

### 极限习题课3(等价无穷小代换)(5题)

1. 设  $x \rightarrow 0$  时,  $e^{\tan x} - e^x$  是  $x^n$  同阶无穷小, 则  $n$  为

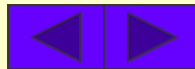
(A) 1      (B) 2      (C) 3      (D) 4

$$2. \lim_{x \rightarrow 0} \frac{\sin x + x^2 \sin \frac{1}{x}}{(1 + \cos x) \ln(1 + x)}$$

$$3. \lim_{x \rightarrow 0} \frac{\ln(\sin^2 x + e^x) - x}{\ln(x^2 + e^{2x}) - 2x}$$

$$4. \lim_{x \rightarrow +\infty} \ln(1 + 2^x) \ln\left(1 + \frac{3}{x}\right)$$

$$5. \text{ 设 } \lim_{x \rightarrow 0} \frac{\ln\left(1 + \frac{f(x)}{\sin x}\right)}{a^x - 1} = A \quad (a > 0, a \neq 1), \text{ 求 } \lim_{x \rightarrow 0} \frac{f(x)}{x^2}.$$



# 答案

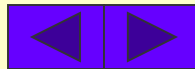
$$1. \text{ 解 } \lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{x^n} = \lim_{x \rightarrow 0} \frac{e^x (e^{\tan x - x} - 1)}{x^n} = \lim_{x \rightarrow 0} \frac{\tan x - x}{x^n}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1 - \cos x}{x^{n-1}} \cdot \frac{1}{\cos x} = \lim_{x \rightarrow 0} \frac{\frac{1}{2} x^2}{x^{n-1}} = \frac{1}{2}$$

所以  $n - 1 = 2$ , 即  $n = 3$  (C)

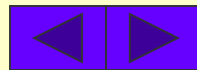
$$2. \lim_{x \rightarrow 0} \frac{\sin x + x^2 \sin \frac{1}{x}}{(1 + \cos x) \ln(1 + x)} = \lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \lim_{x \rightarrow 0} \frac{\sin x + x^2 \sin \frac{1}{x}}{x}$$

$$= \frac{1}{2} \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} + \lim_{x \rightarrow 0} x \sin \frac{1}{x} \right] = \frac{1}{2}.$$



$$\begin{aligned}
 3. \lim_{x \rightarrow 0} \frac{\ln(\sin^2 x + e^x) - x}{\ln(x^2 + e^{2x}) - 2x} &= \lim_{x \rightarrow 0} \frac{\ln[e^x(1 + e^{-x} \sin^2 x)] - x}{\ln[e^{2x}(1 + e^{-2x} x^2)] - 2x} \\
 &= \lim_{x \rightarrow 0} \frac{\ln(1 + e^{-x} \sin^2 x)}{\ln(1 + e^{-2x} x^2)} = \lim_{x \rightarrow 0} \frac{e^{-x} \sin^2 x}{e^{-2x} x^2} = 1
 \end{aligned}$$

$$\begin{aligned}
 4. \lim_{x \rightarrow +\infty} \ln(1 + 2^x) \ln\left(1 + \frac{3}{x}\right) &= \lim_{x \rightarrow +\infty} \left[ \ln 2^x (2^{-x} + 1) \right] \cdot \frac{3}{x} \\
 &= \lim_{x \rightarrow +\infty} \left[ x \ln 2 + \ln(1 + 2^{-x}) \right] \cdot \frac{3}{x} = 3 \ln 2 + \lim_{x \rightarrow +\infty} 2^{-x} \cdot \frac{3}{x} \\
 &= 3 \ln 2.
 \end{aligned}$$



$$5. \text{ 解 因为 } \lim_{x \rightarrow 0} \frac{\ln(1 + \frac{f(x)}{\sin x})}{a^x - 1} = A$$

$$\text{所以 } \frac{\ln(1 + \frac{f(x)}{\sin x})}{a^x - 1} = A + \alpha, \text{ 其中 } \lim_{x \rightarrow 0} \alpha = 0$$

$$\ln(1 + \frac{f(x)}{\sin x}) = (a^x - 1)(A + \alpha) \Rightarrow \frac{f(x)}{\sin x} = e^{(a^x - 1)(A + \alpha)} - 1$$

$$\Rightarrow f(x) = \sin x \cdot [e^{(a^x - 1)(A + \alpha)} - 1]$$

$$\therefore \lim_{x \rightarrow 0} \frac{f(x)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin x \cdot [e^{(a^x - 1)(A + \alpha)} - 1]}{x^2} = \lim_{x \rightarrow 0} \frac{e^{(a^x - 1)(A + \alpha)} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1)(A + \alpha)}{x} = \lim_{x \rightarrow 0} \frac{x \ln a \cdot (A + \alpha)}{x} = \ln a \cdot \lim_{x \rightarrow 0} (A + \alpha)$$

$$= A \ln a$$

