Active Contour Models

a.k.a “Snakes”

CAP5416

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Main Goal:

- Image Segmentation.

What is segmentation?

- Partitioning an image into “meaningful” regions whose union is the image.

Many other methods existed at that time but none as appealing, elegant and flexible.

Problems with earlier methods

- No priors used and thus can't separate image into constituent regions.
- Not effective in the presence of noise and sampling artifacts (e.g. medical images).

Solution

- Use template matching to detect shapes
- But several different kinds of templates are required for these methods.
- We desire a method that looks for any shape in the image that is smooth and forms a closed contour.

Active Contour Models

- Introduced by Kass et al. 1987, has over 10K citations!! Revolutionized Computer Vision, Graphics & other fields.

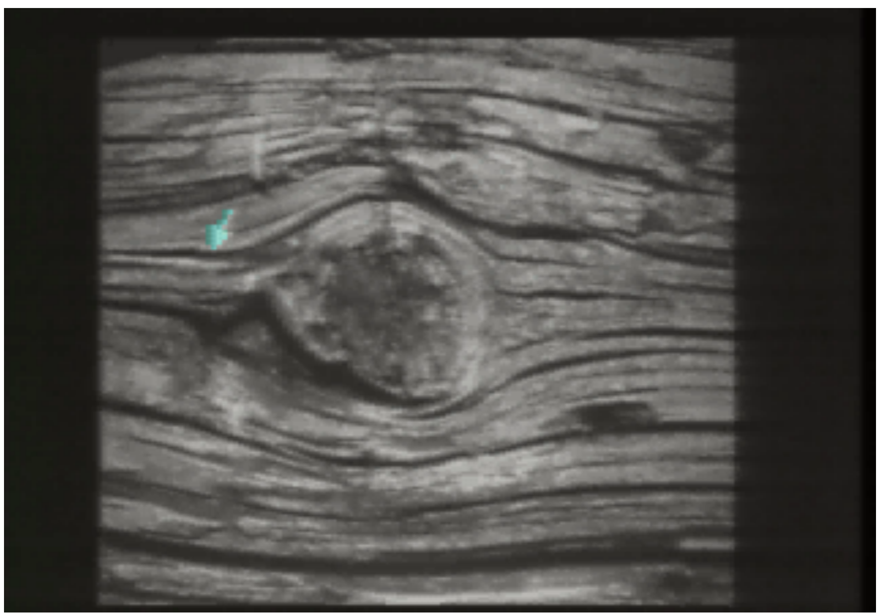
- Active contours are elastically deformable curves whose behavior is based on elasticity theory. They can deform under externally applied forces and come to rest under force balance. Active contours are represented using a **parametric curves.**

Active Contours (Contd.)

- The parametric curve is assigned an internal energy and external energy.
- The problem of segmenting an object boundary is cast in an energy minimization framework.

Framework for snakes

- A higher level process or a user initializes the snake *close to the object boundary to be segmented.*
- The snake then starts *deforming* and moving towards the desired object boundary.
- In the end it sticks to the object boundary or “shrink wraps” the object boundary for closed boundaries.



(Animation: courtesy Demetri Terzopoulos)

Modeling

- The contour is defined in the (x, y) plane of an image as a parametric curve

$$\mathbf{v}(s)=(x(s), y(s))$$

- **Potential energy** (E_{snake}) of the contour is defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- Clever definition of energies required so that the final position of the contour will have a minimum energy (E_{min})

- Hence, problem of segmenting shapes leads to an energy minimization problem.

What energy terms will do the trick for us?

Internal Energy (E_{int})

- Depends on the potential energy of the curve.
- Sum of elastic and bending energy.

Elastic Energy ($E_{elastic}$):

- The curve is assumed to be an elastically deformable rubber band with some potential energy.
- It discourages stretching by introducing tension.

$$E_{elastic} = \frac{1}{2}\int_{s} \alpha(s)\,|\,v_s\,|^2 \,ds \qquad\qquad v_s = \frac{dv(s)}{ds}$$

- Weight $\alpha(s)$ allows us to control elastic energy along different parts of the contour. Considered to be constant α for many applications.
- Responsible for shrinking the Snake.

Bending Energy ($E_{bending}$):

- The snake is also considered to behave like a thin flexible rod giving rise to a bending energy.

- It is defined as sum of squared "curvature" of the contour.

$$E_{bending} = \frac{1}{2}\int_s \beta(s)\,|\,v_{ss}\,|^2\,ds$$

- **β(s)** plays a similar role to **α(s).**
- Bending energy is minimum for a straight line for open contours and a circle for closed contours.

- Total internal energy of the snake can be defined as

$$E_{\text{int}} = E_{elastic} + E_{bending} = \int_s \frac{1}{2}(\alpha\,|\,v_s\,|^2 + \beta\,|\,v_{ss}\,|^2)ds$$

External energy of the contour (E_{ext})

- Derived from the image.
- Define $E_{image}(x,y)$ such that **it has smaller values at** the features of interest, such as object **boundaries.**

$$E_{ext} = \int_s E_{image}(v(s))ds$$

Key is in defining $E_{image}(x,y)$. Some examples

- $E_{image}(x,y) = -|\nabla \mathrm{I}(x,y)|^2$

- $E_{image}(x,y) = -|\nabla (G_\sigma(x,y) * I(x,y))|^2$

Energy and force equations

- The problem then is to find a v(s) that minimizes the energy functional

$$E_{snake} = \int_s \frac{1}{2}(\alpha(s)\,|\,v_s\,|^2 + \beta(s)\,|\,v_{ss}\,|^2) + E_{image}(v(s))ds$$

- Applying Variational Calculus, the Euler-Lagrange equation is:

$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

- Equation can be interpreted as a force balance equation.

