$$

$\frac{M_{Ed}}{M_{c,Rd}} = \frac{238.68 \times 10^6}{1473.96 \times 10^6} < 1 \quad \therefore$ It is ok for bending

Shear: $A_v = A - 2 \times b \times t_f + (t_w + 2 \cdot r) \cdot t_f$

$= 8476 \ mm^2$

$$V_{c,Rd} = \frac{A_v \cdot (f_y / \sqrt{3})}{\gamma_{M0}} = 1698.363 \ kN$$

$$\frac{V_{Ed}}{V_{c,Rd}} < 1 \qquad \therefore \text{OK}$$

LTB: $k = k_w = 1 \qquad C_1 = 1.132 \qquad C_2 = 454$

$$M_{cr} = \frac{C_1 \pi^2 E I_z}{(KL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(kL)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_1 Z_g)^2} - (C_2 \cdot Z_g) \right\}$$

$$= \frac{1.132 \times \pi^2 \times 210000 \times 45050000}{8000^2} \left\{ \sqrt{\frac{3.99 \times 10^{12}}{4505 \times 10^4} + \frac{(8000)^2 \cdot 81000 \times 216 \times 10^4}{\pi^2 \cdot 210000 \times 4505 \times 10^4} + (0.454 \times 308.6)^2} - (0.454 \times 308.6) \right.$$

$$= 557416475.1 \text{ N.mm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = 1.58 \qquad h/b = 2.68 > 2$$

$\therefore$ Curve C $\qquad a_{LT} = 0.49$

$$\phi_{LT} = 0.5\left[1 + a_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2\right] = 1.725$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.36 \begin{cases} \le 1 \\ \le \frac{1}{\lambda_{LT}^2} = 0.4 \end{cases}$$

$$f = 1 - 0.5(1 - k_c)\left[1 - 2(\bar{\lambda}_{LT} - 0.8)^2\right]$$

$$= 1.0065 > 1$$

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} = 0.36$$

$$M_{b,Rd} = \chi_{LT} \cdot W_y \cdot \frac{f_y}{\gamma_{M1}} = 500.7312 \ kN \cdot m$$

$$\frac{M_{Ed}}{M_R} < 1 \qquad \therefore \text{It is Ok}$$

SLS : $UDL = 3.3 \times 3 = 9.9 \ kN/m$

$$\delta = \frac{5 w L^4}{384 \cdot EI} = 2.25 \ mm$$

$$\frac{span}{360} = 22.22 \ mm \qquad \delta = 2.25 < 22.22$$

$\therefore$ This section is OK.

So the section $610 \times 229 \times 140$. is ok for ULS and SLS check.

Q_2 : We try again use the section $610 \times 229 \times 140$. S355 UB

$h = 617.2$ mm $\quad b = 230.2$ mm $\quad t_w = 13.1$ mm $\quad t_f = 22.1$ mm

$r = 12.7$ mm $\quad I_{yy} = 111777 \ cm^4 \quad I_{zz} = 4505 \ cm^4 \quad W_{pl,y} = 4152 \ cm^3$

$I_w = 3.99 \ dm^6 \quad I_T = 216 \ cm^4 \quad A = 178 \ cm^2 \qquad \varepsilon = \sqrt{\frac{235}{f_y}} = 0.8136$

Internal part : $C_w/t_w = 41.8 < 72\varepsilon = 58.6$ class 1

outstand part: $C_f/t_f = 4.34 < 9\varepsilon = 7.32$ class 1

The whole section is class 1.

$$UDL = 9.945 \times 3 = 29.835 \ kN/m$$

$$M_{Ed} = \frac{wL^2}{8} = \frac{29.835 \times 8000^2}{8} = 238.68 \ kN.m$$

$$V_{Ed} = \frac{5wL}{8} = \frac{5 \times 29.835 \times 8000}{8} = 149.2 \ kN$$

Bending: $M_{cRd} = \dfrac{W_{pl,y} \cdot f_y}{\gamma_{M0}} = 1473.96 \ kN.m$

$$\frac{M_{Ed}}{M_{cRd}} < 1 \qquad \text{It is ok}$$

Shear: $A_v = A - 2 \times b \times t_f + (t_w + 2 \cdot r) \cdot t_f$

$= 8476 \ mm$

$$V_{cRd} = \frac{A_v \cdot (f_y/\sqrt{3})}{\gamma_{M0}} = \frac{8476 \times (355/\sqrt{3})}{1} = 1737237 \ N$$

$$\frac{V_{Ed}}{V_{cRd}} < 1 \qquad \therefore \text{ok}$$

LTB: UDL : 9.945 × 3 = 29.835 kN/m

$M_{Ed} = \frac{wL^2}{8} = 23868 \times 10^4$ N·mm

$k = k_w = 1$ $C_1 = 2.21$ $C_2 = 0.88$ $z_g = \frac{h}{2} = 308.6$ mm

$E = 210000$ N/mm² $G = 81000$ N/mm²

$$M_{cr} = \frac{C_1 \pi^2 E I_z}{(kL)^2} \left\{ \sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 \cdot G I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 z_g)^2} - (C_2 \cdot z_g) \right\}$$

$$M_{cr} = \frac{2.21 \times \pi^2 \times 210000 \times 4505 \times 10^4}{8000^2} \times \sqrt{\left(\frac{1}{1}\right)^2 \frac{3.99 \times 10^{12}}{4505 \times 10^4} + \frac{(8000)^2 \times 81000 \times 216 \times 10^4}{\pi^2 \cdot 210000 \times 4505 \times 10^4} + (308.6 \times 0.88)^2} - (308.6 \times 0.88)$$

274325.725 259.6956874

$= 837318858.6$ N·mm

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = \sqrt{\frac{4152 \times 10^3 \times 335}{663321532.8}} = 1.327$$

$h/b = 2.68 > 2 \Rightarrow$ Curve $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \cdot \bar{\lambda}_{LT}^2]$$

$$= 0.5 [1 + 0.49 (1.49 - 0.4) + 0.75 \times 1.49^2]$$

$$= 1.39$$

$$\chi_{LT} = \frac{1}{1.6 + \sqrt{1.6^2 - 0.75 \times 1.6^2}} = 0.4796 \begin{cases} \leq 1 \ ✓ \\ \leq \frac{1}{\bar{\lambda}_{LT}^2} \ 0.568 ✓ \end{cases}$$

$$f = 1 - 0.5 (1 - k_c) [1 - 2 (\bar{\lambda}_{LT} - 0.8)^2] \quad k_c = 0.91$$

$f = 0.98$ $\therefore f = 0.98$

$$\chi_{LT.mod} = \frac{\chi_{LT}}{f} = 0.48$$

$$M_{b.Rd} = \chi_{LT} \cdot W_y \cdot \frac{f_y}{\gamma_{M1}}$$

$$= 681.551 \text{ kN.m}$$

$$\frac{M_{Ed}}{M_{b.Rd}} < 1 \qquad \therefore \text{ ok}$$

For SLS: $UDL = 3 \times 3.3 = 9.9$ kN/m

$$\frac{\text{Span}}{360} = 22.22 \text{ mm}$$

$$\delta = \frac{5wL^4}{384EI} = \frac{5 \times 9.9 \times 8000^4}{384 \times 210000 \times 11777 \times 10^4} = 2.5 < 22$$

So the section is ok for ULS ; SLS