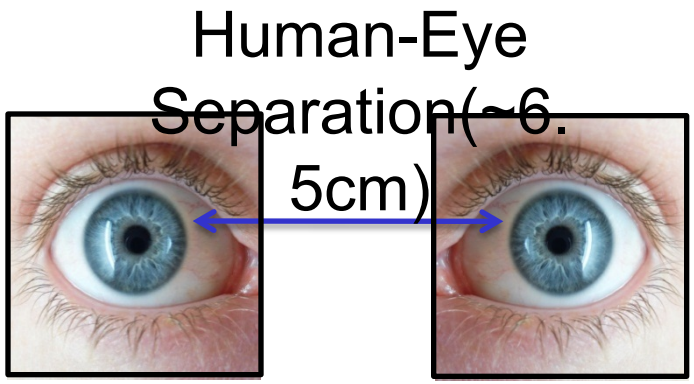
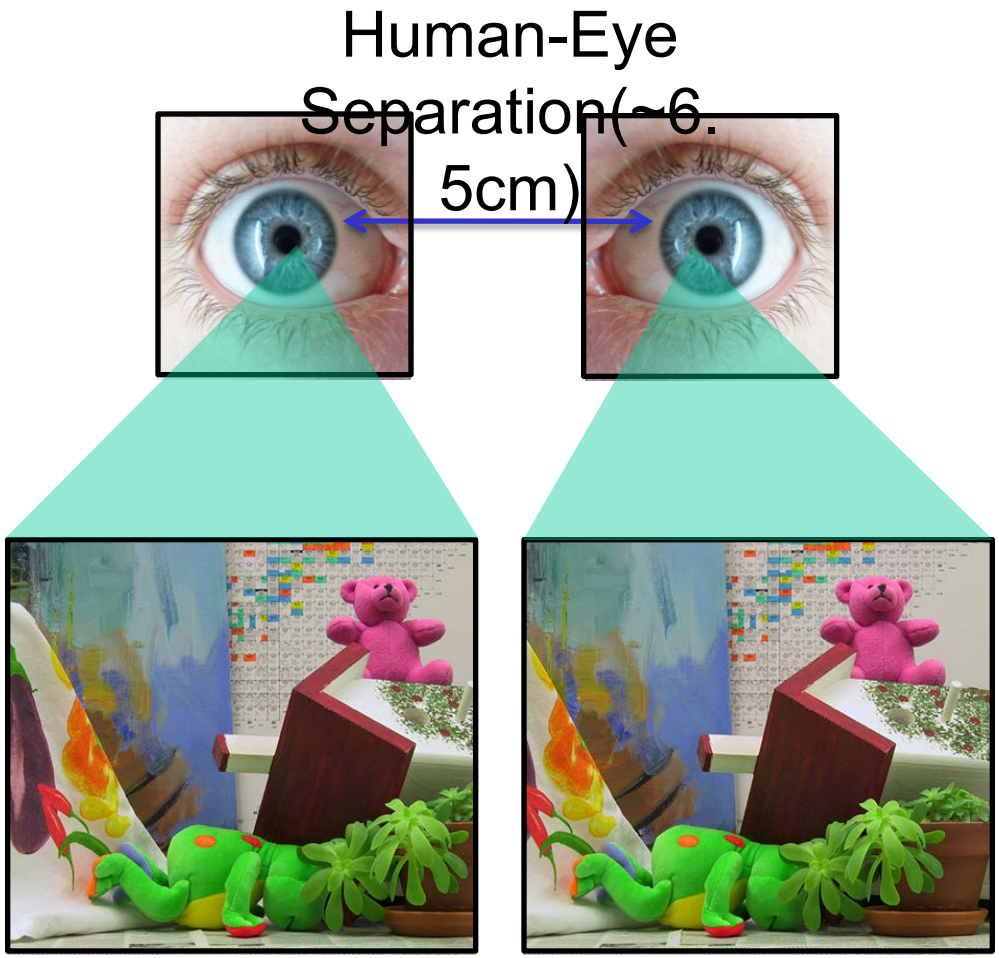
These Slides are from Dr. V. Singh, CS Department, UW Madison