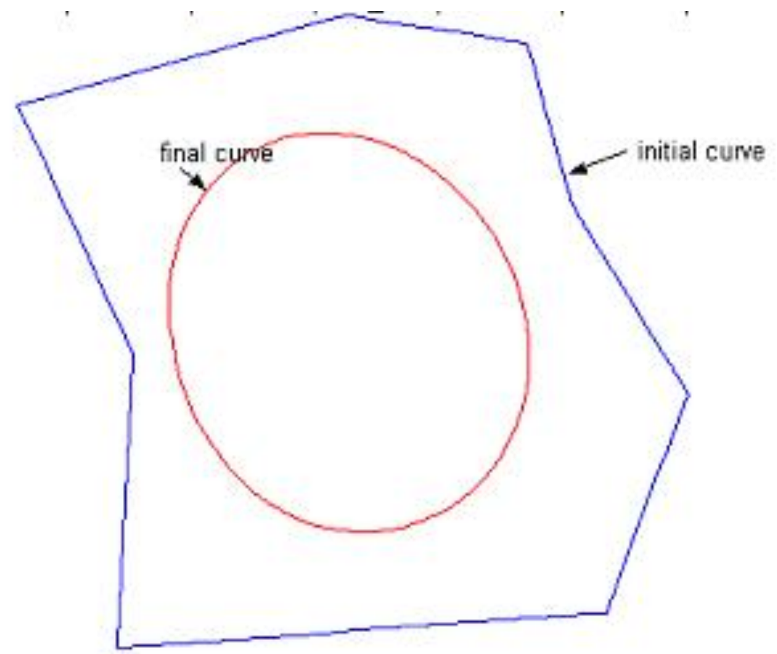
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

Elastic force

- Generated by elastic potential energy of the curve.

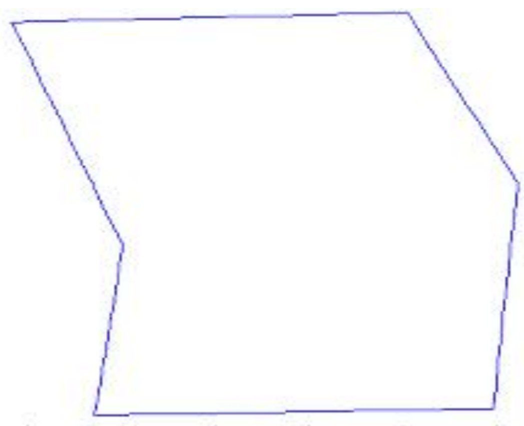
$$F_{elastic} = \alpha v_{ss}$$

- Characteristics (refer diagram)

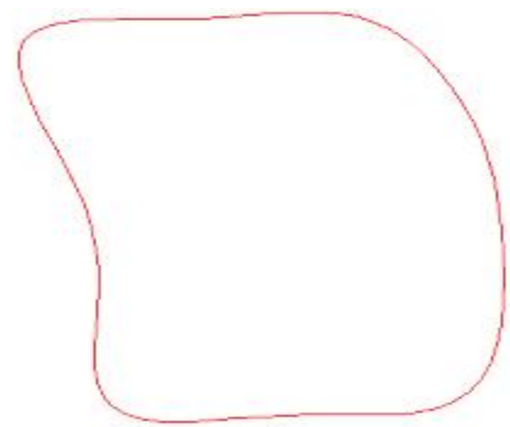


Bending force

- Generated by the bending energy of the contour.
- Characteristics (refer diagram):



Initial curve
(High bending energy)



Final curve deformed by
bending force. (low
bending energy)

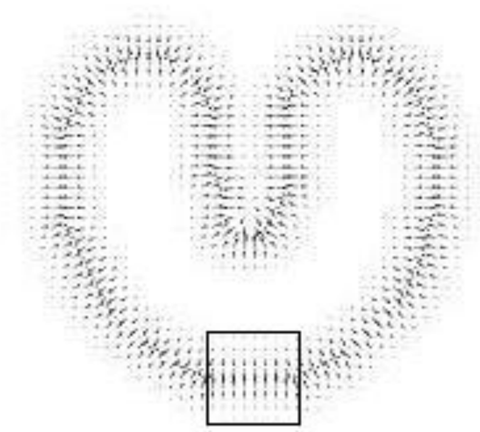
- Thus the bending energy tries to smooth out the curve.

External force

$$F_{ext} = -\nabla E_{image}$$

- It acts in the direction so as to minimize E_{ext}

Image



External force

Zoomed in

Discretizing

- the contour v(s) is represented by a set of discrete points
  $v_0 , v_1 , \ldots\ldots, v_{n-1}$

- Force equations are applied to each point separately.

- Each point is allowed to move freely under the. influence of the forces.

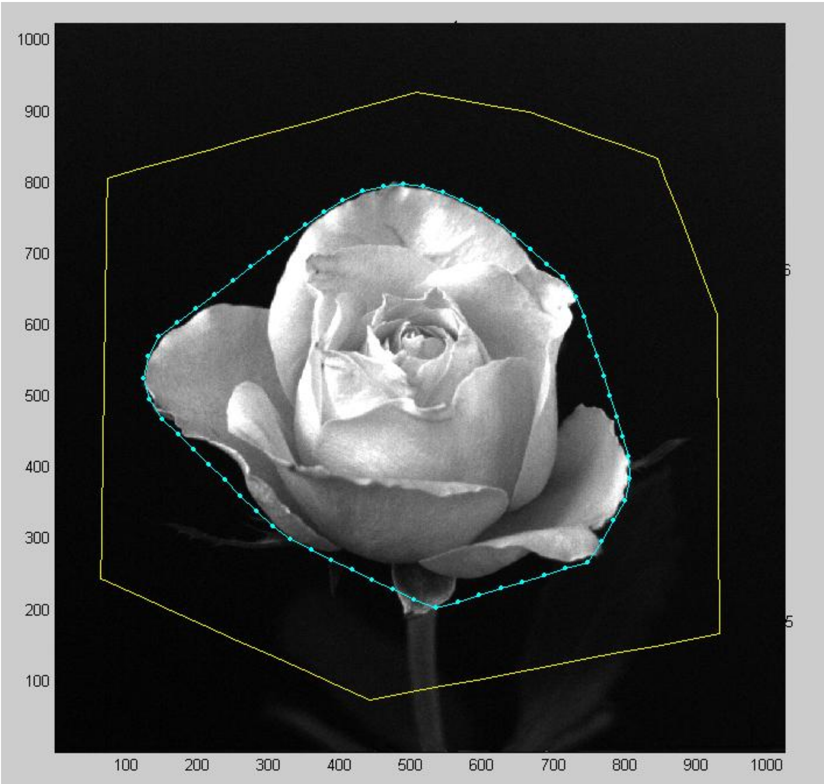
- The force is converted to discrete form with the derivatives substituted by finite differences.

