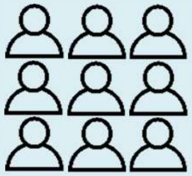
Hypothesis Testing for a Population Proportion

Example - Do you support legalizing marijuana?

In a random sample of 200 Canadians,

59 percent support legalizing marijuana.



59% support

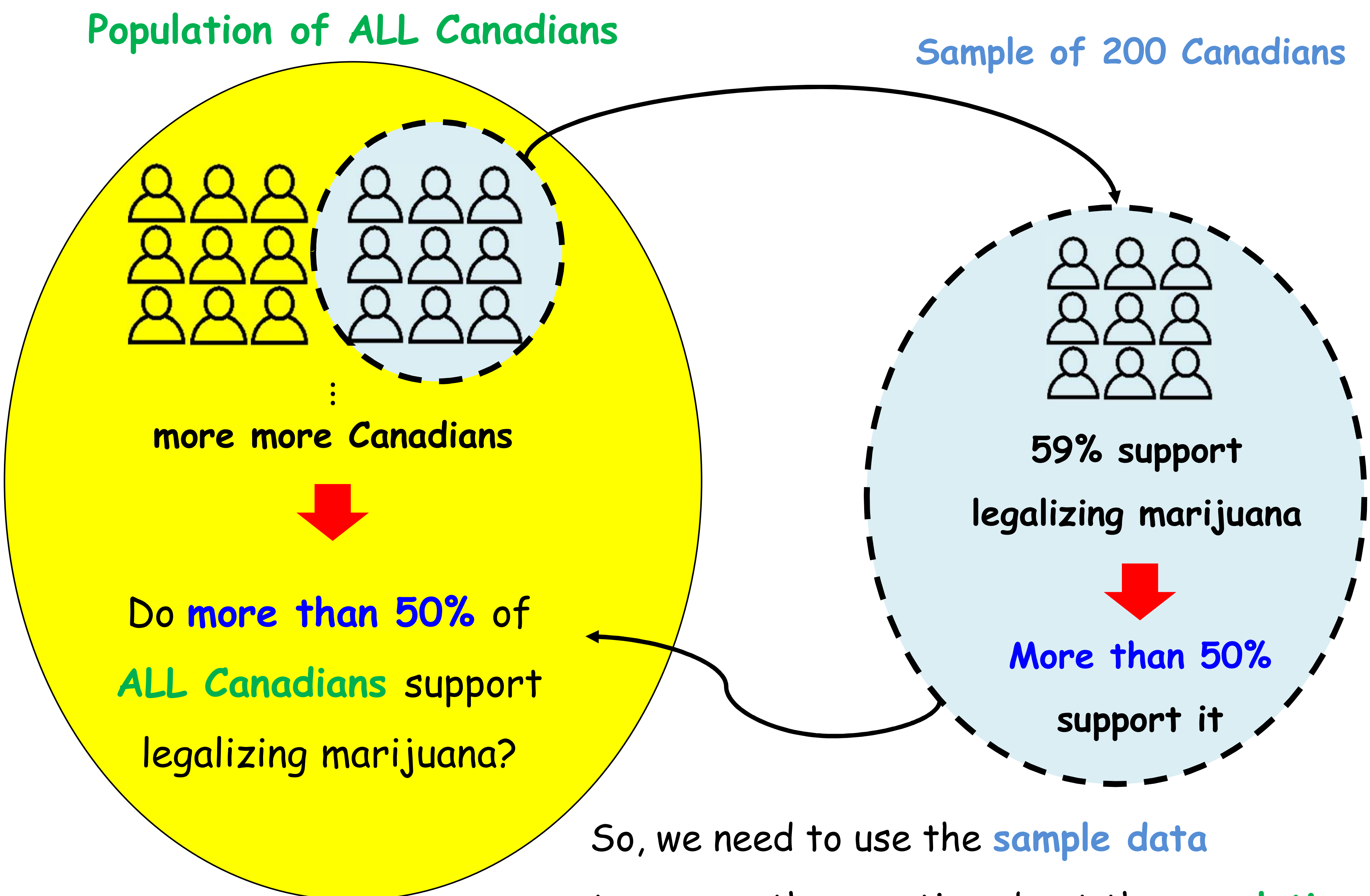
legalizing marijuana

More than 50%

support it

In the sample of 200 Canadians, we have **no doubt** that **more than 50%** support legalizing marijuana.

