On vAMM's unnecessity for liquidity pool

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1 Why vAMM does not need a liquidity pool?

Perpetual Protocol's vAMM uses the same $x * y = k$ constant product formula as Uniswap. One of the very difference between vAMM and AMM is that vAMM does not require a physical liquidity pool. Although the price is still discovered by the AMM curve, traders indeed store their collateral in a smart contract vault. Such mechanism arouses concerns about whether the vault is capable to meet the solvency demands at any time when traders are allowed to long and short in the protocol. Can vAMM still keep its path independency? This literature aims to simplify the solvency problem into several circumstances and prove that vAMM is capable of doing so.

2 Solvency Model

This literature simplifies the market model with only two components, A and B. Both A and B would either long or short the underlying asset. The chronological order of entering or clearing their position always follows that A goes first and B goes second.

In the real world, the solvency of vAMM can be regarded as a multiplication of trading pairs. So if this model can be proved valid in a dual-system, then vAMM can ensure solvency in the real market. The solvency procedure is classified into several catagories. This literature will then prove the solvency of vAMM under each circumstance.

3 Notation

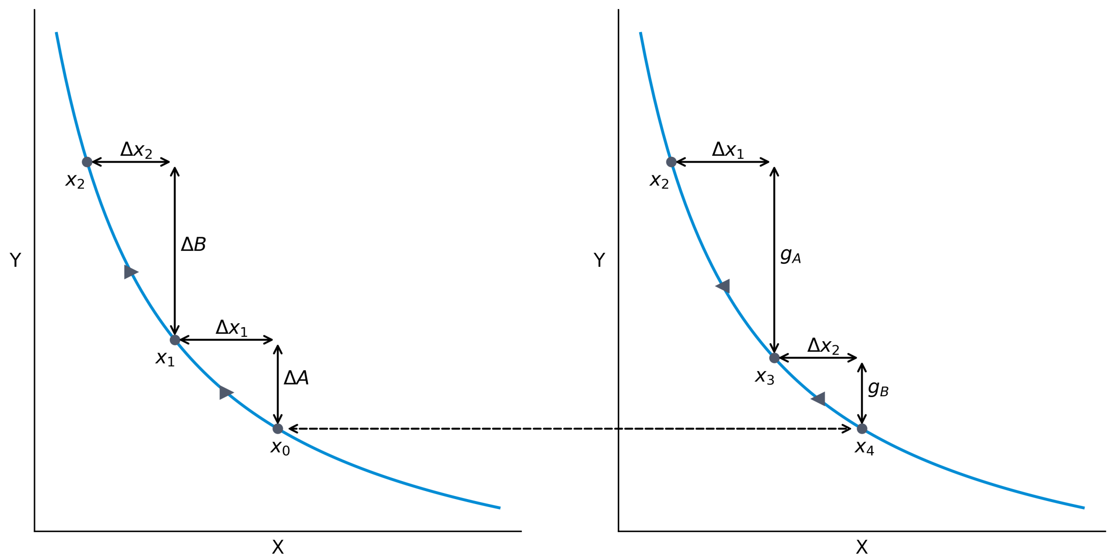
Before deriving model, it is essential to clarify the notation used in this literature:

- **x:** the quote currency, usually referred as an asset, e.g ETH
- **y:** the quote currency, e.g USDC
- ΔA**:** the notional amount of investor A
- ΔB**:** the notional amount of investor B
- x_i**:** the ith step of x during the solvency
- y_i**:** the ith step of y during the solvency
- Δx_i**:** the absolute change of x after the ith step is launched
- Δy_i**:** the absolute change of y after the ith step is launched
- g_A**:** A's capital gain after clearing his position

- g_B: B's capital gain after clearing his position

4 Derivation

4.1 A long, B long ($\Delta A < \Delta B$)



Procedure:

- *The creator* can set an initial amount of x and y on vAMM, respectively with volume x_0 and y_0.
- *Investor A* wants to long the underlying asset x with a certain ratio of leverage. The protocol thus credits ΔA from *Investor A* to the vAMM. In the meantime, we can calculate the amount of x that *Investor A* receives from the protocol. In this step, x_0 moves to x_1.
- *Investor B* also wants to long the underlying asset x. The protocol thus credits ΔB from *Investor B* to the vAMM. In the meantime, the amount of x that *Investor B* receives could be discovered. In this step, x_1 moves to x_2.
- *Investor A* decides to clear his position and receives g_A from the protocol. So x_2 moves to x_3.
- Finally, *Investor B* clears his position and receives g_B from the protocol. x_3 moves to x_4 and the trading ends.

