Computer Vision

Baba Vemuri

CAP5416: Lecture-1*

*Thanks to Prof. Entezari for the slides in this lecture.

Linear Transforms

Reading Assignment

- Image formation tutorial by R. Hartely and A. Zisserman

Today

- Linear transformations
  - Matrix representation of linear transforms

Linear transformation

- Any transformation with the property:

$$T(a\boldsymbol{u} + b\boldsymbol{v}) = aT(\boldsymbol{u}) + bT(\boldsymbol{v})$$

- Can be represented with a matrix -- linear algebra

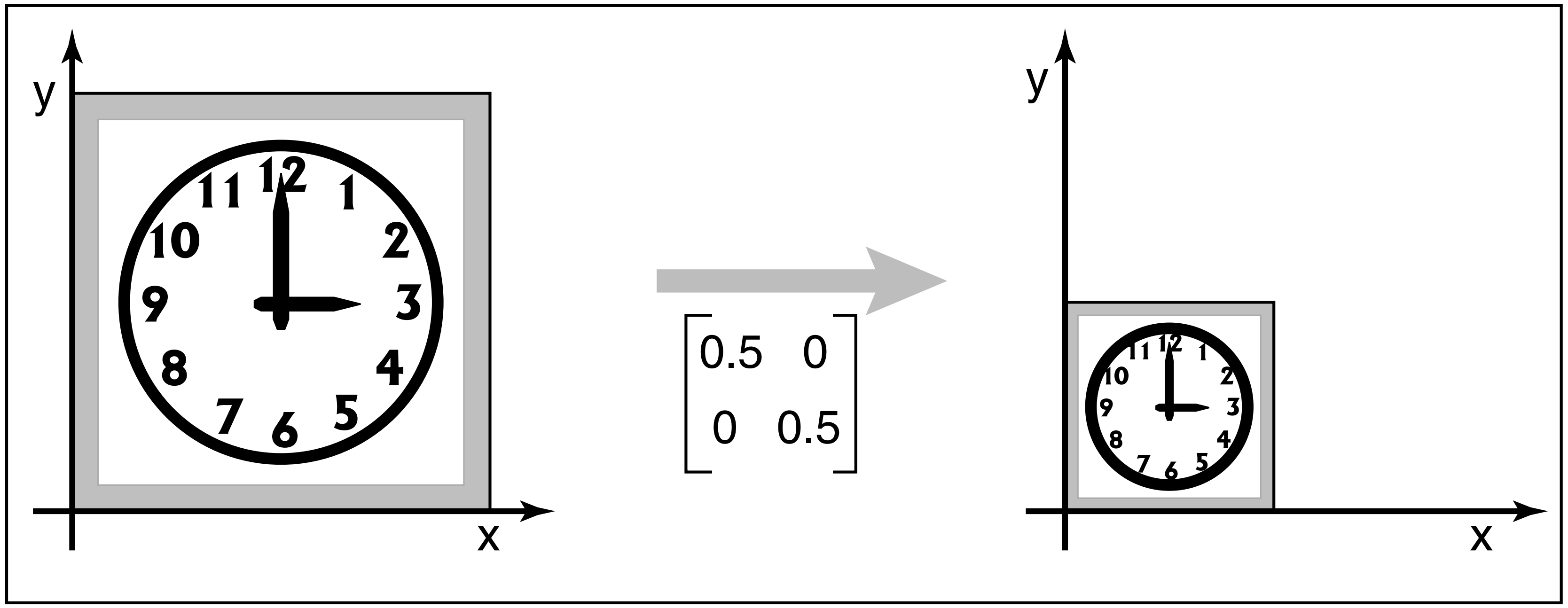
$$T(\boldsymbol{v}) = \boldsymbol{M}\boldsymbol{v}$$

