

# MATH70121 Topics in Derivative Pricing

## PCA Trading Strategy (Github Link)

Seghrouchni Ahmed Reda, Qizhe Cui, Kelvin Wu

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## 1 Introduction

In this report, we explore the construction and implementation of a PCA-based trading strategy utilizing swap prices and rates. Our methodology begins with a detailed comparison of various techniques for building and interpolating curves, focusing on the utilization of PCHIP interpolation applied to log-discount rates. This foundational step ensures that the derived curves accurately represent the underlying financial instruments.

Following the curve construction, we perform Principal Component Analysis (PCA) on the changes in spot rates. Our objective is to extract the first three principal components, aiming to comprehensively understand and articulate their respective influences on the dynamics of spot rate changes, with a particular emphasis on the curvature component as it is less sensitive to overall level and slope movements.

The core of our strategy involves designing a trade, specifically a butterfly, that is largely immune to the first two principal components (level and slope) but are maximally exposed to the third principal component (curvature). This trade design is critical for capitalizing on the unique insights provided by PCA and for achieving specific trading goals.

Additionally, we implement the strategy over a six-month period, meticulously tracking the daily Profit and Loss (PnL) to evaluate the performance of our trading strategy. This analysis is crucial for assessing the effectiveness of the strategy in real-world markets and for understanding the PnL's correlation with the principal components identified earlier. We particularly focus on the PnL's correlation characteristics with the curvature component, expecting a strong correlation as hypothesized in our strategy design.

This comprehensive approach ensures a robust framework for deploying PCA in trading strategies, with a clear focus on practical implementation and performance evaluation.

## 2 Building Curve with GBP OIS

### 2.1 Methodology

After initial data pre-processing, we are given the breakeven swap rates and swap prices of GBP as inputs. Swap price can be written as the linear combination of discount rate

$$1 - V_{\text{swap}} = P(t_n) + \sum_{j=1}^n c\tau P(j\tau) \quad (1)$$

,where  $\tau$  measures the coupon-paying schedule and  $c\tau$  is the coupon payment at times  $\tau, 2\tau, 3\tau, \dots$ . Hence one can build the discount curve  $P(T)$  from the swap prices.

In the following discussion we use lower case  $t_j$  to represent the coupon paying dates of swaps and upper case  $T_i$  for their maturities. The general curve-building methodology can be summarised as follow:

For  $i = 1, 2, \dots, N$ :

1. One has  $P(t_j)$  known for all  $t_j \leq T_{i-1}$
2. Interpolate  $P(t_j)$  for  $T_{i-1} < t_j < T_i$  to ensure  $V_i$  computed from the interpolated  $P(t_j)$  is matched to the market prices

### 2.2 Method of interpolation: PCHIP

The interpolation in step 2 is subject to two choices, parameters to interpolate and interpolation method. Firstly, due to the monotonic decreasing feature and other restrictions on  $P(T)$ , normally one does not directly interpolate  $P(T)$ . Given market swap prices, other options include 1) zero rate curve  $y(T) : P(T) = \exp(-y(T)T)$  and 2) log-discount curve  $l(T) = \log P(T) = y(T)T$ . Secondly, the interpolation method includes a combination of splines with different smoothness.

Given various terms from data inputs, we first found the outcomes from Hermite (quadratic) spline and cubic splines to be similar. Therefore, we settled on the second-order Hermite spline to avoid overfitting. There is a variant of Hermite spline, PCHIP, which ensures the interpolated curve to be monotone when a set of corresponding inputs is monotone. This feature is appreciated, as it can help avoid unwanted zigzag on the interpolated curve which usually can not be explained financially and only caused due to the nature of splines fitting. Hence, we settle with the PCHIP method.

### 2.3 Parameter to interpolate

We then compare the interpolated curves resulting from zero rate and log-discount rate to see which option is optimal.

We select the first and the last day of our dataset, performe PCHIP interpolation on zero rate and log-discount rate, and then plot the resulting zero rate curve and overnight forward rate curve to compare. The plot is given below in Fig 1 and 2.

One sees PCHIP interpolation on log discount rates suppresses zigzags and overshootings further more in overnight forward rates, avoid generating term structures that can not be explained financially. Regarding zero rate curves, both methods give very similar resultant curves. It is worth mentioning that in the four plots, using log discount interpolation blows up the curve on the short tenor side, while using zero rates gives a flat curve. This is due to the reason clarified in Section 2.4. Therefore, one can conclude the interpolation on log-discount rate is better and we eventually settled with **PCHIP method on log-discount rate**.

