In order to expand the validation of the effectiveness of the CFSDAM under different operating conditions, application scenarios, and dataset characteristics, we have correspondingly supplemented real experiments in the manufacturing system scenarios in this explanation part. The example is based on six customized CNC machine tools in a manufacturing system dedicated to producing engine crankshafts, which are respectively composed of CNC milling center $CNC_1$, CNC lathe $CNC_2$, CNC milling machine $CNC_3$, CNC punching machine $CNC_4$, CNC follow-up grinder $CNC_5$, and CNC polishing machine $CNC_6$. Among them, by installing PCB acceleration sensors on the front end of the spindle of the six customized rental CNC machine tools, the real-time monitoring and characterization of the machine's health status during the simplified modeling processing are guaranteed. The sampling frequency of all sensors is set to $1000\ Hz$, and the sampling interval is 1 hour. The sampling length between each interval is 0.5 seconds, and the signal average value between each sampling interval is used to represent the real-time status of the machine tool at the corresponding moment. The set of degradation sensor signal sequences used for verification in this section is collected by repeated simulation of the degradation process of the above-mentioned machine tool.

**Table 1**

The overall setting of the sensor signals in the manufacturing system scenarios.

| Dataset | $CNC_1$ | $CNC_2$ | $CNC_3$ | $CNC_4$ | $CNC_5$ | $CNC_6$ |
|---|---|---|---|---|---|---|
| Component | CNC Center | Lathe | Drilling Machine | Milling Machine | Grinding Machine | Washing Machine |
| Train set | 100 | 100 | 100 | 100 | 100 | 100 |
| Test Set | 100 | 100 | 100 | 100 | 100 | 100 |
| Max/Min Length-train | 3633/2464 | 4155/1552 | 2103/1039 | 2279/69 | 2411/1553 | 4978/408 |
| Max/ Min Length-test | 3691/2356 | 4469/1715 | 1939/1007 | 2290/26 | 2433/1587 | 5543/554 |

In Table 1, we further introduce the overall setting of the degradation sequences collected from machine tools under privacy preservation. This example combines the degraded process sequence fitted by each machine tool in the crankshaft machining process with the random noise disturbance term and constructs a complete sub-dataset of degraded sensor signal sequences through 200 random repeated fittings for every single machine. Among every sub-dataset, 100 degraded sensor sequences are randomly selected from the complete sequence set and defined as the training sensor signal set $\mathcal{H}_{training}$ owned by 5 distributed clients locally, and the remaining 100 degraded sensor sequences are used as the test signal set $\mathcal{H}_{test}$ to verify the example effect of real-time health prediction.

Among them, the degradation signal of $CNC_1$ generally follows a linear degradation trend, and its time-to-failure (TTF) distribution test conforms to a lognormal distribution. The degradation signal of $CNC_2$ primarily follows an exponential degradation trend, with its TTF distribution test conforming to a Weibull distribution. The degradation signal of $CNC_3$ also follows a linear degradation trend, and its TTF distribution test conforms to a normal distribution. The degradation signal of $CNC_4$ follows an exponential degradation trend, with its TTF distribution test conforming to a SEV distribution. The degradation signal of $CNC_5$ follows an exponential degradation trend, and its TTF distribution test conforms to a lognormal distribution. Finally, the degradation signal of $CNC_6$ follows a linear degradation trend, with its TTF distribution test conforming to a Weibull distribution. In this case, each machine is considered to be on a different operation condition and every two machines are paired one by one to generate 30 pairs of source-target mappings. The complete process for scenario design and parameter determination is identical to that of the C-MAPSS dataset. Based on this premise, we present the overall RMSE results comparison as shown in Table 2 and Table 3.

**Table 2**

RUL prediction results in the manufacturing system scenarios.

| Indicators\Method | DANN | CORAL | CADA | CGAN | Proposed |
|---|---|---|---|---|---|
| mean-RMSE | 31.79 | 27.95 | 30.39 | 29.79 | **26.45** |
| Min Distance | 223.83 | 140.67 | 214.08 | 179.29 | **86.49** |

**Table 3**

RUL prediction results in comparison for the different source-target pairs.

| Source | Target | DANN | CORAL | CADA | CGAN | Proposed |
|---|---|---|---|---|---|---|
| $CNC_1$ | $CNC_2$ | 24.94 | 28.61 | 27.45 | 24.63 | **14.47** |
| | $CNC_4$ | 23.91 | 25.25 | 21.49 | 24.99 | **20.87** |
| | $CNC_6$ | 11.49 | 16.06 | 17.88 | 17.7 | **9.66** |
| $CNC_2$ | $CNC_4$ | 46.47 | 51.45 | 45.68 | 43.89 | **27.99** |
| | $CNC_5$ | 5.95 | 13.66 | 5.42 | 10.81 | **5.17** |
| | $CNC_6$ | 26.02 | 15.67 | 14.87 | 12.95 | **7.8** |
| $CNC_3$ | $CNC_1$ | 20.36 | 16.38 | 20.34 | 22.32 | **16.08** |
| | $CNC_6$ | 28.92 | 27.17 | 25.65 | 27.47 | **21.35** |
| $CNC_4$ | $CNC_1$ | 32.94 | 26.03 | 25.83 | 25.68 | **20.3** |
| | $CNC_2$ | 66.88 | 69.53 | 68.5 | 68.1 | **66.31** |
| | $CNC_6$ | 25.78 | 27.82 | 28.31 | 27.14 | **25.29** |
| $CNC_5$ | $CNC_4$ | 48.91 | 44.01 | 41.92 | 42.77 | **38.54** |
| | $CNC_6$ | 32.74 | 31.1 | 29.84 | 24.76 | **19.46** |
| $CNC_6$ | $CNC_1$ | 18.15 | 16.32 | 17.92 | 17.61 | **16.26** |
| | $CNC_5$ | 17.3 | 11.37 | 14.11 | 13.51 | **10.66** |

The comparison results in Table 2 comprehensively present that our CFSDAM can not only protect the privacy of local sensor signals, but also enable accurate RUL predictions for different equipment under various conditions. Compared to the four methods discussed in the manuscript, our method achieves optimal performance with an average RMSE accuracy across 30 overall transfer pairs. Additionally, the calculated aligned distribution distance is also the smallest among all methods, which means CFSDAM effectively reduces distribution shifts under varying operating conditions. Furtherly, Table 3 shows the best performance transfer pairs for each equipment. Because both the C-MAPSS and N-CMAPSS datasets pertain to the same type of equipment with lifespan prediction requirements under different operating conditions, the focus of this explanation part is on a more detailed analysis and numerical experiments of these two publicly available datasets. Since this case study involves different types of equipment, we can demonstrate the effectiveness of the proposed model for different equipment under various degradation processes. In conclusion, all the experiments conducted effectively demonstrate the validity and accuracy of our method in predicting outcomes across different scenarios and device types.