

Algorithm Complexity & Operations Cheat Sheet

1. Union-Find (Disjoint Set) Variations (Week 7)

Variant	Complexity (M ops)	Tree Height	Key Mechanism
Quick Find	$O(MN)$	Flat (≤ 1)	Find is $O(1)$, Union is $O(N)$.
Quick Union	$O(MN)$	Unbounded ($\leq N$)	Lazy union. Find can take $O(N)$.
Weighted Quick Union	$O(M \log N)$	$O(\log N)$	Link smaller tree to larger tree.
Weighted + Path Comp.	$O(M \log^* N) \sim O(M)$	$\sim O(1)$	Flatten path during find.

2. Sorting Algorithms (Weeks 1-3)

Insertion Sort

- Best Case (Sorted): $O(n)$ time. $n-1$ comparisons, 0 swaps.
- Worst Case (Reverse): $O(n^2)$ time. $\sim n^2/2$ comparisons & swaps.
- Average Case: $O(n^2)$ time. $\sim n^2/4$ comparisons & swaps.

Merge Sort

- Complexity: $O(n \log n)$ all cases.
- Comparisons: $\leq n \log n$. Exact worst case: $n \log n - n + 1$.
- Space: $O(n)$ (Not in-place).

Quick Sort

- Best Case: $O(n \log n)$ (Pivot is median).
- Worst Case: $O(n^2)$ (Pivot is min/max). Comparisons: $n(n-1)/2$.
- Average Case: $O(n \log n)$.

Heap Sort

- Heap Construction: $O(n)$.
- Sorting Phase: $n * O(\log n)$.
- Total: $O(n \log n)$ all cases.

3. String Matching (Week 12)

Algorithm	Best Case	Worst Case	Notes
Brute Force	$O(n)$	$O(mn)$	Comps in worst case: $m(n-m+1)$.
Rabin-Karp	$O(n+m)$	$\Theta((n-m+1)m)$	Worst case: Spurious hits (collisions).
Boyer-Moore	$O(n/m)$	$O(mn)$	Best: Skips m chars. Worst: Pattern matches suffix, fails front.

4. Graph Algorithms (Weeks 4-6)

Algorithm	Time Complexity	Key Detail
BFS / DFS	$O(V + E)$	Linear (Adj List).
Topological Sort	$O(V + E)$	Only for DAGs. Uses DFS or Kahn's.
Dijkstra's	$O(E \log V)$	Greedy (Priority Queue). No negative weights.
Prim's (MST)	$O(E \log V)$	Greedy. Grows tree from a node.
Kruskal's (MST)	$O(E \log E)$	Greedy. Sorts edges. Uses Union-Find.

5. Dynamic Programming & Greedy

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- Coin Change (Greedy): $O(n)$. Optimal only for canonical systems.
- Coin Change (DP): $O(S * n)$. S=target, n=denominations.
- LCS (DP): $O(nm)$. Space $O(nm)$.
- 0/1 Knapsack (DP): $O(nW)$. Pseudo-polynomial.
- Matrix Chain Multiplication: Time $O(n^3)$, Space $O(n^2)$.