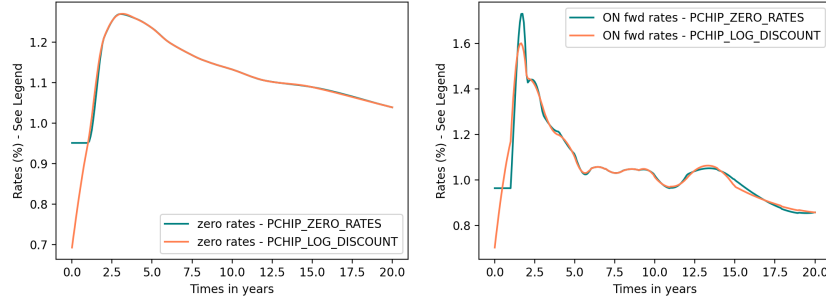


Figure 1: Comparison of zero rate and log-discount rate interpolation on 2022-01-27

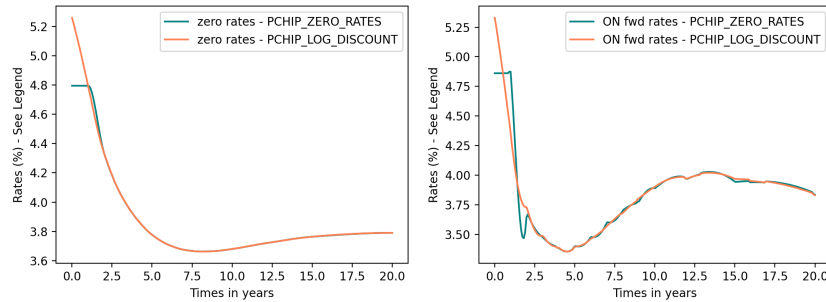


Figure 2: Comparison of zero rate and log-discount rate interpolation on 2023-02-14

## 2.4 Clarification on the usage of swap data with small tenors

In the programming infrastructure implemented in ‘financepy’, swaps with monthly coupons can not be aligned with those with annual coupons when interpolating. To avoid this error we only used data of swaps with annual coupons. This is the reason why there is a flat part in the resultant curves built in section 2.3, and should lead to errors in the following parts.

If not for time constraints, one should rebuild the interpolation method in order to obtain more accurate curves and trade analysis.

## 3 PCA

### 3.1 Review of PCA

PCA is a method to explain a high-dimensional dataset use several basis vectors called principal components. Consider a normalised dataset written as an  $m \times n$  matrix  $X$  (typically  $m > n$ ), first one conducts a Singular Value Decomposition (SVD) to obtain

$$X = U\Sigma V^T \quad (2)$$

where  $U$  ( $m \times m$ ) contains the eigenvectors of  $XX^T$  and  $V$  ( $n \times n$ ) contains the eigenvectors of  $X^T X$ , namely left and right singular vectors.  $\Sigma$  is a  $m \times n$  matrix containing eigenvalues of  $X^T X$  and trailing zero rows.

Then the  $i$ -th principal components (PC) of  $X$  is defined as  $Xv_i$ , where  $v_i$  is the  $i$ -th column of  $V$ , i.e. the  $i$ -th right singular vectors. Then one is able to approximate the datasets using first several PCs and measure the power of approximation using variance explained in the PCs used.

### 3.2 Conduct PCA on the changes of spot rates

Firstly we discuss whether to conduct PCA on spot or forward rates. We have data inputs in swap rates and prices and the interpolation discussed in the last section was on log-discount rate. Therefore, the forward rates can only be calculated through a finite-difference fashion from the interpolated curve, which induces errors. Hence for the accuracy of the trading strategy and simplicity, we chose to conduct PCA on spot swap rates.

We then discuss whether to do PCA on levels or changes. In reality, market reacts much more to the dynamics of the rates than the levels, so starting statistical analysis right on rate changes appears more reasonable. Also, PCA behaves poorly on time series that are highly correlated and auto-correlated, while changes on the rates have much less correlation. This is because the levels is a time series with trends while the changes do not necessarily have trends. Here we plot the correlation heatmap in Fig 3 to verify this argument, and proceed to use the changes of curve instead of levels. Notice in the plot the colour bars are set at the same range to depict the high correlation of levels compared to changes.

As will be discussed in Section 4, we will construct a trade on 5Y, 10Y and 20Y butterfly. Given this and the reason discussed in Section 2.4, we conduct PCA on the part of the curve between 1Y and 20Y tenors.