2D linear transforms

- uniform scaling
- non-uniform scaling
- rotation
- shear
- reflection

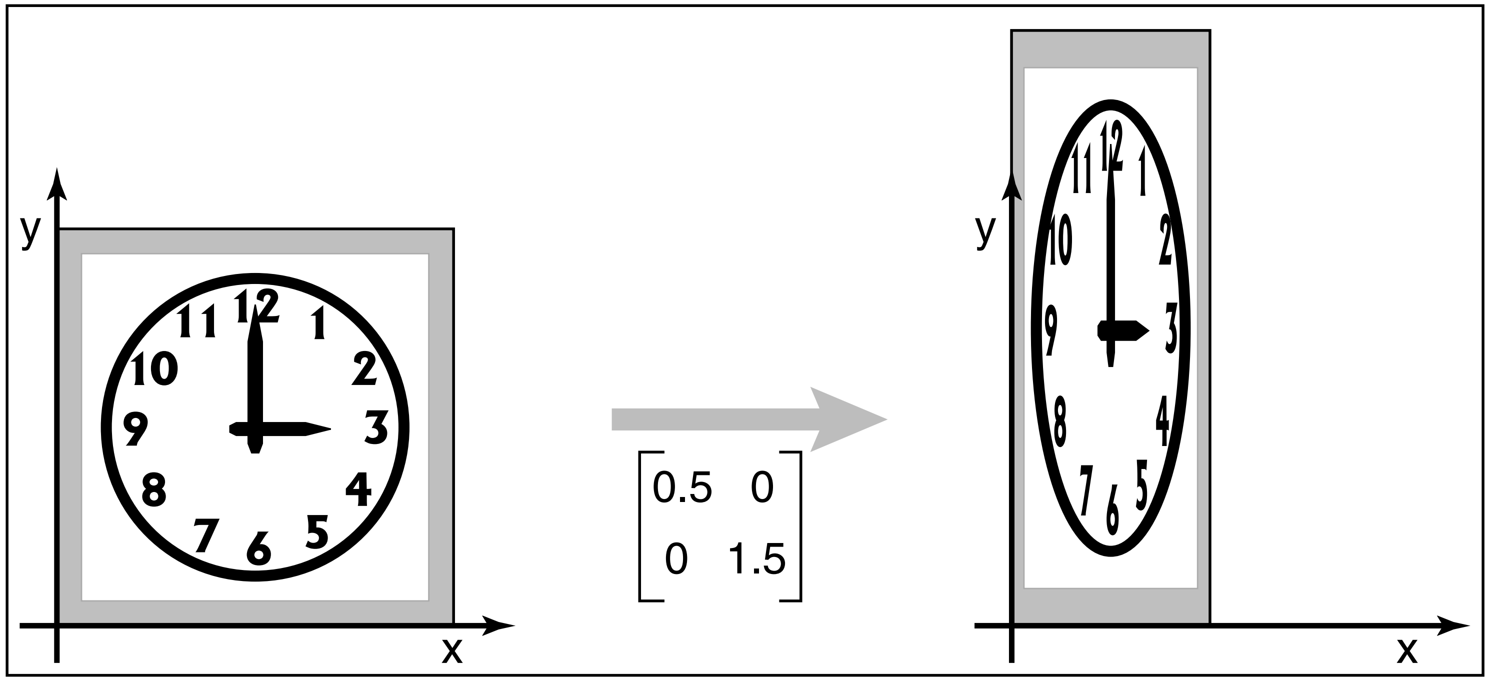
Uniform scaling

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$$



