Fluid Mechanics Principles for Sediment Transport

Executive Summary

This briefing document synthesizes the foundational principles of fluid mechanics as they apply to the theory and practice of sediment transport. The central thesis is that sediment transport is fundamentally governed by fluid mechanics, with specific hydraulic variables being of primary importance.

The most critical takeaway is the shift in focus from flow velocity to **bed shear stress (τ_0)** and **turbulence** as the principal drivers of sediment motion. Bed shear stress, defined as the force per unit area exerted by the fluid on the channel bed, is derived directly from a force balance in steady, uniform open-channel flow, resulting in the seminal equation: $\boldsymbol{\tau_0 = \gamma RS}$. Nearly all sediment transport formulas, from incipient motion criteria to transport rate equations, are based on this variable.

Flow resistance, which links hydraulics to sediment movement, is another core concept. It is quantified by parameters like the Darcy-Weisbach friction factor (f) or the empirical Manning's roughness coefficient (n). The behavior of flow resistance is determined by the flow regime, with natural sediment-transporting rivers operating almost exclusively in turbulent conditions.

Within turbulent flow, the interaction between the bed's physical roughness (e.g., sediment grain size, k_s) and the thin viscous sublayer near the bed defines the hydraulic conditions. This leads to three distinct regimes: hydraulically smooth, transitional, and hydraulically rough. The prevailing regime dictates whether resistance is controlled by viscosity, roughness, or both, which has profound implications for sediment transport rates.

Finally, while steady, uniform flow provides the basis for fundamental concepts, it is insufficient for describing dynamic river systems. **Unsteady, non-uniform flow**, governed by the Saint-Venant equations, is essential for analyzing phenomena such as flood waves, sediment pulses, and flood-driven morphological changes.

I. Foundational Concepts in Open-Channel Flow

Purpose and Rationale

A review of fluid mechanics is essential because sediment transport theory is built directly upon its principles. Key fluid dynamic concepts—including flow resistance, shear stress, turbulence, and unsteadiness—directly control the fundamental processes of sediment transport:

- Initiation of motion
- Transport rate
- Bedform development and morphodynamics

Governing Principles

Fluid mechanics is based on three fundamental conservation laws. In the context of sediment transport, these laws translate into specific hydraulic drivers:

1. **Mass Conservation (Continuity):** Ensures that mass is conserved within the flow system.
2. **Momentum Conservation (Newton's Second Law):** The momentum balance is the direct origin of the concept of bed shear stress.
3. **Energy Conservation (Bernoulli):** The principle of energy loss in a system is directly related to flow resistance.

Classification of Open-Channel Flow

Open-channel flow is classified based on its variation in time and space:

- **By Time:**
  - **Steady Flow:** Flow properties do not change with time ($\partial/\partial t = 0$).
  - **Unsteady Flow:** Flow properties change with time ($\partial/\partial t \neq 0$).
- **By Space:**
  - **Uniform Flow:** Flow properties do not change along the flow path ($\partial/\partial x = 0$).
  - **Non-uniform Flow:** Flow properties change along the flow path ($\partial/\partial x \neq 0$).

This review begins with the simplest case—steady, uniform, one-dimensional flow—before extending the principles to more complex unsteady flow.

II. Bed Shear Stress: The Primary Driver of Sediment Motion

The Central Argument

A core principle in modern sediment transport is that **"velocity is secondary; bed shear stress and turbulence are primary."** This paradigm shift focuses on the forces directly responsible for dislodging and moving sediment particles.

Force Balance in Uniform Flow

In steady, uniform flow, the system is in equilibrium. The driving force, which is the downslope component of the water's weight, is perfectly balanced by the resisting force generated by friction at the flow boundary (bed and banks). This fundamental balance leads directly to the formulation of bed shear stress.