However, it is not my concern



So, we need to use the sample data to answer the question about the population.

To do that, we can perform a statistical hypothesis test.

Step 1 – State the Hypotheses

- The hypotheses are the statements about a population.
- There are two hypotheses we need to state at the beginning called
  - Null Hypothesis **(denoted by Ho)**
  - Alternative Hypothesis **(denoted by Ha)**

Alternative Hypothesis (Ha):

The **Alternative Hypothesis** is the **research hypothesis** or **claim** we are going to test

When testing a population proportion, the **alternative hypothesis** says that

the population proportion is **less than**, **greater than or not equal** to a specific value.

In this example, we want to test whether

more than 50% of ALL Canadians support legalizing marijuana.

Alternative Hypothesis (Ha):

The Null Hypothesis

- In general, the Null Hypothesis states the opposite of the Alternative Hypothesis
- More importantly, **when we perform the hypothesis test, the null hypothesis is assumed to be true as our starting point**.
- If the null hypothesis is rejected, we will support the **alternative hypothesis**.

It is just like in the criminal court,

- the defendant is assumed to be NOT guilty and
- police need to find/show evidence to support that **the defendant is guilty.**

The Null Hypothesis

- For testing a population proportion, the null hypothesis says that
- the population proportion equals to the hypothesized value.

Null Hypothesis (Ho):

The null and alternative hypotheses are usually written symbolically

Let p represent a **population proportion.** In this example,

p is the **proportion of all Canadians** who support legalizing marijuana.

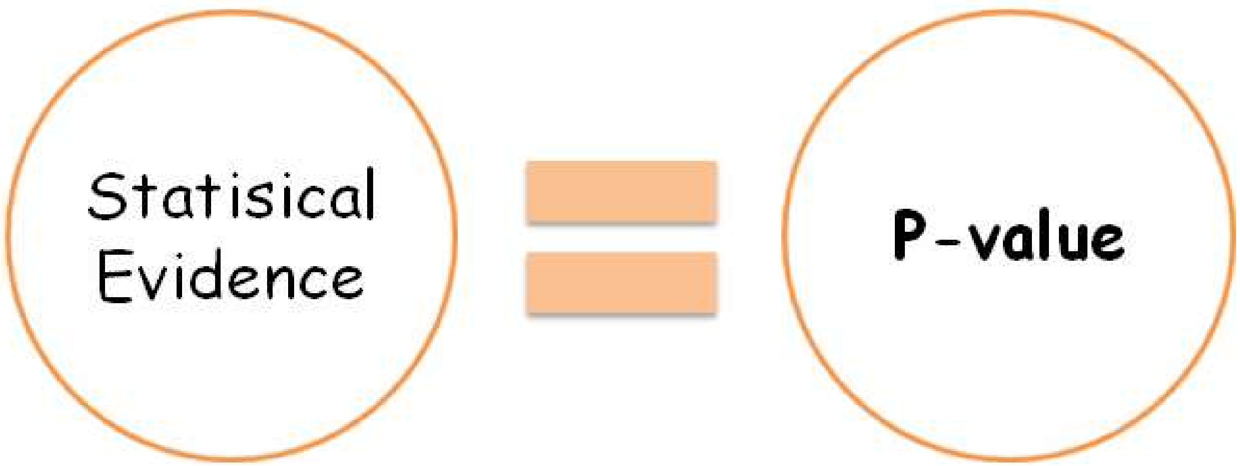
Null Hypothesis (Ho): $\underbrace{p}$

The proportion of all Canadians
who support legalizing marijuana

Alternative Hypothesis(Ha):