3D Perception

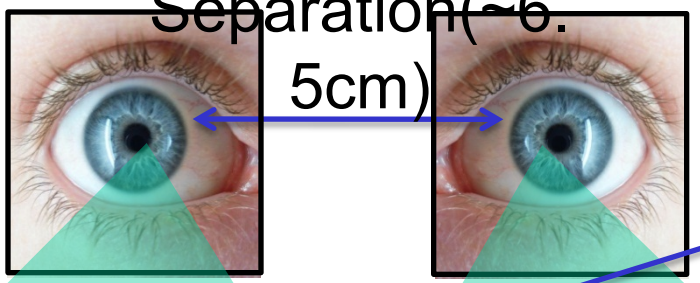


Many stereo slides from Michael Bleyer

3D Perception

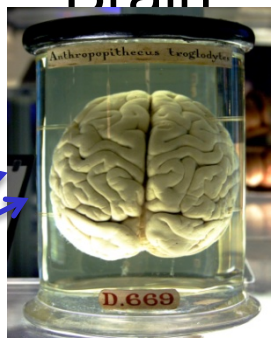


3D Perception



Human-Eye Separation(~6.5cm)

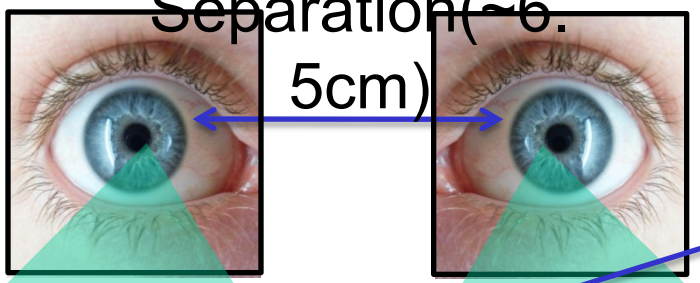
Brain



Left 2D Image

Right 2D Image

3D Perception



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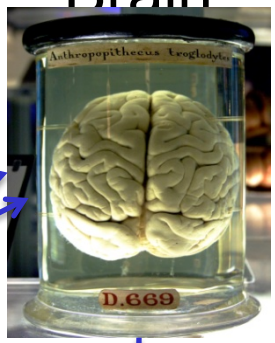