Solution and Results

Method 1:

$$\alpha v_{ss} - \beta v_{ssss} - \gamma \nabla E_{image} = 0$$

- γ is a constant to give separate control on external force.
- Solve iteratively.



Method 2:

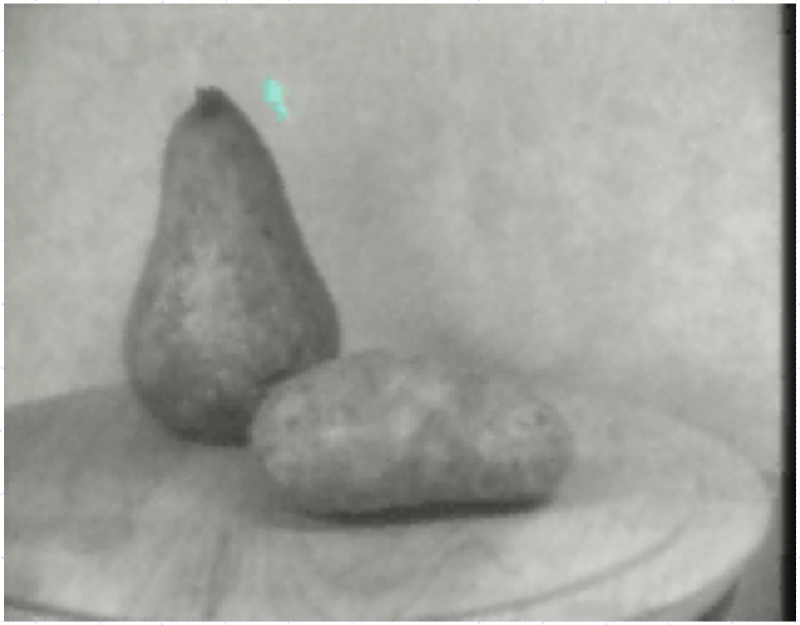
- Consider the snake to also be a function of time i.e. $v_t(s,t)$

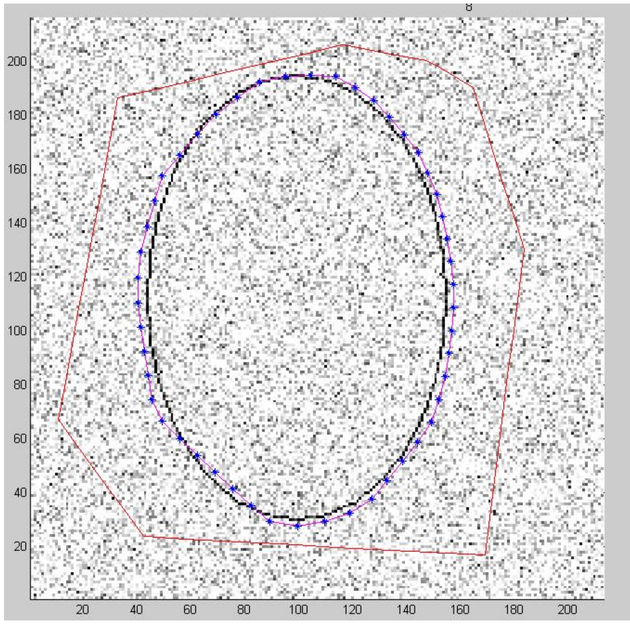
$$\alpha v_{ss}(s,t) - \beta v_{ssss}(s,t) - \nabla E_{image} = v_t(s,t) \qquad v_t(s,t) = \frac{\partial v(s,t)}{\partial t}$$

- If the curve is stable and does not move, we have reached the solution.
- In every iteration update the points only if new position has a lower external energy.

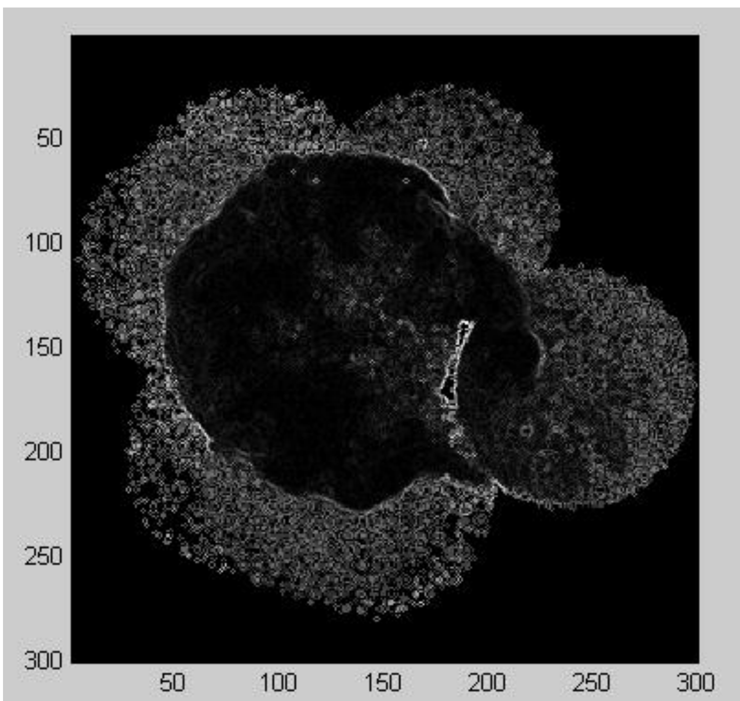
- Snakes are very sensitive to false local minima which leads to wrong convergence. Note that they were never meant to be stand alone segmentation tools.