Step 2 – Test the null hypothesis using the p-value

- First, we **assume that the null hypothesis is true** that the **proportion of all Canadians** who support marijuana is **0.5 (or 50%)**.
- Based this assumption, we need to determine the amount of evidence **against** the **null hypothesis**.
- The amount of statistical evidence is given the **P-value.**



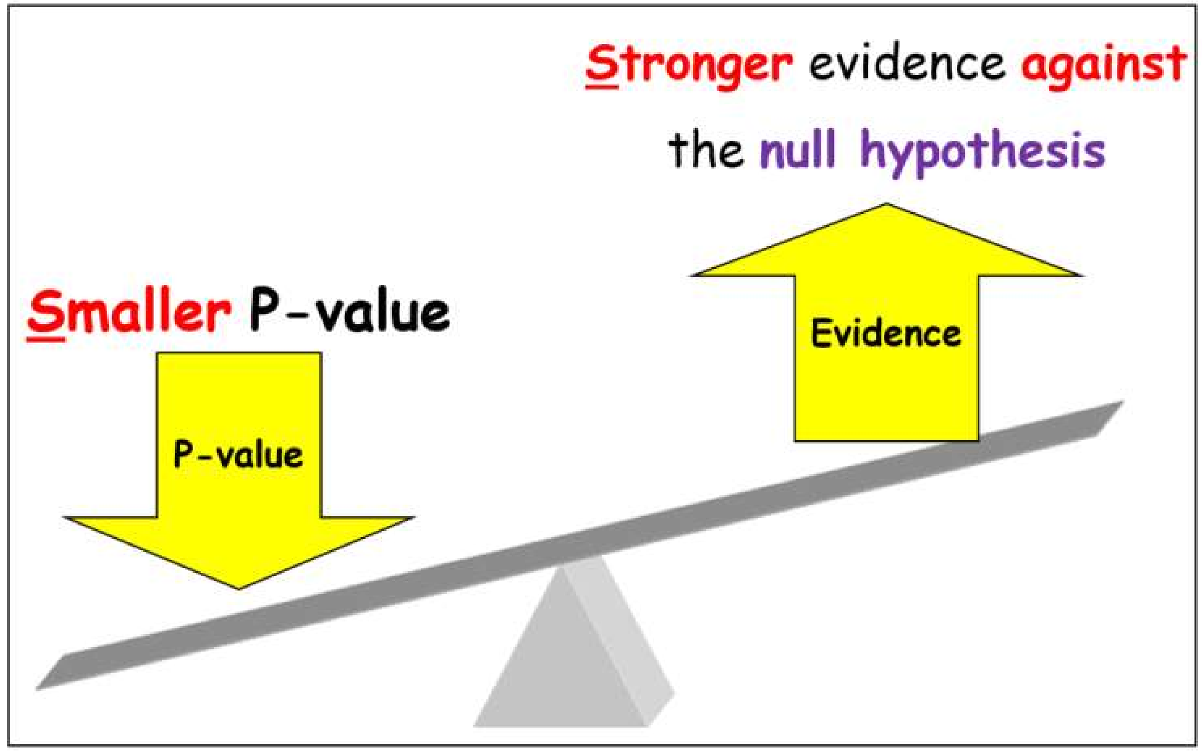
The P-value **(stands for** "Probability-value"**)**

is a number **between 0 and 1** that tells you:

