

## 极限习题课1 (极限四则运算法则) (9题)

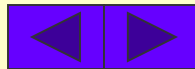
$$1. \lim_{n \rightarrow \infty} [\sqrt{1 + 2 + \cdots + n} - \sqrt{1 + 2 + \cdots + (n-1)}]$$

$$2. \lim_{x \rightarrow +\infty} x(\sqrt{x^2 + 1} - x)$$

$$3. \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{1 + 2 + 3 + \cdots + n}$$

$$4. \lim_{n \rightarrow \infty} \left( \frac{1^3 + 2^3 + \cdots + n^3}{n^3} - \frac{n}{4} \right)$$

$$5. \text{设 } \lim_{x \rightarrow 2} \frac{x^2 + ax + b}{x^2 - x - 2} = 2, \text{ 求常数 } a, b.$$



6. 设  $\lim_{x \rightarrow -1} \frac{x^3 - ax^2 - x + 4}{x + 1}$  有有限极限值  $b$ ,

试求常数  $a$  及极限值  $b$ .

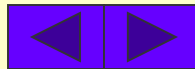
7. 设  $\lim_{x \rightarrow \infty} \left( \frac{x + 2a}{x - a} \right)^x = 8$ , 则  $a = \underline{\hspace{2cm}}$ .

8. 已知  $n$  为正整数,  $a$  为某常数,  $a \neq 0$

且  $\lim_{x \rightarrow +\infty} \frac{x^{1999}}{x^n - (x - 1)^n} = \frac{1}{a}$ , 求  $n$  和  $a$ .

9. 设  $\lim_{x \rightarrow 0} \frac{a \tan x + b(1 - \cos x)}{c \ln(1 - 2x) + d(1 - e^{-x^2})} = 2$ , 其中  $a^2 + c^2 \neq 0$ , 则

(A)  $b = 4d$  (B)  $b = -4d$  (C)  $a = 4c$  (D)  $a = -4c$



# 答案

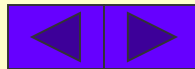
$$1. \text{ 原式} = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{1+2+\cdots+n} + \sqrt{1+2+\cdots+(n-1)}}$$

$$= \lim_{n \rightarrow \infty} \frac{n}{\sqrt{\frac{n(n+1)}{2}} + \sqrt{\frac{n(n-1)}{2}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{2}}{\sqrt{1+\frac{1}{n}} + \sqrt{1-\frac{1}{n}}} = \frac{\sqrt{2}}{2}$$

$$2. \text{ 解} \quad \lim_{x \rightarrow +\infty} x(\sqrt{x^2+1}-x) = \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2+1}+x} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}+1} = \frac{1}{2}$$

$$3. \text{ 解} \quad \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{1+2+\cdots+n} = \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{2}{n(n+1)} = 2 \lim_{N \rightarrow \infty} \left(1 - \frac{1}{N+1}\right) = 2.$$

$$4. \lim_{n \rightarrow \infty} \left( \frac{1^3 + 2^3 + \cdots + n^3}{n^3} - \frac{n}{4} \right) = \lim_{n \rightarrow \infty} \frac{\frac{n^2(n+1)^2}{4} - \frac{n^4}{4}}{n^3} = \frac{1}{4} \lim_{n \rightarrow \infty} \frac{2n^3 + n^2}{n^3} = \frac{1}{2}.$$



$$5. \text{ 解 } \lim_{x \rightarrow 2} \frac{(x-2)(x+k)}{(x-2)(x+1)} = \lim_{x \rightarrow 2} \frac{(x+k)}{(x+1)} = \frac{2+k}{3} = 2 \Rightarrow k = 4$$

$$\therefore x^2 + ax + b = (x-2)(x+4) = x^2 + 2x - 8 \Rightarrow a = 2, b = -8$$

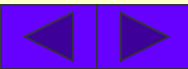
$$6. \lim_{x \rightarrow -1} (x^3 - ax^2 - x + 4) = -1 - a + 1 + 4 = 4 - a = 0 \quad \text{得} \quad a = 4$$

$$\begin{aligned} b &= \lim_{x \rightarrow -1} \frac{x^3 - 4x^2 - x + 4}{x + 1} = \lim_{x \rightarrow -1} \frac{(x+1)(x-1)(x-4)}{x+1} \\ &= \lim_{x \rightarrow -1} (x-1)(x-4) = 10 \end{aligned}$$

$$7. \text{ 左 } = \lim_{x \rightarrow \infty} \left( 1 + \frac{3a}{x-a} \right)^{\frac{x-a}{3a} \cdot 3a+a}$$

$$= \left[ \lim_{x \rightarrow \infty} \left( 1 + \frac{3a}{x-a} \right)^{\frac{x-a}{3a}} \right]^{3a} \cdot \lim_{x \rightarrow \infty} \left( 1 + \frac{3a}{x-a} \right)^a = e^{3a}$$

$$\because e^{3a} = 8 \quad \therefore a = \ln 2$$



$$8. \text{ 解 } (x-1)^n = x^n - nx^{n-1} + \frac{n(n-1)}{2}x^{n-2} - \cdots + (-1)^n$$

$$\frac{1}{a} = \lim_{x \rightarrow +\infty} \frac{x^{1999}}{nx^{n-1} - \frac{n(n-1)}{2}x^{n-2} + \cdots + (-1)^{n+1}}$$

$$\therefore n-1 = 1999 \quad n = 2000$$

$$\frac{1}{a} = \frac{1}{n} \therefore a = 2000$$

$$9. \text{ 左} = \lim_{x \rightarrow 0} \frac{a \frac{\tan x}{x} + b \frac{1 - \cos x}{x}}{c \cdot \frac{\ln(1-2x)}{x} + d \frac{1 - e^{-x^2}}{x}} = \frac{a \cdot \lim_{x \rightarrow 0} \frac{\tan x}{x} + b \cdot \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}}{c \cdot \lim_{x \rightarrow 0} \frac{\ln(1-2x)}{x} + d \cdot \lim_{x \rightarrow 0} \frac{1 - e^{-x^2}}{x}}$$

$$= \frac{a \cdot \lim_{x \rightarrow 0} \frac{x}{x} + b \cdot \lim_{x \rightarrow 0} \frac{\frac{1}{2}x^2}{x}}{c \cdot \lim_{x \rightarrow 0} \frac{-2x}{x} + d \cdot \lim_{x \rightarrow 0} \frac{x^2}{x}} = -\frac{a}{2c} \quad \text{由 } -\frac{a}{2c} = 2 \text{ 得 } a = -4c \quad (\text{D})$$