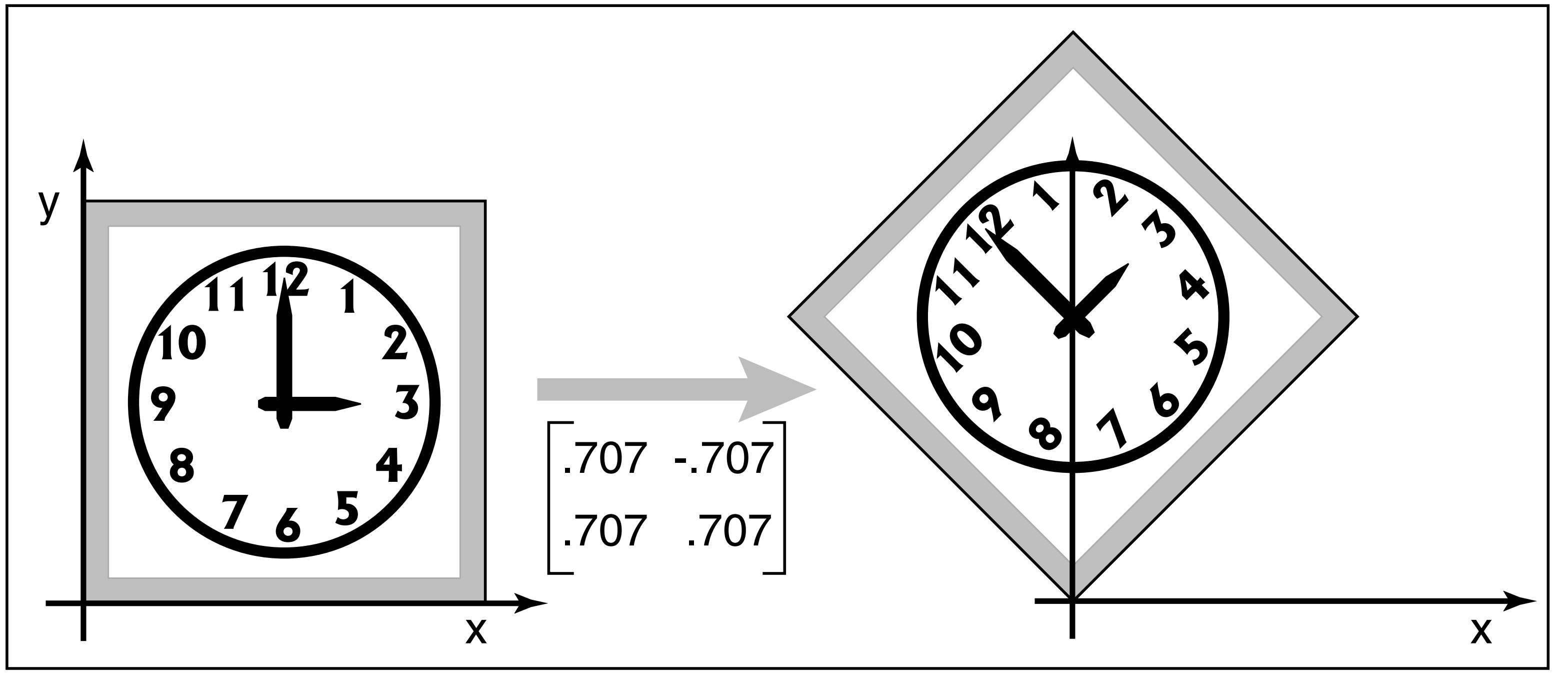
Non-uniform scaling

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$



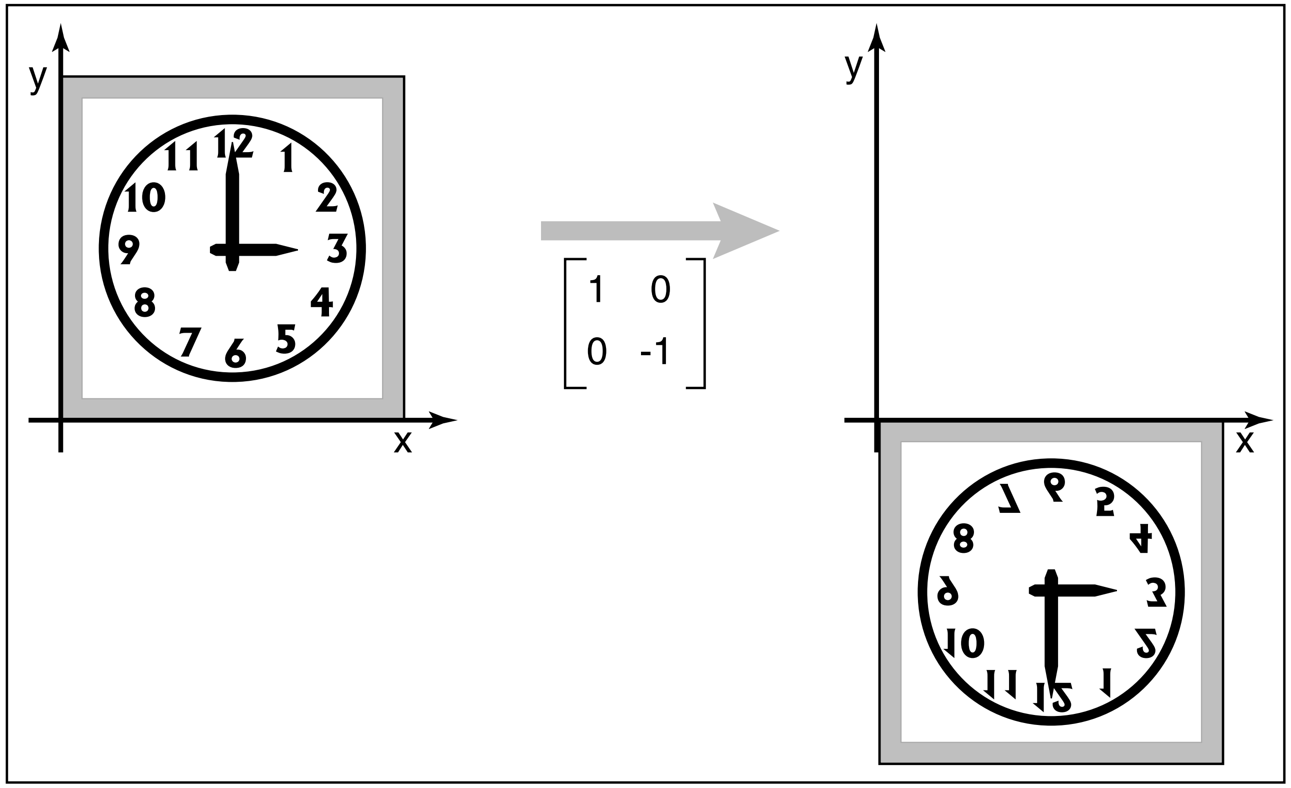
Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$



