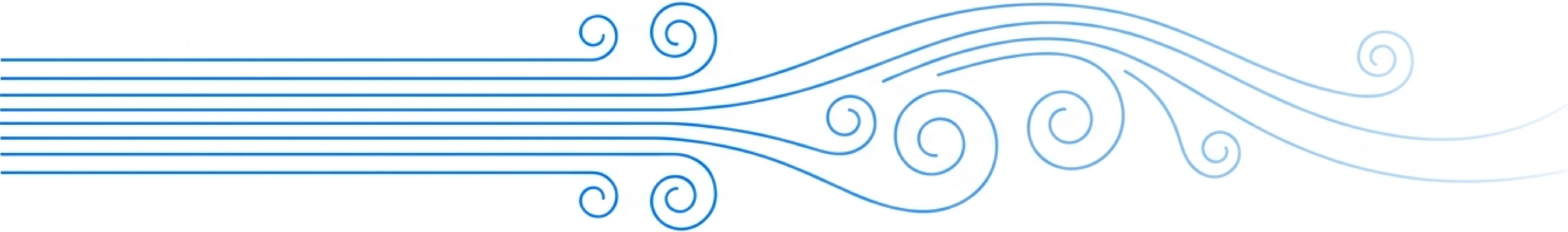
Review of Fluid Mechanics: The Physics of Sediment Transport

