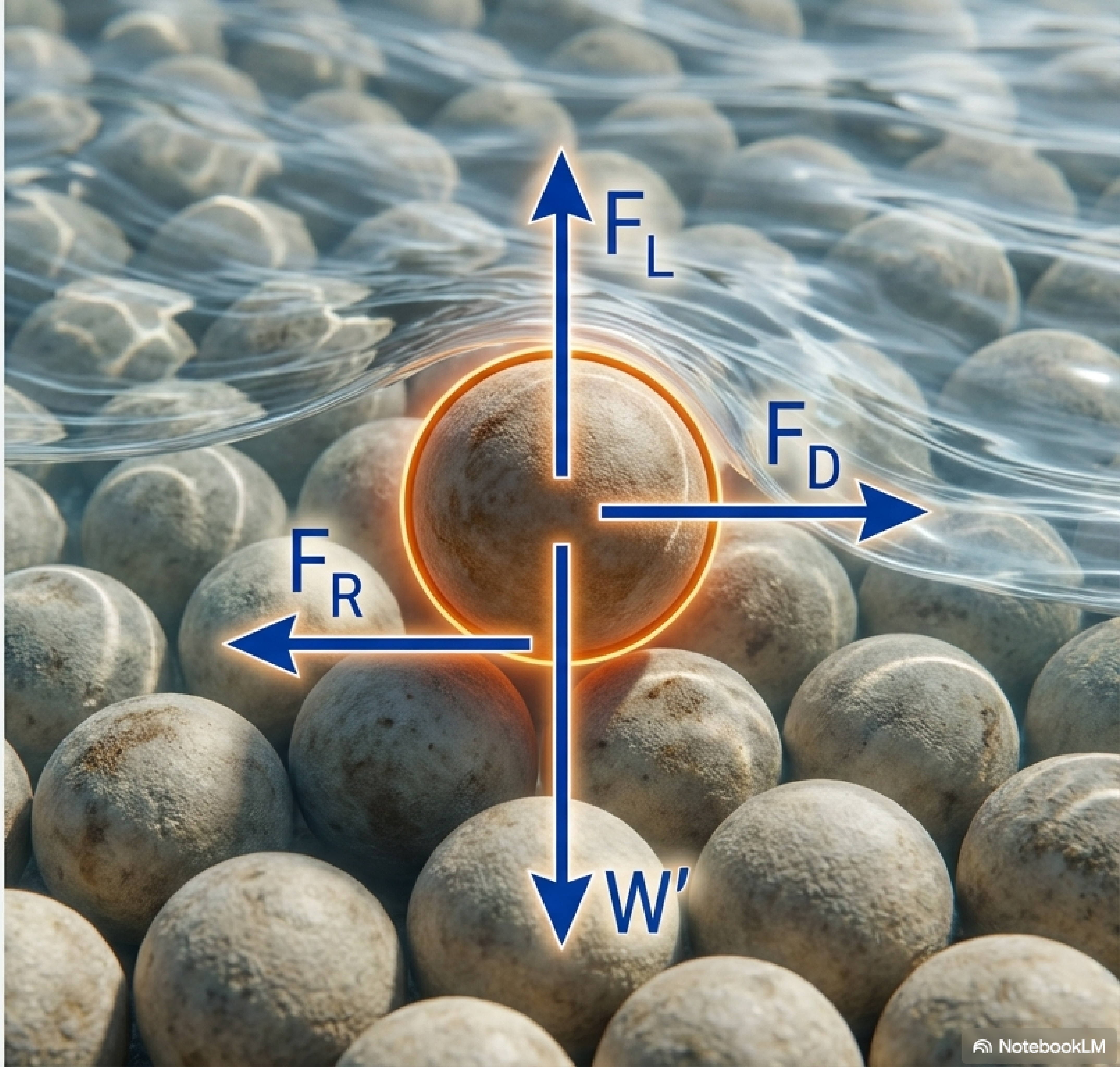
Scour Criteria & Stable Channel Design

From Particle Physics to Hydraulic Geometry

- **The Goal:** To derive the geometry of a stable channel where the sediment is at the threshold of motion but does not scour.
- **The Logic Chain:** Particle Mechanics → Dimensionless Threshold → Stable Channel Geometry.
- **The Scope:** Non-cohesive sediment, uniform particle diameter (d), clear-water threshold, steady uniform flow.
- **Key Definition:** Incipient motion occurs when applied boundary shear stress equals critical shear stress.