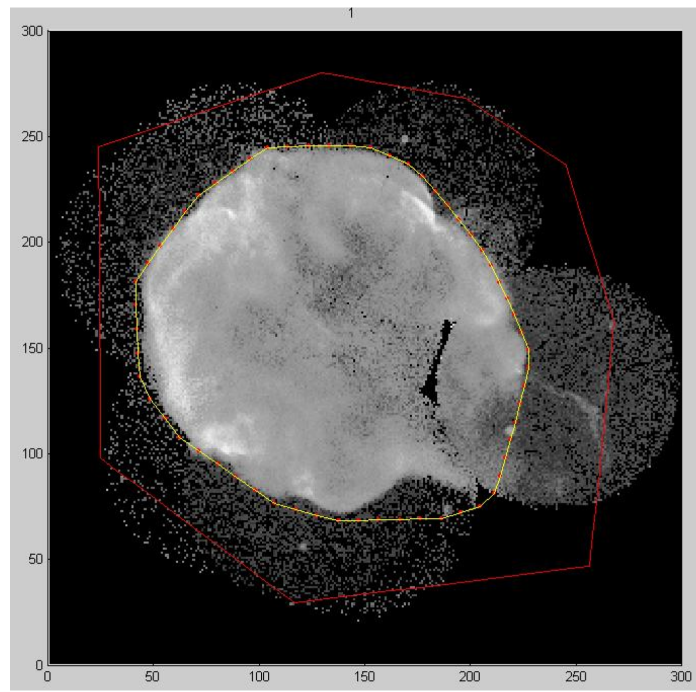
Examples





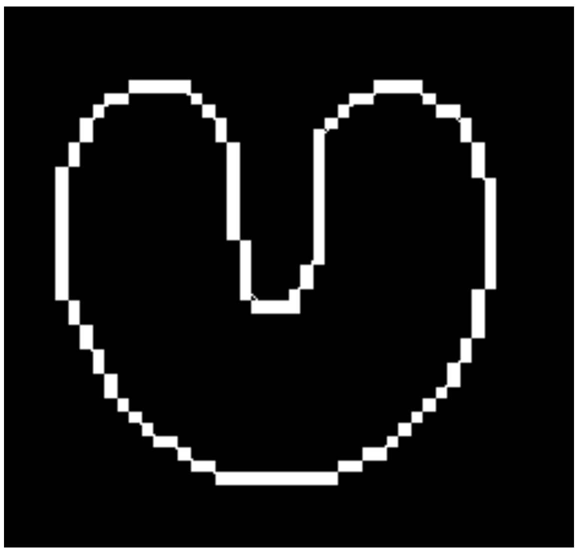
- Noisy image with many local minimas
- sigma=0.1
- Threshold=15

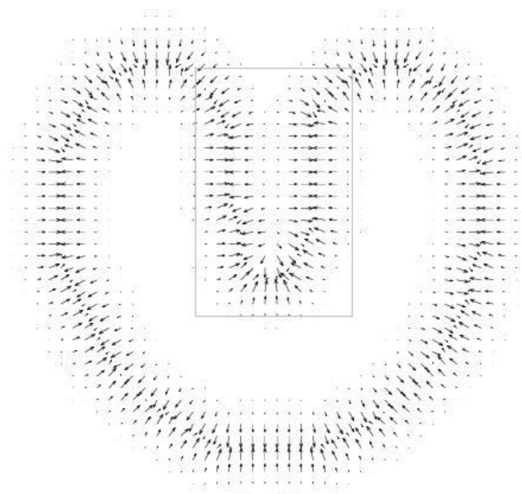




Weakness of traditional snakes (Kass model)

- Extremely sensitive to parameters.
- Small capture range.

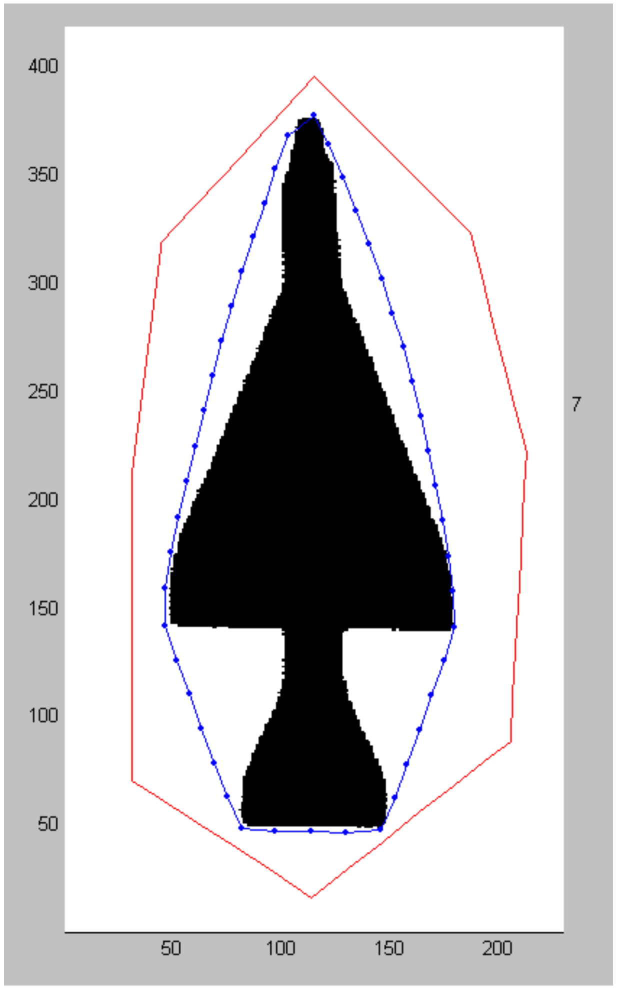




- No external force acts on points which are far away from the boundary.
- Convergence is dependent on initial position.

Weakness (contd…)

- Difficult to detect concave boundaries. External force cant pull contour points into boundary concavity.



Gradient Vector Flow (GVF)

(Alternative external force for snakes)

- Detects shapes with boundary concavities.
- Large capture range.

Model for GVF snake

- The GVF field is defined to be a vector field

  V(x,y) = $(u(x,y), v(x,y))$
- Force equation of GVF snake

$$\alpha v_{ss} - \beta v_{ssss} + V = 0$$

- V(x,y) is defined such that it minimizes the energy functional

$$E = \int\int \mu(u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \, dxdy$$

f(x,y)= -E_{ext}(x,y), e.g. it is the edge map of the image.