From Conservation Laws to Bed Shear Stress

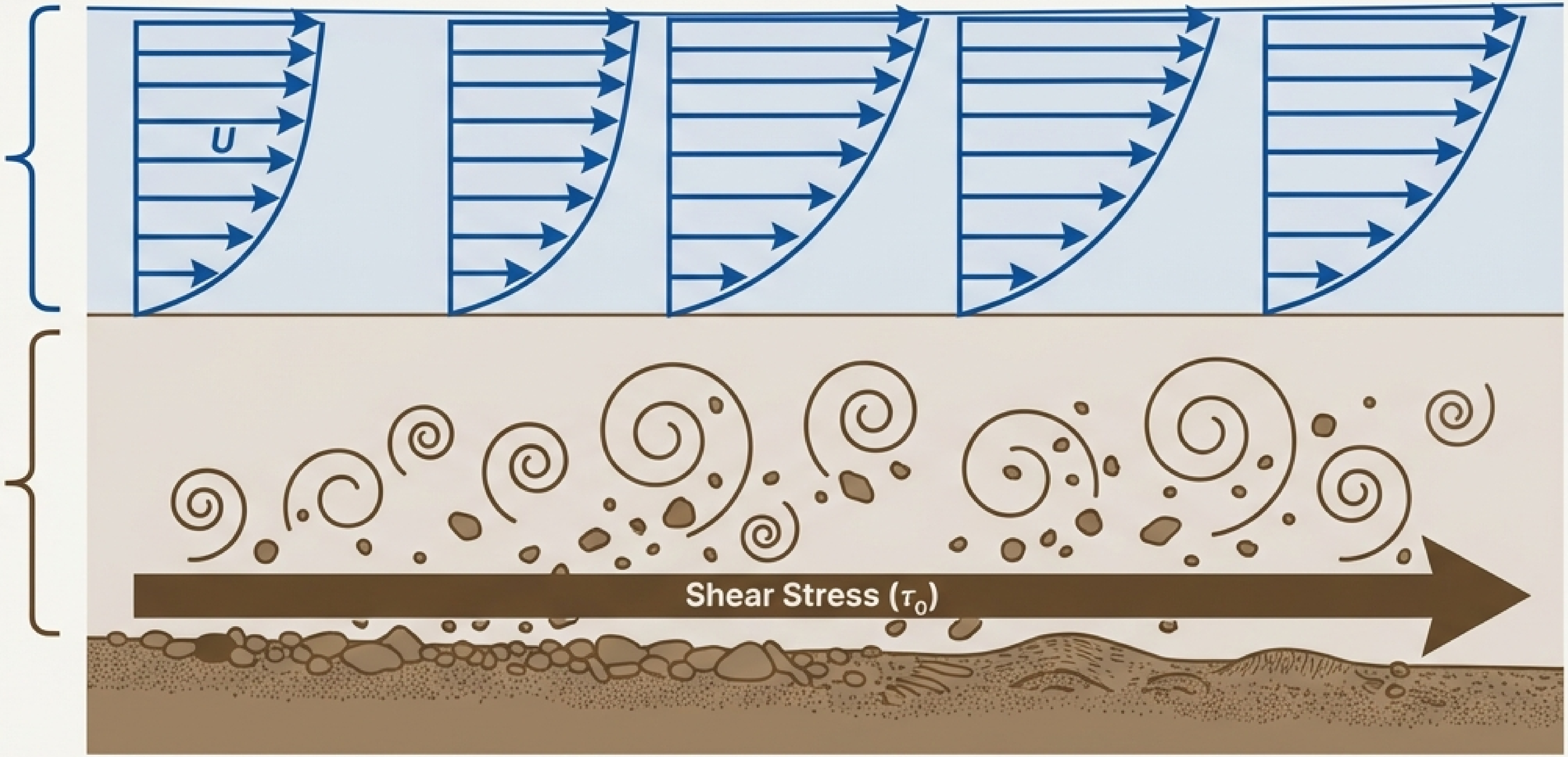


FOCUS: OPEN-CHANNEL FLOW & PRACTICAL APPLICATIONS

The Complete Picture: Flow vs. Force

What we see:
Velocity (U)
controls Discharge

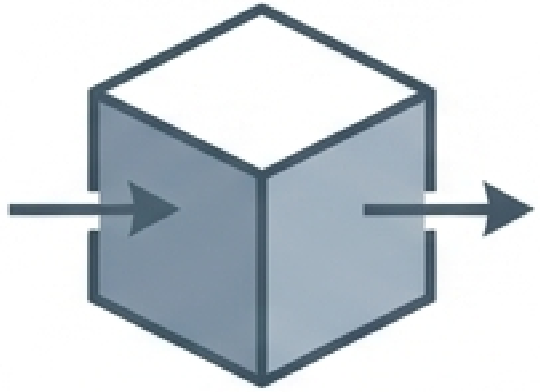
What moves the bed:
Shear Stress (τ_0)
controls Transport



In Sediment Transport, Velocity is Secondary.
Bed Shear Stress (τ_0) and Turbulence are Primary.

The Governing Principles

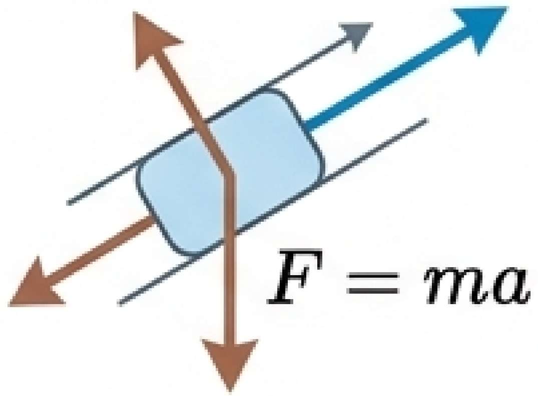
Three Fundamental Conservation Laws



1. Mass Conservation

(Continuity)

Input = Output. Ensures water doesn't disappear.

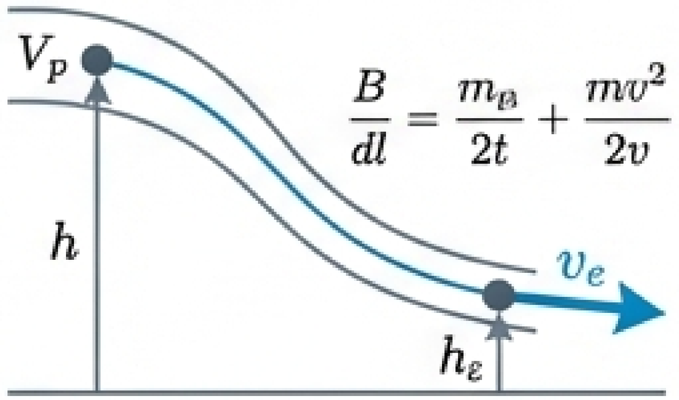


2. Momentum Conservation

(Newton's Second Law)

Balances gravity and friction.

Defines Bed Shear Stress.



3. Energy Conservation

(Bernoulli)