$$\tau_0 = \tau_c$$



The Micro Scale: Force Balance on a Single Grain

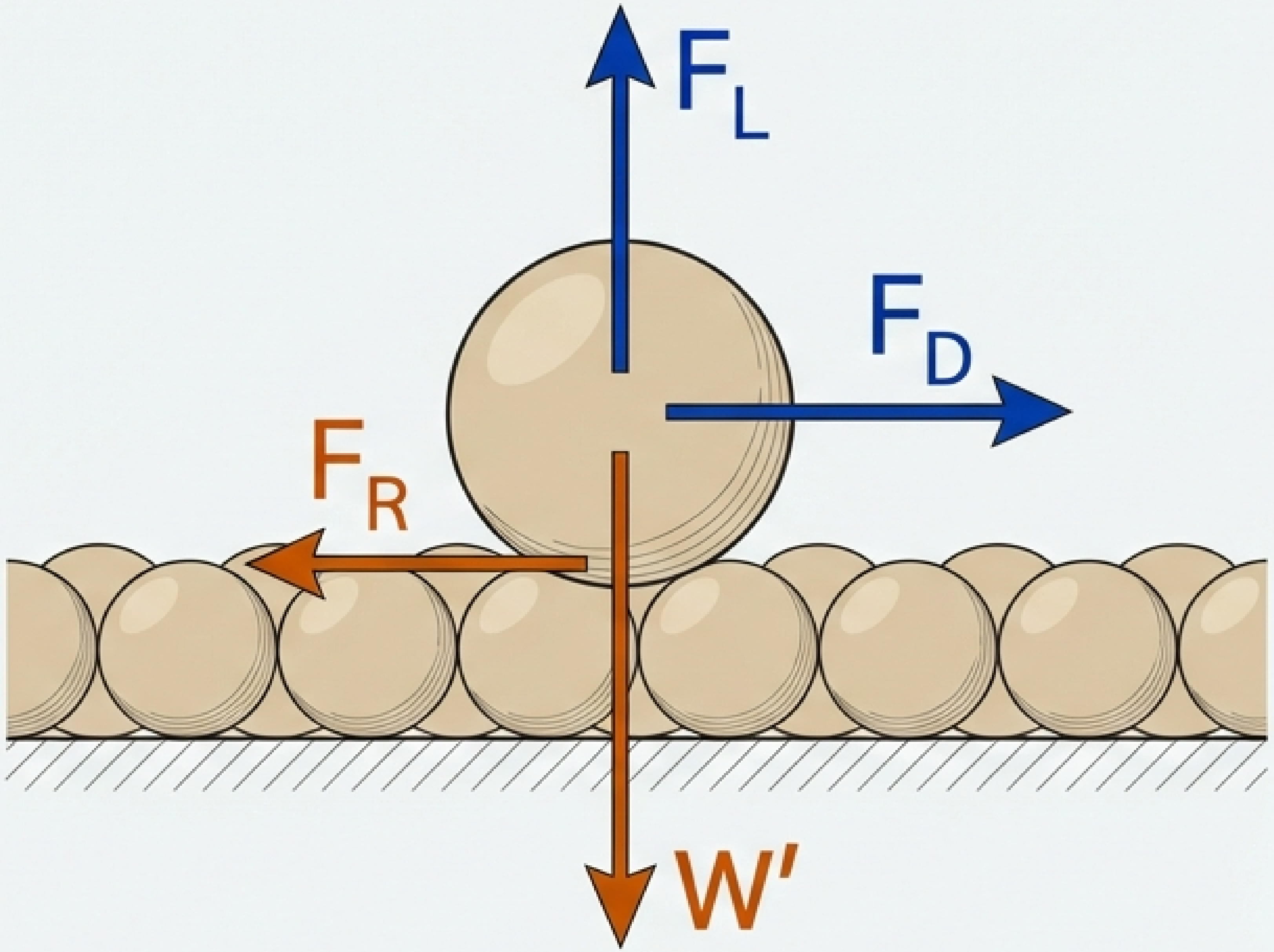
The stability of a channel begins with a single grain.

Hydrodynamic Forces:

The flow exerts Drag (F_D) pushing the particle downstream and Lift (F_L) attempting to raise it from the bed.

Resisting Forces:

The particle is anchored by its Submerged Weight (W') and the resulting Frictional Resistance (F_R).



Threshold Condition: $F_D = \mu W' - F_L$

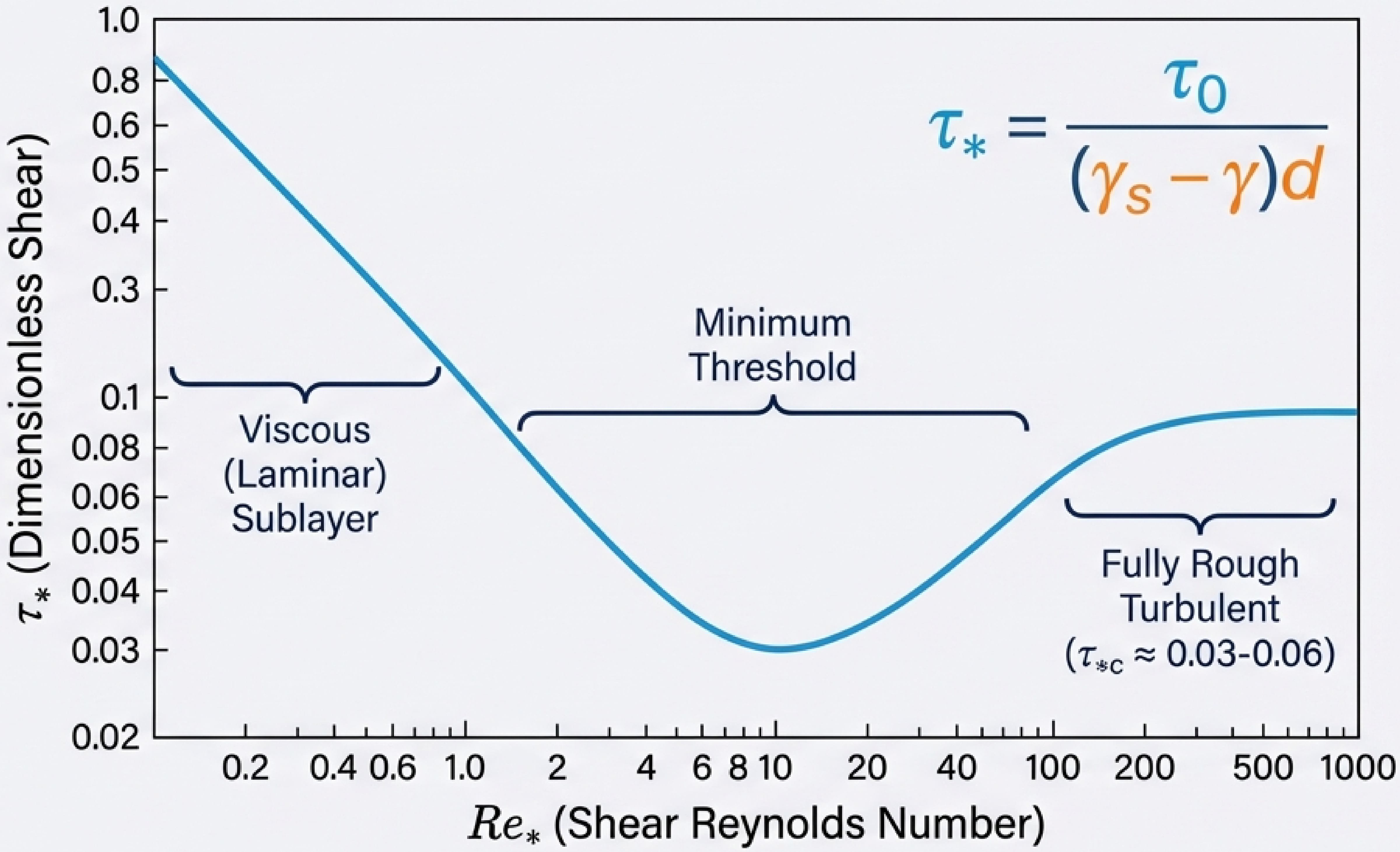
Friction Coefficient: $\mu = \tan(\phi)$

Typical ϕ values: Sand (28–34°), Gravel (up to 40°)

The Gold Standard: The Shields Parameter

Albert F. Shields (1936) normalized particle stability into a dimensionless threshold.