- **Driving Force = Resisting Force**

The Fundamental Equation

The force balance derivation yields what is described as "the most important equation in sediment transport":

$\boldsymbol{\tau_0 = \gamma RS}$

Where:

- $\boldsymbol{\tau_0}$ is the boundary (bed) shear stress.
- $\boldsymbol{\gamma}$ is the specific weight of water (ρg, where ρ is density and g is gravity).
- **R** is the hydraulic radius (cross-sectional area / wetted perimeter).
- **S** is the energy slope (approximated by the bed slope in uniform flow).

Derivation of the Force Balance Equation

The derivation for uniform flow is based on the conservation of momentum in the flow direction (x-direction) under several key assumptions:

- **Steady Flow:** $\partial V/\partial t = 0$
- **Uniform Flow:** $\partial V/\partial x = 0$ (implying constant slope and depth)
- **Hydrostatic Pressure Distribution**

Under these conditions, the pressure forces at the beginning and end of a control volume cancel out. The remaining force balance is between the friction force and the weight component of the water. By substituting geometric definitions and assuming small slopes (where $\sin \theta \approx S_0$), the final equation is derived.

Relevance to Sedimentology

Bed shear stress (τ_0) is the foundational variable for nearly all quantitative analyses of sediment dynamics. It is used to determine:

- Incipient motion criteria (e.g., the Shields parameter)
- Bedload and suspended load transport formulas
- Bedform regimes and stability
- Channel stability and scour potential

III. Flow Resistance and Turbulent Regimes

Linking Resistance and Shear Stress

Flow resistance describes the forces that oppose flow. The Darcy-Weisbach equation, originally developed for pipe flow, provides an alternative expression for shear stress based on velocity:

$$\boldsymbol{\tau_0 = (f/8)\rho U^2}$$

Where f is the dimensionless Darcy-Weisbach friction factor and U is the cross-sectional mean velocity. By equating this with the force balance equation ($\tau_0 = \gamma RS$), a direct relationship between velocity and the friction factor is established, explicitly demonstrating how flow resistance controls velocity.

Flow Regimes: Laminar vs. Turbulent

- **Laminar Flow:** Characterized by smooth, orderly layers of fluid with no velocity fluctuations. This regime is rare in natural rivers.
- **Turbulent Flow:** Characterized by irregular, chaotic velocity fluctuations and strong vertical mixing. This is the dominant regime in nearly all natural sediment-transporting channels.

The Reynolds number (Re) for open-channel flow characterizes the flow regime:

$Re = 4UR/\nu$ (where ν is the kinematic viscosity)

- **$Re < 500$:** Laminar
- **$500 < Re < 2000$:** Transitional
- **$Re > 2000$:** Turbulent

Structure and Characteristics of Turbulent Flow

Turbulent flow near a boundary has a distinct structure:

- **Viscous (Laminar) Sublayer:** A very thin region adjacent to the bed where viscous forces dominate and flow is smooth.
- **Fully Turbulent Region:** The rest of the flow depth where inertial forces dominate and velocity fluctuations are significant.

The **shear velocity (u_*)**, defined as **$u_* = \sqrt{(\tau_0/\rho)}$**, is a critical parameter that controls both turbulence intensity and the entrainment of sediment particles from the bed.

Roughness-Based Turbulent Regimes

The interaction between the height of roughness elements on the bed (k_s, the equivalent sand roughness) and the thickness of the viscous sublayer determines the nature of flow resistance. This leads to three distinct turbulent flow regimes.

Regime	Criterion	Description	Friction Factor (*f*) Dependency
Hydraulically Smooth	$k_s u^*/\nu < 5$	Roughness elements are fully submerged within the viscous sublayer.	Depends on Reynolds number (Re).
Transitional	$5 \le k_s u^*/\nu \le 70$	Roughness elements partially protrude through the viscous sublayer.	Depends on both Re and relative roughness.
Hydraulically Rough	$k_s u^*/\nu > 70$	Roughness elements fully protrude through the sublayer, disrupting it.	Independent of Re; depends only on relative roughness (R/k_s).

