$$H = \frac{1}{2\mu}(\vec{p} - \frac{q}{c}\vec{A})^2 + q\phi$$

理由如下： $\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}$ ， $\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}$

一维 $\dot{x} = \frac{\partial H}{\partial p_x}$ ， $\dot{p}_x = -\frac{\partial H}{\partial x}$

$$\dot{x} = \frac{1}{\mu}(p_x - \frac{q}{c}A_x)$$

$$p_x = \mu\dot{x} + \frac{q}{c}A_x$$

$$\text{or } \vec{p} = \mu\vec{v} + \frac{q}{c}\vec{A}$$

正则　机械

↙代入 $H = \sum_{i=1}^{3}\frac{1}{2\mu}(p_i - \frac{q}{c}A_i) - q\phi$ ， p_i 与 x 无关的

$$\mu\ddot{x} = (\dot{p}_x - \frac{q}{c}\dot{A}_x) = -\frac{\partial H}{\partial x} - \frac{q}{c}\dot{A}_x = \underline{\frac{1}{\mu}\sum_{i=1}^{3}(p_i - \frac{q}{c}A_i)\frac{q}{c}\frac{\partial A_i}{\partial x} - q\frac{\partial \phi}{\partial x}} - \frac{q}{c}\dot{A}_x$$

$$= \frac{q}{c}\sum_{i=1}^{3}\dot{x}_i\frac{\partial A_i}{\partial x} - q\frac{\partial\phi}{\partial x} - \frac{q}{c}(\dot{A}_x + \sum_{i=1}^{3}\dot{x}_i\frac{\partial A_x}{\partial r_i})$$

$$= -q(\frac{\partial\phi}{\partial x} + \frac{1}{c}\frac{\partial A_x}{\partial t}) + \frac{q}{c}[\dot{x}\frac{\partial A_x}{\partial x} + \dot{y}\frac{\partial}{\partial x}A_y + \dot{z}\frac{\partial}{\partial x}A_z - \dot{x}\frac{\partial}{\partial x}A_x - \dot{y}\frac{\partial}{\partial y}A_x - \dot{z}\frac{\partial}{\partial z}A_x]$$

$$= -q(\nabla\phi + \frac{1}{c}\frac{\partial}{\partial t}\vec{A}) + \frac{q}{c}[\vec{v}\times(\nabla\times\vec{A})]$$

$$= q(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{p} \to \hat{\vec{p}} = -i\hbar\nabla$$

$$H = \frac{1}{2\mu}(\hat{\vec{p}} - \frac{q}{c}\vec{A})^2 + q\phi$$

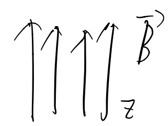
薛定谔方程:

$$i\hbar\frac{\partial}{\partial t}\psi = [\frac{1}{2\mu}(\hat{\vec{p}} - \frac{q}{c}\hat{\vec{A}})^2 + q\phi]\psi$$

$$[\hat{\vec{p}}, \vec{A}] = -i\hbar\nabla\cdot\vec{A}$$

Landau能级:

考虑 $\vec{B} = B\vec{e}_z$ ， $\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$ （ $\nabla\times\vec{A} = \vec{B}$ ， $\nabla\cdot\vec{A} = 0$ ）



此时 $A_x = -\frac{1}{2}By$ ， $A_y = \frac{1}{2}Bx$ ， $A_z = 0$

$$H = \frac{1}{2\mu}[(\hat{p}_x - \frac{eB}{2c}y)^2 + (\hat{p}_y - \frac{eB}{2c}x)^2 + \hat{p}_z^2]$$

$$= \frac{1}{2\mu}(\underbrace{\hat{p}_x^2 + \hat{p}_y^2}_{=|\hat{\vec{p}}|^2 = \hbar^2\nabla^2 = \hbar^2(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2})} + \underbrace{\hat{p}_z^2}_{\text{忽略}}) + \frac{e^2B^2}{8\mu c^2}(\underbrace{x^2 + y^2}_{\rho^2}) + \frac{eB}{2\mu c}(\underbrace{x\hat{p}_y - y\hat{p}_x}_{= -i\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}) = -i\hbar\frac{\partial}{\partial\varphi} = l_z})$$

$$\because [H, \hat{l}_z] = [H, -i\hbar(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x})] = 0$$

$$\psi(\rho,\varphi) = R(\rho)e^{im\varphi}$$

$$[-\frac{\hbar^2}{2\mu}(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}) + \frac{1}{2}\quad w_L^2\rho^2]R(\rho) = (E - m\hbar w_L)R(\rho)$$

$w_L = eB/2\mu c$ 拉莫尔频率

$$\hat{l}_z = -i\hbar\frac{\partial}{\partial\varphi}$$

$$\therefore E = (N+1)\hbar w_L$$

$$N = (2n_\rho + |m| + m) = 0, 2, 4\cdots, \quad n_\rho = 0, 1, 2, \cdots$$

$$\therefore R_{n_\rho,|m|} \sim \rho^{|m|}F(-n_\rho, |m|+1, \alpha^2\rho^2)e^{-\frac{\alpha^2\rho^2}{2}}$$

$$\alpha = \sqrt{\mu w_L/\hbar} = \sqrt{eB/2\hbar c}$$

能级简并度为∞

若取 $A_x = -By,\ A_y = A_z = 0$

Landau规范

$H = \frac{1}{2\mu}\left[(\hat{p}_x - \frac{eB}{c}y)^2 + \hat{p}_y^{\,2}\right]$ $\qquad \psi(x,y) = e^{ip_x x/\hbar}\chi(y)$

$\Rightarrow -\frac{\hbar^2}{2\mu}\chi''(y) + \frac{1}{2}\mu\omega_c^2(y-y_0)^2\chi(y) = E\chi(y)$

$\omega_c = eB/\mu c = 2\omega_L$ 回旋

$E = (n+\frac{1}{2})\hbar\omega_c,\ n = 0,1,2,\cdots$

$= (N+1)\hbar\omega_L,\ N = 0,2,4,\cdots$

$E = \mu_z B = (n+\frac{1}{2})\frac{|e|\hbar}{mc}B$

对二维电子气：

$e^{ik_x(x+L_x)} = e^{ik_x x},\quad e^{ik_x L_x} = 1,\quad k_x L_x = 2n_x\pi$

$\Delta k_x = \frac{2\pi}{L_x}\Delta n_x = \frac{2\pi}{L_x},\quad \Delta y_0 = \frac{\hbar c}{|e|B}\Delta k_x = \frac{hc}{|e|BL_x}$

$G = \frac{L_y}{\Delta y_0} = \frac{|e|BL_xL_y}{hc} = \frac{\Phi}{\Phi_0}$

$\Phi_0 = \frac{hc}{|e|}$ 为元磁通量子