The curve represents the boundary between "Stable" (below) and "Motion" (above).



The Mechanism: The Coleman–Ikeda–Iwagaki (C–I–I) Model

The Problem

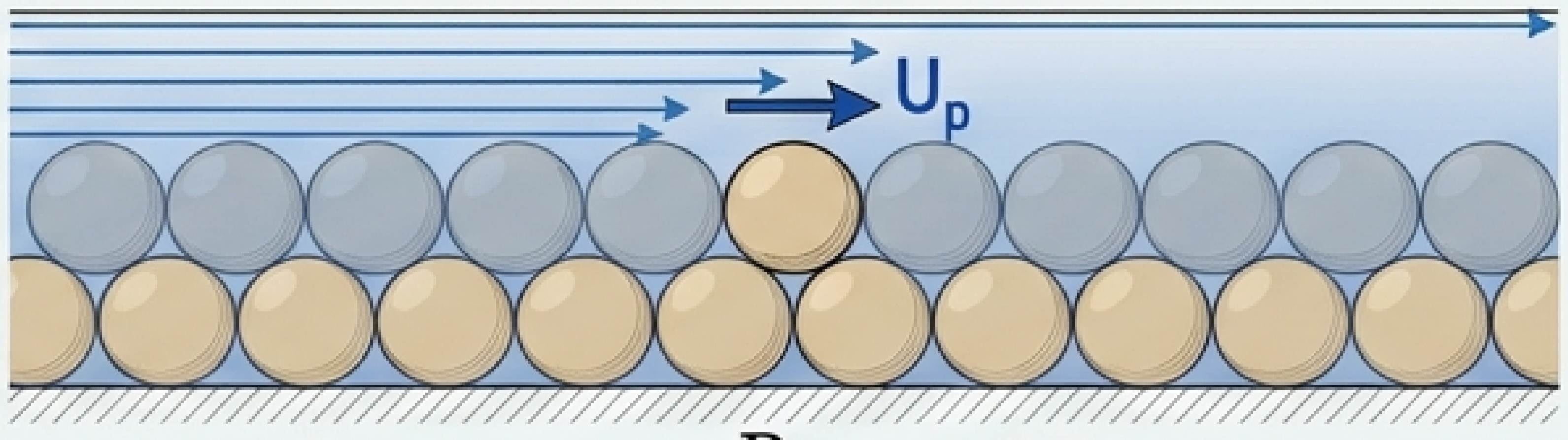
Shields is empirical. To understand the physics, we must look at the specific velocity acting on the particle (U_p), not the depth-averaged velocity.

The Hiding Factor:

- Small Re_*^*: The particle hides in the viscous sublayer. The velocity it feels is small, making it harder to move (Curve rises).
- Large Re_*^*: The particle protrudes into the turbulent layer. It is fully exposed to flow (Curve levels off).

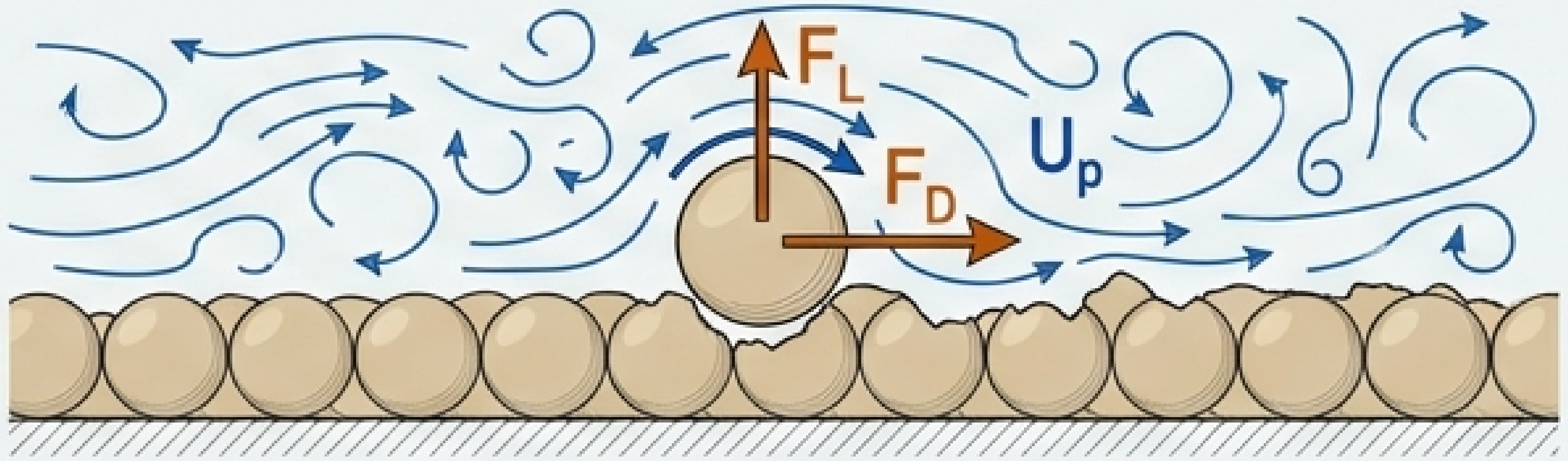
Visual Definition

Viscous Limit ($\mathrm{Re}_* \lesssim 5$)



$$f(\mathrm{Re}_*) = \frac{\mathrm{Re}_*}{2} \quad \text{(Linear)}$$

Rough Limit ($\mathrm{Re}_* \gtrsim 70$)



$$f(\mathrm{Re}_*) \approx 6.7 \text{ (Log-law)}$$

Mathematical Derivation of the Critical Threshold

Step 1: Force Balance:

$$\frac{1}{2}\rho C_D A U_p^2 = \frac{\mu W'}{1+\alpha_L \mu}$$

(Driving Drag = Resisting Friction (adjusted for Lift))