This behavior is conceptually illustrated by the Moody Diagram, which shows that for fully rough flow (labeled "Complete Turbulence"), the friction factor lines become horizontal, indicating independence from the Reynolds number. In sediment transport, the sediment size itself often controls the effective roughness k_s.

IV. Empirical and Practical Formulations

Manning's Equation

For practical engineering applications in open channels, flow resistance is commonly calculated using the empirical Manning's equation.

- **Discharge Form (Metric): $Q = (1/n)AR^{2/3}S^{1/2}$**
- **Velocity Form (Metric): $V = (1/n)R^{2/3}S^{1/2}$**

The **Manning's roughness coefficient (n)** is an aggregate, dimensional parameter that accounts for all sources of flow resistance, including:

- Bed material size
- Bedforms (dunes, ripples)
- Vegetation
- Channel irregularity and sinuosity

Crucially, in the context of sediment transport, **Manning's *n* is not a constant**; it changes as bedforms evolve with flow conditions.

Typical Manning's *n* Values

Channel Surface / Description	Typical Manning's *n* Value(s)
Concrete, trowel finish	0.012 - 0.015
Earth, straight and uniform	0.018 - 0.022
Natural Streams, clean and straight	0.025 - 0.035
Natural Streams, winding with pools	0.033 - 0.045
Flood Plains, light brush and trees	0.040 - 0.060
Heavy timber, flood stage below branches	0.080 - 0.120

Hydraulic Geometry

Solving flow equations requires defining geometric relationships for the channel cross-section. Formulas for area (A), wetted perimeter (P), and hydraulic radius (R) as a function of flow depth (y) are established for common shapes like rectangles, trapezoids, and circles. These relationships are necessary for numerical solutions of unsteady flow.

V. Unsteady, Non-Uniform Flow Dynamics

Limitations of the Uniform Flow Assumption

While essential for deriving core concepts, the steady, uniform flow assumption is a simplification. It cannot represent dynamic phenomena common in real rivers, such as:

- Flood waves
- Dam breaks
- Rapidly varied flow (e.g., hydraulic jumps)
- Morphodynamic adjustments over time

The Saint-Venant Equations

Unsteady, non-uniform open-channel flow is governed by the Saint-Venant equations, a pair of non-linear hyperbolic partial differential equations derived from mass and momentum conservation.

1. **Continuity Equation (1-D):** $\partial A/\partial t + \partial(UA)/\partial x = 0$
2. **Momentum Equation (1-D):** Accounts for changes in momentum due to local and convective acceleration, pressure, gravity (bed slope S_o), and friction (friction slope S_f).

To solve these equations, a **friction slope closure** is required to relate S_f to flow variables. Common closures include the Manning, Darcy-Weisbach, or Chezy equations. Solutions typically require numerical methods like finite difference or finite volume schemes.

Significance for Sediment Transport

Modeling unsteady hydraulics is critical for understanding and predicting complex sediment dynamics, including:

- The propagation of sediment pulses through a river system.
- Hysteresis effects, where sediment transport rates differ for the same discharge on the rising and falling limbs of a flood.
- Flood-driven channel morphology changes.
- Reservoir sedimentation patterns.
- Scour processes during extreme events.

VI. Key Takeaways

- **Bed shear stress** is the central hydraulic variable that drives sediment transport, superseding mean velocity in importance.
- **Flow resistance** provides the essential link between channel hydraulics and sediment movement, governed by the properties of turbulent flow near the bed.
- The **interaction between bed roughness and the viscous sublayer** determines the turbulent flow regime (smooth, transitional, or rough), which dictates resistance behavior.
- **Manning's equation** is a convenient and widely used empirical tool for calculating flow, but its roughness coefficient (n) is not a fixed constant in channels with mobile beds.
- **Unsteady flow analysis**, using the Saint-Venant equations, is indispensable for capturing the dynamic behavior of real river systems and their sediment transport processes.