Interpretation of GVF eqn.

- When |grad f| is small, total energy is dominated by the regularizer yielding a slowly varying field.
- When |grad f| is large, second term dominates the integrand and is minimized by setting V= grad f.

- GVF field can be obtained by solving following equations

$$\mu\nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0$$

$$\mu\nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0$$

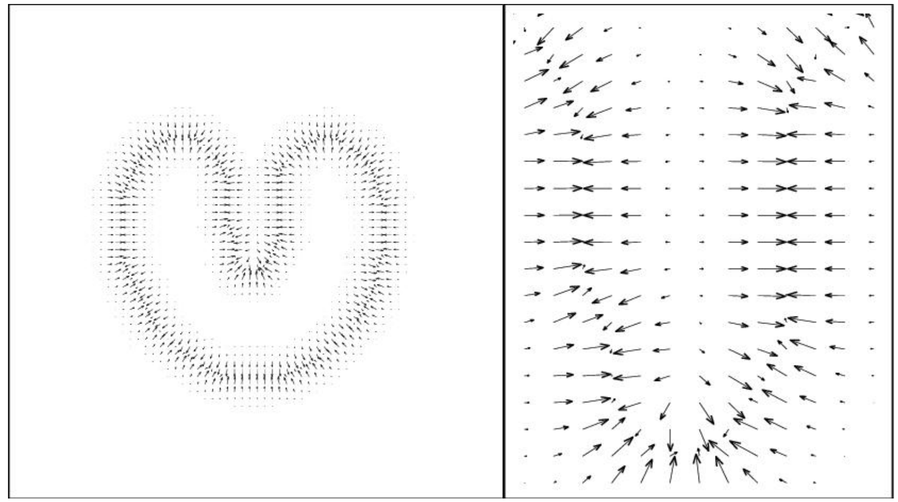
∇^2 Is the Laplacian operator.

- Reason for detecting boundary concavities.

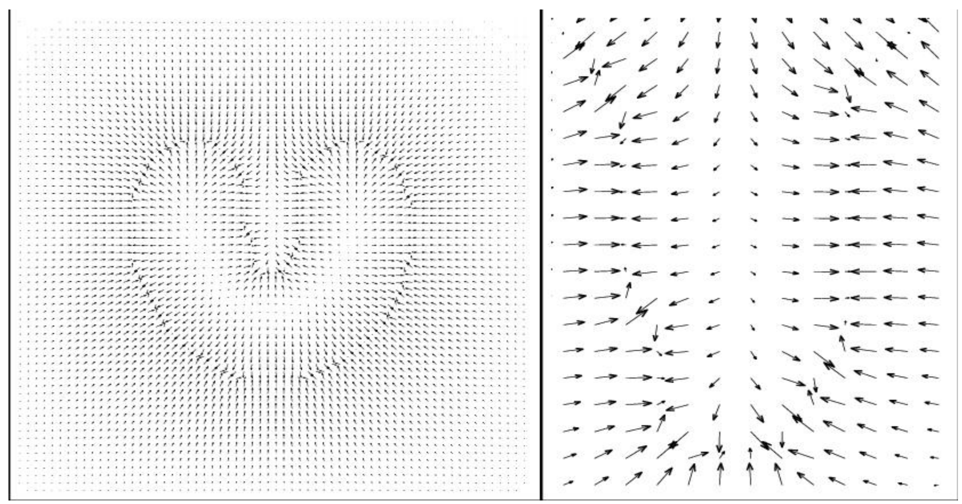
- The above equations are solved iteratively using time derivative of u and v.

Traditional external force field v/s GVF field

Traditional force

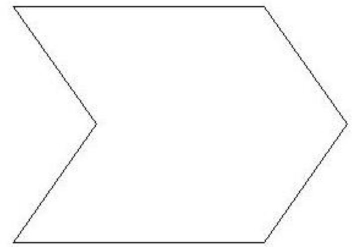


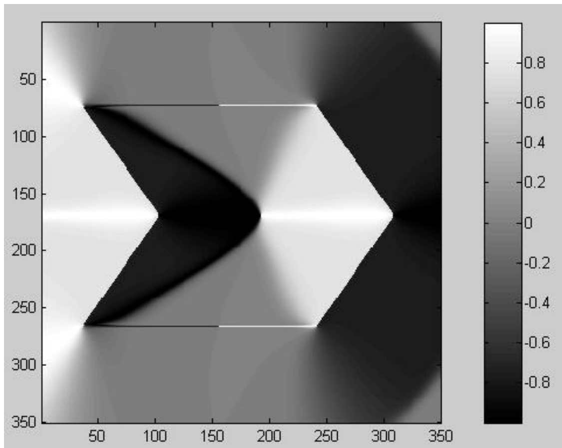
GVF force



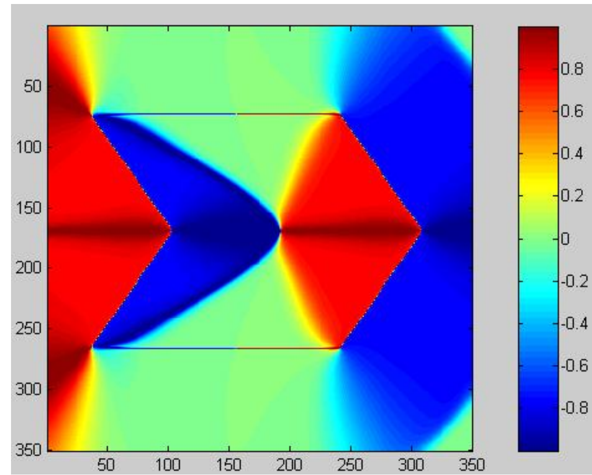
(Diagrams courtesy “Snakes, shapes, gradient vector flow”, Xu, Prince)