Brain

Left 2D Image

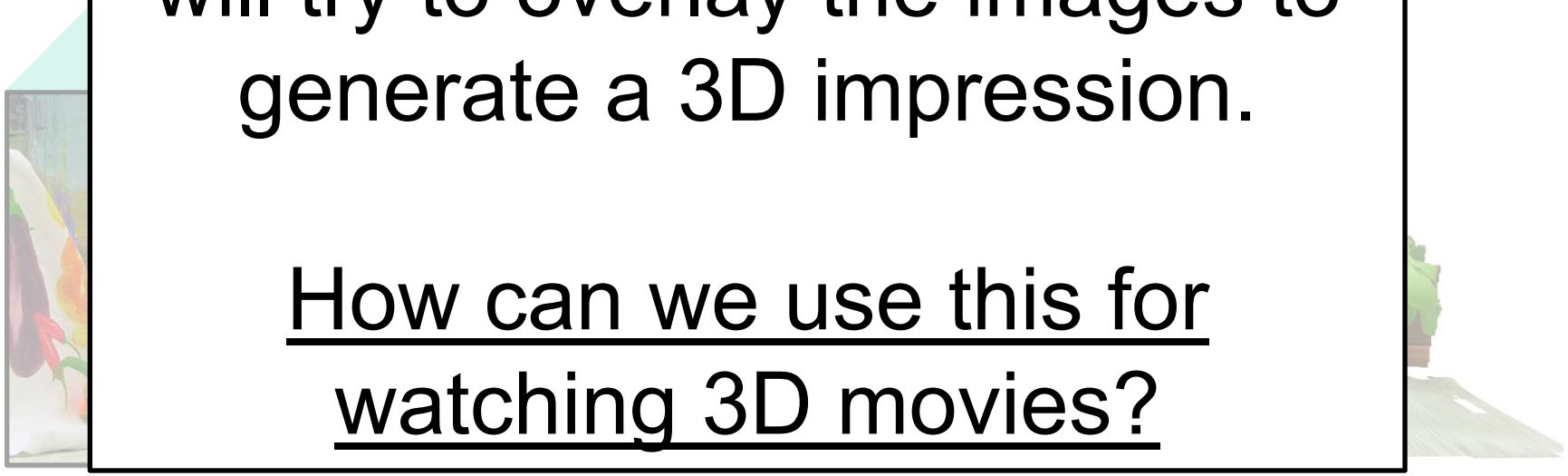
Right 2D Image

3D View

3D Perception

If we ensure that the left eye sees a 2D image and the right eye sees another one, our brain will try to overlay the images to generate a 3D impression.

<u>How can we use this for watching 3D movies?</u>



Left 2D Image

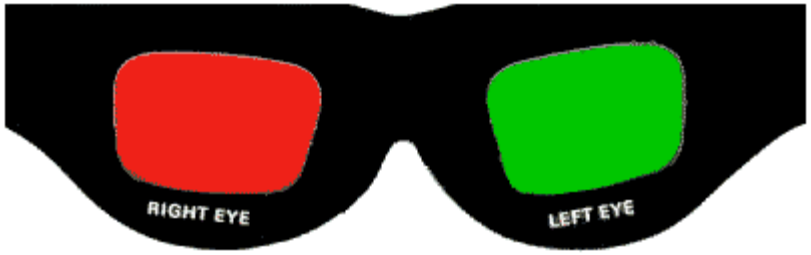
Right 2D Image

3D View

Anaglyphs

- Two images of complementary color are overlaid to generate one image.
- Glasses required (e.g. red/green)
- Red filter cancels out red image component, green filter cancels out green component
- Each eye gets one image => 3D impression
- Current 3D cinemas use this principle. However, use polarization filters

(Anaglyph



(Red/Green Glasses)

Shutter Glasses

- Display flickers between left and right image (i.e. each even frame shows left image, each odd frame shows right image)

