Motion Field and Optical Flow

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Motion field

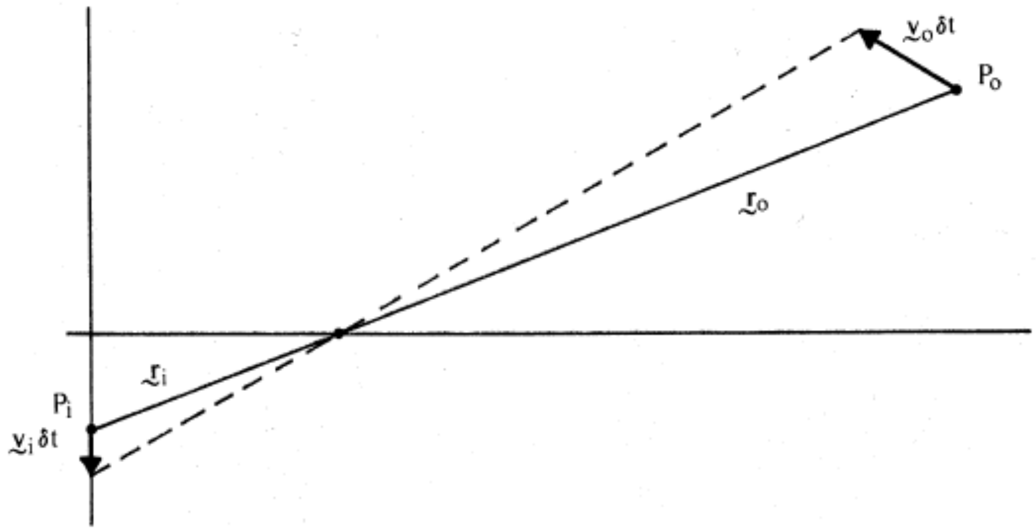
Whenever there is a relative motion between camera and the object, there are corresponding changes in the image.

Problem: Use these changes to recover the relative motions as well as the shapes of the objects.

Motion fields and Optical Flows

- Motion field: projection of 3D motion vectors on image plane

Object point P_0 has velocity $\mathbf{v}_0$, induces $\mathbf{v}_i$ in image

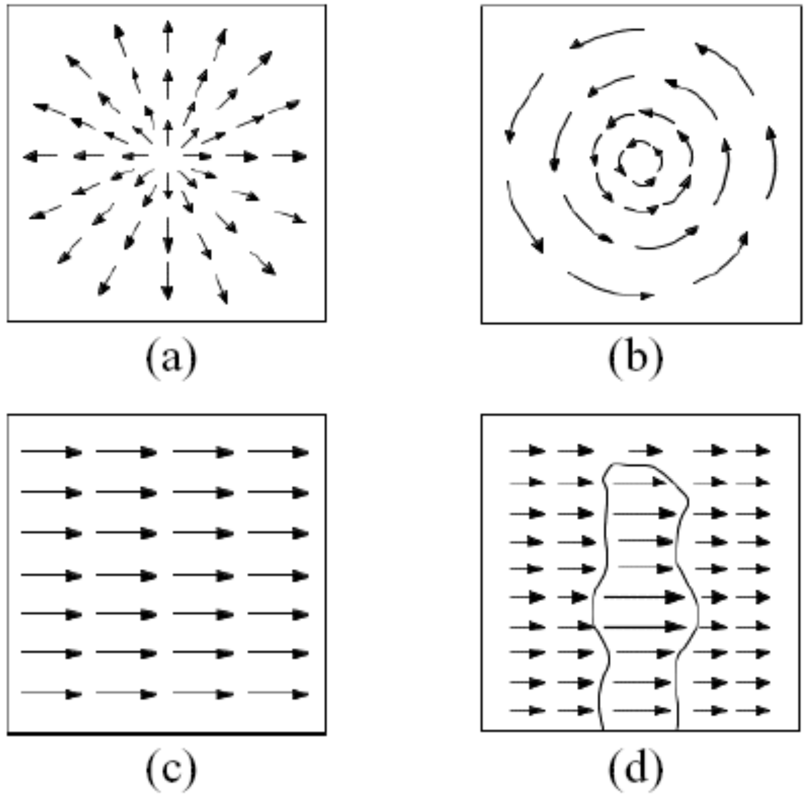


$$\mathbf{v}_0 = \frac{d\mathbf{r}_0}{dt} \quad \mathbf{v}_1 = \frac{d\mathbf{r}_i}{dt}$$

$\mathbf{r}_0$ related to $\mathbf{r}_i$ by $\dfrac{\mathbf{r}_i}{f} = \dfrac{\mathbf{r}_0}{\mathbf{r}_0 \cdot \hat{\mathbf{z}}_0}$