↓

Step 2: Solving for Particle Velocity (U_p):

$$U_p^2 = \frac{4\mu}{3C_D(1+\alpha_L \mu)} Rgd$$

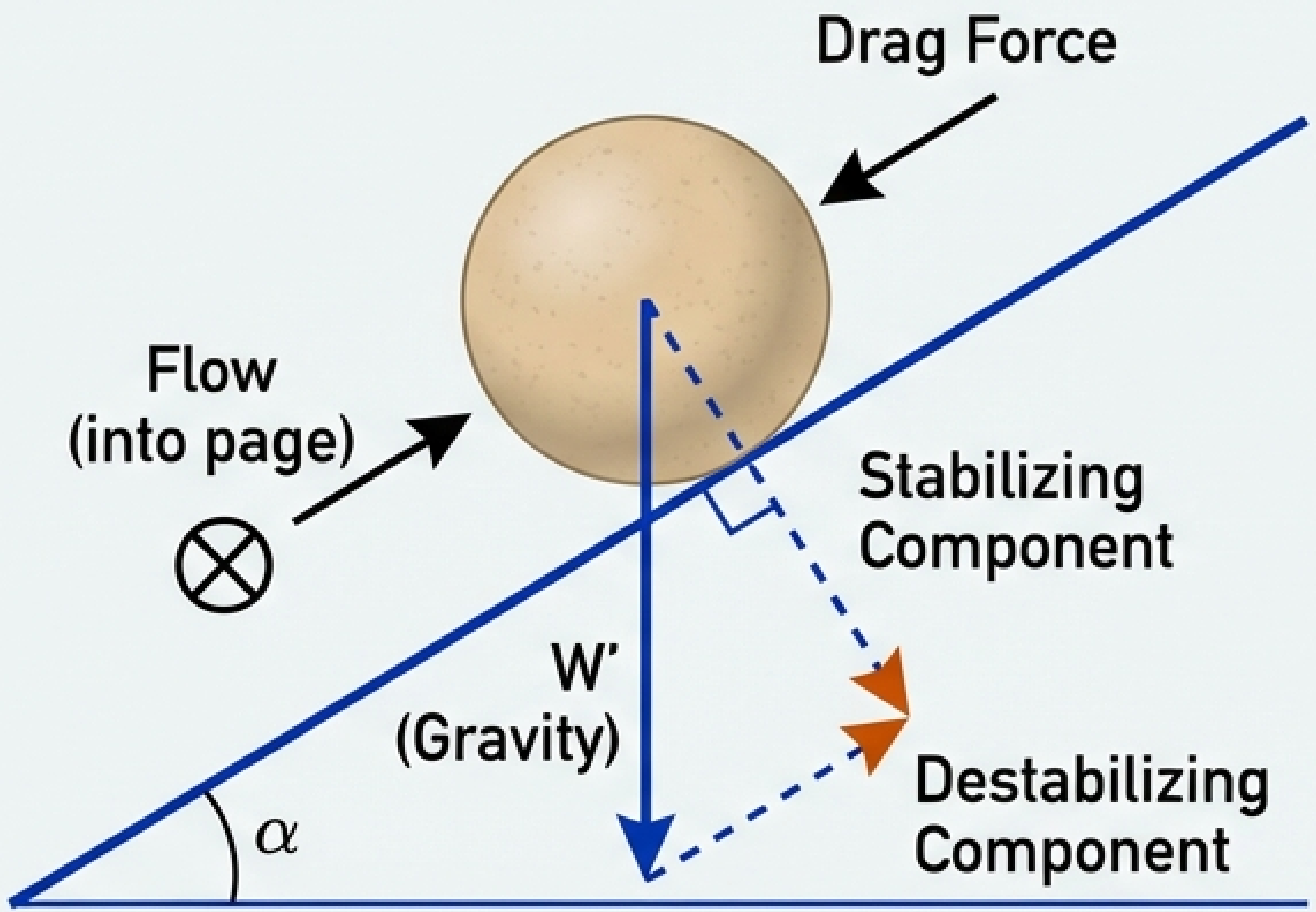
↓

Step 3: The C-I-I Relationship (Substituting $U_p = f u_*$):

$$\tau_{*c} = \frac{u_*^2}{Rgd} = \frac{4\mu}{3C_D(1+\alpha_L \mu) f^2}$$

Since the velocity function '*f*' changes with Re_*, the critical shear τ_{*c} must change.
This model mathematically predicts the dip and rise of the Shields curve.

The Meso Scale: Stability on Side Slopes



On a flat bed, gravity (W') acts purely to resist motion.

On a bank with slope angle α, gravity splits into two components:

1. Downslope component: Pulls the particle down the bank (Destabilizing).
2. Perpendicular component: Holds the particle against the bank (Stabilizing).

Result: A particle on a bank moves more easily than a particle on the bed.