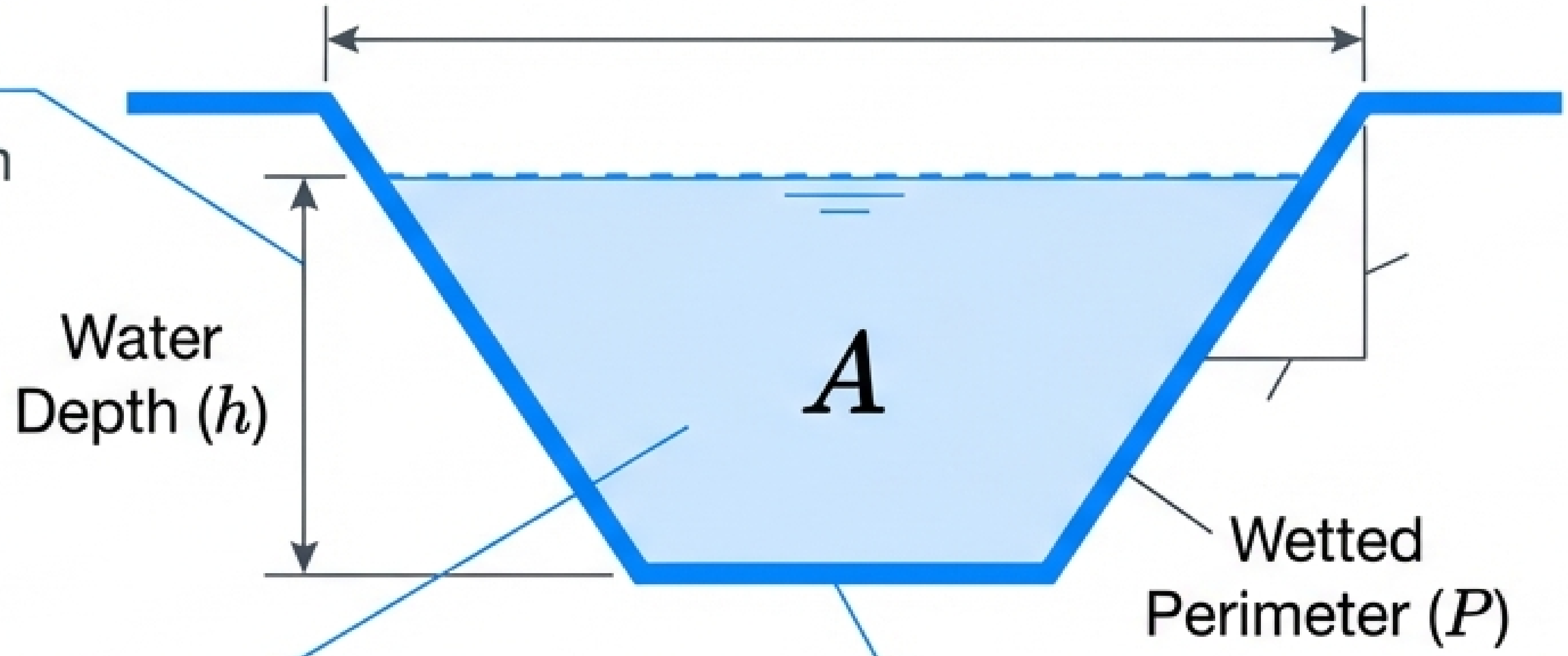
Tracks energy head along the channel.

Explains Flow Resistance.

Key Insight: We use these laws to derive the force balance that dictates whether sediment stays still or moves downstream.

The Geometry of Open-Channel Flow

Flow Depth (h): Vertical distance from bed to surface.



Area (A): Total cross-sectional area of the flow.

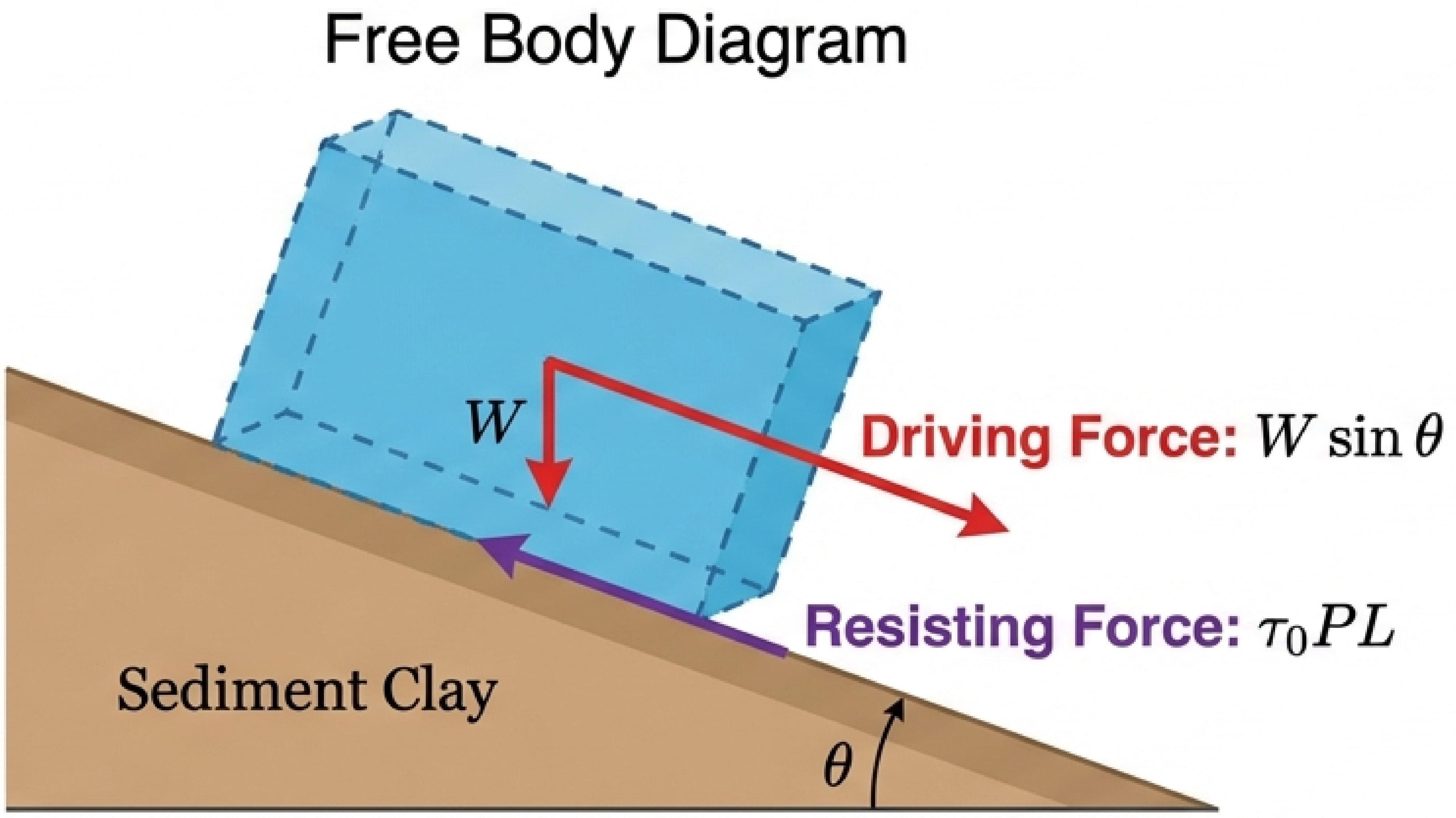
Wetted Perimeter (P): Length of the boundary in contact with fluid.

