Problem 1.

(i) From eq.(1):

$$L_{\pi_{\theta_1}}(\pi_{\theta_1}) = J(\pi_{\theta_1}) + \underbrace{\sum_s d_M^{\pi_\theta}(s)\left(\sum_a \pi_{\theta_1}(a|s) A^{\pi_{\theta_1}}(s,a)\right)}_{0}$$

$$\therefore L_{\pi_{\theta_1}}(\pi_{\theta_1}) = J(\pi_{\theta_1})$$ ... 得證

(ii)

① $$\nabla_\theta L_{\pi_{\theta_1}}(\theta) = \underbrace{\cancel{\nabla_\theta J(\pi_{\theta_1})}}_{0} + \sum_s d_M^{\pi_{\theta_1}}(s) \cdot \sum_a \left(\nabla_\theta \pi_\theta(a|s)\right) A^{\pi_{\theta_1}}(s,a)$$

$$\nabla_\theta L_{\pi_{\theta_1}}(\theta) = \sum_s d_M^{\pi_{\theta_1}}(s) \cdot \sum_a \left(\nabla_\theta \pi_\theta(a|s)\right) A^{\pi_{\theta_1}}(s,a)$$

② Original: $$J(\pi_\theta) = J(\pi_{\theta_1}) + \sum_s d_M^{\pi_\theta}(s) \sum_a \pi_\theta(a|s) \cdot A^{\pi_{\theta_1}}(s,a)$$

$$\therefore \nabla_\theta J(\pi_\theta) = \underbrace{\cancel{\nabla_\theta J(\pi_{\theta_1})}}_{0} + \left[\sum_s d_M^{\pi_{\theta_1}}(s) \sum_a \left(\nabla_\theta \pi_\theta(a|s)\right) A^{\pi_{\theta_1}}(s,a)\right]$$

$$\Rightarrow \nabla_\theta J(\pi_\theta)\Big|_{\theta=\theta_1} = \sum_s d_M^{\pi_{\theta_1}}(s) \left(\sum_a \nabla_\theta \pi_\theta(a|s)\Big|_{\theta=\theta_1}\right) A^{\pi_{\theta_1}}(s,a)$$

③ From ② to ①

$$\therefore \nabla_\theta L_{\pi_{\theta_1}}(\pi_\theta)\Big|_{\theta=\theta_1} = \nabla_\theta J(\pi_\theta)\Big|_{\theta=\theta_1}$$ # 得證

Problem2

(a) (i) Since $D(\lambda) = \min_{\theta \in \mathbb{R}^d} \mathcal{L}(\theta, \lambda)$, $\Rightarrow$ θ^* from $\nabla_\theta \mathcal{L}(\theta,\lambda) = 0$

$$\Rightarrow 0 = -\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right) + \lambda \cdot H(\theta - \theta_k)$$

$$\Rightarrow \theta^* - \theta_k = \frac{1}{\lambda} H^{-1}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right) \quad — ①$$

代入到 eq.(*) : $\mathcal{L}(\theta,\lambda)$:

$$\Rightarrow D(\lambda) = -\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)^T \cdot \frac{H^{-1}}{\lambda}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)$$

$$\underbrace{+\frac{\cancel{\lambda}}{2}\left[\frac{H^{-1}}{\cancel{\lambda}}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right]^T \cancel{H}\left[\frac{\cancel{H}^{-1}}{\lambda}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right]}_{= \frac{1}{2\lambda}\left(\nabla_\theta L_{\theta_k}(\theta)\big|_{\theta=\theta_k}\right)^T \underbrace{(H^{-1})^T}_{= H^{-1}}\left(\nabla_\theta L_{\theta_k}(\theta)\big|_{\theta=\theta_k}\right)} - \lambda \cdot \delta$$

$$\therefore D(\lambda) = \frac{-1}{2\lambda}\left[\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)^T \times H^{-1} \times \left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right] - \lambda\delta.$$

(ii) Find λ^* By Solving $\frac{dD(\lambda)}{d\lambda} = 0$

可得 $$\frac{\delta D(\lambda)}{d\lambda} = \frac{1}{2\lambda^2}\left[\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)^T \times H^{-1} \times \left(\nabla_{\theta_k} L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right] - \delta = 0$$

$$\Rightarrow \lambda^* = \sqrt{\frac{1}{2\delta}\left[\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)^T \times H^{-1} \times \left(\nabla_{\theta_k} L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right]}$$

(b) Since (a)–① $\Rightarrow$ $$\theta^* = \theta_k + \frac{1}{\lambda^*} H^{-1}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)$$

$$= \theta_k + \alpha \cdot H^{-1}\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)$$

$$\therefore \alpha = \frac{1}{\lambda^*} = \sqrt{2\delta} \cdot \left[\left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)^T \cdot H^{-1} \cdot \left(\nabla_\theta L_{\theta_k}(\theta)\Big|_{\theta=\theta_k}\right)\right]^{-\frac{1}{2}}$$