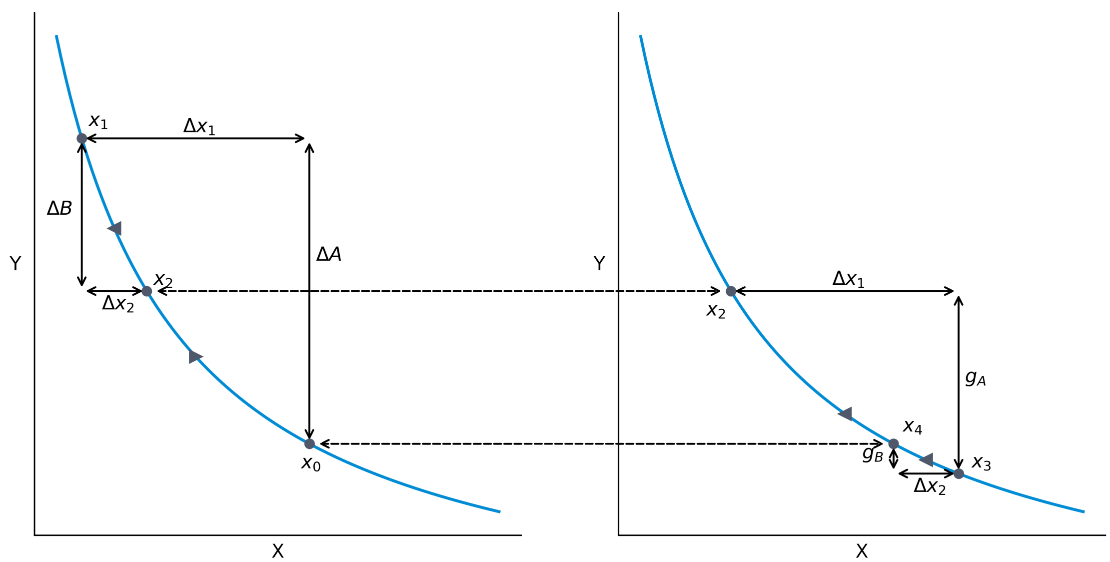
The equation system goes as follows:

$$(x_0 - \Delta x_1)(y_0 + \Delta A) = k \tag{1}$$
$$(x_1 - \Delta x_2)(y_1 + \Delta B) = k \tag{2}$$
$$(x_2 + \Delta x_1)(y_2 - g_A) = k \tag{3}$$
$$(x_3 + \Delta x_2)(y_3 - g_B) = k \tag{4}$$

Conclusion:

- From the plot, it is easy to find $g_A + g_B = \Delta A + \Delta B$. Thus, $g_A - \Delta A = \Delta B - g_B$. A's gain covers B's loss. So the vault always has enough collateral to pay back every trader under this circumstance.
- And so forth it's easy to deduce that one trader's gain will cancel out another trader's loss when A short and B short, no matter the size of their notional.

4.2 A long, B short ($\Delta A > \Delta B$)



Procedure:

- *The creator* can set an initial amount of x and y on vAMM, respectively with volume x_0 and y_0.
- *Investor A* wants to long the underlying asset x with a certain ratio of leverage. The protocol thus credits ΔA from *Investor A* to the vAMM. In the meantime, we can calculate the amount of x that *Investor A* receives from the protocol. In this step, x_0 moves to x_1.
- *Investor B* wants to short the underlying asset x. The protocol thus credits $-\Delta B$ from *Investor B* to the vAMM. In the meantime, the amount of x that *Investor B* shorts could be discovered. In this step, x_1 moves to x_2.
- *Investor A* decides to clear his position and receives g_A from the protocol. So x_2 moves to x_3.
- Finally, *Investor B* clears his position and redelivers Δx_2 unit of x to the protocol. x_3 moves to x_4 and the trading ends.