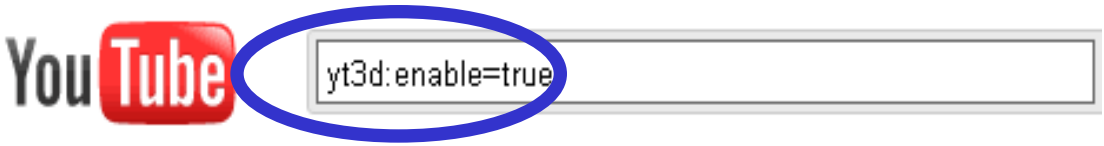
(Shutter Glasses and 120 Hz

- When left frame is shown, shutter glasses close right eye and vice versa

- Requires new displays of high frame rate

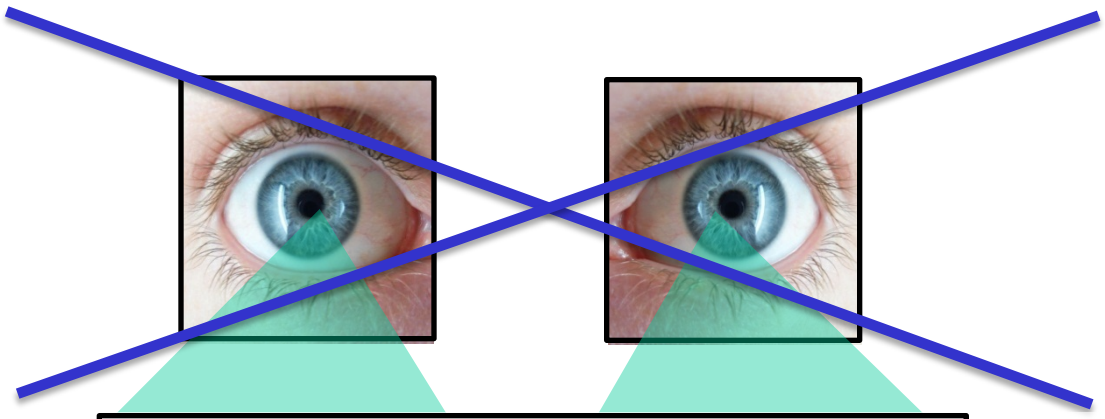
(Nvidea Artwork)

3D on YouTube

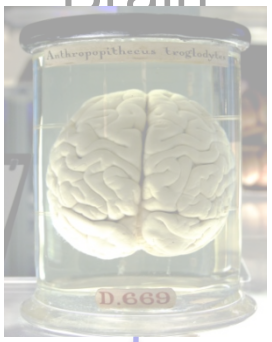


Computational Stereo

Computational Stereo



Brain



Replace human eyes with a ~~pair of slightly displaced~~ cameras.

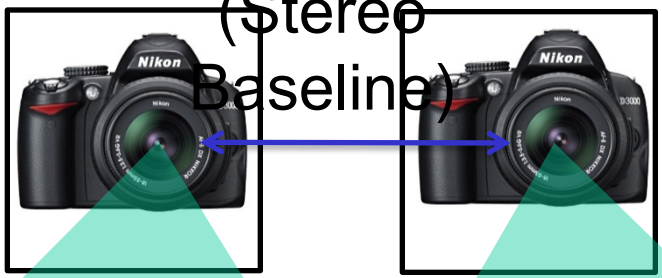
Left 2D Image

Right 2D Image

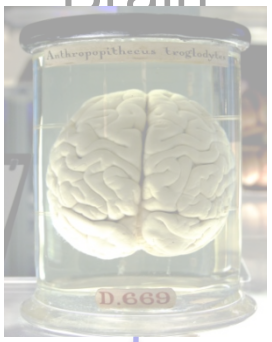
3D View

Computational Stereo

Displacement (Stereo Baseline)



Brain



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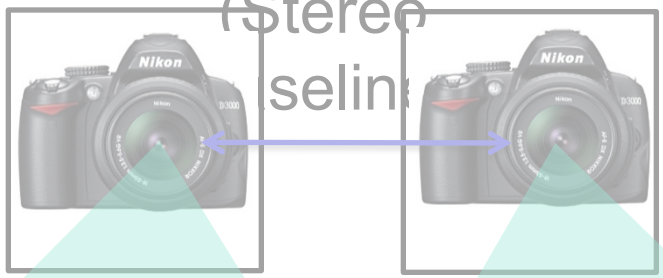
Left 2D Image

Right 2D Image

3D View

Computational Stereo

Displacement
(Stereo
(seli



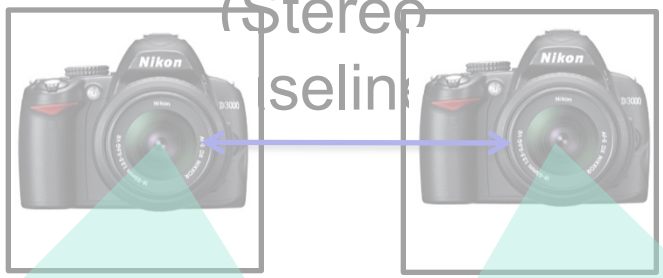
Left 2D Image

Right 2D
Image

Brain

Computational Stereo

Displacement
(Stereo
baseline)

