Particle Settling Velocity: Physical Principles and Dynamics

Executive Summary

Particle settling velocity (V_s) is the terminal vertical velocity reached by a sediment particle falling through a fluid under the influence of gravity. It is a fundamental property in sediment transport, serving as the primary determinant for whether particles remain in suspension or deposit. The settling process is governed by a precise balance between three vertical forces: the particle’s weight, the fluid's buoyant force, and the resisting drag force.

Because the drag force is influenced by the flow regime—characterized by the Reynolds number (Re)—settling behavior varies significantly across different particle sizes and fluid conditions. While very small particles (clays and fine silts) follow the predictable, viscous-dominated Stokes’ law, larger natural sediments often fall within transitional or inertia-dominated regimes. In these cases, because settling velocity and the Reynolds number are interdependent, determining V_s requires iterative calculation or empirical approximation. Accurately determining V_s is critical for engineering and morphodynamic modeling, as errors propagate directly into predictions of suspended loads and deposition rates.

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1. Physical Significance in Sediment Transport

Settling velocity is not merely a measure of downward motion; it is a governing parameter for the following processes:

- **Deposition vs. Suspension:** Determining the threshold at which particles settle out of the water column.
- **Vertical Concentration Profiles:** Influencing how sediment is distributed throughout the depth of a fluid.
- **Selective Transport:** Facilitating grain-size sorting during transport.
- **Residence Time:** Defining how long a particle remains in the water column before reaching the bed.

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2. Fundamental Force Balance

A particle settling in a quiescent (still) fluid is acted upon by three primary vertical forces:

Force	Formula	Variables
Weight (W)	$W = \rho_s g V_p$	ρ_s: particle density; V_p: particle volume
Buoyant Force (F_B)	$F_B = \rho g V_p$	ρ: fluid density; g: gravity
Drag Force (F_D)	$F_D = \frac{1}{2} C_D \rho A V_s^2$	C_D: drag coefficient; A: projected area normal to motion

Terminal Settling Concept

Immediately after release, a particle accelerates because its submerged weight ($W - F_B$) is greater than the drag force. As velocity increases, drag increases until it balances the submerged weight. At this point, acceleration becomes negligible, and the particle reaches **terminal settling velocity**, where: $F_D = W - F_B$

For a spherical particle of diameter d, the general foundation for all settling relationships is:
$\frac{1}{2} C_D \rho \frac{\pi d^2}{4} V_s^2 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 (\rho_s - \rho) g$

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3. Hydrodynamic Flow Regimes

The settling behavior is characterized by the **Reynolds number (Re)**, defined as: $Re = \frac{V_s d}{\nu}$ (where ν is the kinematic viscosity of the fluid)

The definition of Re creates a coupling effect: the Reynolds number depends on V_s, which is the unknown value being sought. Consequently, settling calculations are regime-dependent.

3.1 Low Reynolds Number (Stokes / Viscous Regime)

- **Range:** $Re \lesssim 1$
- **Characteristics:** Fully laminar flow; boundary layers remain attached; no flow separation or wake.
- **Physics:** Drag is dominated by viscous shear.
- **Governing Law:** $C_D = 24/Re$.
- **Stokes' Law:** $V_s = \frac{(\rho_s - \rho)gd^2}{18\mu}$ (where μ is dynamic viscosity). V_s is proportional to d^2.

3.2 Transitional Reynolds Number

- **Range:** $1 < Re < 10^3$
- **Characteristics:** Flow begins to separate; a steady wake forms; boundary layers remain laminar.
- **Physics:** Drag transitions from viscous-dominated to pressure-dominated. C_D decreases rapidly as Re increases.

3.3 High Reynolds Number (Inertia-Dominated)

- **Range:** $Re \gtrsim 10^3$
- **Characteristics:** Drag is dominated by **form drag** (pressure drag) caused by flow separation and a low-pressure wake behind the particle.
- **Physics:** C_D becomes approximately order-one and is weakly dependent on viscosity.
- **Note:** For smooth spheres, a "drag crisis" occurs near $Re \approx 2 \times 10^5$ due to boundary-layer turbulence, though this is generally not relevant for natural sediment in water.

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4. Effect of Particle Shape and Fluid Properties

Natural sediments are rarely perfectly spherical, which significantly impacts settling:

- **Increased Drag:** Non-spherical particles (angular or platy) experience higher drag than spheres of the same nominal diameter.
- **Reduced Velocity:** As particle irregularity increases, settling velocity decreases.
- **Temperature Sensitivity:** Because viscosity changes with temperature, V_s (particularly in the Stokes regime) is highly sensitive to fluid temperature.
- **Shape Factors:** Empirical charts for naturally worn quartz sand use Shape Factors (S.F.) typically ranging from 0.5 to 0.9 to adjust fall velocity calculations.

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5. Calculation Methodologies

The Iteration Procedure

Outside of the Stokes regime, no closed-form analytical solution exists because V_s, Re, and C_D are all interdependent. The standard iterative procedure is as follows:

1. **Guess** an initial C_D (e.g., 1.0).
2. **Compute** V_s using the force balance equation.
3. **Compute** Re using the derived V_s.
4. **Update** C_D by reading from an empirical C_D–Re curve.
5. **Repeat** until V_s changes negligibly (e.g., < 1%).

Example Case: A 1.0 mm spherical quartz particle in 20°C water converges to a settling velocity of approximately **15.8 cm/s** with a Reynolds number of approximately **160**.

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6. Scope and Limitations

The physical principles outlined here assume:

- A **single particle** in clear, quiescent water.
- **Steady terminal conditions** (ignoring initial acceleration and history forces).

In complex natural environments, other factors—not covered in this fundamental framework—become important:

- **Hindered Settling:** Effects of high sediment concentrations.
- **Turbulence:** Interactions between the particle and turbulent flow.
- **Flocculation:** The clustering of cohesive sediments (clays).

7. Key Findings

- Settling velocity is a result of a physical force balance, not a material constant.
- Flow regimes (Viscous vs. Inertial) dictate which physics govern the particle's descent.
- Particle shape is as critical as size when evaluating natural sediments.
- Precise V_s values are essential, as errors propagate into all aspects of morphodynamic and sediment transport modeling.