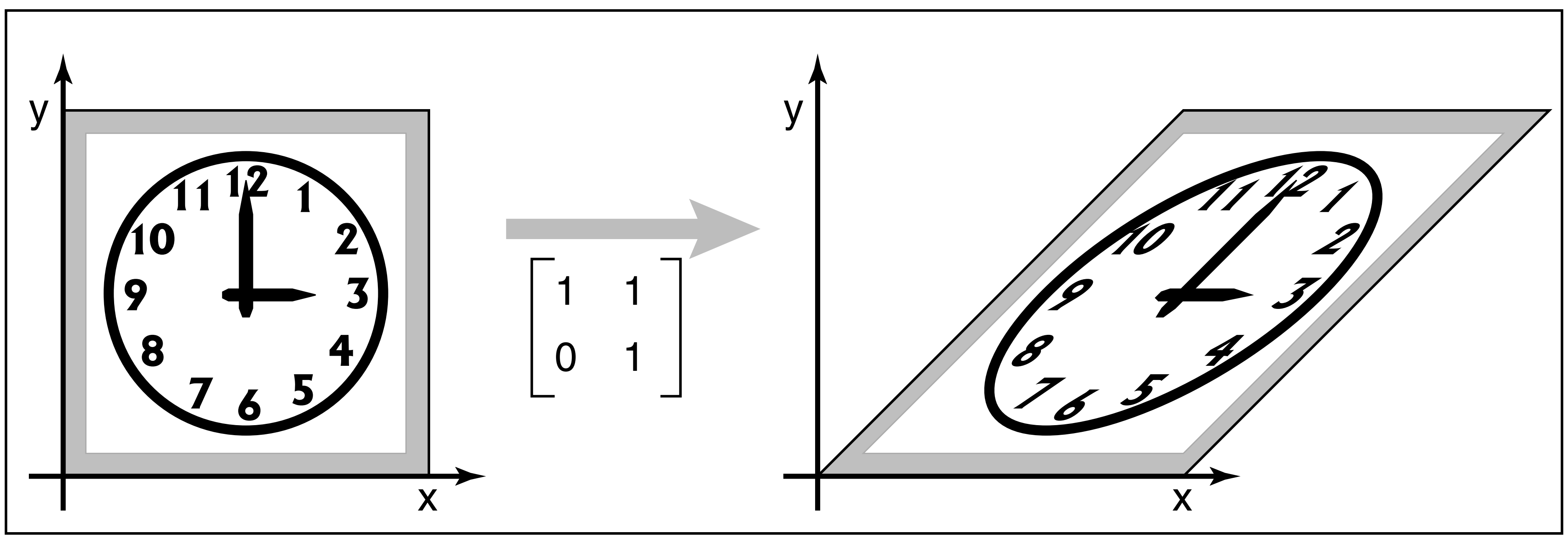
Reflection

$$\begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \pm x \\ \pm y \end{bmatrix}$$



Shear

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$$



Linear transformations

- Scaling
- Shear
- Rotate
- Reflect

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Properties

- Satisfies linearity
- origin is mapped to origin
- maps lines to lines
- parallel lines remain parallel
- ratios are preserved
- closed under composition