$$R = \frac{A}{P}$$

Hydraulic Radius (R): A measure of channel efficiency.

Sediment Insight: For wide rivers, $R \approx h$.

The Force Balance in Uniform Flow



Downslope Gravity = Boundary Resistance

$$W \sin\theta = \tau_0 PL$$

Assumption: Steady, Uniform Flow (Accel

The Most Important Equation in Sediment Transport

Specific Weight. (ρg). The weight density of water.

$$\tau_0 = \gamma R S$$

Bed Shear Stress. The force per unit area acting on the bed. This moves the sediment.

Energy Slope. The gradient driving the flow.

Hydraulic Radius. The shape efficiency (A/P).

DERIVATION NOTE: Since $\sin\theta \approx S$ for small slopes, the weight component simplifies directly to this expression.

Key Takeaway: Almost all sediment formulas (Shields parameter, bedload rates) start here.

Linking Shear Stress to Velocity

The Darcy–Weisbach Equation

$$\tau_0 = \frac{f}{8}\rho U^2$$

Shear stress is proportional to the square of velocity.

f

Darcy-Weisbach Friction Factor

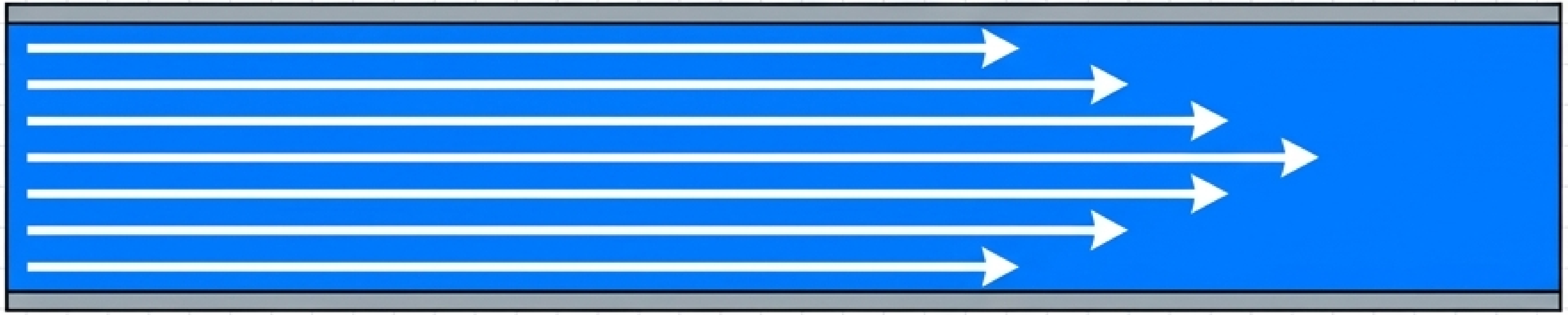
This variable represents the “roughness” or “drag” of the channel. A higher f means more resistance, which results in slower velocity for the same slope.

$$U = \sqrt{\frac{8gRS}{f}}$$

This equation proves that Velocity is a result of the balance between Gravity (Slope) and Resistance (f).

