In order to expand the validation of the effectiveness of the CFSDAM under different operating conditions, application scenarios, and dataset characteristics, we have correspondingly supplemented real experiments in the manufacturing system scenarios in this explanation part. The example is based on six customized CNC machine tools in a manufacturing system dedicated to producing engine crankshafts, which are respectively composed of CNC milling center CNC_1, CNC lathe CNC_2, CNC milling machine CNC_3, CNC punching machine CNC_4, CNC follow-up grinder CNC_5, and CNC polishing machine CNC_6. Among them, by installing PCB acceleration sensors on the front end of the spindle of the six customized rental CNC machine tools, the real-time monitoring and characterization of the machine's health status during the simplified modeling processing are guaranteed. The sampling frequency of all sensors is set to $1000\ Hz$, and the sampling interval is 1 hour. The sampling length between each interval is 0.5 seconds, and the signal average value between each sampling interval is used to represent the real-time status of the machine tool at the corresponding moment. The set of degradation sensor signal sequences used for verification in this section is collected by repeated simulation of the degradation process of the above-mentioned machine tool.

Table 1

The overall setting of the sensor signals in the manufacturing system scenarios.

Dataset	CNC_1	CNC_2	CNC_3	CNC_4	CNC_5	CNC_6
Component	CNC Center	Lathe	Drilling Machine	Milling Machine	Grinding Machine	Washing Machine
Train set	100	100	100	100	100	100
Test Set	100	100	100	100	100	100
Max/Min Length-train	3633/2464	4155/1552	2103/1039	2279/69	2411/1553	4978/408
Max/ Min Length-test	3691/2356	4469/1715	1939/1007	2290/26	2433/1587	5543/554

In Table 1, we further introduce the overall setting of the degradation sequences collected from rental machine tools under privacy preservation. This example combines the degraded process sequence fitted by each rental machine tool in the crankshaft machining process with the random noise disturbance term, and constructs a complete set of degraded sensor signal sequences through 200 random repeated fittings. Among them, 100 degraded sensor sequences are randomly selected from the complete sequence set and defined as the private sensor signal set $\mathcal{H}_O'$ owned locally by the machine tool manufacturing enterprise. Furthermore, 90 degraded sensor sequences in the remaining degraded sensor sequences are set on average to the local private sensor signal set $\mathcal{H}_m'$ owned by 9 distributed customer lessees, and the remaining 10 degraded sensor sequences are used as the test signal sequence of the machine tool to verify the example effect of real-time health prediction. At the same time, it is worth noting that no direct signal sharing and data interaction can be performed between any $\mathcal{H}_m'$ and $\mathcal{H}_O'$.

Among them, the degradation signal of CNC_1 generally follows a linear degradation trend, and its time-to-failure (TTF) distribution test conforms to a lognormal distribution. The degradation signal of CNC_2 primarily follows an exponential degradation trend, with its TTF distribution test conforming to a Weibull distribution. The degradation signal of CNC_3 also follows a linear degradation trend, and its TTF distribution test conforms to a normal distribution. The degradation signal of CNC_4 follows an exponential degradation trend, with its TTF distribution test conforming to a SEV distribution. The degradation signal of CNC_5 follows an exponential degradation trend, and its TTF distribution test conforms to a lognormal distribution. Finally, the degradation signal of CNC_6 follows a linear degradation trend, with its TTF distribution test conforming to a Weibull distribution. In this scenario, each machine is on a different operation condition and the signals obtained from different machines have diverse distributions. The complete process for scenario design and parameter determination is identical to that of the C-MAPSS dataset. Based on this premise, we present the overall RMSE results comparison as shown in Table 2 and Table 3.

Table 2

RUL prediction results in comparison in the manufacturing system scenarios.

Indicators\Method	DANN	CORAL	CADA	CGAN	Proposed
mean-RMSE	31.79	27.95	30.39	29.79	**26.45**
Min Distance	223.83	140.67	214.08	179.29	**86.49**

Table 3

RUL prediction results in comparison for the specific machines.

Source	Target	DANN	CORAL	CADA	CGAN	Proposed
CNC_1	CNC_2	24.94	28.61	27.45	24.63	**14.47**
	CNC_4	23.91	25.25	21.49	24.99	**20.87**
	CNC_6	11.49	16.06	17.88	17.7	**9.66**
CNC_2	CNC_4	46.47	51.45	45.68	43.89	**27.99**
	CNC_5	5.95	13.66	5.42	10.81	**5.17**
	CNC_6	26.02	15.67	14.87	12.95	**7.8**
CNC_3	CNC_1	20.36	16.38	20.34	22.32	**16.08**
	CNC_6	28.92	27.17	25.65	27.47	**21.35**
CNC_4	CNC_1	32.94	26.03	25.83	25.68	**20.3**
	CNC_2	66.88	69.53	68.5	68.1	**66.31**
	CNC_6	25.78	27.82	28.31	27.14	**25.29**
CNC_5	CNC_4	48.91	44.01	41.92	42.77	**38.54**
	CNC_6	32.74	31.1	29.84	24.76	**19.46**
CNC_6	CNC_1	18.15	16.32	17.92	17.61	**16.26**
	CNC_5	17.3	11.37	14.11	13.51	**10.66**

The comparison results in Table 2 comprehensively present that our CFSDAM can not only protect the privacy of local sensor signals, but also enable accurate RUL predictions for different equipment under various conditions. Compared to the four methods discussed in the manuscript, our method achieves optimal performance with an average RMSE accuracy across 30 overall transfer pairs. Additionally, the calculated aligned distribution distance is also the smallest among all methods, which means CFSDAM effectively reduces distribution shifts under varying operating conditions. Furtherly, Table 3 shows the best performance transfer pairs for each equipment. Since this case study involves different types of equipment, we can demonstrate the effectiveness of the proposed model for different equipment under various degradation processes. In conclusion, both the C-MAPSS and N-CMAPSS datasets pertain to the same type of equipment with lifespan prediction requirements under different operating conditions. So, the focus of this explanation part is on a more detailed analysis and numerical experiments of these two publicly available datasets. Nonetheless, all the experiments conducted effectively demonstrate the validity and accuracy of our method in predicting outcomes across different scenarios and device types.