

Projective Geometry

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What are Vanishing Points?

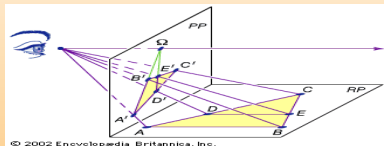


Figure: Vanishing Points

- Vanishing point of a line is the intersection of the image plane with a ray parallel to the world line and passing through the camera center.
- Vanishing point depends only on the direction of a line and not its position.

- **Algebraically:** Vanishing point is the limit point. Points on a line in 3D through a point \mathbf{A} and direction $\mathbf{D} = (\mathbf{d}^t, 0)^t$ are written as

$$\mathbf{X}(\lambda) = \mathbf{A} + \lambda \mathbf{D}$$

- As λ varies from $0 \rightarrow \infty$, point $\mathbf{X}(\lambda)$ varies from \mathbf{A} to \mathbf{D} at ∞ .
- Under a projective camera, $P = K(I|0)$, point $\mathbf{X}(\lambda)$ is imaged at,

$$\begin{aligned} \mathbf{X}(\lambda) &= P\mathbf{X}(\lambda) = P\mathbf{A} + \lambda P\mathbf{D} \\ &= \mathbf{a} + \lambda K\mathbf{d} \end{aligned}$$

$$\mathbf{v} = \lim_{\lambda \rightarrow \infty} \mathbf{X}(\lambda) = \lim_{\lambda \rightarrow \infty} (\mathbf{a} + \lambda K\mathbf{d}) = K\mathbf{d}$$

- \Rightarrow Vanishing point back projects to a ray with direction \mathbf{d} .
- Note that \mathbf{v} is independent of position of the line specified i.e., A .
- In projective geometry, this result can be trivially obtained. In 3D, the plane at infinity is the plane of directions and all lines with the same directions intersect Π_∞ in the same point.
- Vanishing point is simply the image of this intersection.
- Let \mathbf{d} be the direction of a line, it intersects Π_∞ in the point $\mathbf{X}_\infty = (\mathbf{d}^t, 0)^t$

- Then \mathbf{v} is the image of \mathbf{X}_∞ ,

$$\mathbf{v} = P\mathbf{X}_\infty = K(I|\mathbf{0}) \begin{pmatrix} \mathbf{d} \\ 0 \end{pmatrix} = K\mathbf{d}$$

- **Summary:** Vanishing point of lines with direction \mathbf{d} in 3D is the intersection \mathbf{v} of the image plane with the ray through the camera center with direction \mathbf{d} , namely $\mathbf{v} = K\mathbf{d}$.

Vanishing Lines

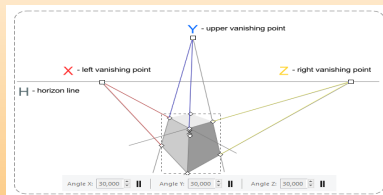


Figure: Vanishing Line

- Parallel planes in 3D intersect Π_∞ in a common line and image of this line is the vanishing line of the plane.
- Vanishing line depends only on the orientation of the scene plane and not on its position.