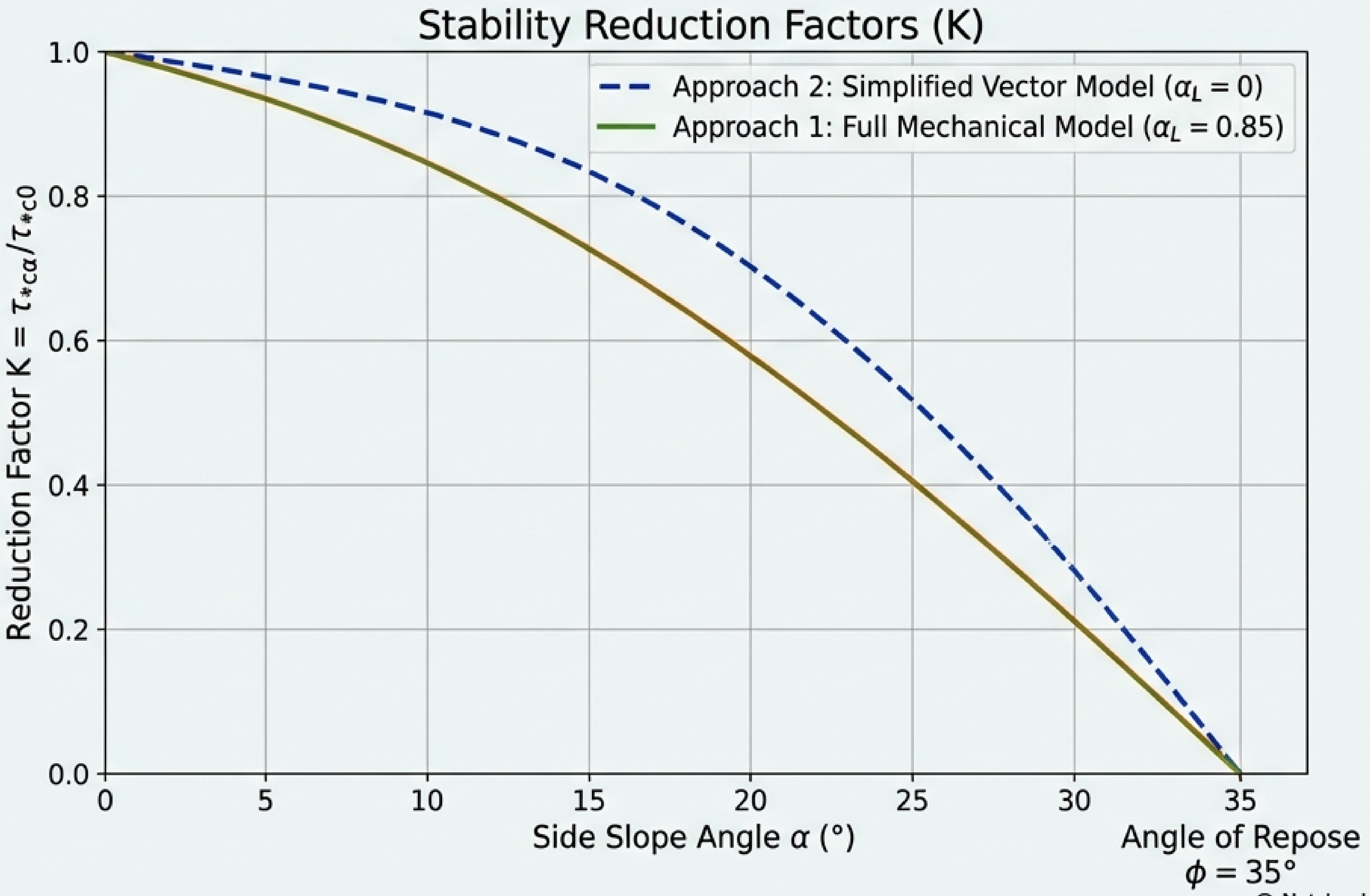
Limit Condition: As $\alpha \to \phi$ (Angle of Repose), the stable shear stress approaches zero.

The Reduction Factor (K)

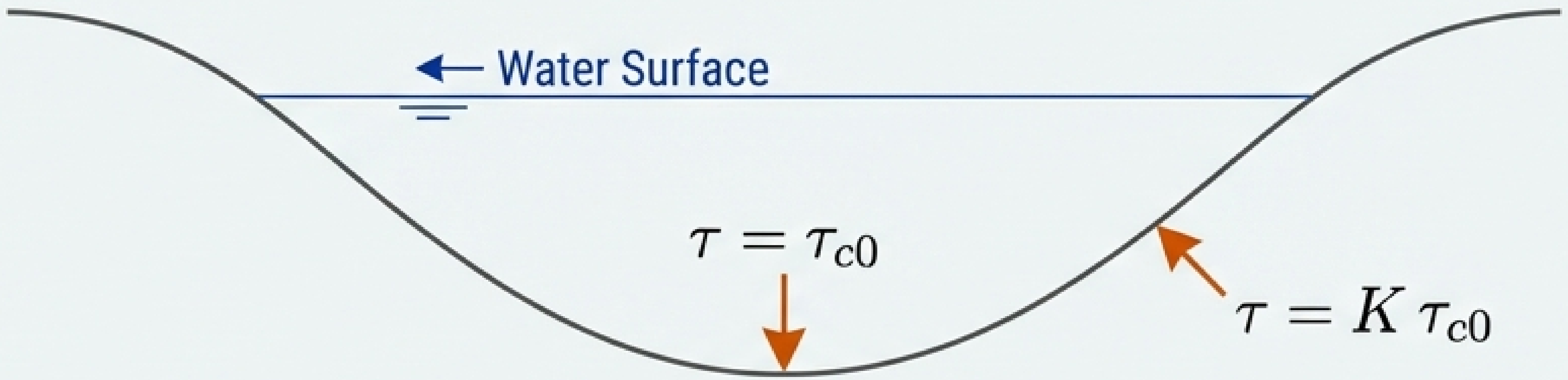
We define K as the ratio of critical shear on a slope to critical shear on a flat bed: $K < 1$.

The Plot:
Approach 1 (Solid Line) includes the Lift Force. It shows a steeper reduction in stability, meaning the bank is more susceptible to scour than simplified models predict.

Design Implication: Using Approach 1 leads to safer, more conservative channel geometries.



The Macro Scale: Simultaneous Incipient Motion



The 'Holy Grail' of stable channel design.
An ideal stable channel is not a box or a trapezoid. It is a shape where every point on the perimeter is exactly at the threshold of motion.
Efficiency: If shear < critical: Over-designed (too wide).
If shear > critical: Erosion (scour).
Result: Nature dictates a continuous curved bank profile.