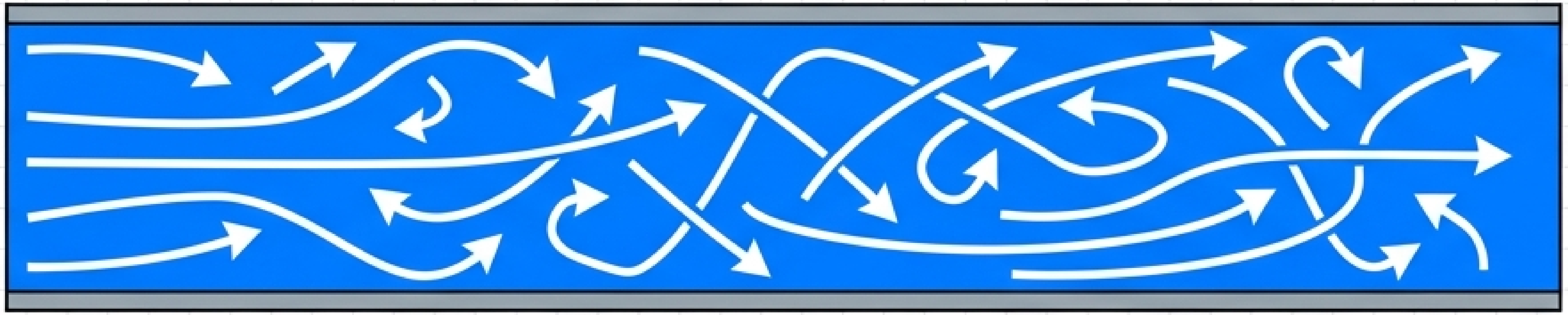
The Reality of Flow Regimes: Laminar vs. Turbulent

Laminar (*Re* < 500)



Fluid moves in smooth layers. Viscosity dominates. Rare in natural rivers.

Turbulent (*Re* > 2000)



Strong vertical mixing. Inertial forces dominate. The rule for rivers.

Reynolds Number

$$Re = \frac{4UR}{\nu}$$

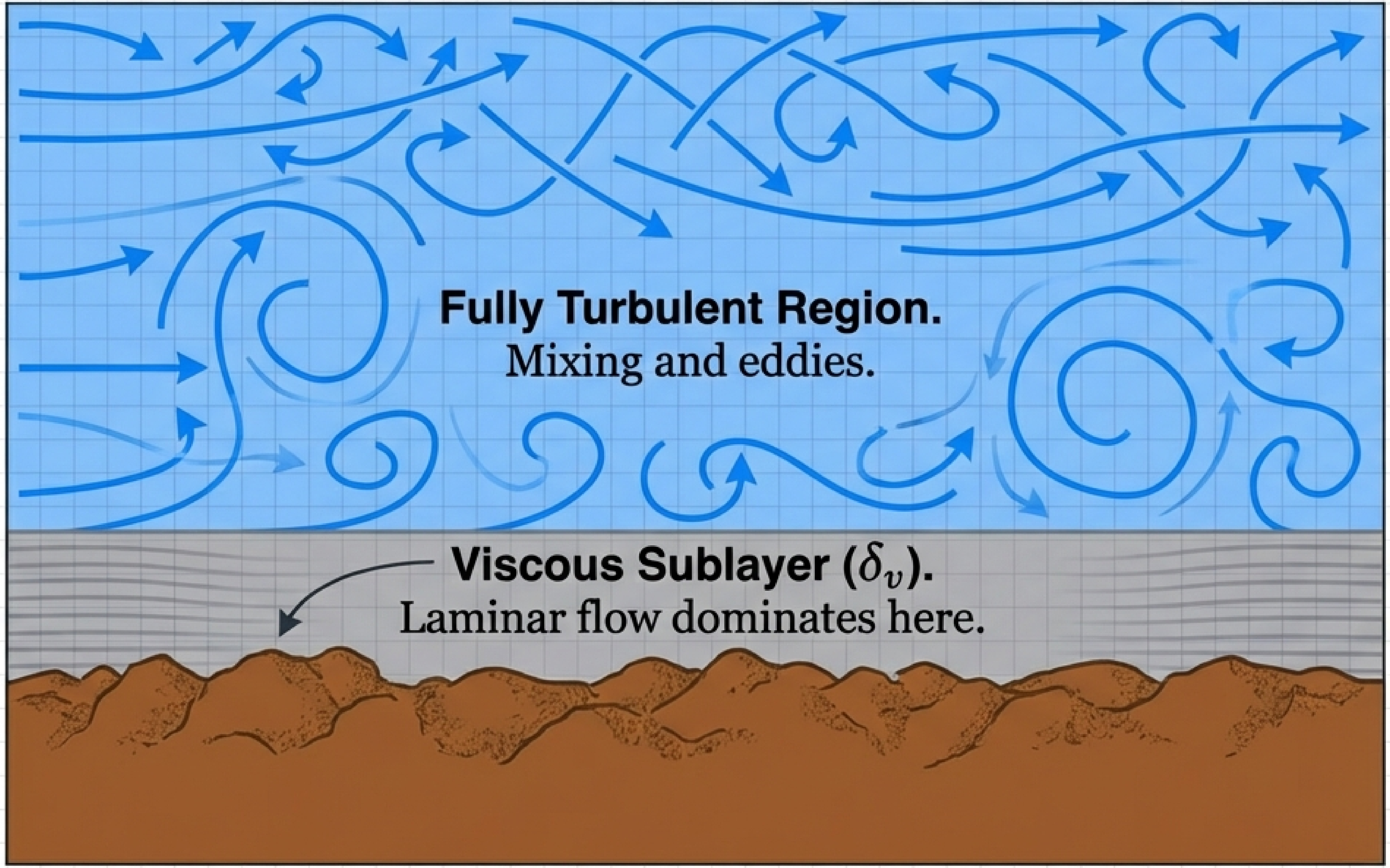
Ratio of Inertial Forces to Viscous Forces.