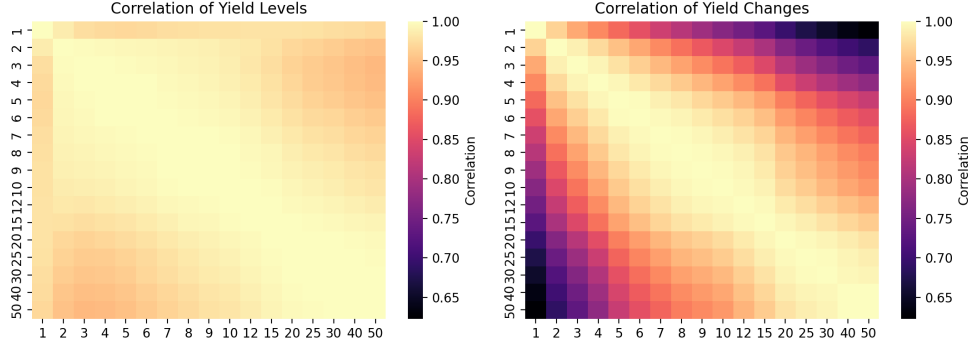


Figure 3: High correlation in levels unmanageable for PCA

### 3.3 Results

We conducted PCA on the changes of swap spot rates using data for tenors between 1Y and 20Y. The resultant first 3 PCs explain respectively 94.6%, 4.5% and 0.6% of variance. See the following plot for visual depiction.

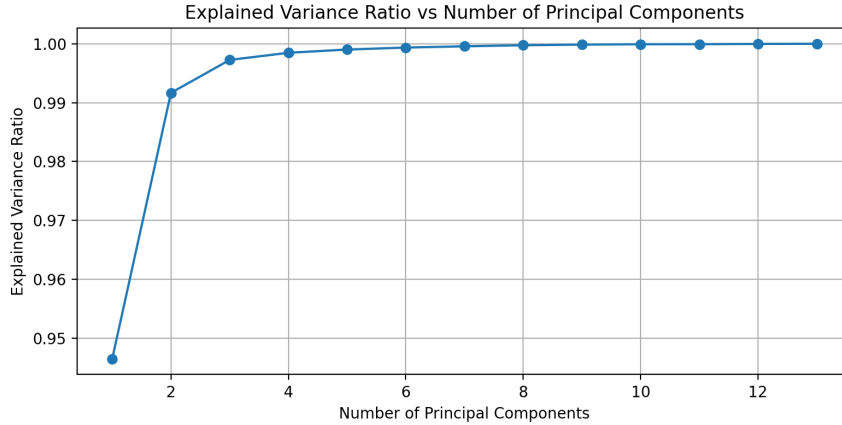


Figure 4: Variance explained by PCs

Then we move on to examine what the three PCs represent by computing their correlations with level, butterfly and spread of 2Y, 5Y and 10Y rates, as summarised in Table 1.

	PC2		PC3	
PC1	2s5s10s spread	-0.125	2s5s10s spread	0.664
10s level   0.985	5s10s20s spread	-0.588	5s10s20s spread	-0.113
	5s10s spread	-0.926	5s10s spread	-0.305
	2s10s spread	-0.977	2s10s spread	-0.065

Table 1: Summary of correlations with the first three PCs.

We can see that PC1 exhibits a strong representation of the 10Y rate changes, which is consistent with the theory. To determine the best tenors for our butterfly trade, we use a 2s5s10s spread to represent short to medium-term curvature, and a 5s10s20s spread to represent medium-to-long-term curvature, both with a simple ratio of 1:-2:1 for quick estimate. PC2 is highly correlated with both the 2s10s and 5s10s spreads, supporting the theory that the slope is the second principal component. However, we also find a reasonable correlation between PC2 and the 5s10s20s butterfly, suggesting that medium-to-long-term variations occasionally contribute to the dynamics of the curve as the second principal component. To further validate this hypothesis, we demonstrate that the third PC is only weakly correlated with the 5s10s20s spread, but more so with the short-term 2s5s10s spread. This suggests that a butterfly trade based on medium-term curvature might be a good strategy. And we will now go into the details on how to construct such trade to capture such deviation from a normal curve.