Translation

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

Translation

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

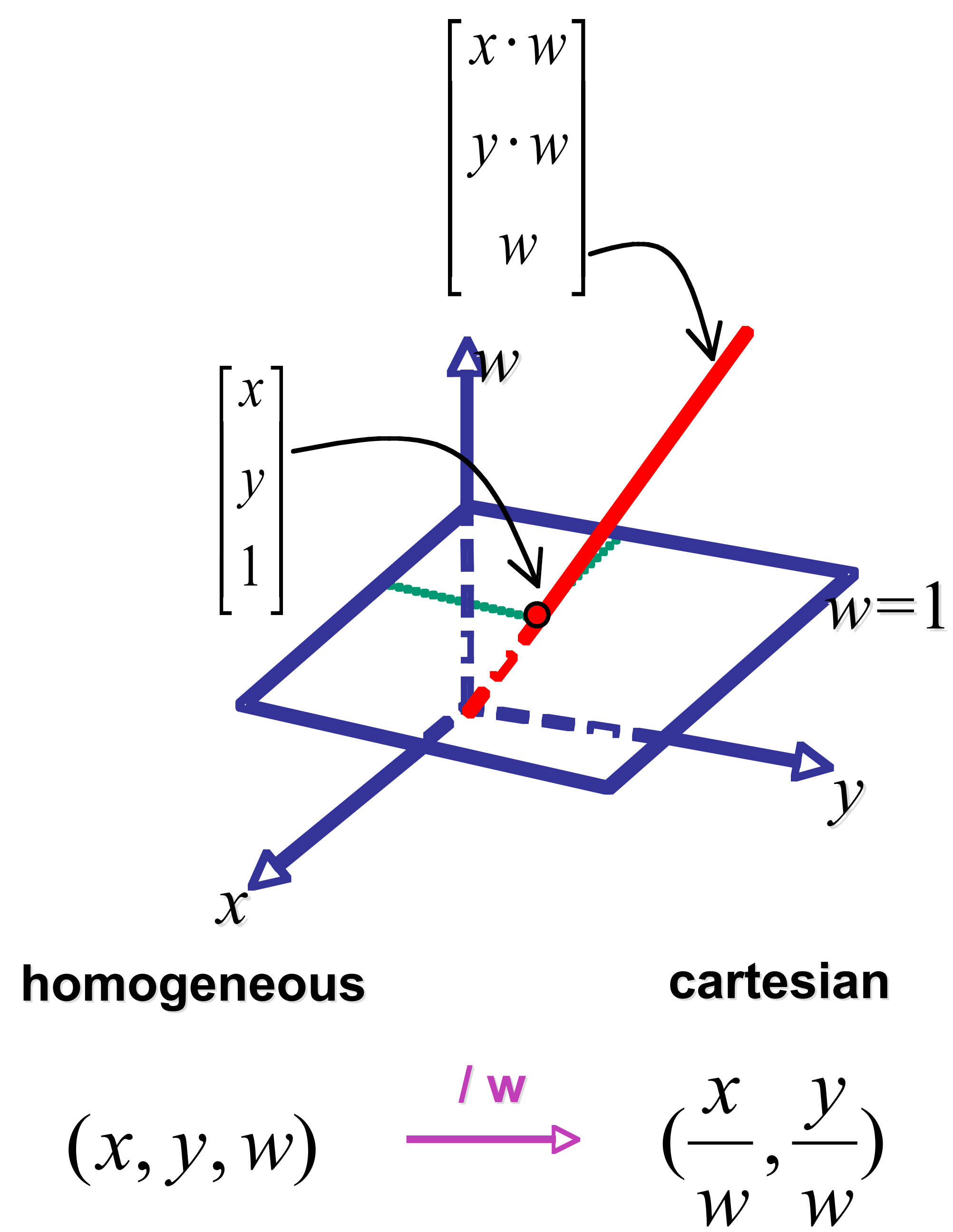
Is this linear?

Homogeneous coordinates

- Represent (x,y) with a 3-D vector: (x,y,1)
- A trick to represent translation as a linear transform!
- Calculate the transform in homogeneous coordinates, then project (i.e., remove the extra coordinate)

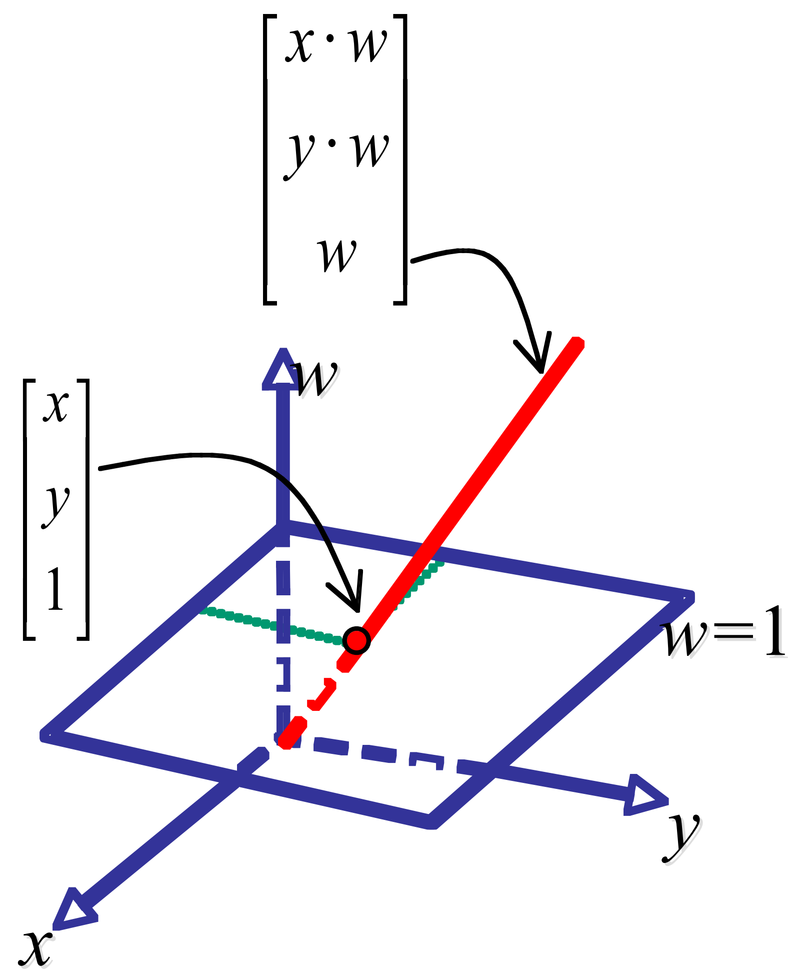
Homogeneous coords geometrically

- Point in 2D augmented by w = point in 3D homog. coords
- Multiples of (x,y,w)
  - form a line L in 3D
  - all homog. points on L represent the same (x,y) in 2D: (2,2,1), (4,4,2) and (1,1,.5)