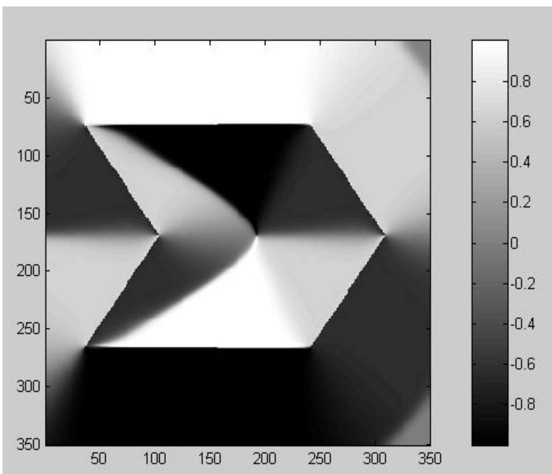
A look into the vector field components



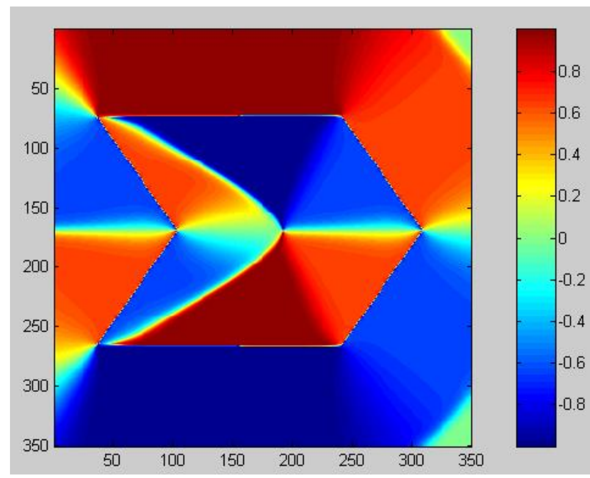


← u(x,y) →



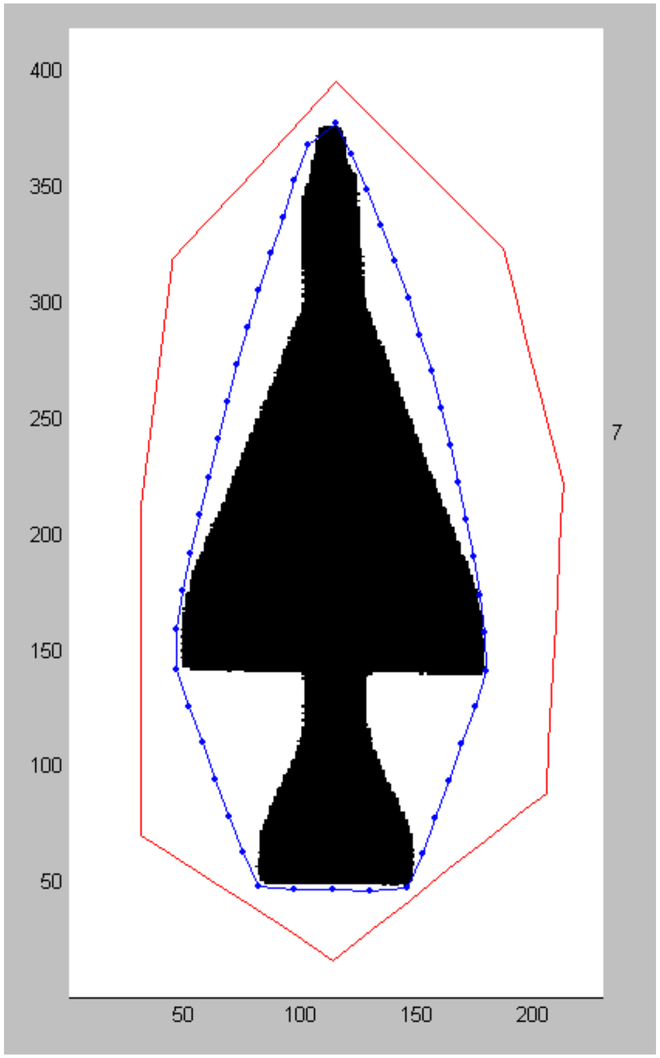


← v(x,y) →

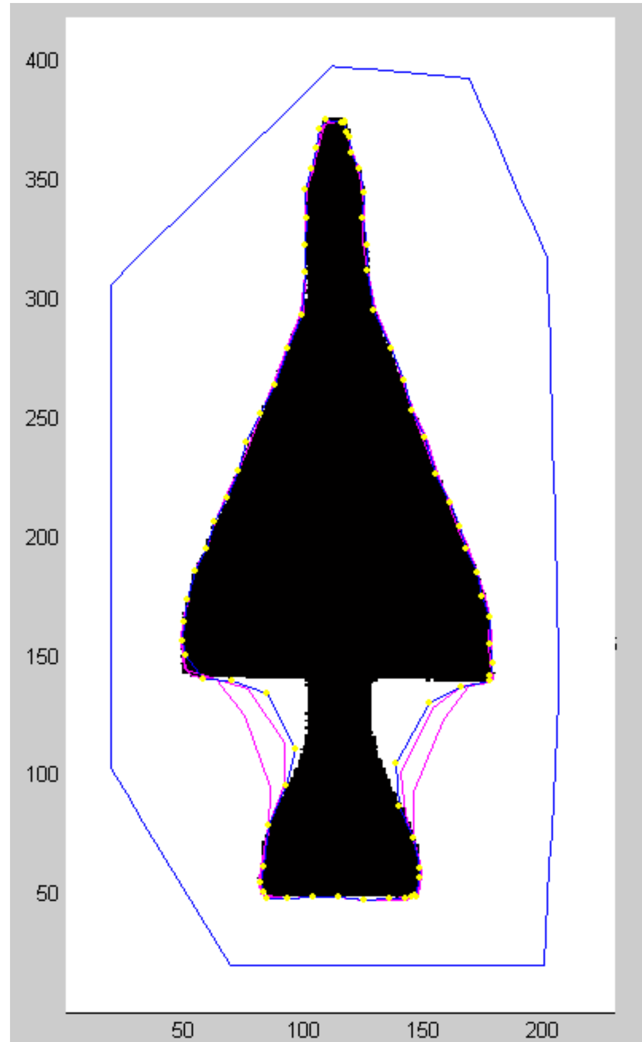


Note forces also act inside the object boundary!!

