Chapter 2

Date: November 14, 2022

2.1 SK model

Question: Use cavity method to calculate the free energy and magnetization of the Sherrington-Kirkpatrick (SK) model, and compare the result with numerical enumeration.

Answer: The Hamiltonian of the SK model is

$$H = -\frac{1}{2}\sum_{i\neq j} J_{ij}\sigma_i\sigma_j \tag{1}$$

where the spins σ_i are binary and the couplings J_{ij} follow a Gaussian distribution $J_{ij} \sim \mathcal{N}\left(\frac{J_0}{N}, \frac{J}{N}\right)$. However, in cavity case, we don't need to take the quenched average over J_{ij} into consideration, which will be seem as known constants in this question.

Here is the factor graph of 4-spins SK model.

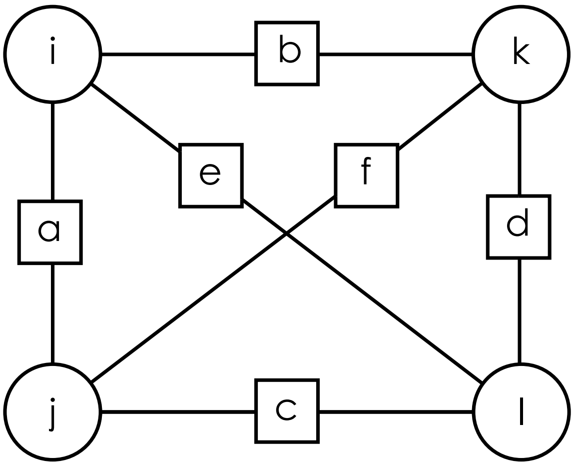

Figure 1: 4-spins SK modelaa

The only difference between SK model and Sourlas code system (main text) is that every variable node has $N-1$ factor node neighbourhoods (labelled a, b, c, d, $\cdots$ below) and every factor node has only two variable node neighbourhoods (labelled i, j below). Therefore, we can use the free energy and AMP equations in the main text with slight alteration.

Here we rewrite the J_{ij} as J_a to conform to the notion in main text and the eq. (2.14) is rewritten as:

$$Z^{new} = Z^{old}\cosh(\beta J_a)\left(1+\tanh(\beta J_a)m_{i\to a}m_{j\to a}\right) \tag{2}$$

Thus the eq. (2.15) is altered into:

$$-\beta\Delta F_a = \ln\left[\cosh(\beta J_a)\left(1+\tanh(\beta J_a)m_{i\to a}m_{j\to a}\right)\right] \tag{3}$$

Similarly, the eq. (2.18) and eq. (2.19) are changed into:

$$\Gamma^{+}_{b\to i} = \cosh(\beta J_b)\left(1+\tanh(\beta J_b)m_{j\to b}\right) \tag{4}$$

$$\Gamma^{-}_{b\to i} = \cosh(\beta J_b)\left(1-\tanh(\beta J_b)m_{j\to b}\right) \tag{5}$$

and eq. (2.20) keeps the same form. Finally, the total free energy can be calculated by eq. (2.21).

Beside the equations about free energy, the iterative formulae (or AMP equations) eq. (2.30), eq. (2.31) about cavity magnetication are need to altered as well:

$$\begin{aligned} m_{i\to a} &= \tanh\Big(\sum_{b\in\partial i/\ a}\tanh^{-1}(\hat{m}_{b\to i})\Big) \\ \hat{m}_{a\to i} &= \tanh(\beta J_a)m_{j\to a}, \quad j=\partial a/\ i \end{aligned} \tag{6}$$

In the experimental part, we have publish the code in the github and here are some results for reference only.

Here, we set the parameters $J_0 = 0$, $J = 1.0$, $N = 15$, $\beta = 1.0$ in the experiment and the result of cavity method and enumeration method are in fig 2

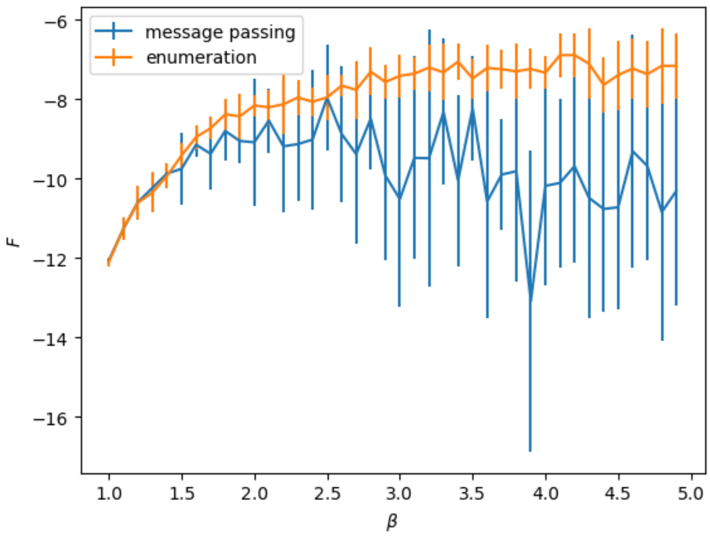

Figure 2: Cavity and enumeration method

2.2 Sourlas code

Question: a: Write a program to implement the encoding and decoding scheme in Sourlas code system ($K \equiv |\partial a| = 3$, $R \equiv \frac{N}{M} = 0.5$). b:Then show how the decoding performance changes with the flipping rate p at a special temperature $\beta_p = \frac{1}{2}\ln\frac{1-p}{p}$

Answer: This is a fully programming exercise and the iterative equations has been obtain in main text.

Here we show present the main result in fig 3 and the details of codes will be shown in github.

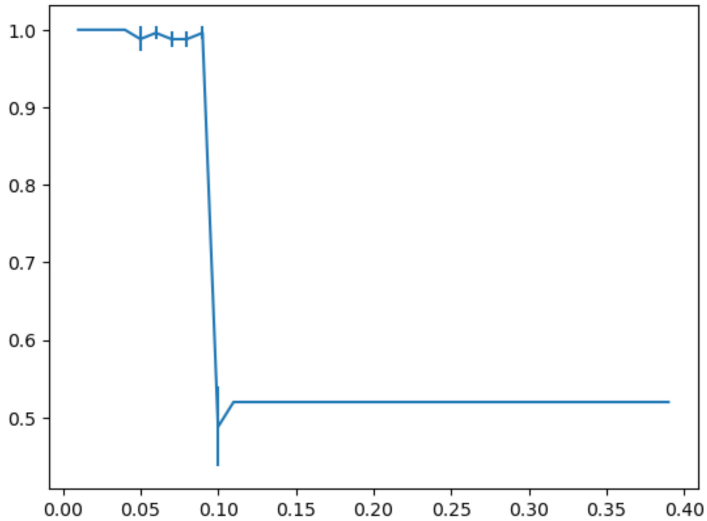

Figure 3: Sourlas Code decoding performance

Github code: https://github.com/Qjbtiger/SMNN-exercise/tree/main/Chapter%202