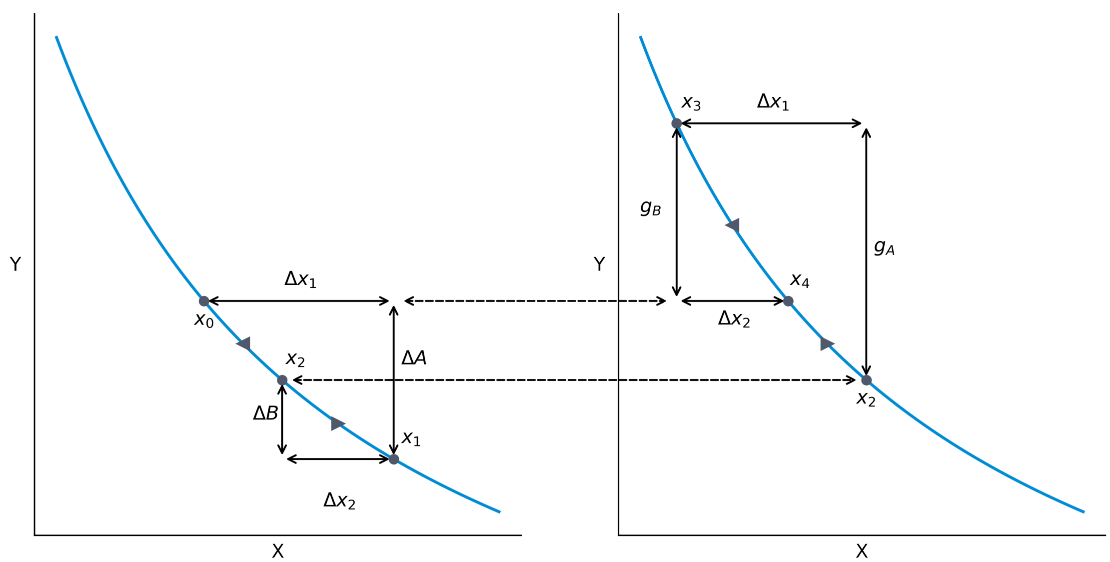
The equation system goes as follows:

$$(x_0 - \Delta x_1)(y_0 + \Delta A) = k \tag{5}$$
$$(x_1 + \Delta x_2)(y_1 - \Delta B) = k \tag{6}$$
$$(x_2 + \Delta x_1)(y_2 - g_A) = k \tag{7}$$
$$(x_3 - \Delta x_2)(y_3 + g_B) = k \tag{8}$$

Conclusion:

- From the plot, it is easy to find $g_A - \Delta A = \Delta B - g_B$. A's loss is covered by B's profit. So the vault always has enough collateral to pay back every trader under this circumstance.
- And so forth it's easy to deduce that one trader's gain will cancel out another trader's loss when $\Delta A < \Delta B$.

4.3 A short, B long($\Delta A > \Delta B$)



Procedure:

- *The creator* can set an initial amount of x and y on vAMM, respectively with volume x_0 and y_0.
- *Investor A* wants to short the underlying asset x with a certain ratio of leverage. The protocol thus credits $-\Delta A$ from *Investor A* to the vAMM. In the meantime, we can calculate the amount of x that *Investor A* shorts from the protocol. In this step, x_0 moves to x_1.
- *Investor B* wants to short the underlying asset x. The protocol thus credits ΔB from *Investor B* to the vAMM. In the meantime, the amount of x that *Investor B* receives could be discovered. In this step, x_1 moves to x_2.
- *Investor A* decides to clear his position and redelivers Δx_1 unit of x to the protocol. So x_2 moves to x_3.

- Finally, *Investor B* clears his position and receives g_B from the protocol. x_3 moves to x_4 and the trading ends.

The equation system goes as follows:

$$(x_0 + \Delta x_1)(y_0 - \Delta A) = k \tag{9}$$
$$(x_1 - \Delta x_2)(y_1 + \Delta B) = k \tag{10}$$
$$(x_2 - \Delta x_1)(y_2 + g_A) = k \tag{11}$$
$$(x_3 + \Delta x_2)(y_3 - g_B) = k \tag{12}$$

Conclusion:

- From the plot, it is easy to find $g_A - \Delta A = \Delta B - g_B$. A's loss is covered by B's profit. So the vault always has enough collateral to pay back every trader under this circumstance.
- And so forth it's easy to deduce that one trader's gain will cancel out another trader's loss when $\Delta A < \Delta B$.