## 4 Trade Construction

### 4.1 Instruments

In the previous section, we selected GBP OIS as our benchmark instrument for curve construction. While intuitively, constructing a butterfly trade with SONIA swap seems reasonable post-Libor Reform, we believe both IborSwap and OIS are largely similar in FinancePy package, with `ibor_swap.py` providing much more functionalities than `ois.py`. Therefore, we will stick with GBP IborSwap and IborCurve for the trade, which should provide close proximity to a trade based on SONIA rate. We will also use annual payment frequency for both floating and fixed leg, ACT\_365F day convention to ensure it largely resembles the GBP OIS market.

The formula for the value of a fixed-for-floating Libor swap at time  $t$ , with a fixed rate  $k$ , Libor rate fixing dates  $0 \leq T_0 < T_1 < \dots < T_N$ , tenor  $\tau_n = T_{n+1} - T_n$ , forward Libor rate  $L_n(t) = L_n(t, T_n, T_{n+1})$  and T-maturity zero-coupon bond  $P(t, T)$ , is given as follows:

$$V_{swap}(t) = \sum_{n=0}^{N-1} \tau_n P(t, T_{n+1}) (L_n(t) - k) \quad (3)$$

By simple algebra, we can rewrite the formula as

$$V_{swap}(t) = A(t)(S(t) - k) \quad (4)$$

where

$$A(t) \triangleq A_{0,N}(t) = \sum_{n=0}^{N-1} \tau_n P(t, T_{n+1}) \quad (5)$$

$$S(t) \triangleq S_{0,N}(t) = \frac{\sum_{n=0}^{N-1} \tau_n P(t, T_{n+1}) L_n(t)}{\sum_{n=0}^{N-1} \tau_n P(t, T_{n+1})}, \quad (6)$$

$A(t)$  is the annuity/PVBP of the swap, which often referred to as the PV01 by practitioners, and we could easily obtain  $DV01 = PV01 * Notional$  which will be useful for evaluating

swap sensitivity.  $S(t)$  is the forward swap rate that make the swap to have value 0 for at time  $t$ , which will be important later for PnL calculation.

We will construct a 5s10s20s butterfly using spot swaps that are trading with value 0 at the start. We choose 5s10s20s tenor as we observe strong correlation between the 5s10s20s spread and the second principle component, which is usually the slope of a yield curve. This means that in the first 6 month of the dataset, the second dominant move is the curvature, especially among 5s10s20s tenors, not the slope of the curve. We chose spot swap as the PCA is conducted on the spot rate, not the forward rate, and this allows us to simplify the trade construction and later PnL calculation.

## 4.2 Notional and Direction

In terms of the notional and direction, we receive  $w_{5y} \approx 0.652$  notional of 5y IborSwap, pay  $w_{10y} = 1$  notional of 10y IborSwap, and receive  $w_{20y} \approx 0.396$  notional of 20y IborSwap. In the trade, we will scale by 10000 for numerical simplicity purpose.

While we fix  $w_{10y} = 1$ , we decide  $w_{5y}$  and  $w_{20y}$  such that the trade is immune to the moves in the level/slope PCs. We can solve the ratio by either:

- Constraint Optimization: maximize the third exposure while constraining the first two to be zero.
- Solving the linear system for the first two PC2s

In fact, the ratio of notional for the two methods match exactly, according to our results.

### 4.2.1 Constraint Optimization

Let  $\mathbf{L}_1, \mathbf{L}_2$ , and  $\mathbf{L}_3$  be the first three eigenvectors (or loadings) for the covariance matrix, respectively. Then let  $\mathbf{l}_1, \mathbf{l}_2$ , and  $\mathbf{l}_3$  be the first three eigenvectors but only with tenors 5,10,20 years, which are the tenors for our butterfly. We construct the vector for DV01s as  $\mathbf{DV01s} = [DV01_{5y}, DV01_{10y}, DV01_{20y}] = [PV01_{5y} * w_{5y}, DV01_{10y} * 1, DV01_{20y} * w_{20y}]$ . We need to ensure the portfolio's sensitivity to PC1 and PC2 is neutralized, and the portfolio's sensitivity is maximized along PC3. Therefore, we have the following constraint optimization problem:

$$\underset{w_{5y}, w_{20y}}{\text{maximize}} \quad \mathbf{DV01s} \cdot \mathbf{l}_3 \quad \text{subject to} \quad \mathbf{DV01s} \cdot \mathbf{l}_1 = 0 \quad \mathbf{DV01s} \cdot \mathbf{l}_2 = 0 \quad (7)$$

We can solve the above constraint optimization through SLSQP in the `scipy` library.

