DANA 4800
Midterm Test 2
Thursday July 17th, 2025

Student Name (Please circle your last name): _____

Student Number: _____

Read the following instructions carefully:

- Please turn off your cell phone and other communication device (including but not limited to laptop computers and tablets etc.) Any noise (ringing or noise from vibration) any device of yours makes during the exam will result in an automatic zero in the midterm.

- This is a closed book examination, but you are allowed to use a hand-held calculator.

- You are allowed to bring a one-sided hand-written letter-size cheat sheet.

- Any form of communication (like peeking over neighbour's paper, reading the questions out loud to yourself etc. in any languages) is underline not permitted during the exam, including sharing of calculator or stationery etc. Offender's midterm paper will be given a mark of zero and the incident will be reported to the Student Conduct and Academic Integrity Office.

- You could use either pens or pencils as long as your handwriting is legible. Remember, if I cannot read it, I cannot mark it.

- The criteria of marking are based on what you have provided, not what the marker thinks you know about the material.

- Marks might be deducted from wrong spelling or incorrect/incomplete names of proper statistical terminologies/jargons.

- Full credits might not be given for unsupported answers.

- Please answer all questions in plain language and in the context of the question.

- You can leave all probability answers in 4 decimal places.

- The duration of this midterm examination is 60 minutes.

Q1	Q2	Q3	Q4	Total
				/30

1. **[9 Total Marks]** John is planning a trip to Toronto in August. Last week, the average price of round-trip plane tickets from Vancouver to Toronto was $450. He wanted to see if the price would drop and decided to wait for a week. A random sample of 40 round-trip plane tickets was drawn today and the average price is $445 and a standard deviation of $30. It is believed that round-trip plane ticket price does not follow a Normal distribution.

   a) Provide a description of the parameter of interest. **[2 marks]**

   Define μ as the average price of all round-trip plane tickets from Vancouver to Toronto in August.

   b) Set up the null hypothesis and alternative hypothesis using the parameter defined in part (b). **[2 marks]**

   $$H_0: \mu \geq 450 \text{ vs. } H_a: \mu < 450$$

   c) Calculate the test statistic and find the p-value. **[3 marks]**
   Note: Checking conditions is part of the procedure.

   $$TS = \frac{445 - 450}{24/\sqrt{40}} = -1.05$$
   p-value = 0.1469

   The p-value is only an approximate value because the variable of interest does not have a Normal distribution. Here, the $\overline{X}$ has approximately a Normal distribution because the sample size is 40, which is bigger than the requirement of 30 (according to the Central Limit Theorem).

   d) Draw an appropriate conclusion with 5% level of significance. **[2 marks]**

   Since the p-value is bigger than the 5% significance level, we do not have enough statistical evidence to reject the null hypothesis and conclude that the average price of all round-trip plane tickets from Vancouver to Toronto in August is not significantly lower than $450.

2. **[6 Total Marks]** Suppose the number of customers arriving at a local bank per hour has an average of 5 customers per hour.
   a) Briefly explain the meaning of the independence assumption, in the context of the question. **[2 marks]**

   It means that the number of customers arriving at the local bank in one hour does not have an effect on the number of customers arriving at the same bank in another hour.

   b) What is the probability that there are exactly 5 customers arriving at the local bank during lunch hour? **[2 marks]**

   Answer: $f(5) = \dfrac{e^{-5} \cdot 5^5}{5!} = 0.1755$

   c) What is the probability that there are at least 2 customers arriving at the local bank during lunch hour? **[2 marks]**

   Answer: $1 - f(0) - f(1) = 1 - \dfrac{e^{-5} \cdot 5^0}{0!} + \dfrac{e^{-5} \cdot 5^1}{1!} = 1 - e^{-5} - 6 \cdot e^{-5} = 0.9596$

3. **[5 Total Marks]** A recent study reported that college students spent an average of 95 minutes and a standard deviation of 14 minutes using Generative AI Tools (like Grammarly, ChatGPT etc.) per day. Suppose the amount of time using Generative AI Tools follows a Normal distribution.
   a) When a college student is randomly selected, what is the probability that they spend more than 80 minutes using Generative AI Tools per day? **[2 marks]**

   Z = (80 – 95)/14 = -1.07
   From the Z-table, the cumulative area is 0.1423.
   Answer = 1 – 0.1423 = 0.8577

   b) Find the 22nd percentile of daily time usage Generative AI Tools among college students. **[3 marks]**

   22nd percentile means cumulative area of 0.22.
   From the Z-table, the closest cumulative area is 0.2206, which gives Z = -0.77.
   X = mu + Z*sigma = 95 – 0.77*14 = 84.22

4. **[10 Total Marks]** Consider the game of American Roulette. There are total of 38 slots where the ball can fall in when the roulette wheel stops: 36 winnable slots (number 1 to 36) and 2 non-winnable slots (0 and 00). In most American Roulette games, when the ball falls in the 0 or 00 slot, the house takes your money or you lose your bet.

a) Consider betting $1 on a "Corner Bet". If the ball falls in any of the four adjacent spots, you win $8 (or have a winning amount is +$8, or positive $8). Otherwise, you lose the $1 bet (or have a winning amount of -$1, or negative $1).

Define the random variable X as the winning amount. Construct the probability distribution table of the random variable X. **[3 marks]**

CORNER BET PAYOUT 8 TO 1

AMERICAN ROULETTE

	Win	Loss
X (winning amount, in $)	+8	-1
Probability	4/38	34/38

b) Calculate the expected value of X. **[2 marks]**

$$E(X) = (+8) * \left(\frac{4}{38}\right) + (-1) * \left(\frac{34}{38}\right) = \frac{32-34}{38} = -\frac{2}{38} \text{ or -0.053}$$

c) Provide an interpretation of the expected value from the gambler's point of view. Briefly explain to a casual gambler why they will always lose money in the long run. **[2+1 marks]**

Again, it has three components: 1) single value, 2) random variable, 3) population.
The average amount of dollar that a gambler loses per game is $0.052, in a long run.

d) Suppose a player is going to play this "Corner Bet" game 10 times. Find the probability that the player will win one or more games. **[2 marks]**

Given: $n = 10, p = 4/38$
Answer: $1 - f(0) = 1 - \frac{10!}{(10-0)! \cdot 0!} \left(\frac{4}{38}\right)^0 \left(1 - \frac{4}{38}\right)^{10-0} = 1 - \left(\frac{34}{38}\right)^{10} = 0.6712$