Particle Settling Velocity in Sediment Transport

Executive Summary

Particle settling velocity (V_s) is the terminal vertical velocity reached by a sediment particle falling through a fluid under the influence of gravity. It is a fundamental property in sediment transport that determines whether particles remain in suspension or deposit, dictates vertical concentration profiles, and influences grain-size sorting and residence times in the water column.

The settling process is defined by a dynamic force balance between particle weight, buoyancy, and fluid drag. Because the drag coefficient (C_D) depends on the Reynolds number (Re), which in turn depends on the settling velocity, V_s calculations are inherently regime-dependent and often require iterative solutions. While Stokes' Law provides an analytical solution for very small particles in laminar flow, larger particles in transitional or inertia-dominated regimes require empirical data and account for "form drag" caused by flow separation and pressure imbalances.

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1. Physical Significance of Settling Velocity

Settling velocity is not a material constant; it is a variable emerging from the interaction between particle characteristics (size, shape, density) and fluid properties (density, viscosity). In sediment transport, it governs:

- **Deposition vs. Suspension:** The threshold at which particles settle out of the flow.
- **Vertical Concentration:** The distribution of sediment throughout the depth of the water column.
- **Selective Transport:** The mechanism behind grain-size sorting during transport.
- **Residence Time:** The duration a particle remains within the water column before reaching the bed.

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2. Fundamental Mechanics: Force Balance

For a single particle settling in a quiescent (still) fluid, three primary vertical forces must be considered:

2.1 Primary Forces

Force	Formula	Description
Weight (W)	$W = \rho_s g V_p$	Downward force due to gravity; ρ_s is particle density, V_p is volume.
Buoyant Force (F_B)	$F_B = \rho g V_p$	Upward force equal to the weight of the displaced fluid; ρ is fluid density.
Drag Force (F_D)	$F_D = \frac{1}{2} C_D \rho A V_s^2$	Resistance force opposing motion; C_D is the drag coefficient, A is the projected area.

2.2 Terminal Velocity Concept

Immediately upon release, a particle accelerates because its submerged weight exceeds the drag force. As velocity increases, drag increases until a state of equilibrium is reached. At this point, acceleration becomes negligible, and the particle reaches its terminal settling velocity (V_s). The force balance is expressed as: $F_D = W - F_B$

For a spherical particle of diameter d, the balanced equation is: $\frac{1}{2} C_D \rho \frac{\pi d^2}{4} V_s^2 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 (\rho_s - \rho) g$

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3. Hydrodynamic Regimes and the Reynolds Number

The behavior of a settling particle is characterized by the Reynolds number (Re), which relates inertial forces to viscous forces: $Re = \frac{V_s d}{\nu}$ where ν is the kinematic viscosity of the fluid. Because Re depends on the unknown V_s, determining the correct flow regime is critical for accurate calculation.

3.1 Low Reynolds Number: Stokes Regime ($Re \lesssim 1$)

- **Flow Characteristics:** Fully laminar; boundary layers remain attached; no flow separation or wake formation.
- **Drag Source:** Dominated by viscous shear.
- **Drag Coefficient:** $C_D = 24/Re$.
- **Stokes' Law:** $V_s = \frac{(\rho_s - \rho) g d^2}{18\mu}$, where μ is dynamic viscosity.
- **Applicability:** Limited to small, smooth, spherical particles such as clay and silt. V_s is proportional to d^2.

3.2 Transitional Regime ($1 < Re < 10^3$)

- **Flow Characteristics:** Flow begins to separate from the particle surface, creating a steady wake.
- **Drag Source:** Transitions from viscous-dominated to pressure-dominated.

- **Drag Coefficient:** C_D decreases rapidly as Re increases; values must be obtained from experimental curves.

3.3 High Reynolds Number: Inertia-Dominated ($Re \gtrsim 10^3$)

- **Flow Characteristics:** Drag is dominated by "form drag" (pressure imbalance). The wake structure controls resistance.
- **Drag Source:** Pressure differences between the front and rear surfaces.
- **Drag Coefficient:** C_D becomes approximately order-one and is weakly dependent on viscosity.
- **Note:** For smooth spheres, a "drag crisis" occurs near $Re \approx 2 \times 10^5$ as boundary layers become turbulent, but this is rarely relevant for natural sediment in water.

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4. Physical Origin of Form Drag

Form drag arises when fluid accelerates around the sides of a particle, causing velocity to increase and pressure to decrease along the sides. This results in a low-pressure wake behind the particle. The resulting pressure difference between the front (high pressure) and rear (low pressure) surfaces produces a net force that opposes motion. This mechanism causes the sharp increase in drag observed upon flow separation.

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5. Particle Shape and Natural Sediments

Natural sediment particles are rarely spherical, which significantly impacts settling behavior:

- **Increased Drag:** Non-spherical particles experience higher drag than spheres of equivalent nominal diameter.
- **Shape Factors:** More angular or platy particles (e.g., mica) settle more slowly.
- **Correction:** Shape effects are represented via the drag coefficient or specific shape factors (S.F.). Common S.F. values for quartz sand include 0.5, 0.7, and 0.9.
- **Temperature Sensitivity:** Settling velocity is sensitive to temperature through changes in fluid viscosity, particularly in the Stokes regime.

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6. Iterative Calculation Procedure

For particles outside the Stokes regime, no closed-form analytical solution exists because V_s, C_D, and Re are all interdependent.

6.1 Steps for Iteration

1. **Write the general force balance:** $V_s = \sqrt{\frac{4gd(s-1)}{3C_D}}$, where $s = \rho_s/\rho$.
2. **Guess an initial C_D** (e.g., 1.0).
3. **Compute V_s** based on the assumed C_D.
4. **Compute Re** using the calculated V_s.
5. **Update C_D** by reading the value from a C_D–Re curve for the given Re.
6. **Repeat** until V_s converges (change is $< 1\%$).

6.2 Worked Example: 1.0 mm Quartz Sphere

For a particle with d = 1.0 mm, $\rho_s = 2650$ kg/m³, and $\rho = 1000$ kg/m³ in 20°C water ($\nu = 1.0 \times 10^{-6}$ m²/s):

- **Pre-computed term:** $V_s = \sqrt{0.021582 / C_D}$.
- **Initial Guess (C_D=1.0):** $V_s = 0.1469$ m/s, Re = 147.
- **Iteration 1:** Updated $C_D \approx 0.90$ (from curve) $\rightarrow V_s = 0.1548$ m/s, Re = 155.
- **Iteration 2:** Updated $C_D \approx 0.88 \rightarrow V_s = 0.1566$ m/s, Re = 157.
- **Convergence:** $V_s \approx 0.158$ m/s (15.8 cm/s) and Re ≈ 160.

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7. Framework and Scope

Settling behavior is governed by two primary nondimensional parameters: the **Reynolds number** (Re) and the **Density contrast** ($\mathcal{R} = \rho_s/\rho - 1$).

7.1 Assumptions

The physical principles outlined here assume:

- A single particle (no interaction with other particles).
- Clear, quiescent water.
- Steady terminal conditions.

7.2 Excluded Factors (Real-world Complexity)

In natural environments, several factors not covered in this basic framework can alter settling:

- **Hindered Settling:** Effects of high sediment concentration.
- **Turbulence:** Interactions between fluid eddies and particles.
- **Flocculation:** Cohesive sediment clumping.
- **Acceleration:** The initial phase before terminal velocity is reached.

Errors in calculating settling velocity propagate directly into suspended load predictions, deposition rates, and morphodynamic modeling, making correct interpretation essential for engineering applications.