U = mean flow velocity

R = hydraulic radius

ν = kinematic viscosity

Anatomy of the Turbulent Boundary Layer



Shear Velocity:

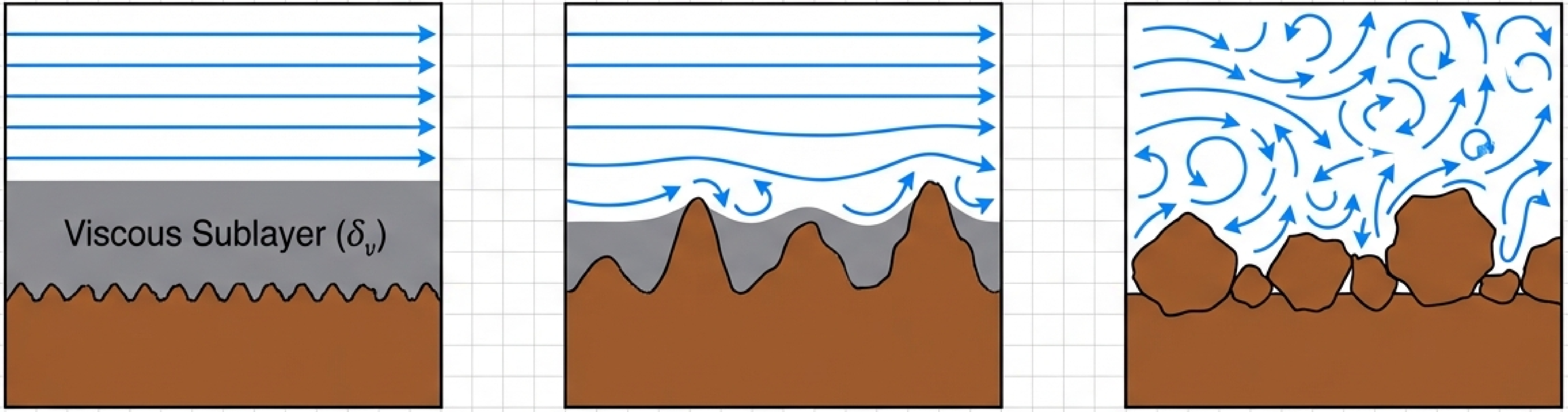
$$u_* = \sqrt{\frac{\tau_0}{\rho}}$$

A velocity scale representing the intensity of turbulence and shear at the bed. This is what lifts particles.