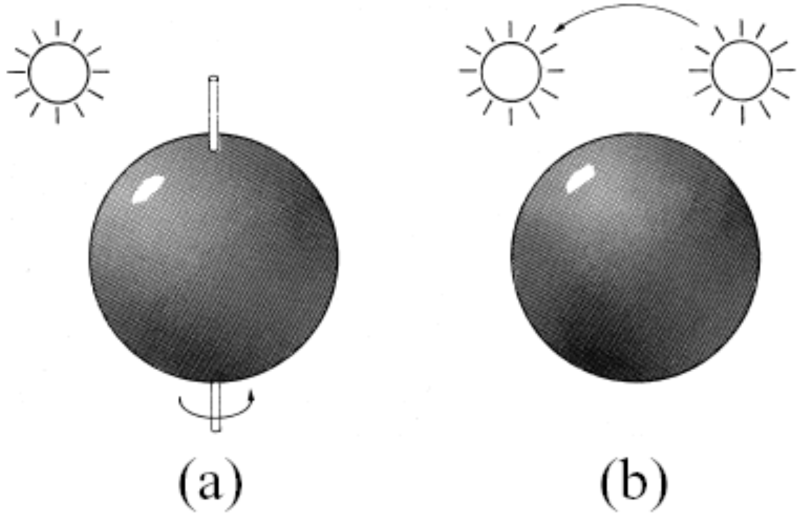
- Optical flow field: apparent motion of brightness patterns
- We equate motion field with optical flow field

Examples of motion fields



(a) Translation perpendicular to a surface. (b) Rotation about axis perpendicular to image plane. (c) Translation parallel to a surface at a constant distance. (d) Translation parallel to an obstacle in front of a more distant background.

2 Cases Where this Assumption Clearly is not Valid

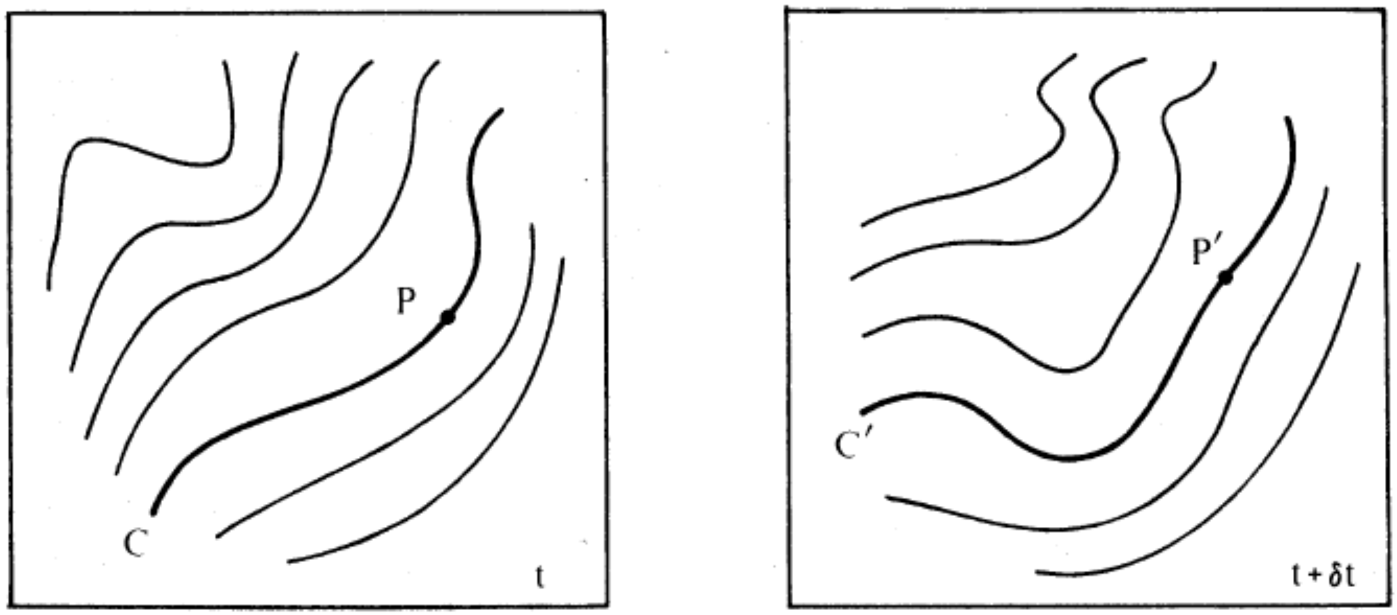


(a) A smooth sphere is rotating under constant illumination. Thus the optical flow field is zero, but the motion field is not.