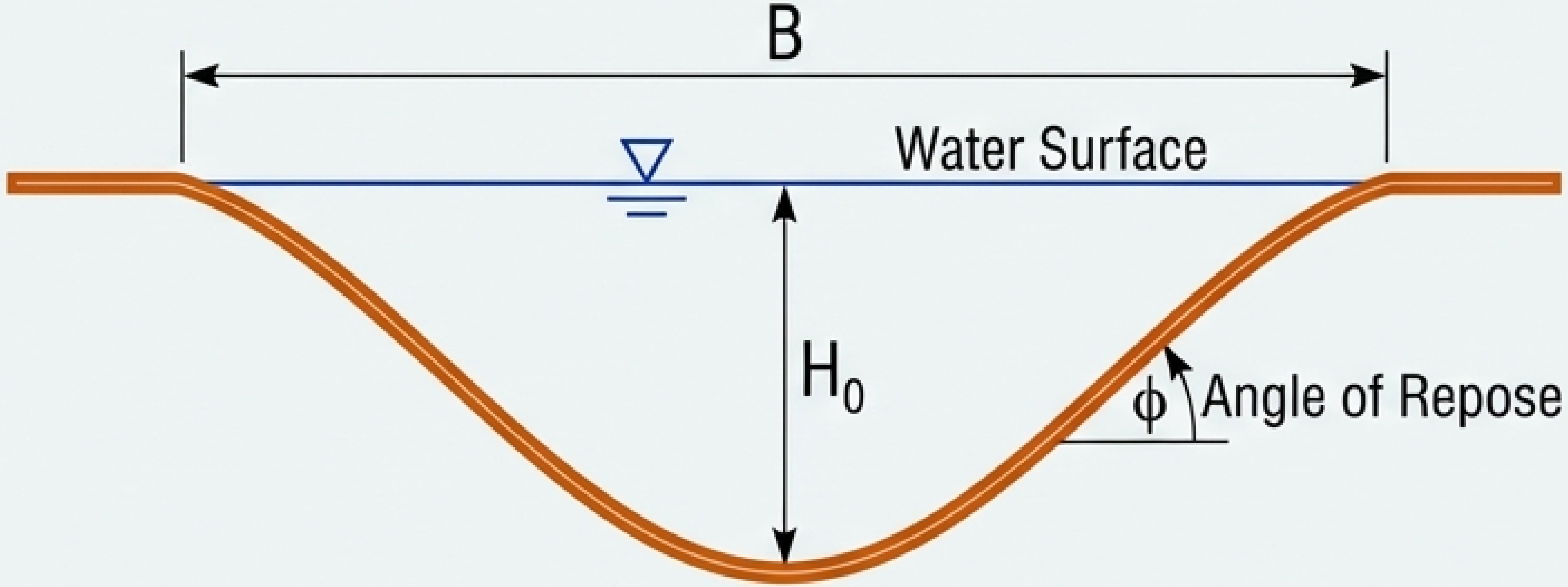
The Glover-Florey-Lane (G-F-L) Profile

Deriving the Shape:

By assuming local shear is proportional to depth ($\tau \propto$ depth) and applying the K-factor reduction, we solve the differential equation for stability. The solution is a trigonometric cosine curve.

$$H(y) = H_0 \cos\left(\frac{y \tan(\phi)}{H_0}\right)$$

$$B = \frac{\pi H_0}{\tan(\phi)}$$



The Shifted Cosine: Impact of Lift on Geometry

When Lift is included (Approach 1), the stable profile changes. The channel effectively widens. The cosine frequency scales to account for reduced effective friction on the bank.

Practical Approximation: For quick calculations, engineers use power-power-law approximations derived from the exact shape:

$$\boxed{\frac{B}{H_0} \approx 3.14\,\phi^{-1.038}}$$

The Exact Equation (Approach 1):

$$H(y) = H_0\Big[(1 - \alpha_L\mu)\cos\left(\left(\frac{\mu}{1-\alpha_L\mu}\sqrt{\frac{1-\alpha_L\mu}{1+\alpha_L\mu}}\frac{y}{H_0}\right)\right) - \alpha_L\mu\Big]$$

NotebookLM

Practical Design Workflow

1 

Material Analysis

Determine particle size (d) and Angle of Repose (ϕ).

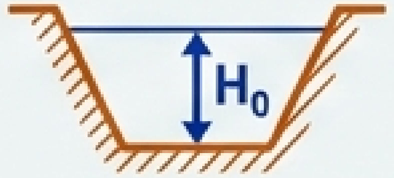
d, ϕ

2 

Threshold Calculation

Calculate Critical Shields Stress (τ_{*c}) for the flat bed.

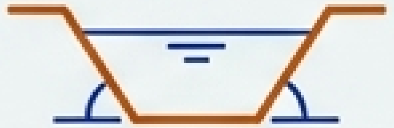
τ_{*c}

3 

Centerline Depth (H_0)

Solve using maximum allowable shear:

$$H_0 = \frac{R d \tau_{*c}}{S}$$

4 

Side Geometry

Use the G-F-L relation (B/H_0) to find the stable width or side slope.