Roughness Regimes

Interaction of Sediment (k_s) and the Viscous Sublayer (δ_v)

Hydraulically Smooth | **Transitional** | **Hydraulically Rough**



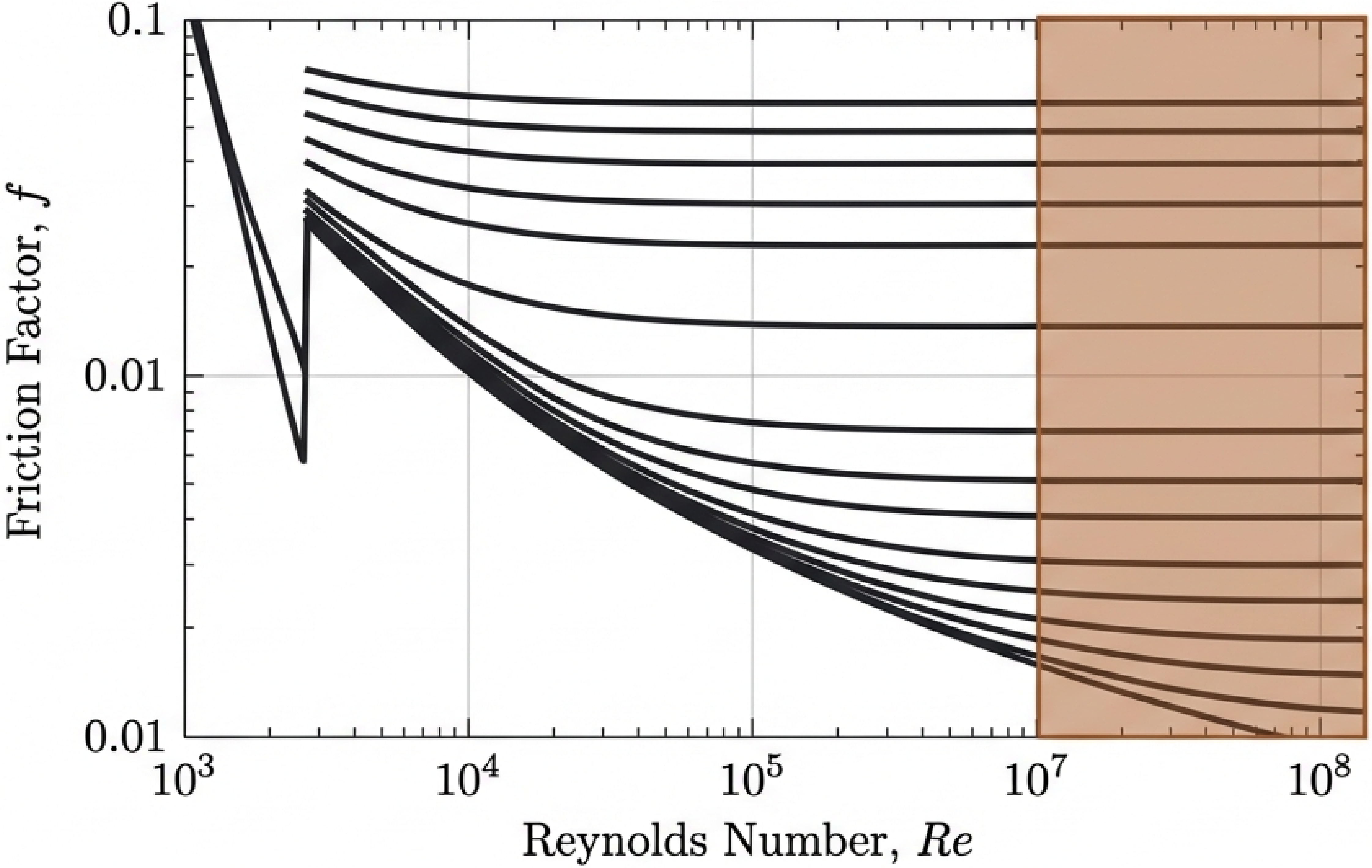
Roughness is hidden. Resistance is viscous.

Roughness begins to disrupt the sublayer.

Fully Rough. Resistance is purely form drag on the rocks. Viscosity is irrelevant.

Most gravel-bed rivers operate in the Fully Rough regime.

The Moody Diagram



Rough flow formula:

$$f = 8.5 + 2.5 \ln\left(\frac{12R}{k_s}\right)$$

Notice: The Friction Factor depends ONLY on relative roughness (R/k_s), not viscosity.

In rivers (Rough Turbulent), f is constant regardless of Re.

From Theory to Practice: Manning's Equation

The Engineer's Tool

Discharge (m^3/s)

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

Manning's Roughness Coefficient. An empirical factor.

Geometry and Slope.

Cool Slate border

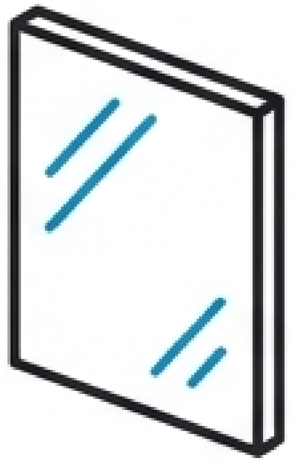
Context

While Darcy-Weisbach is theoretically superior, Manning's equation is the standard for open channel engineering.