### 4.2.2 Linear System

Alternatively, we could just solve the linear system for the two constraints. If the trade is orthogonal to both PC1 and PC2, then it should by construction be maximally exposed to PC3. This allows us to construct the linear system below:

We define  $A$  as the coefficient matrix,  $\mathbf{w}$  as the vector of unknowns (weights), and  $\mathbf{b}$  as the vector of constants (results of the system). The linear system can be expressed as:

$$A\mathbf{w} = \mathbf{b}$$

where

$$A = \begin{bmatrix} \mathbf{l}_1[5y] \cdot PV01_{5y} & \mathbf{l}_1[20y] \cdot PV01_{20y} \\ \mathbf{l}_2[5y] \cdot PV01_{5y} & \mathbf{l}_2[20y] \cdot PV01_{20y} \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_{5y} \\ w_{20y} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -\mathbf{l}_1[10y] \cdot PV01_{10y} \\ -\mathbf{l}_2[10y] \cdot PV01_{10y} \end{bmatrix}$$

The matrix equation can be solved by:

$$\mathbf{w} = A^{-1}\mathbf{b}$$

assuming that  $A$  is invertible. If  $A$  is singular (not invertible), then we need to use other techniques such as pseudoinverse.

This matrix representation is the system of equations that needs to be solved to find the weights  $w_{5y}$  and  $w_{20y}$  that make the butterfly trade immune to both parallel shifts and slope changes in the yield curve, which corresponds to the first two PCs.

## 4.3 Carry and Roll-Down

### 4.3.1 Carry

Given that we are trading a butterfly with LiborSwap instruments, carry is defined as the value of all known cashflows over the next few months depending on our trading period. In our case, with the trade starting at  $T_0$  and pays at  $T_1, \dots, T_N$ , the payment at  $T_1$  is known at  $T_0$  and therefore the carry of our butterfly is just the sum of individual carry for each swap.

$$\begin{aligned} \text{carry} &= \text{carry}_{5y} + \text{carry}_{10y} + \text{carry}_{20y} \\ &= w_{5y}\tau_0(k_{5y} - L_0(0)) + w_{10y}\tau_0(L_0(0) - k_{10y}) + w_{20y}\tau_0(k_{20y} - L_0(0)) \end{aligned} \quad (8)$$

where  $k_{5y}$ ,  $k_{10y}$ ,  $k_{20y}$  are the fixed rate for each swap, which are also the forward swap rate in our trade since the swaps have value 0 at start. We can annualize it by dividing the tenor  $\tau_0$  to get:

$$\text{annualized carry} = w_{5y}(k_{5y} - L_0(0)) + w_{10y}(L_0(0) - k_{10y}) + w_{20y}(k_{20y} - L_0(0)) \quad (9)$$

It is common to express carry in annualized rate terms, which is simply:

$$\text{annualized rate carry} = \frac{w_{5y}(k_{5y} - L_0(0)) + w_{10y}(L_0(0) - k_{10y}) + w_{20y}(k_{20y} - L_0(0))}{(w_{5y} + w_{10y} + w_{20y})} \quad (10)$$

Intuitively, we can interpret the carry as the sum of the individual difference between each fixed rate and expected floating rate, weighted by the notional. Note that this is the calculation for carry of **IborSwap**. For **OIS**, the calculation will be more complicated as we need to have the projected rate over the next few months using forward term OIS rate  $R_0(0)$ .



### 4.3.2 Roll-Down

Roll-down, as the name suggest, could be intuitively understood as the gain or loss from the movement of the swap rate down (or up) the yield curve as time passes. In a typical upward-sloping curve assuming the curve is unchanged over time, a bond will gain value as it slope down the curve. In the world of Hedge Fund, it is preferred to use spot delta rather than forward delta to measure roll-down, where constant forwards theta assume the curve roll along its forward as time passes, and constant spot theta assumes yield curve stay the same as time passes.

In our example, since we are using spot starting swap, the roll-down for a single swap is:

$$\text{rolldown}(\text{swap}, 1, N) = V_{\text{swap},1,N-1} - V_{\text{swap},2,N} \quad (11)$$

Where  $V_{\text{swap},n,m}$  is the value of the swap with the first payment date  $T_n$  and the last payment date  $T_m$ . To obtain the portfolio cash roll-down, we just sumup the individual roll-down. So, the formula suggests that the roll-down value is equal to the difference in value between a swap with  $N - 1$  periods remaining and a swap with  $N$  periods remaining that starts one period later. Intuitively, this means that as we move closer to the swap's maturity (as time "rolls down"), the value of the swap changes due to the decrease in the remaining time to maturity. If interest rates remain unchanged, the value of the swap with fewer remaining periods ( $V_{\text{swap},1,N-1}$ ) is compared with the value of the swap with more periods remaining ( $V_{\text{swap},2,N}$ ) but starting one period in the future.