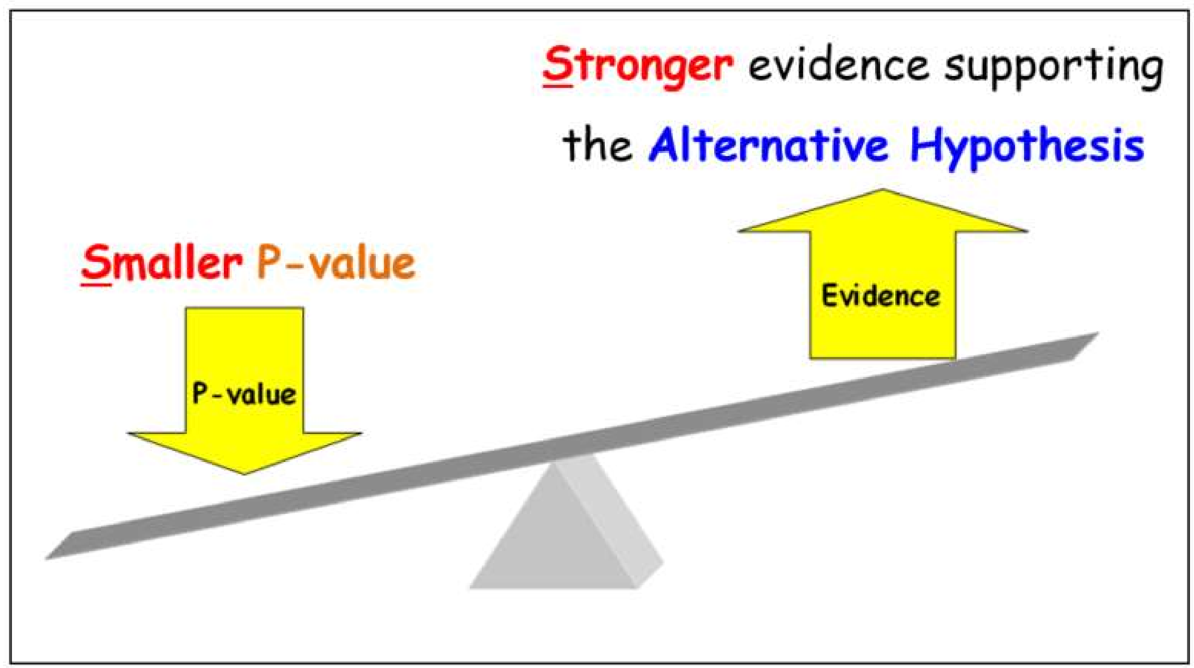
how much evidence the sample data provide **against** null hypothesis.

Please Remember:

A Smaller P-value provides Stronger evidence **against** the null hypothesis.



It is also equivalent to say:



Calculating the Test Statistic (z-statistic)

- To calculate the **p-value**, we need a stepping-stone, **test statistic**
- The **test statistic** is a number calculated from the sample data.
- Then we use the **test statistic** to calculate the **p-value**.

Calculating the Test Statistic (z-statistic)

For testing a population proportion, we use z-statistic.

The z-statistic is defined as

Sample proportion

Population proportion assuming Ho is true

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

Sample Size

The z-statistic measures how far a sample proportion differs from its population proportion by the **number of standard deviation**.

Important Note:
Recall what you learned about the distribution of the sample proportion in the last lecture

If a sample is random selected from a population,

their sample proportion should be somewhat close to its population proportion.

How close?

Typically, the sample proportion is usually within 2 standard deviation of population proportion.

Therefore, if Ho is true,

- the sample proportion should be within 2 standard deviation of the hypothesized proportion
- In other words, the **z-statistic** should be less than or equal 2.

Otherwise if Ha is true,

- the **z-statistic** is likely to be larger than 2.
- The larger z-statistic gives stronger evidence against Ho.

Example - In a random sample of 200 Canadians, 59 percent support legalizing marijuana.

