4.5 Data processing

4.5.1 Measuring muon impact

Given the aforementioned setup of the detector plates [^4.1.1], we get a certain time measurement t_i, for each detector i_n (with $n = 1, 2, 3, 4$). t_i can be theoretically expressed as the time when a muon hit the plane, plus the time it took the photon to travel the distance to the detector. We can mathematically express this using the standard formula for velocity (3) as the following:

$$t_i = t_0 + t_{m\to i}$$

$$t_i = t_0 + \frac{s_{m\to i}}{v_\gamma} \tag{1}$$

With t_i being the time measurement from the detector, t_0 the time the muon hit the plane, $s_{m\to i}$ the distance from impact to the detector and v_γ is the speed of the photon. The speed of the photon is not the speed of light, because the light is not travelling in a vacuum, it's travelling through a medium, the scintillator material. This alters the speed at which the photon travels, according to the following equation.

$$v_\gamma = \frac{c}{n} \tag{2}$$

In this scenario, n is the degree at which a photon is slowed down by the material—the refractive index. For our scintillator material it is approximately $n = 1.58$.

We centre our plane, with the middle orientated on the origin of a graph (0,0). In this case the detectors are orientated at

$(\pm L_x/2, \pm L_y/2)$ with L being the length of the plate in it's respective direction (x or y) and in short the coordinate of a detector is $((x_i, y_i)$. The point of impact is (x_μ, y_μ). Abstractly this looks like the following.

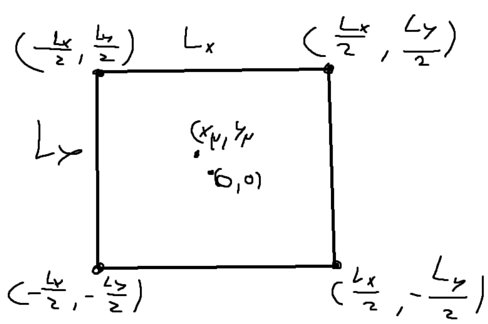

Figure 2*: Schematic view of the coordinate plane.*

In the aforementioned equation 1, the term $s_{m\rightarrow i}$ is not directly available. So we must derive it. This is easily done by the Pythagorean theorem. In our situation, this equation is mathematically defined as the following.

$$s_{m\rightarrow i} = \sqrt{(x_{\mu} - x_{i})^{2} - (y_{\mu} - y_{i})^{2}} \quad (3)$$

If we substitute this into equation 1 we get the following equation.

$$t_{i} = t_{0} + \frac{1}{v_{\gamma}}\sqrt{(x_{\mu} - x_{i})^{2} - (y_{\mu} - y_{i})^{2}} (4)$$

In this equation t_0 remains undefined because we do not know the time at which the muon collided with the plane. This is not a problem because we have 4 of these time equations, $i = 1 ... 4$ and we have 3 unknowns, x_{μ}, y_{μ}, t_0.So we can solve for all 3.

The most obvious and theoretical way to get these variables, is to just solve for x_{μ}, y_{μ}, t_0. But this approach relies on the fact that the 4 equations intersect at a singular point, (x_{μ}, y_{μ}). However, this is not the case due to noise in the readings, due to scattering, energy loss, detector inaccuracy and electronic jitter. This is why we use a method called χ^2 minimisation.

In our case χ^2 minimisation finds the most likely point of impact (POI) according to the time readings it has been given. This works in the following manner. As mentioned previously our time readings will have a probabilistic error following a Gaussian curve. The measurement error (r_i) can be mathematically defined as the following.

$$r_i(x_{\mu}, y_{\mu}, t_0) = t_{measured, i} - t_{predicted, i}(x_{\mu}, y_{\mu}, t_0) \quad (5)$$

By subtracting the predicted time, we get the error. In this equation $t_{predicted, i}(x_{\mu}, y_{\mu}, t_0)$ is the predicted t_i as a function of x_{μ}, y_{μ}, t_0. We defined this prediction as equation 4 earlier. We define these 3 parameters as θ.

Because each error in our measurements, as mentioned earlier, can be approximated as a Gaussian curve, the total error in the measurement will also follow a Gaussian curve. This causes r_i to also follow a Gaussian curve.