$$\frac{B}{H_0}$$

5 

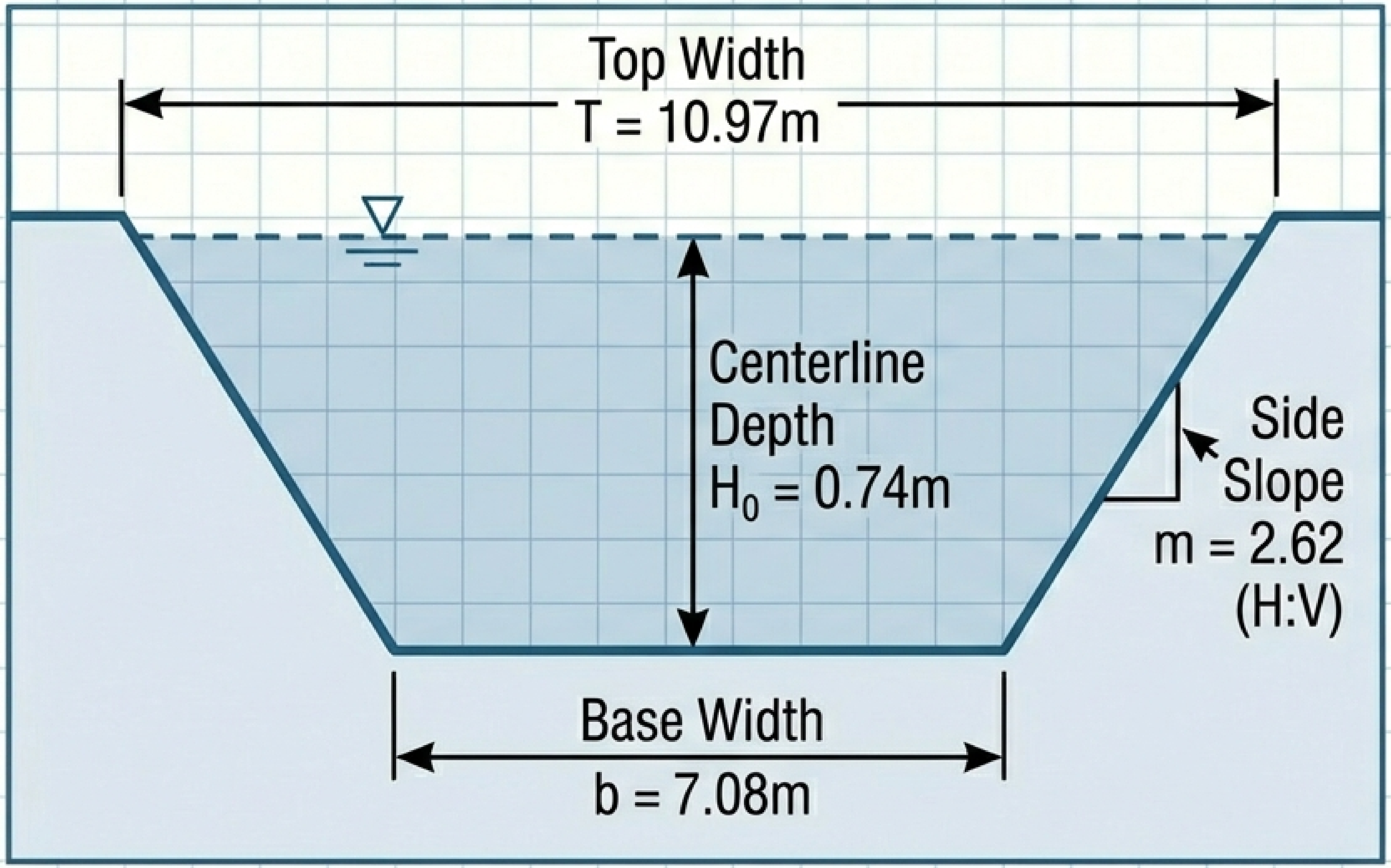
Capacity Check

Use Manning's Equation to solve for the required bottom width (b) to carry the design discharge (Q).

Q, b

Worked Example: Method A (Trapezoidal)

Scenario Text: Design a channel for $Q = 5.0\ m^3/s$, $d = 0.01m$, $Slope = 0.001$.



Calculation Summary

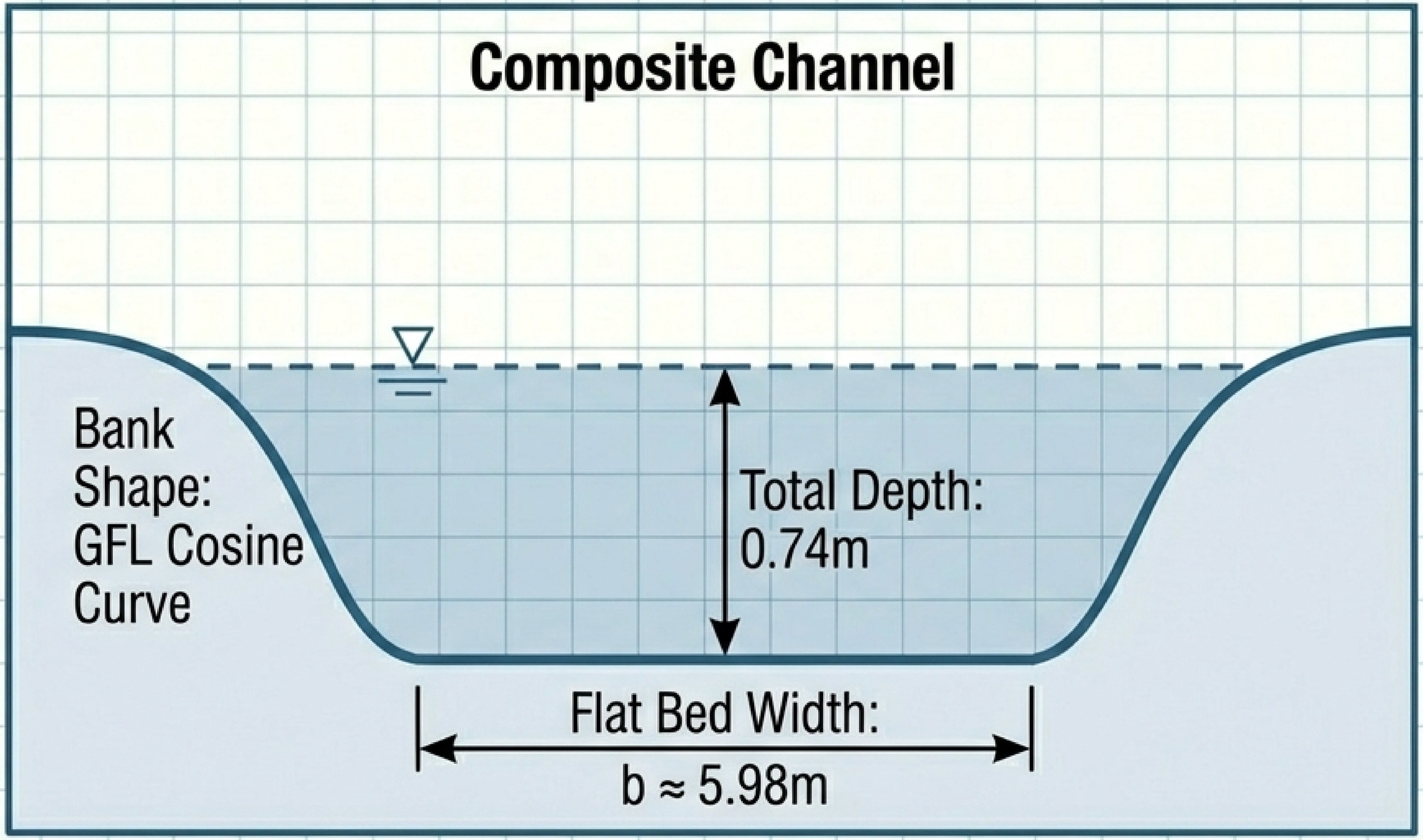
- **Step 1:** H_0 derived from critical shear.
- **Step 2:** Side slope m derived from GFL approximation.
- **Step 3:** Base width b solved via Manning's Equation iteration.