n = Sample Size

=

$\overline{p}$ = Sample Proportion

=

p_0 = Population Proportion if Ho is true

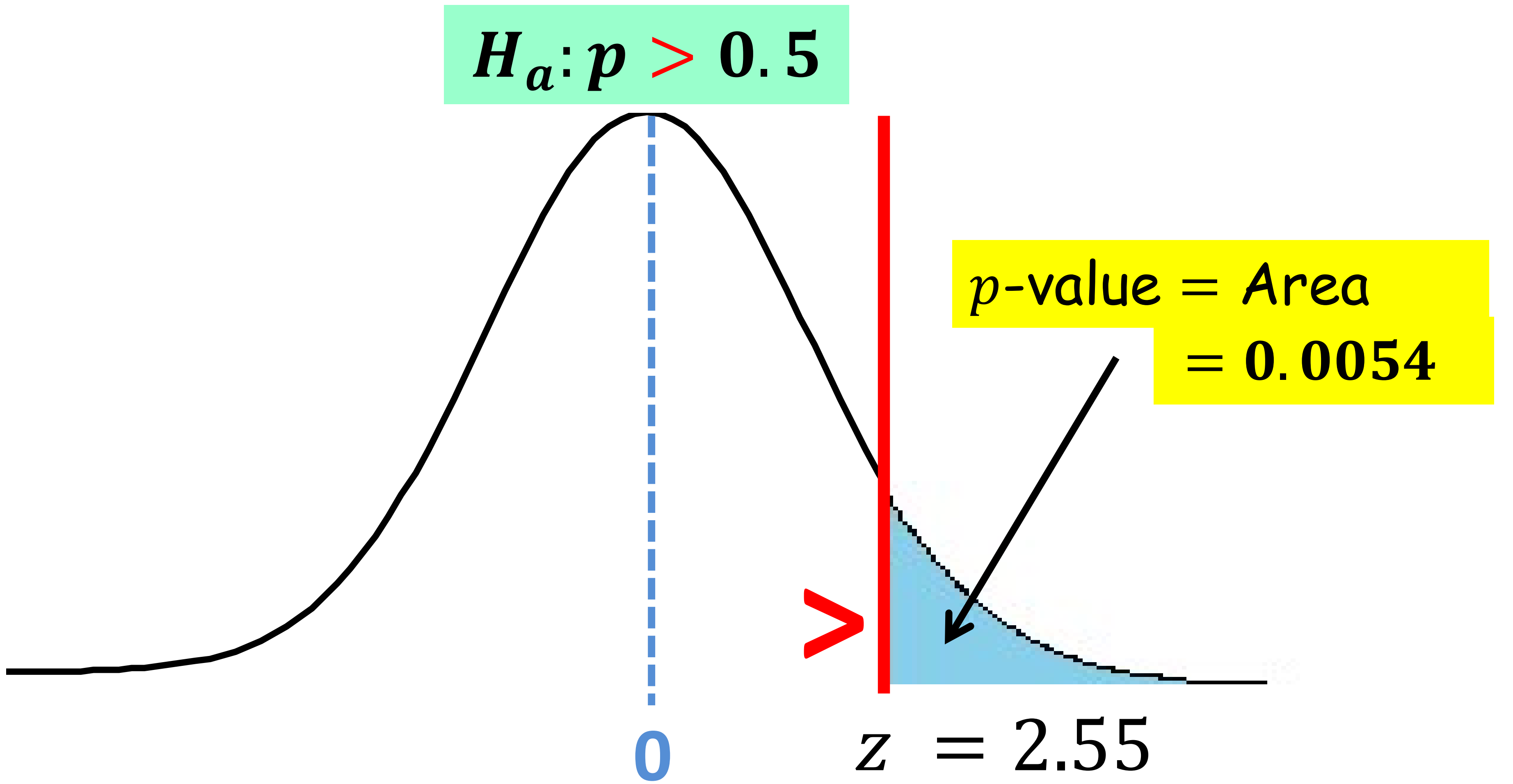
=

$$z = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Interpretation

- The **z-statistic (2.55)** tells you:
- The sample proportion we observed (0.59) is above **the population proportion under Ho (0.5)**
- by 2.55 standard deviation