When we use this idea for all our detectors we have 4 values for r_i. When we add these multiple Gaussian curves to each other and square the total, we get a χ^2 curve. If we also divide r_i by our aforementioned error due to noise (σ_i) we also account for the noise and inaccuracy caused by scattering, energy loss and electronic jitter. To mathematically define our χ^2 curve we write it as following.

$$\chi^2(\theta) = \sum_i \frac{(r_i(\theta))^2}{\sigma_i^2} \quad (6)$$

This equation calculates for the certain parameters θ the χ^2 curve, we divide by σ_i^2 , because we must account for the inaccuracy in the measurement and the value is squared due to the definition of a χ^2 curve, a sum of Gaussian curves added and then squared.

Now that we defined the χ^2 curve, we must find the θ which describes our system the best. We want to find the best predicted t_i measurement and those are our most optimal coordinates and time measurement. Because we subtract the $t_{predicted, i}$ from $t_{measured, i}$ to obtain r_i in order to then find the best θ we must look for the smallest difference in $t_{measured, i}$ and $t_{predicted, i}$ so the smallest r_i. If we look at equation 6 we see that the smaller r_i gets, the smaller χ^2 becomes. So we must find the smallest value for χ^2. There is a mathematical function for this called arg min. It ''tries'' different possibilities for θ and calculates the corresponding χ^2 and it does this until it finds the smallest value for χ^2 possible. This gives the following definition for the optimal θ measurements.

$$\theta_{best} = arg\ min\ \chi^2(\theta)$$

The reason using χ^2 minimisation is so effective, is because it uses all 4 measurements for the detector and thus reduces the error in the calculation and thus provides the most accurate θ measurements.

So once we do this for both plates we get the following data:

$$(x_\mu, y_\mu, t_{0,})_{Plate\ 1}$$

$$(x_\mu, y_\mu, t_0)_{Plate\ 2}$$

4.5.2 Reconstructing the muon's trajectory

Now that we have the coordinates for our muon. We have $(x_\mu, y_\mu, t_0)_i$, we also know the z coordinate of the muon passing through each plate, because we know the distance with which we distanced the planes. So we define z_1 as 0 and z_2 as the distance between the plates. So mathematically we can define this as.

$$z_1 = 0 \text{ and } z_2 = h$$

This track can be defined by a vector, using multiple variables. Because there is not any real applied magnetic field, the trajectory would not be curved making it quite an easy, linear vector.

This linear vector can be defined by 3 parameters; it's (x, y, z) coordinates as a function of time. This results in a vector like the following.

$$\vec{r} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$$

This vector tells us for each time measurement the corresponding $(x,\ y,\ z)$ coordinate.

We can express $x(t)$, $y(t)$ and $z(t)$ as a starting position of the respective coordinate and the change in this position. For example, we can solve for the changing x using the derivative function of speed in the x direction;

$$v_x = \frac{dx}{dt}$$

$$dx = v_x dt$$

If we now integrate both sides, we get a function of x with variable t. We get $v_x t$ and a constant, the constant equates to the starting position of x thus x_0.

$$x(t) = x_0 + v_x t$$

This equation explains the x position with respect to time. We can use this same principle for the y and z direction so we get the following equations.

$$x(t) = x_0 + v_x t$$

$$y(t) = y_0 + v_y t$$

$$z(t) = z_0 + v_z t$$

In these scenarios, t is the time measurement, which we can approximate using the earlier found values for t_0 for both plates. Our detectors consist of two plates so we can generalize the values as $t_{0,\,i}$, where $i = 1,\ 2$ represent the separate detector plates. This value is the time when the light particle got emitted by the scintillator plate, thus the approximate time when the muon hit the plane.

We get the following vector when we substitute in our equations.

$$\vec{r} = \begin{pmatrix} x_0 + \mathrm{v}_x\,\mathrm{t}_0 \\ y_0 + \mathrm{v}_y\mathrm{t}_0 \\ z_0 + \mathrm{v}_z\mathrm{t}_0 \end{pmatrix}$$

In this situation our unknown parameters are the following.

$$\theta = (x_0,\ y_0, z_0,\ v_x,\ v_y,\ v_z)$$

We can solve for these variables using χ^2 minimisation as well. In our scenario, our noise comes from the inaccuracy in the earlier measurement; this error was due to the Gaussian noise of the aforementioned factors. So we can use the following, basic, equation for χ^2 minimisation.

$$\chi^2 = \sum_i \frac{(measured - predicted)^2}{\sigma^2}$$

With σ being the uncertainty in our measurement. In order to find our residual, we must look at the residual of each axis, x and y. For example, the residual for the x-axis would be the measured value minus the predicted value; the predicted value is the aforementioned equation for x. Mathematically, this looks like the following.

$$r_{x,i} = x_{measured,i} - (x_0 + v_x t_{0,i})$$

We can do the same thing for the z and y-axis.

$$r_{y,i} = y_{measured,i} - (y_0 + v_y t_{0,i})$$

$$r_{z,i} = z_{measured,i} - (z_0 + v_z t_{0,i})$$

The measured z is in this equation attained using the known height difference between the plates.