To obtain the annualized rate roll-down, we just simply decide the cash roll-down by its DV01 and year-fraction of the trade. The annualized rate roll-down for a trade/portfolio is not very clear in general, This is because each instrument in the trade has different maturity and duration, and when expressed in rate terms, the sum of individual DV01s doesn't capture the non-linear and non-additive effects of interest rate changes across different swaps. Hence we won't expand further on annualized rate roll-down.

### 4.3.3 Results

In the trade, we calculate and plot the portfolio cash roll-down and overall DV01 using the `plot_metric_for_strategy` function. The results are summarized in the table. Since we only would only put the trade for around 1.5 year, which correspond to the remaining dataset, we only need the DV01 and Roll on 0Yx1Y and 1Yx1Y.

Bucket Label	DV01 (Total)	ROLL (Total)
0Yx1Y	-0.048703	9.100481
1Yx1Y	-0.046224	0.266217

Table 2: Cash DV01 and Roll-Down of the butterfly trade through the lifespan of the trade

We also provide a full graphical demonstration below for the entire lifespan. Based on the calculation, the total DV01 is -0.537 and total roll-down is 8.562008. This means our trade is immune to the level change, as expected in the above calculation, and the trade

has positive roll-down, which will be very important for hedge fund manager as they obtain static gains through the trade.

To summarize, we successfully show that our trade is DV01 neutral and has positive carry&roll-down over the trade period, which makes it a more attractive trade to hedge fund managers.

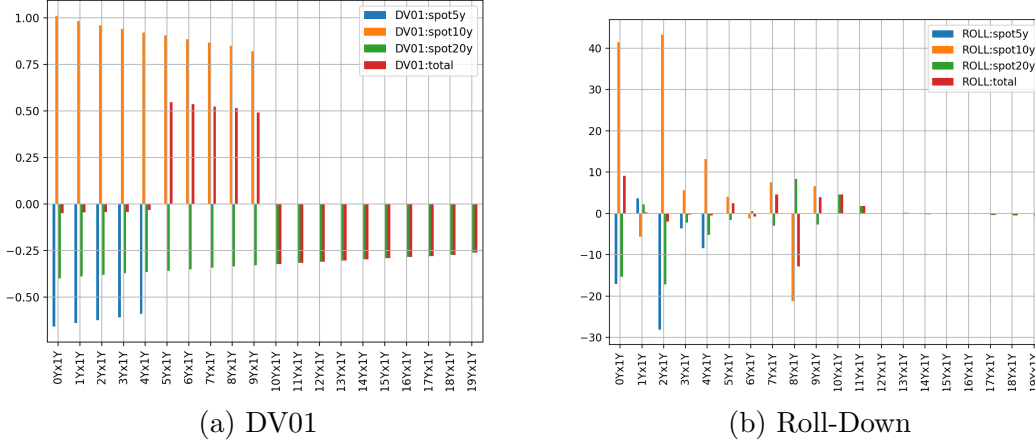


Figure 5: DV01 and Roll-Down of the trade through the entire lifespan

## 5 PnL Analysis

In the assessment of our trading strategy's profitability, we conducted a thorough analysis of the Profit and Loss (PnL) series calculated on a daily basis from the inception of the trade until the end of the dataset. The PnL's behavior over time is crucial to understand the strategy's sensitivity to the underlying market factors. Specifically, we are interested in the PnL's correlation with the first two principal components, level and slope, and its strong correlation with the third component, curvature.

The PnL for a given day  $i$  is computed as the difference in the value of the swap portfolio between two consecutive valuation dates. Mathematically, it is expressed as follows:

$$\text{PnL}(i) = V(i) - V(i - 1) \quad (12)$$

where  $V(i)$  denotes the portfolio value on day  $i$ , derived from the valuation of the swap given the current curve. The portfolio value  $V(i)$  for a swap is determined by the valuation of the fixed leg minus the valuation of the floating leg, adjusted for the first fixing rate based on the prevailing LIBOR curve:

$$V(i) = \text{PV}_{\text{fixed leg}}(i) - \text{PV}_{\text{floating leg}}(i) \quad (13)$$

The present value of the fixed leg is calculated using the formula:

$$PV_{\text{fixed leg}}(i) = \sum_{t=1}^T C_t \cdot (1 + r_t)^{-t} \quad (14)$$

where  $C_t$  is the cash flow from the fixed leg at time  $t$ , and  $r_t$  is the discount rate at time  $t$  for the corresponding tenor.