(b) A fixed sphere is illuminated by a moving source—the shading of the image changes. Thus the motion field is zero, but the optical flow field is not.

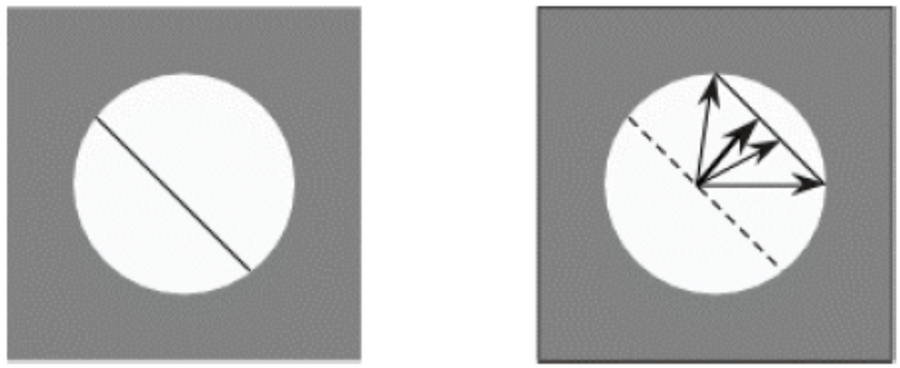
Apparent motion of brightness patterns

Brightness Constancy Assumption



The apparent motion of brightness patterns is an awkward concept. It is not easy to decide which point P' on a contour C' of constant brightness in the second image corresponds to a particular point P on the corresponding contour C in the first image.

Aperture Problem



(a) (b)

(a) Line feature observed through a small aperture at time t.

(b) At time $t+\delta t$ the feature has moved to a new position. It is not possible to determine exactly where each point has moved. From local image measurements only the flow component perpendicular to the line feature can be computed.

Normal flow: Component of flow perpendicular to line feature.

The Barber Pole Illusion

http://en.wikipedia.org/wiki/Barber's_pole

The Optical Flow Constraint Equation

Let $E(x, y, t)$ be the irradiance and $u(x, y)$, $v(x, y)$ the components of optical flow.

$$E(x + u\delta t, y + v\delta t, t + \delta t) = E(x, y, t)$$

Taylor expansion

$$E(x, y, t) + \delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} + e = E(x, y, t)$$

dividing by δt and taking limit $\delta t \to 0$

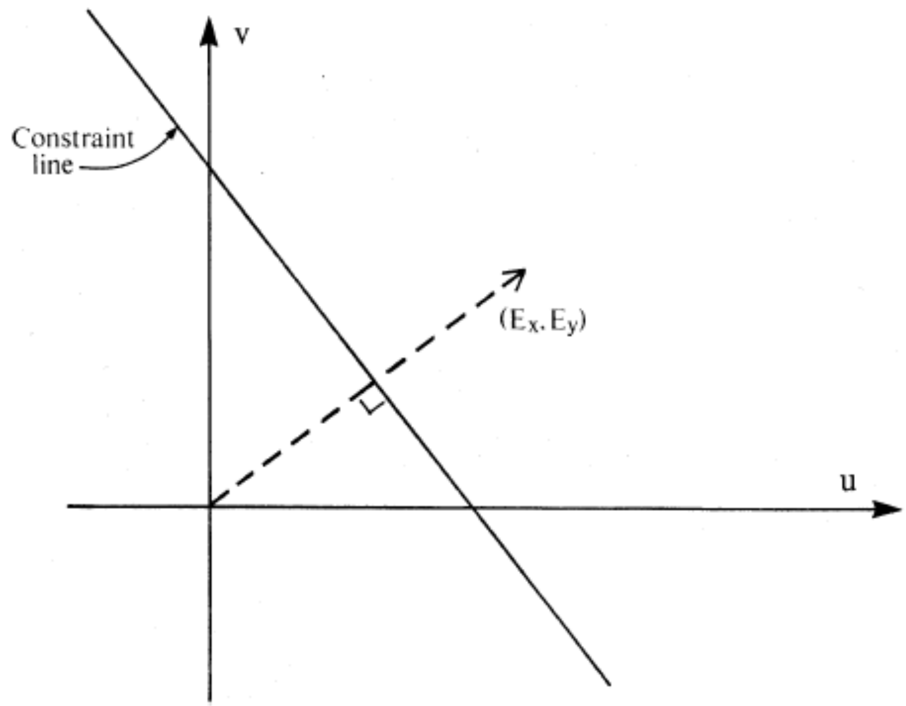
$$\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

which is the expansion of the total derivative

$$\frac{dE}{dt} = 0$$

short: $\boxed{E_x u + E_y v + E_t = 0}$

Interpretation



Values of (u, v) satisfying the constraint equation lie on a straight line in velocity space. A local measurement only provides this constraint line (aperture problem).