5 Benefits & Risks of LP

Project	Category	Yield Farming Mechanism	Liquidity Providers	Token Utility
Compound	Deposits & Borrow	Use the Protocol	COMP, Interests	Governance
Aave	Deposits & Borrow	Use the Protocol	AAVE, Interests, Flash Loan Fees	Staking, Governance
UniSwap V2	Spot Trading	Add Liquidity	Trading Fees, UNI	Governance
SushiSwap	Spot Trading	Add Liquidity	Swap Fees, KMP & SLP	Staking, Earn SUSHI & XSUSHI
PancakeSwap	Spot Trading	Add Liquidity	Trading Fees, Cake	Staking, Governance
Bancor	Spot Trading	Add Liquidity	Swap Fees, BNT	Staking
Balancer	Spot Trading	Add Liquidity	Trading Fees, BAL	Governance
Curve	Spot Trading	Add Liquidity	Trading Fees, CRV	Governance
MCDEX	Margin Trading	Add Liquidity	Trading, Funding and Liquidation Fees, MCB	Staking, Governance

5.1 Benefits

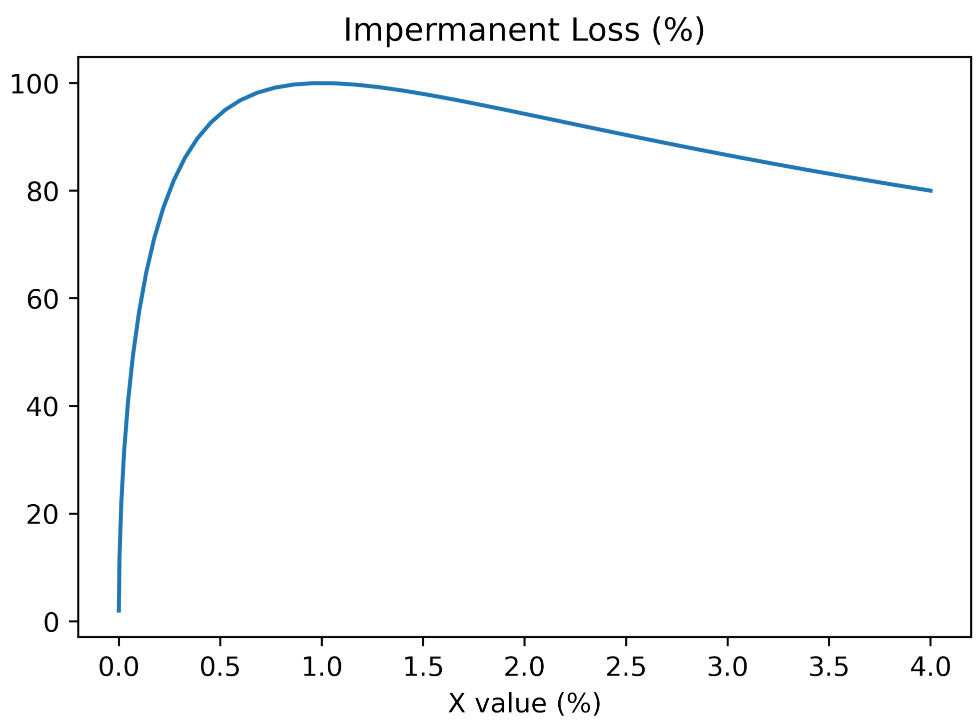
- **Provide Initial Liquidity**
  The DeFi protocol needs to initialize the liquidity pool in order to provide services with a larger amount of funds (thus it can attract more demanders), so the liquidity pool just plays this role, and distributes the potential benefits to the early-stage liquidity provider.

- **Attract Users**
  For the DeFi protocol itself, liquidity pool can attract more users to participate in the use of the protocol to earn yields. Specifically, a soaring TVL could boost engagement in social media and accelerate the convergence of consensus , which attracts more users to the protocol in return. Besides, the yield farming also attract an amount of long-term crpytocurrency investors since the liquidity pool could offer substantial profits during their holding period. In a word, liquidity pool is a recycling system for the protocol's promotion and recycling.

- **Promote Decentralized Governance** The protocol can allocate its governance authority to its long-term investors and solid believers, rather than speculators or the development team, in order to real a true decentralized autonomous organization.

5.2 Risks

- **Impermanent Loss**
  Impermanent loss is an inevitable topic when coming to the liquidity pool of AMM. It refers to the difference between holding the tokens directly and injecting the tokens into an AMM liquidity pool. As long as the price changes with respect to your deposit price, an impermanent loss will be incurred.



- **Market Risk** If you use a Deposits & Borrow platform, like Compound or Aave, when your collateral is no longer sufficient to cover your loan amount due to the volatility of the borrowed assets or collateral, the automatic liquidation of the collateral will be triggered. You will be also imposed on a liquidation fee.
- **Smart Contract Risk** The smart contract code is immutable and operates exactly as specified. However, for this reason, if a smart contract has man-made or non-man-made loopholes, it can be exploited without recourse. The contract cannot be guaranteed to be 100% safe even if the contract code is audited. Such risks must be taken into account when considering investments on yield farming.