For the floating leg, the valuation adjusts after each payment date by resetting the first fixing rate:

$$PV_{\text{floating leg}}(i) = \sum_{t=1}^T F_t \cdot (1 + \text{LIBOR}_t)^{-t} \quad (15)$$

where  $F_t$  is the cash flow from the floating leg at time  $t$ , and  $\text{LIBOR}_t$  is the LIBOR rate fixed at the start of each interest period.

The cumulative PnL is the sum of daily PnL values, which can be represented as:

$$\text{Cumulative PnL}(i) = \sum_{j=1}^i \text{PnL}(j) \quad (16)$$

The daily PnL and cumulative PnL are plotted over time, as shown in Figure 6. The blue line represents the PnL per day, while the orange line illustrates the cumulative PnL. Through visual inspection, we can observe the volatile nature of the daily PnL and a trend in the cumulative PnL.

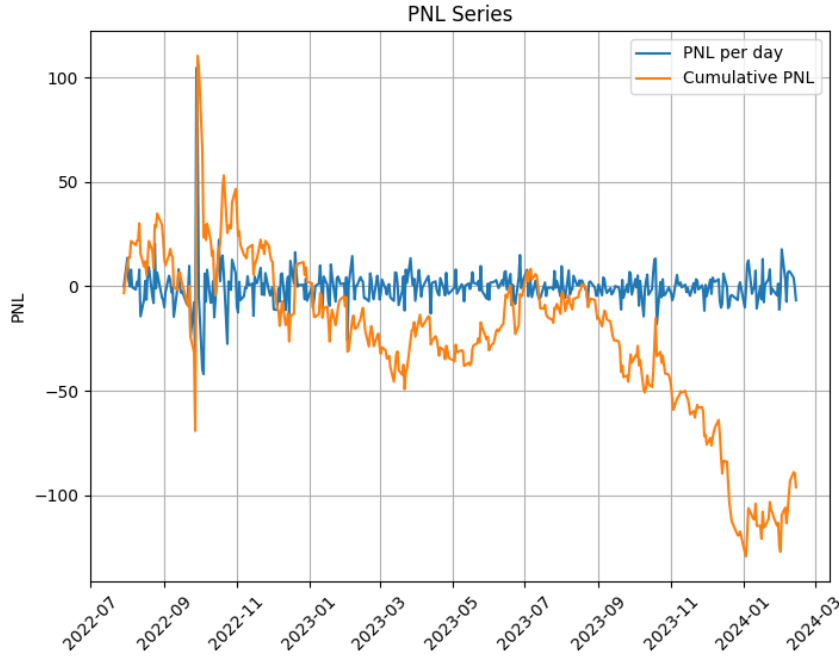


Figure 6: Daily and Cumulative PnL Series

Correlations between the daily PnL series and various yield curve spreads were investigated to discern the PnL's sensitivities to the first two principal components (level and slope) and its expected strong correlation with the third component (curvature):

- Correlation between PnL and 10-year yield: 0.0159, indicating negligible correlation.
- Correlation between PnL and 5-10 year spread: 0.1771, suggesting a mild positive correlation.
- Correlation between PnL and 2-10 year spread: 0.2383, showing a slightly stronger correlation than the 5-10 spread.
- Correlation between PnL and the butterfly spread (5s10s20s): 0.5399, which shows a significant positive correlation and aligns with our strategy's premise of being most responsive to curvature movements.

The correlations were computed using the Pearson correlation coefficient, defined by the formula:

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \quad (17)$$

where  $\text{cov}(X,Y)$  is the covariance of variables  $X$  and  $Y$ , and  $\sigma_X$  and  $\sigma_Y$  are their respective standard deviations.

The last day of trade presents a curve that elucidates the observed PnL behavior, depicted in Figure 7. The inversion of the curve around the 2s10s section in July 2023, followed by a reversion to inversion in February 2024, likely contributes to the oscillation seen in the PnL.