In these scenarios, i equates to the respective value corresponding to either of the 2 plates, so plate 1 or 2.

We can define noise in these 3 axes as well, so we get σ_x, σ_y and σ_z. We can now plug this into the χ^2 minimisation equation, we get the following.

$$\chi^2 = \sum_i \left(\left(\frac{r_{x,i}}{\sigma_x}\right)^2 + \left(\frac{r_{y,i}}{\sigma_y}\right)^2 + \left(\frac{r_{z,i}}{\sigma_z}\right)^2 \right)$$

Now, in order to get the right measurements for θ, we utilise the $arg\ min$ function again, and we get the following.

$$\theta_{best} = arg\ min\ (\chi^2)$$

After we do this we get the right parameters for our vector and we thus have the correct vector corresponding to the trajectory of the muon.

4.6 Quantifying our detector

In order to correctly utilise the equations mentioned in the earlier chapters to calculate the muon's trajectory, we must know the error, also known as σ. We can obtain this error using Geant4 simulations.

Geant4 is a simulation toolkit written in C++, that simulates a beam of particles, in our case the cosmic ray, and it then simulates a detector measuring the particles. The amount of detected particles can then be used to calculate the error and efficiency of the detector.

In our previous calculations, 2 values for σ are used, the one in the hit reconstruction and the one in the track reconstruction. These 2 values are inherently different, yet they closely resemble each other. The only difference between the two is that in the σ for track reconstruction, the error due to multiple scattering between plates must also be added [^]. We can express this as the following.

$$\sigma_{track} = \sigma_{hit} + \sigma_{ms}$$

The inaccuracy due to multiple scattering in our case is so negligible in comparison to the resolution of our detector, so we can approximate the σ_{track} as σ_{hit}.

So in order to find our accuracy for the detector we must use Geant4 to measure the ‘’shot’’ particles against the detected particles of our detector.

4.6.1 Running the simulation

In order to properly simulate the cosmic ray hitting our detector and doing measurements on this, we must define a set of variables.

These variables fall under 3 “user initialization classes”: *G4VUserDetectorConstruction, G4VUserPhysicsList and G4VUserPrimaryGeneratorAction*. In these classes we define the variables associated with their respective class.

For the detector construction, we define the world in which the reactions take place and what the geometry and materials are of our detector.

For the physicslist we define the physical processes which are simulated in our detector, things like Bremsstrahlung and multiple scattering.

For the primarygeneration, or ‘’gun’’, we define what particles are generated and ‘’shot’’ at the detector. For us this is a cosmic ray.

4.6.2 Simulating the cosmic ray

In order to correctly simulate the cosmic ray, the *G4VUserPrimaryGeneratorAction* , we must define the cosmic rays properties. The properties we must define are the following: energy spectrum, angular distribution, charge ratio, spatial distribution and time structure.

We look at muons in our scenario, thus we only simulate the muons.

For the energy spectrum, a cosmic ray muon at sea level generally follows a spectrum like the following.

$$\frac{dN}{dE} \alpha\, E^{-2.7}$$

This tells us that the amount of muons per energy “slice’’ are proportional to the energy to the power of -2.7.

For the angular distribution of the cosmic ray muons. We use a formula for the intensity as a function of the zenith angle, θ. This function looks like the following.

$$I(\theta)\ \alpha\ cos^2\theta$$

This means that not all muons are shot straight from the z-axis onto the plane, but rather come from a lot of different angles.

The flux of our cosmic ray follows a regular amount of 1 $muon\ cm^{-3}\ minute^{-1}$.

We must also define the charge ratio between the 2 types of muons, the positive and negative ones. These obey a charge ratio of around 1.2-1.3, so this looks like the following.

$$\frac{\mu^+}{\mu^-} \approx 1.2 - 1.3$$

The muons also obey a spatial distribution, meaning they do not all get produced at the same x, y, z coordinates. To solve this problem we simulate the muons coming down from a plane above the detector, this plane is larger than the detector to simulate a realistic scenario.

4.6.3 Simulating the detector

We must also define the parameters of the detector itself, the *G4VUserDetectorConstruction.*

We place the detector in the middle of the created world volume, causing the simulation to run more symmetrically.