Normal flow $\mathbf{u}_n$

$$(E_x, E_y) \cdot (u, v) = -E_t$$

$$\text{Let } \mathbf{n} = \frac{(E_x E_y)^T}{\left\| (E_x, E_y)^T \right\|}$$

$$\mathbf{u}_n = (\mathbf{u} \cdot \mathbf{n})\mathbf{n} = \left(\frac{-E_x E_t}{\sqrt{E_x^{\ 2} + E_y^{\ 2}}}, \frac{-E_y E_t}{\sqrt{E_x^{\ 2} + E_y^{\ 2}}} \right)^T$$

- Additional constraints are necessary to estimate optical flow, for example, constraints on size of derivatives, or parametric models of the velocity field.
- Horn and Schunck (1981): global smoothness term

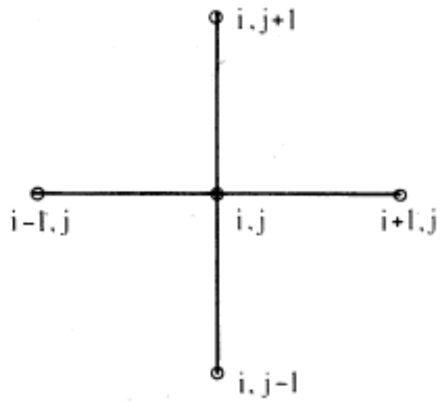
$$e_s = \iint_D \left(u_x^{\ 2} + u_y^{\ 2}\right) + \left(v_x^{\ 2} + v_y^{\ 2}\right) dx\, dy : \text{departure from smoothness}$$

$$e_c = \iint_D \left(E_x u + E_y v + E_t\right)^2 dx\, dy : \text{error in optical flow constraint equation}$$

Let $\nabla A = \left(A_x, A_y\right)^T$ denote the gradient of A

$$\iint (\nabla E \cdot \mathbf{u} + E_t)^2 + \lambda\left(\left\|\nabla u\right\|_2^{\ 2} + \left\|\nabla v\right\|_2^{\ 2}\right) dx\, dy \to \min$$

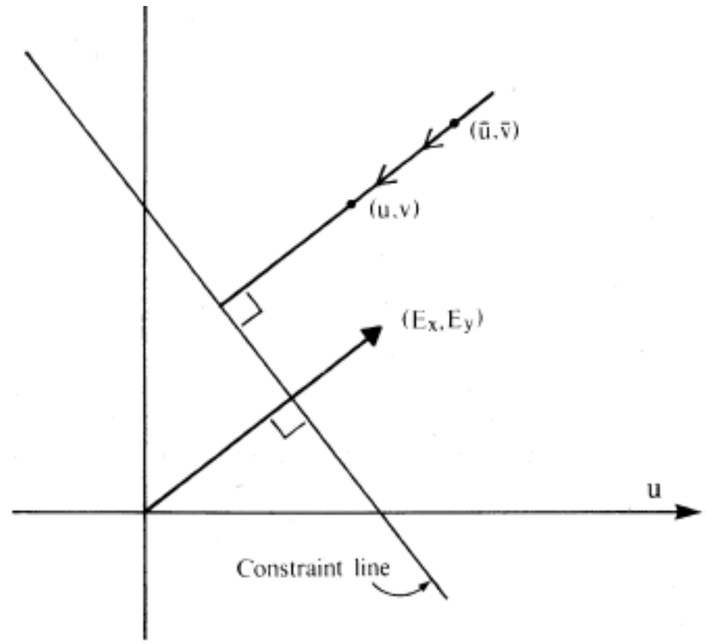
- This approach is called regularization.
- Solve by means of calculus of variation.



$\bar{u}, \bar{v}$ denotes local averages of u and v

$$u^{n+1} = \bar{u}^{n} - \frac{\left(E_x \bar{u}^{n} + E_y \bar{v}^{n} + E_t\right)}{\frac{1}{\lambda} + E_x^{\,2} + E_y^{\,2}} E_x$$

$$v^{n+1} = \bar{v}^{n} - \frac{\left(E_x \bar{u}^{n} + E_y \bar{v}^{n} + E_t\right)}{\frac{1}{\lambda} + E_x^{\,2} + E_y^{\,2}} E_y$$



In the iterative scheme for estimating the optical flow, the new value (u, v) at a point is the average of the values of the neighbors $(\bar{u}, \bar{v})$, minus an adjustment in the direction toward the constraint line.