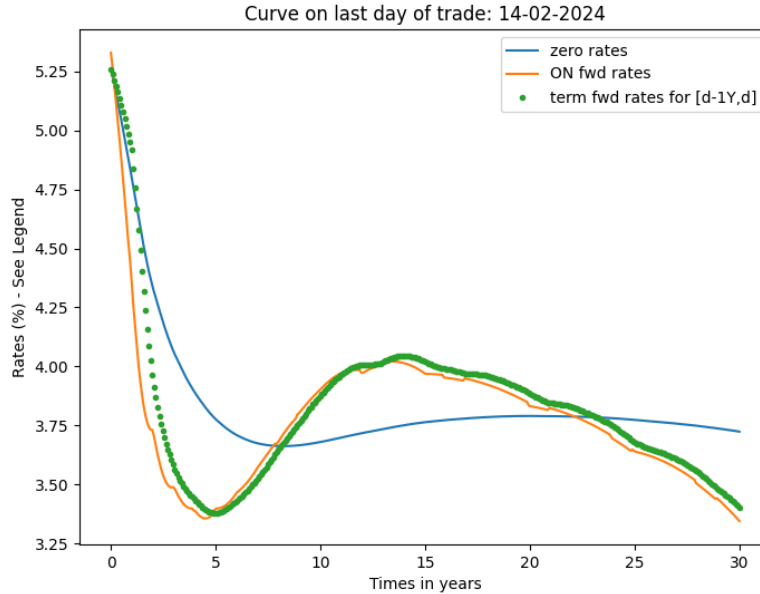


Figure 7: Curve on the Last Day of Trade

This inversion phenomenon is critical to our PnL outcome. Initially, the curve’s less inverted nature signaled a decrease in curvature, which would be expected to reduce PnL for a strategy that profits primarily from curvature movements. As the curve inverts again, the strategy’s sensitivity to curvature is once again activated, reflected in the PnL’s subsequent increase. This reinforces the inherent connection between curve dynamics and the profitability of our curvature-focused trading strategy.

## 5.1 Post-Trade PnL Correlation with Principal Components

Following the establishment of the trading position, a Principal Component Analysis (PCA) was performed on subsequent market data to discern the primary influences on PnL variation. The resulting principal components provide a perspective on the market’s dimensional behavior. Of particular interest is the relationship between our trade’s PnL and these components.

Our analysis revealed the following post-trade correlations between the PnL and the principal components:

- Correlation with the first principal component (PC1):  $-0.04982$
- Correlation with the second principal component (PC2):  $0.10276$
- Correlation with the third principal component (PC3):  $-0.53464$

Notably, the strongest correlation is observed with the third principal component. Despite the negative sign, which is a mathematical artifact of the PCA process and not indicative of directionality, this correlation is in line with the strategic design of the trade. The third principal component, typically associated with the curvature of the yield curve, was expected to be the most significant driver of PnL given our strategy’s exposure to curvature movements.

The weaker correlations with the first two principal components are consistent with the intended design of the trade to be less sensitive to level and slope movements. The substantial magnitude of the correlation with the third principal component underscores the effectiveness of the strategy in isolating the curvature as the key factor influencing PnL.

## 6 Conclusion

In summary, this report has presented a comprehensive analysis and implementation of a PCA-based trading strategy using swap rates. Our meticulous approach in curve construction and interpolation has provided a robust foundation for further analysis. The PCHIP interpolation method applied to log-discount rates has proven effective in maintaining the monotonicity of the curve, a critical aspect of accurate financial representation.

The PCA conducted on the changes in spot rates has led to the extraction of principal components, with a specific focus on the curvature component. The trading strategy developed from these findings involved a butterfly trade structured to be resilient to movements

in the level and slope, while being sensitive to changes in curvature. Our evaluation through a six-month implementation period revealed that the third principal component indeed had the most substantial influence on the PnL, validating the strategic emphasis placed on the curvature of the yield curve.

The correlations observed post-trade provide pivotal insights. The negligible correlation with the first PC and the mild positive correlation with the second PC aligned with the trade's design to minimize sensitivity to level and slope. The significant correlation with the third PC highlights the effectiveness of the trade in capturing gains primarily through curvature movements. Despite the negative sign of the correlation with the third PC, the substantial magnitude points towards the curvature being the dominant factor influencing the PnL, as strategically intended.

This trading strategy, through the application of PCA, has demonstrated the potential for sophisticated techniques to isolate and capitalize on specific market movements. The observed performance underscores the capacity for PCA to serve as a valuable tool in constructing trading strategies that can withstand and exploit particular market dynamics.

Overall, our findings contribute valuable knowledge to the field of derivative pricing and trading strategies. The successful implementation and analysis of the PCA-based trading strategy not only exhibit its practical viability but also reinforce the significance of mathematical tools in financial decision-making.