Decoding Manning's n

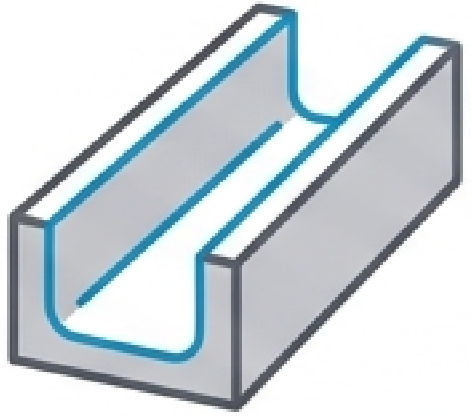
More than just a number: An aggregate of chaos.

Value 0.010



Glass/Plastic

Value 0.012



Finished Concrete

Value 0.030

Clean Natural Stream

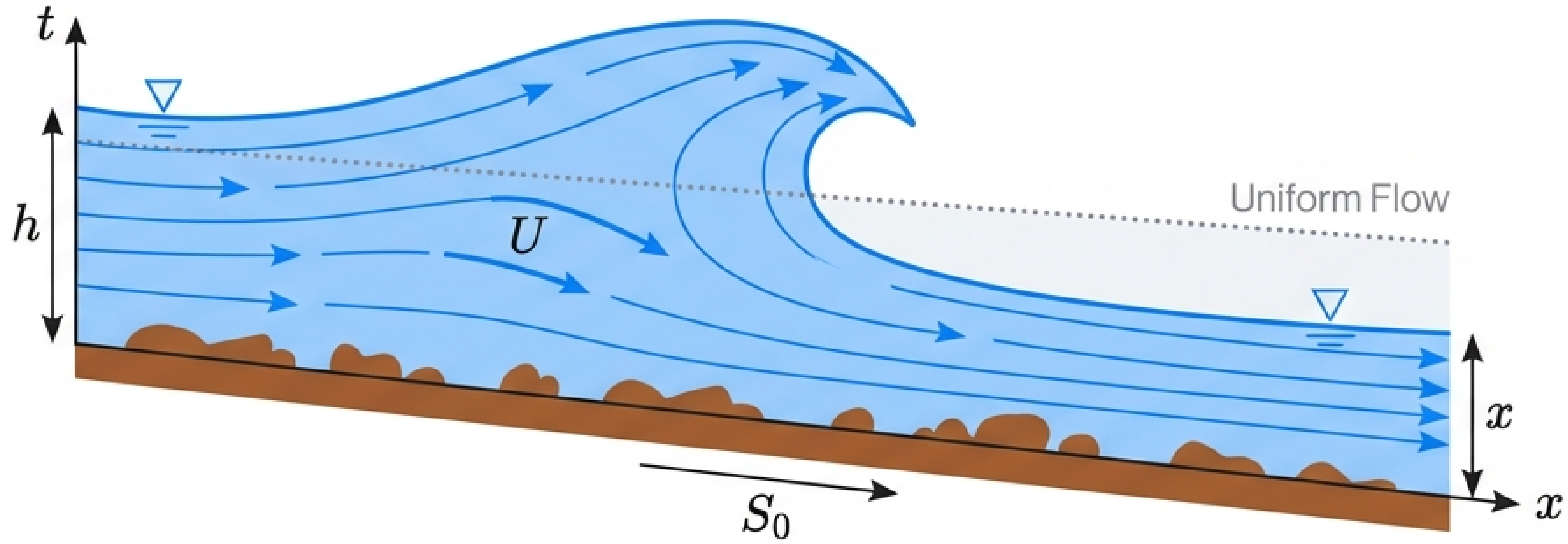
Value 0.040+

Weedy / Winding

Critical Insight:

In sediment transport, n varies dynamically. As ripples and dunes form on the bed, 'n' increases, slowing the flow.

Beyond Uniform Flow: The Unsteady Reality



1. Continuity (Mass)

$$\frac{\partial A}{\partial t} + \frac{\partial (UA)}{\partial x} = 0 \quad \frac{\partial (UA)}{\partial t} = 0$$

2. Momentum

$$\frac{\partial (UA)}{\partial t} + \frac{\partial}{\partial x}(UA^2 + gA\bar{h}) - gA(S_0 - S_f) = 0$$

These differential equations are required to model floods, sediment pulses, and rapid scour events.

Summary & Key Takeaways

Bed Shear Stress (τ_0)
The primary force moving sediment. Velocity is secondary.

Flow Resistance (f, n)
Connects energy loss to the boundary. Controlled by roughness.

Rough Turbulent Flow
Nature's default. Viscosity matters less; sediment size (k_s) dominates.

Unsteady Flow
Real rivers change in time and space. We need Saint-Venant equations for morphology.

"Hydraulics and sediment transport are inseparable."