

1. Toronto Food Bank has been receiving an average of \$30 (worth of food) per donor per month. The head of Vancouver Food Bank (VFB) would like to see if the donation is lower than the Toronto counterpart. A random sample of 60 VFB donors in was drawn and it was found that the average donation was \$28 and a standard deviation of \$10. From past experience, it is believed that the amount of donation follows a right-skewed distribution.

Subjects: VFB donors

Variable: the amount of donations (in \$)

Type: numerical variable

- a) Identify the parameter of interest. [2 marks]

Define μ as the average amount of donations among all VFB donors

- b) Set up the null hypothesis and the alternative hypothesis. [2 marks]

$H_0: \mu \geq 30$ vs. $H_a: \mu < 30$

Note: μ_0 has a value of 30, under the null hypothesis (or under the assumption that the null hypothesis is true)

- c) Calculate the test statistic and find the corresponding p-value. [4 marks]

Note: Checking conditions is part of the procedure.

Given: $n = 60$, $\bar{X} = 28$, and $s = 10$

$$TS = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Note: We didn't know the value of σ , but we could use s (a statistic) to estimate σ (an unknown parameter).

$$TS = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{28 - 30}{10 / \sqrt{60}} = -1.55$$

-> p-value is the left-hand area because H_a points to the left-hand side, i.e. p-value = 0.0606

Note: CLT says that when X doesn't have a Normal distribution, \bar{X} will still have approximately a Normal distribution when $n \geq 30$.

Here, X has a right-skewed shape but the sample size is 60, which is bigger than the requirement of 30. Hence, the p-value is only an approximation (but a good one).

- d) Draw an appropriate conclusion, in the context of the question using 5% significance level. [2 marks]

Since the p-value is bigger than the significance level of 0.05, we do not have enough statistical evidence to reject the null hypothesis and conclude that the average amount of donations among all VFB donors is not significantly less than \$30.

2. In the past, 80% of ABC Airlines flights have been on-time departure. A new bag checking system has been implemented 3 months ago. The CEO of ABC Airlines would like to see if the on-time departure rate has improved since. A random sample of 500 flights in the last 3 months was drawn and 415 of them had on-time departure.

Subjects: (ABC Airlines) flights

Variable: whether or not an ABC Airlines flight will have a on-time departure

Type: categorical variable

- a) Identify the parameter of interest. [2 marks]

Define p as the proportion of all ABC Airlines flights that have a on-time departure

- b) Set up the null hypothesis and the alternative hypothesis. [2 marks]

$H_0: p \leq 0.80$ vs. $H_a: p > 0.80$

- c) Calculate the test statistic and find the corresponding p-value. [4 marks]

Note: Checking conditions is part of the procedure.

$$\bar{p} = \frac{x}{n} = \frac{415}{500} = 0.83$$

$$TS = \frac{\bar{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.83 - 0.8}{\sqrt{0.8 \times (1-0.8)/500}} = 1.68 \text{ (keep 2 decimal places for Z-table)}$$

-> p-value is the right-hand area because H_a points to the right-hand side,

i.e. p-value = $1 - 0.9535 = 0.0465$

Note: The p-value is always an approximation

Here, the approximation is good (enough) because all of the following 3 conditions are met:

1) $n = 500$, which is bigger than the requirement of 30

2) $n \times p_0 = 500 \times 0.8 = 400$, which is bigger than the requirement of 5

3) $n \times (1 - p_0) = 500 \times (1 - 0.8) = 100$, which is bigger than the requirement of 5

- d) Draw an appropriate conclusion, in the context of the question using 5% significance level. [2 marks]

Since the p-value is (slightly) less than the significance level of 5%, we have enough statistical evidence to reject the null hypothesis and conclude that the proportion of all ABC Airlines flights that have a on-time departure is significantly bigger than 80%.