Homogeneous coords geometrically

- Homogenize: to convert 3D homog. coord to 2D Cartesian point:
  - Divide by w: (x/w, y/w, 1)
  - When w=0, consider it as a direction: points at infinity
  - (0,0,0) undefined



homogeneous **cartesian**

$$(x, y, w) \xrightarrow{/\,w} \left(\frac{x}{w}, \frac{y}{w}\right)$$

Translation

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

Translation

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Homogeneous coords

- Our 2D transformations matrices are now 3x3:

$$Rot = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Translation:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Affine transformations

- Are combinations of:
  - linear transformation
  - translation

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

Affine transformations

- Properties:
  - origin does not necessarily map to origin
  - lines map to lines
  - parallel lines remain parallel
  - ratios are preserved
  - closed under composition

Homogeneous coords

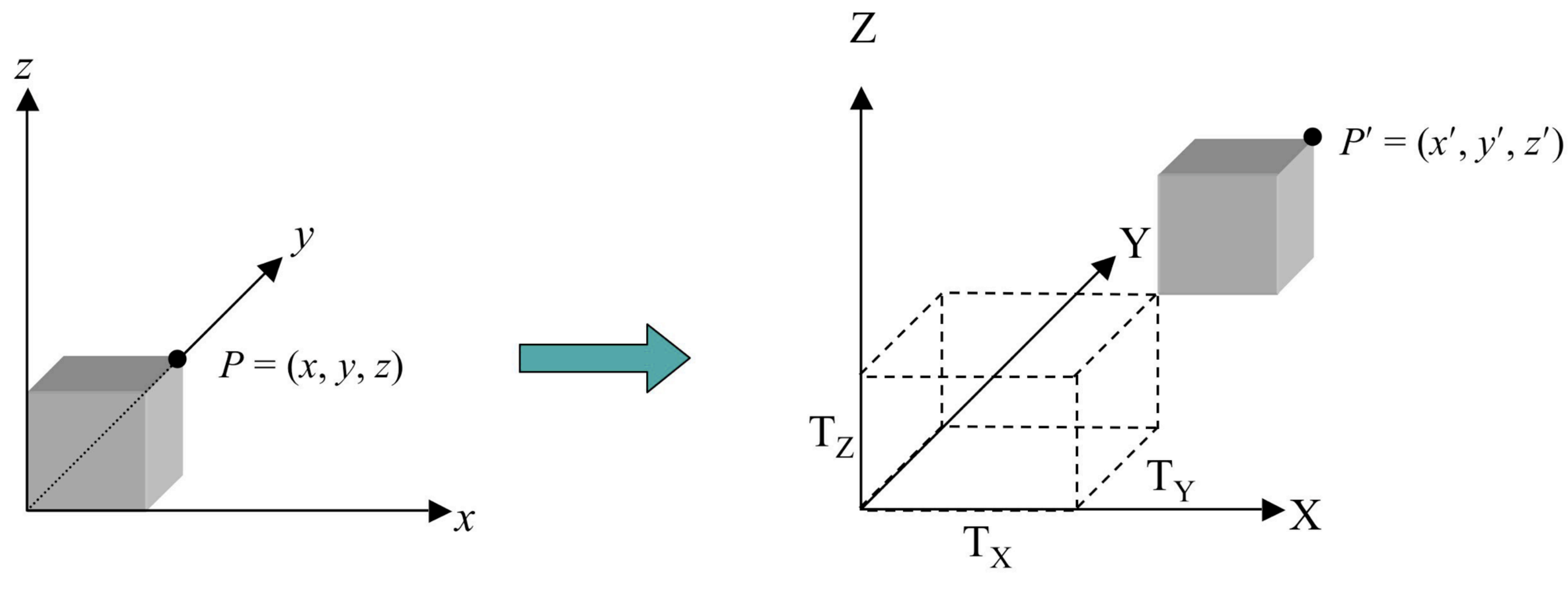
- May seem unintuitive, but make life easier
- Allow all affine transformations to be represented by matrix multiplications
- Just one dimension higher in any-D

3D Transformations

- Similar to 2D -- employ homogeneous coordinates!