Results

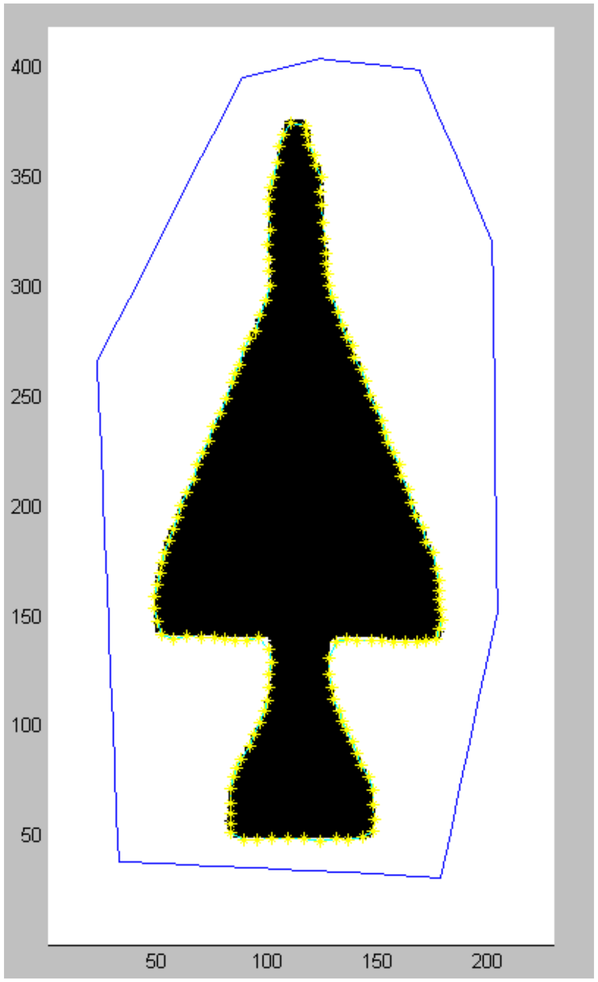


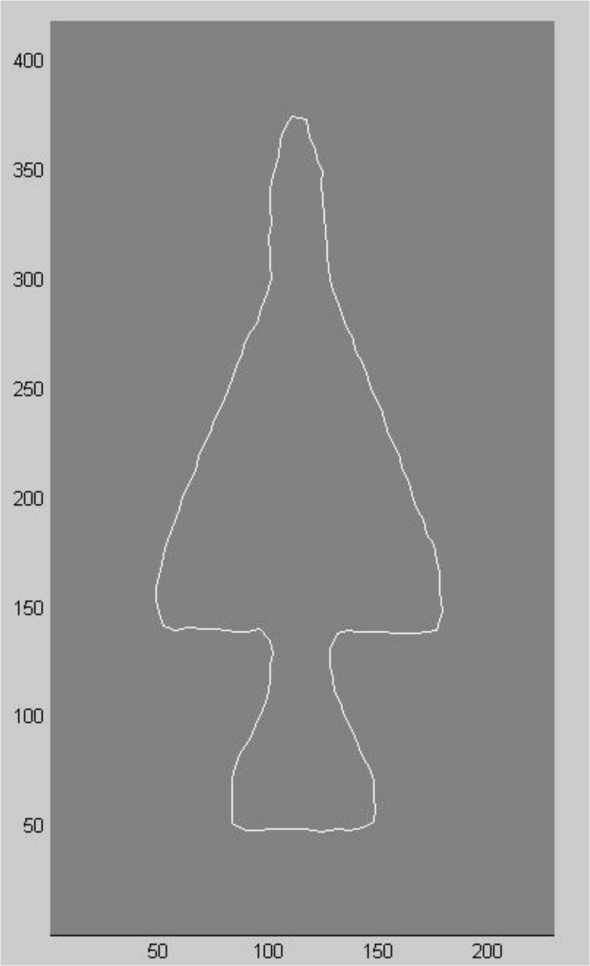
Traditional snake



GVF snake

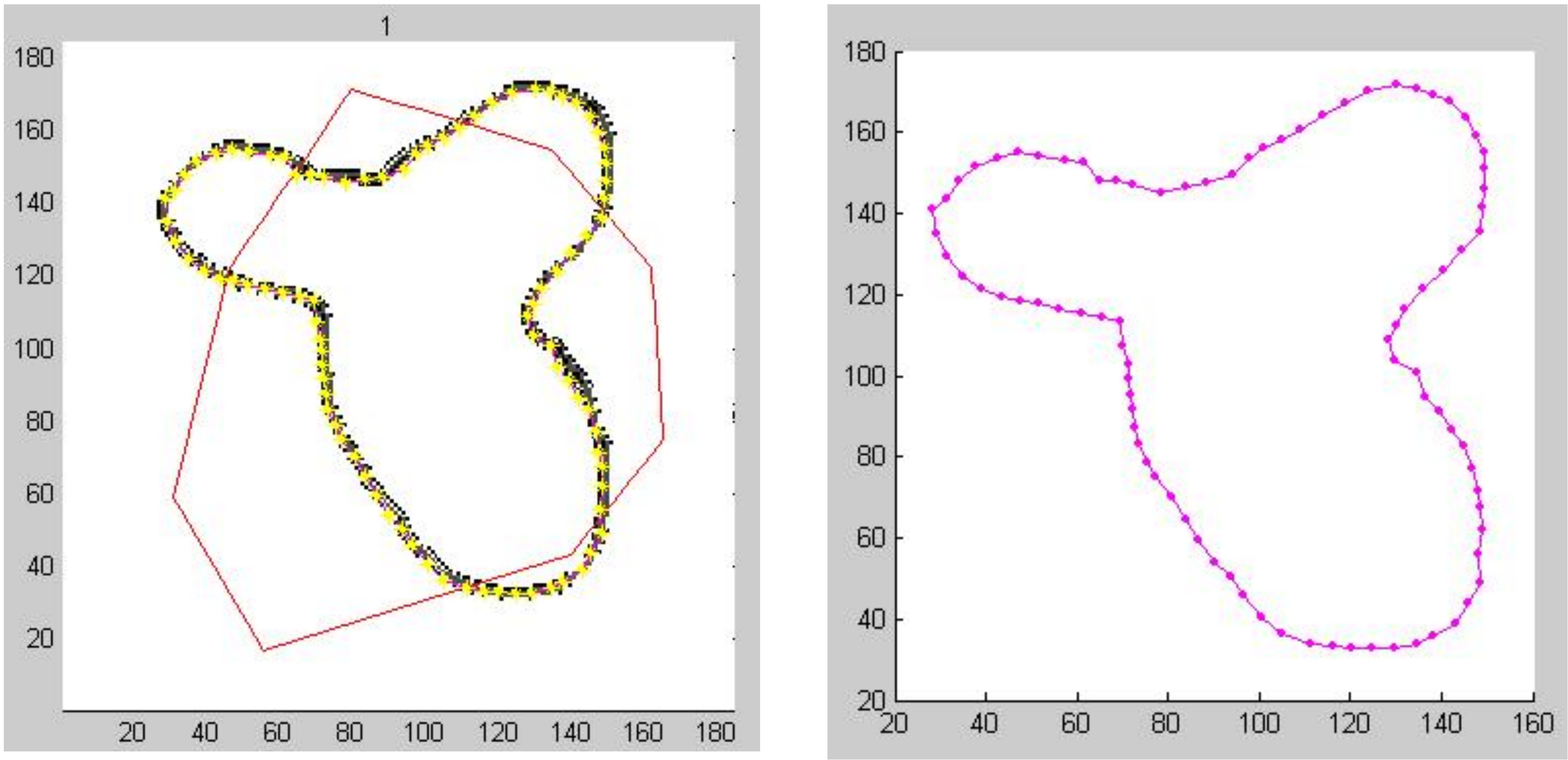
Cluster and reparametrize the contour dynamically.