Worked Example: Method B (The Truly Ideal)

Problem/Solution Text:

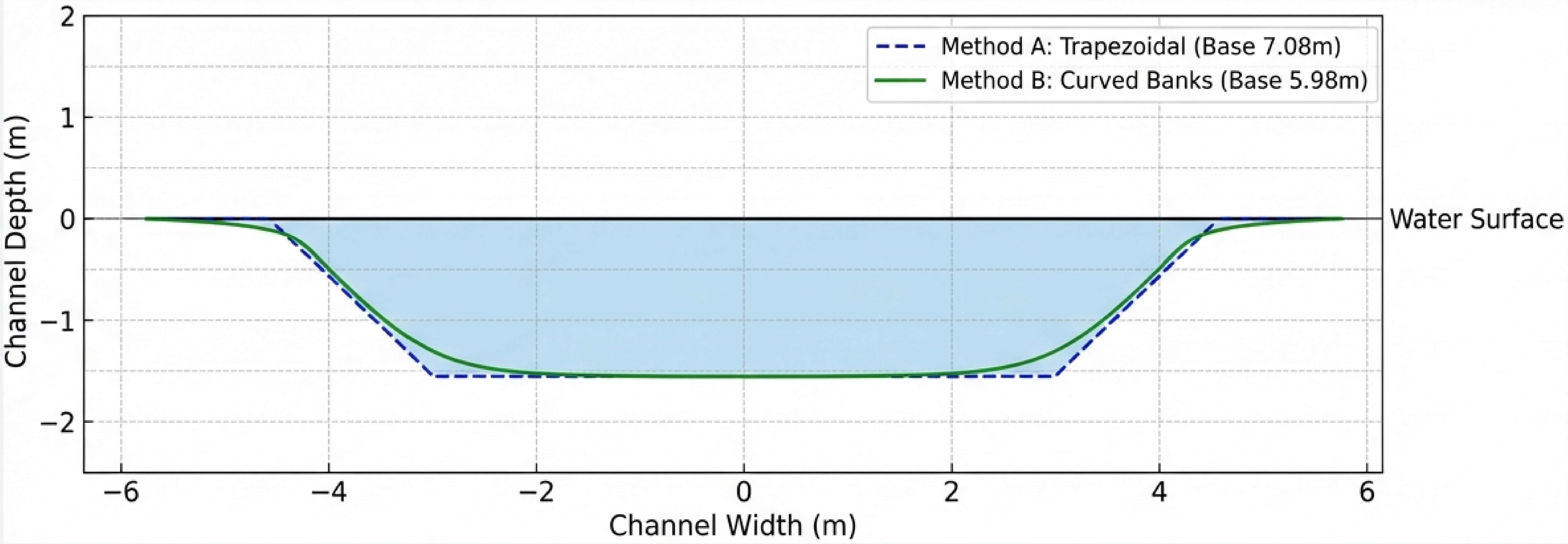
Problem: A pure cosine shape defined by GFL often lacks capacity. (Calculated $Q \approx 1.21\ m^3/s$ vs Target $5.0\ m^3/s$).

Solution: Composite Design. Insert a flat floor to carry the extra load while keeping curved curved banks for stability.



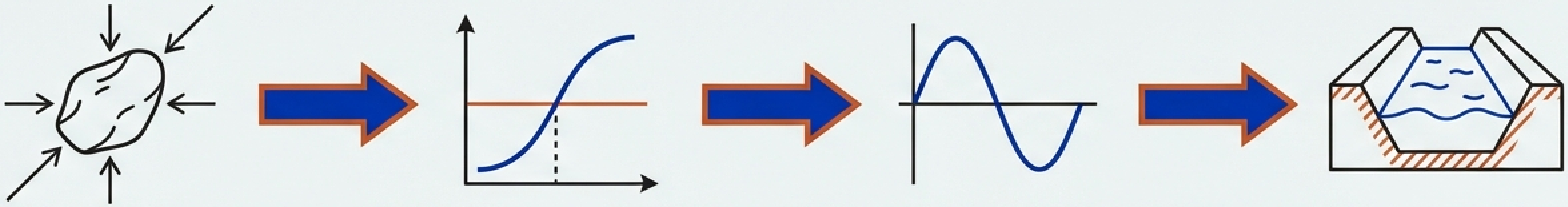
"Physics defines the curve; Engineering defines the width."

Design Comparison: Trapezoid vs. Ideal Curve



Takeaway: Method B is more efficient (less excavation area for same Q), but Method A A is easier to construct. The side slopes align closely, validating the trapezoidal approximation.

The Physical Foundation of Design



Grain Mechanics — K-Factor / Threshold — Ideal Shape — Hydraulic Design

Limitations & Context:

- Probabilistic Nature: Scour is a statistical probability, not instant.
- Mixed Beds: Hiding/exposure effects modify the simple Shields threshold.
- Cohesion: This workflow applies only to non-cohesive sediments (sand/gravel).

Conclusion: Understanding the physics of the single grain is the only way to reliably design the geometry of the channel.