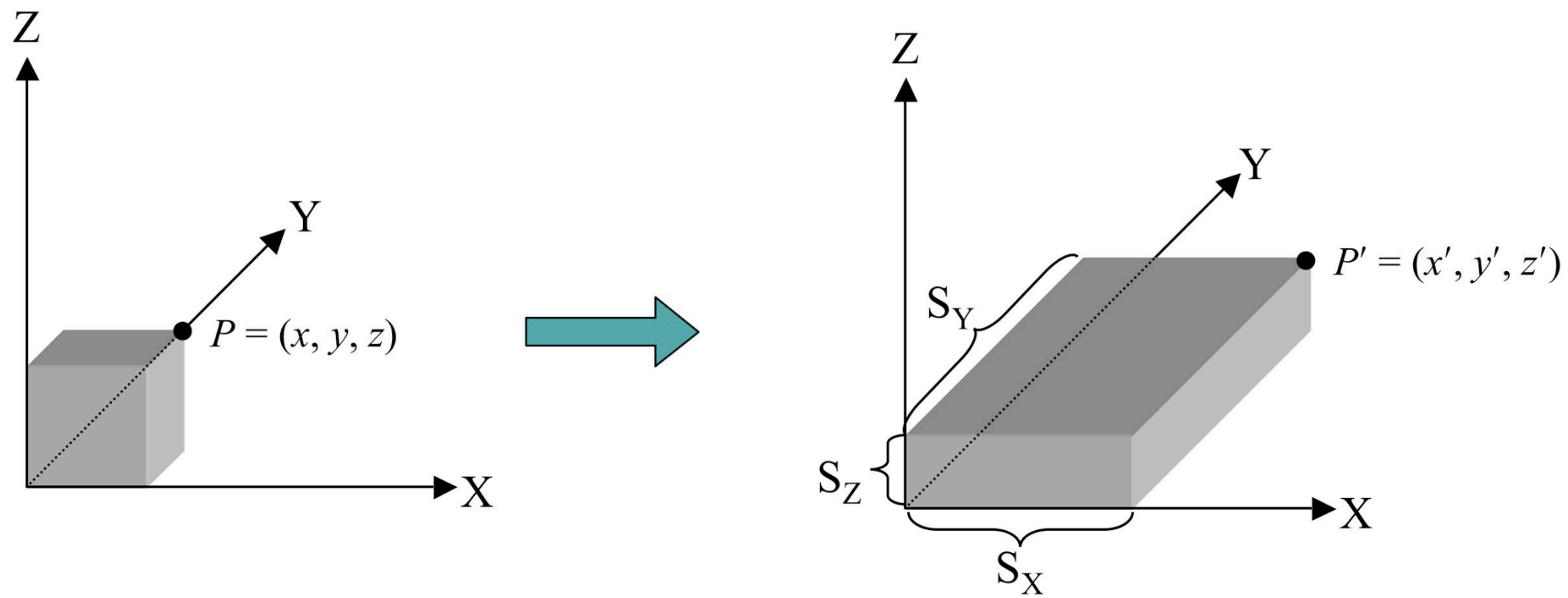
Translation

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix}$$



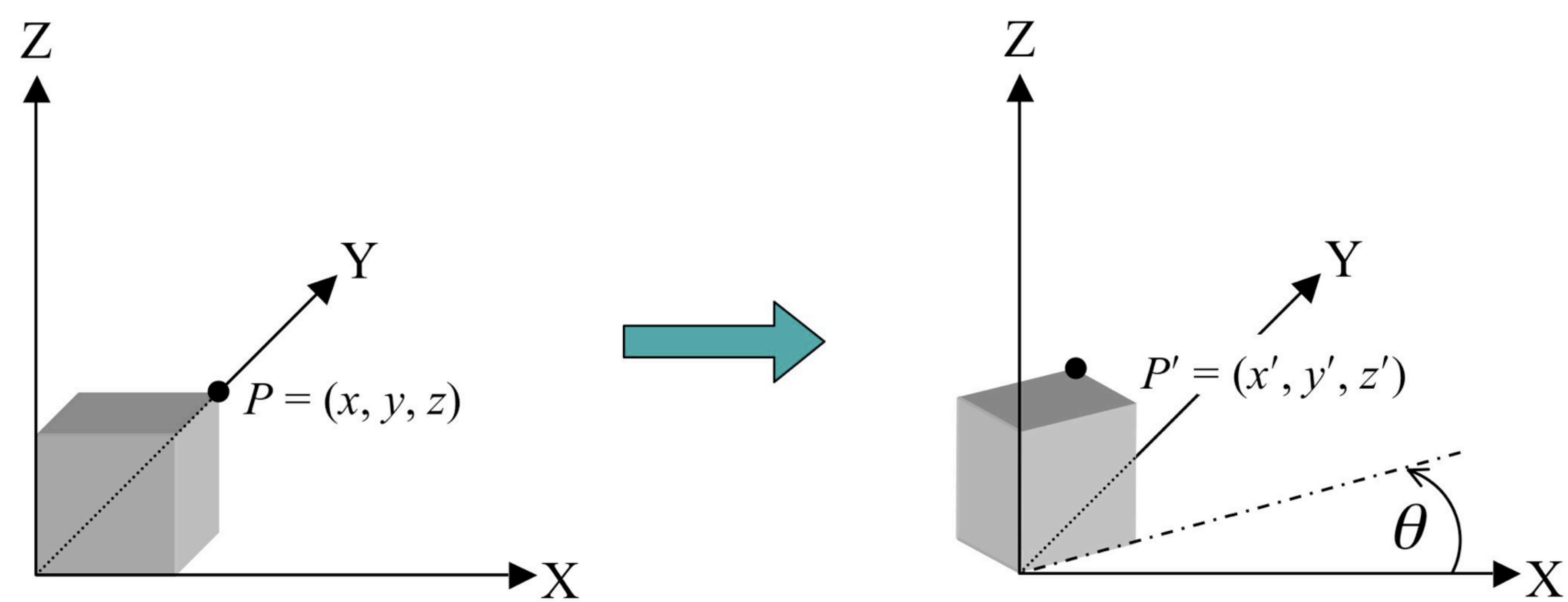
Scaling

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} xs_x \\ ys_y \\ zs_z \\ 1 \end{bmatrix}$$



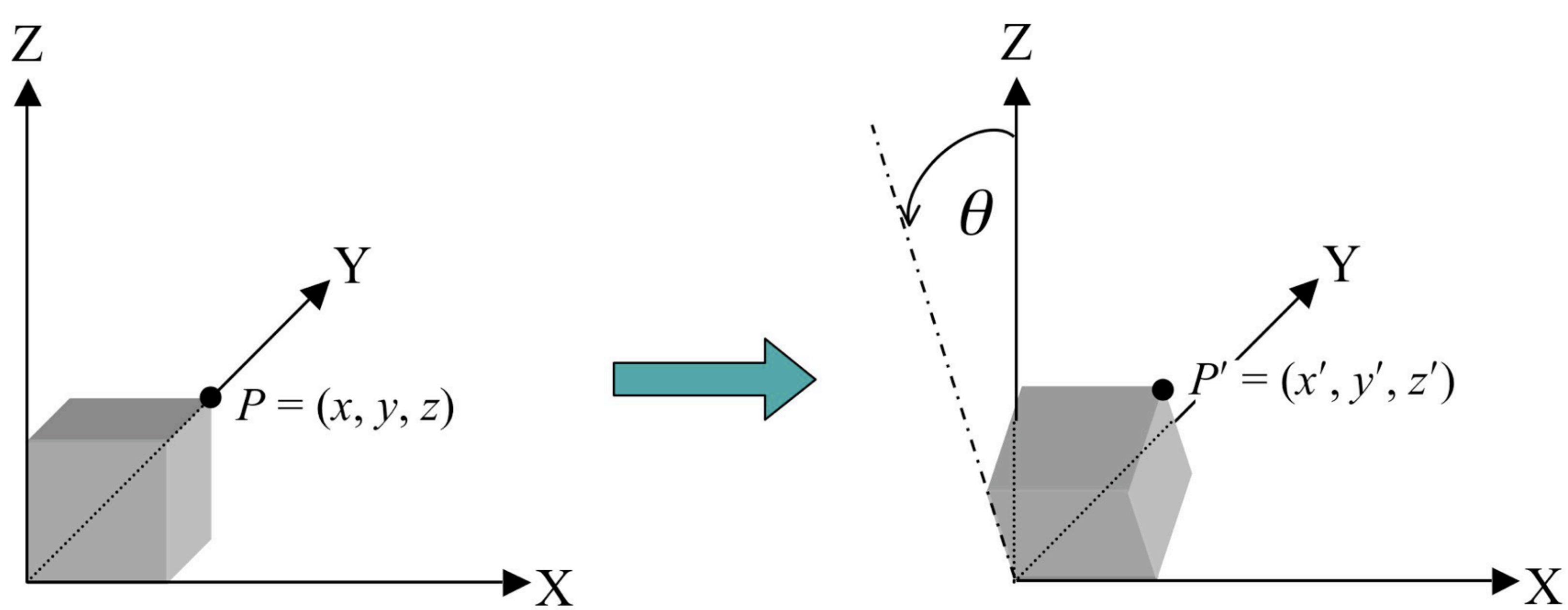
Rotation about z axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$



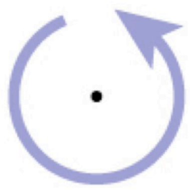
Rotation about x axis

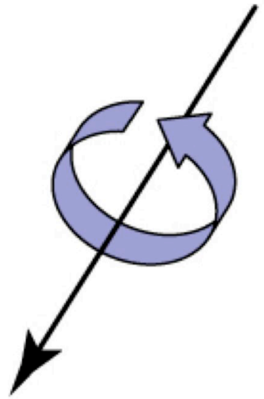
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$



General rotations

- Rotation in 2D -- around a point
- 3D rotations -- around an axis





General Rotations

- How can we build a general rotation matrix?
- Compute by **composing** elementary transforms
  - transform to align with x axis
    - apply x axis rotation
  - inverse transform back into original