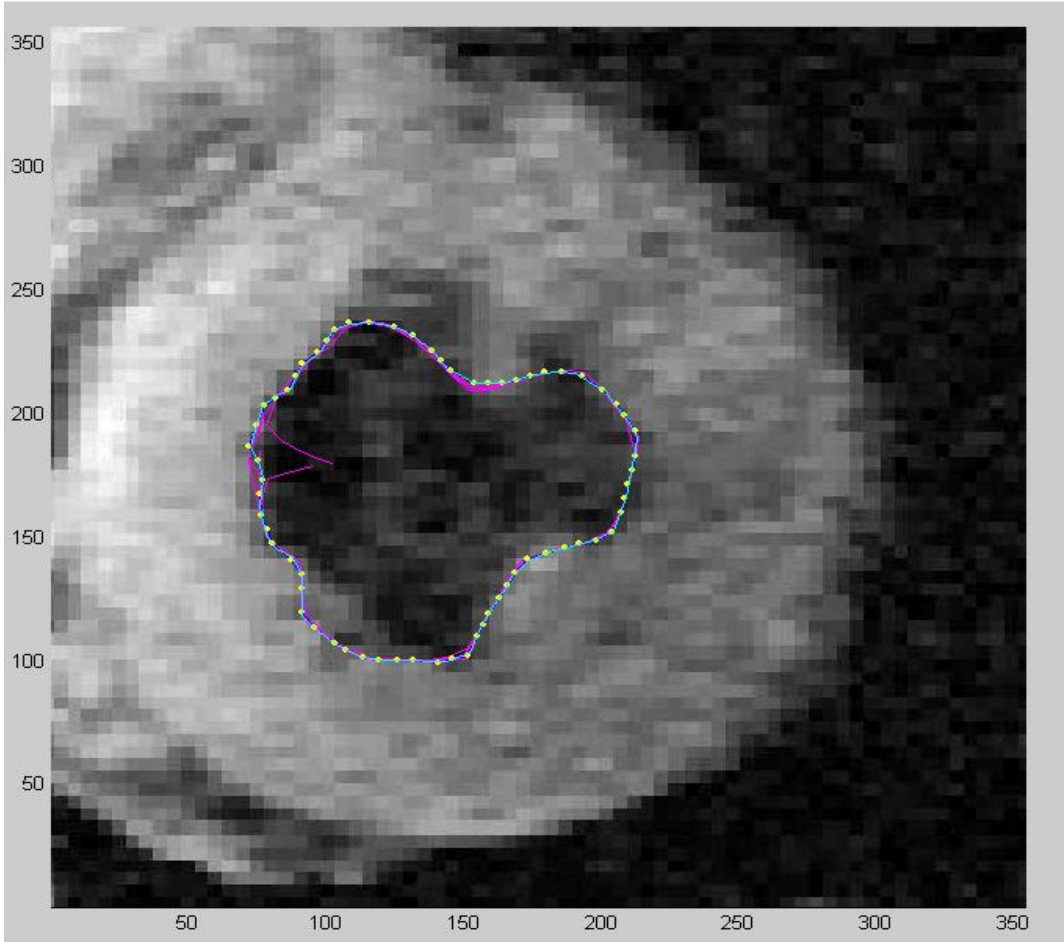


Final shape detected

The contour can also be initialized across the boundary of object!!
Something not possible with traditional snakes.



Medical Imaging



Magnetic resonance image of the left ventricle of human heart

Notice that the image is poor quality with sampling artifacts

Problem with GVF snake

- Very sensitive to parameters.
- Slow. Finding GVF field is computationally expensive.

Applications of snakes

- Image segmentation particularly medical imaging community (tremendous help).

- Motion tracking.

- Stereo matching (Kass, Witkin).

- Shape recognition.

References

- M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models.“, International Journal of Computer Vision. v. 1, n. 4, pp. 321-331, 1987.

- Chenyang Xu and Jerry L. Prince , "Snakes, Shape, and Gradient Vector Flow“, IEEE Transactions on Image Processing, 1998.

- C. Xu and J.L. Prince, “Gradient Vector Flow: A New External Force for Snakes”, Proc. IEEE Conf. on Comp. Vis. Patt. Recog. (CVPR), Los Alamitos: Comp. Soc. Press, pp. 66-71, June 1997.

Questions??