4.6.3.1 Simulating the electronics

Our SiPM detectors have an intrinsic noise due to electronic jitter and other quantum processes. This noise is incredibly hard to define in a simulation, so we use a process called smearing. Smearing ‘’adds’’ a gaussian probability to our measurements after the initial simulation has been done, this acts and uses the same values as the regular noise would, except it is way easier to define. There are three main variables contributing to the smearing effect, among these are PDE, gain fluctuation and timing resolution.

As aforementioned, see figure 17, the PDE of our SiPM detector is 41% and thus we use this value as the PDE value in the class for the smearing inaccuracy.

The gain in a detector can be primarily recognised as being proportional to the difference between the bias voltage and the breakdown voltage, thus it is proportional to the residual, the overvoltage. The primary contributor to a change in this gain, the gain fluctuation, is temperature. This changes the 2 voltage values causing an uncertainty in the overvoltage measurement and thus in the measured energy value.

In our SiPM the gain fluctuation due to the temperature variability is $-\ 0.8\%\ /°C$. This means that a 1 degree change in temperature from baseline, $21°C$, results in a 0.8% change in voltage output. So a change to 22 degrees would result in a voltage measurement 0.8% lower than at 21 degrees for the same input energy.

The timing resolution of our system is defined by the timing resolution of the SiPM and the timing resolution of other electronics. The rise time of our SiPM is 300ps so this also becomes the timing resolution of our SiPM, because during the rise time, it is uncertain of the exact time thus creating the timing resolution. The electronics used in amplifying and measuring the signal also add a certain time inaccuracy. The average uncertainty in time for the other electronics is 150ps. Both of these resolutions follow a Gaussian curve when measured. So in order to add the two we must square everything so we get the following timing resolution.

$$\sigma_{time,\ electronics} = \sqrt{300^2 + 150^2} = 335.4ps$$

We must also define the trigger threshold, which in our SiPM is around 10 PE, this means that it reads an impact when it measures 10 photoelectron hits. This is a regular value for cosmic ray detectors.

4.6.3.2 Simulating the scintillator planes

For the planes we use a scintillator material called polyvinyltoluene. For the simulation, we use the regular values mentioned by the facturer.

The density of the material is $1.023g/cm^3$ and it has a refractive index of 1.58. The specific scintillator we use is the BC-408. This material has a H:C atom ratio of 1.104.

This scintillator material has a light output of 64% anthracene, this is approximately 10.000 photons per MeV. The rise time of the scintillator is $0.9ns$ and the decay time is $2.1ns$. The peak emission wavelength of the scintillator material is $425nm$. This value is only the peak, and for a simulation an emission spectrum is necessary, in order to make this a spectrum we approximate it as a gaussian distribution with a peak at $425nm$. The width of the pulse is 2.5ns The light attenuation length is $210cm$.

4.6.3.3 Simulating detector geometry

In order to run the simulation we define the geometry of the world in which we do the experiment as well. For this we must define the size of the world and what substance this world is made up of. Because we're simulating a cosmic ray in a non-vacuum scenario, we define the substance of the world as regular air so primarily nitrogen (78%) and oxygen (21%). Our detector must fit in the world in which we place it, we must also make sure the simulation happens according to a real life scenario. This is why we use quite a large simulation world of $500cm\ x\ 500cm\ x\ 500cm$ in the respective $x,\ y,\ z$ dimensions.

In this world we position our detector in the center of the 3 dimensional system. So the center of mass of the detector is placed at $x = 0,\ y = 0,\ z = 0$. The scintillator planes are each 63cm by 63 cm and are 10 mm thick. The planes are each separated by 63cm. This means that the center of the first plane is located at $(0,\ 0,\ 0\ )$, for the second detector this is $(0,\ 0,\ 63)$ in their respective directions.

Our SiPM's are set up at each corner of the planes to measure the photons. So in total 8 SiPM's. These SiPM's are the parts actually doing the measurement. These SiPM's each have a sensor size of $6mm$ and a microcell size of 35μ. In our simulation, these SiPM's are optically coupled with EJ-550, which have a refractive index of 1.48.

4.6.4 Defining the physical processes

The physical processes must also be added to the simulation. These processes will then be used in the calculations and measurements in the geant4 application.

In our simulation, multiple scattering, Bremsstrahlung and energy loss are most important. And thus these are used. Also in order to run the simulation we must also import photon processes, because we're working with a scintillator based detector.