Calculating the P-value



$$z = 2.55 \quad \rightarrow \quad Area(left) = 0.9946$$

$$Area(right) = 1 - 0.9946 = 0.0054$$

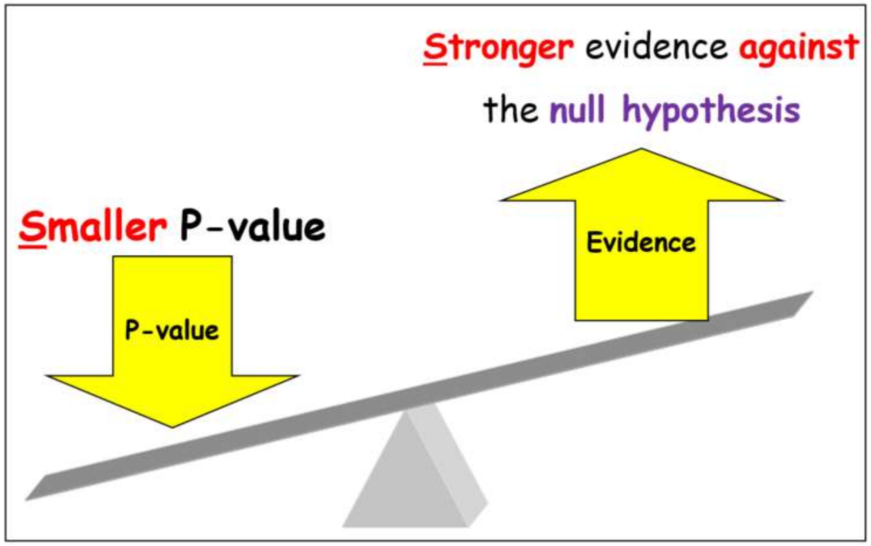
Interpretation of the p-value

- Assume the **null hypothesis is true** that

  the **proportion of all Canadians** who support legalizing marijuana is **0.5**.
- When a random sample of 200 Canadians is selected from the population,
- there is **0.0054 probability** that **the proportion of the sampled Canadians** support legalizing marijuana is above the **hypothesized proportion (0.5)** by **2.55 Standard Deviations** or even **more**.

Step 3 - State the Conclusion

- Recall, a smaller p-value provides strong evidence against the null hypothesis.
- In most cases, we only care whether the sample data provide "sufficient" evidence against the null hypothesis



Question: How small is the p-value required in order to provide "sufficient" **evidence** against the null hypothesis

Answer: We need to a standard, called significance level

Conclusion

- If the p-value is smaller than the significance level we reject the null hypothesis
- At __% significance level, the sample data provide sufficient evidence to support the alternative hypothesis.

Conclusion

- If the p-value is **larger than the significance level** we **DO <u>NOT</u> reject the null hypothesis.**
- At ____% significance level, the sample data **DO <u>NOT</u> provide sufficient** to support the **alternative hypothesis**.

Step 3 – State the Conclusion

Let's use the 5% significance level (0.05) to state the conclusion.

P-value		Significance level
0.0054	$<$	0.05

Since the p-value (0.0054) is **smaller than** the significance level (0.05),

we **reject the null hypothesis (Ho)**.

At the 5% significance level, the sample data **provide sufficient evidence** to conclude that the **proportion of ALL Canadians** who support legalizing marijuana is **higher than 0.5**.

Therefore, **more than 50%** of ALL Canadians support legalizing marijuana.

Assumptions / Conditions Required for valid Hypothesis Test for a Population Proportion

- First, **not all datasets can be used to estimate a population proportion with a confidence interval**.
- The data must satisfy certain conditions; otherwise, any conclusions drawn from the confidence interval will be **invalid**.
- To obtain **valid** conclusions from a hypothesis test for a population proportion, the sample data must meet specific conditions.
- **What are these required conditions?**

2. The sample is **sufficiently large.** When we perform a <u>**hypothesis test**</u>, the **expected number** of individuals in a random sample who

- fall into the category of interest and
- do not fall into the category of interest

are both **at least 5** assuming the **null hypothesis is true.**

In the example, we want to test whether **more than 50%** of **ALL Canadians** support legalizing marijuana.

Here are null and alternative hypotheses.

$$H_0: p = 0.5 \quad vs \quad H_a: p > 0.5$$

Suppose the null hypothesis is true that 50% of Canadians support legalizing marijuana.

In a random sample of 200 Canadians, we expect

- 50% of 200 Canadians support legalizing marijuana.

	Expected Number of Canadians	
Support		
Do not support		

Since both expected numbers are at least 5, the sample is sufficiently large.

Since the large-sample condition is satisfied,
any conclusion drawn from the hypothesis test are valid and can be trusted.