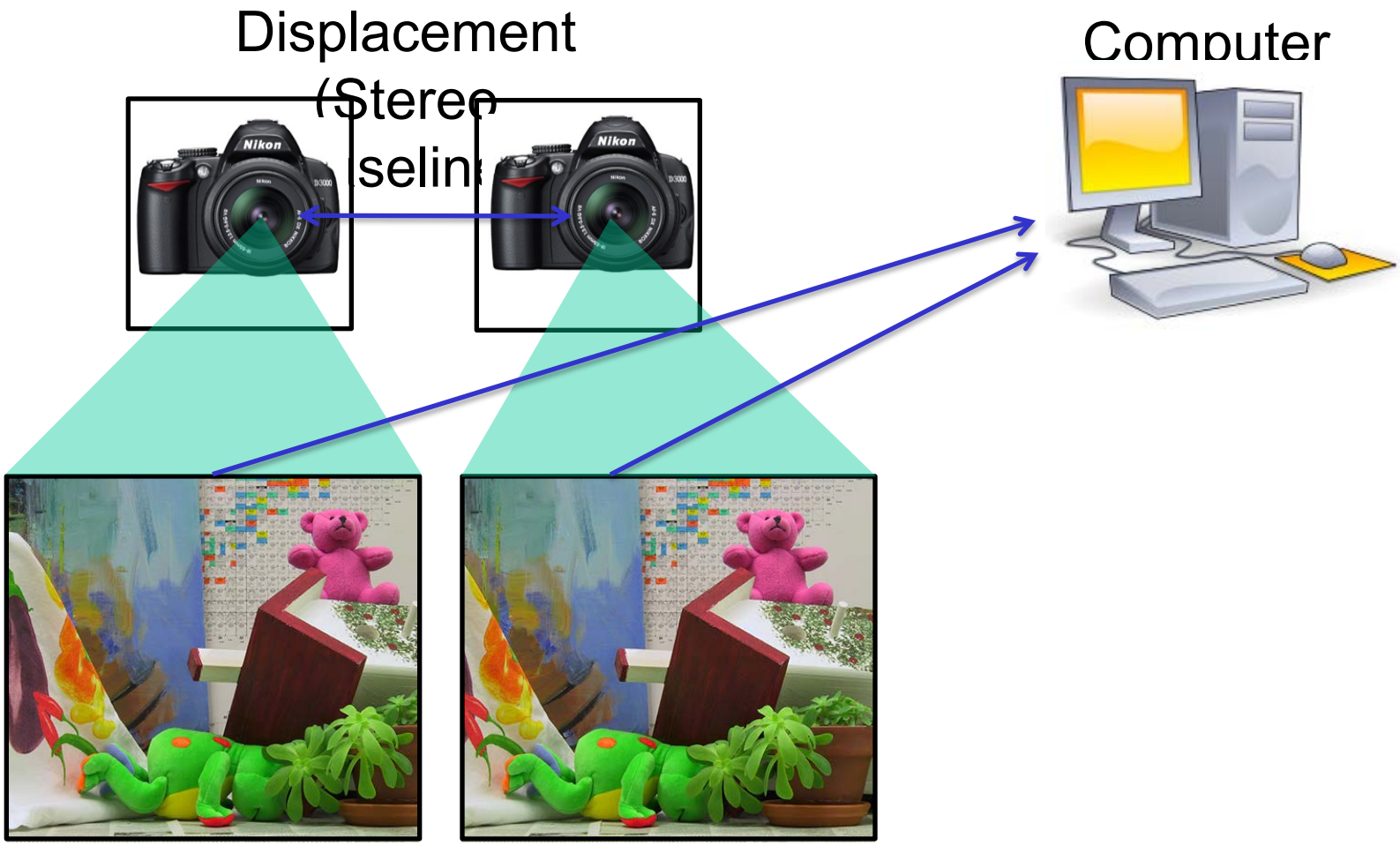


Computer

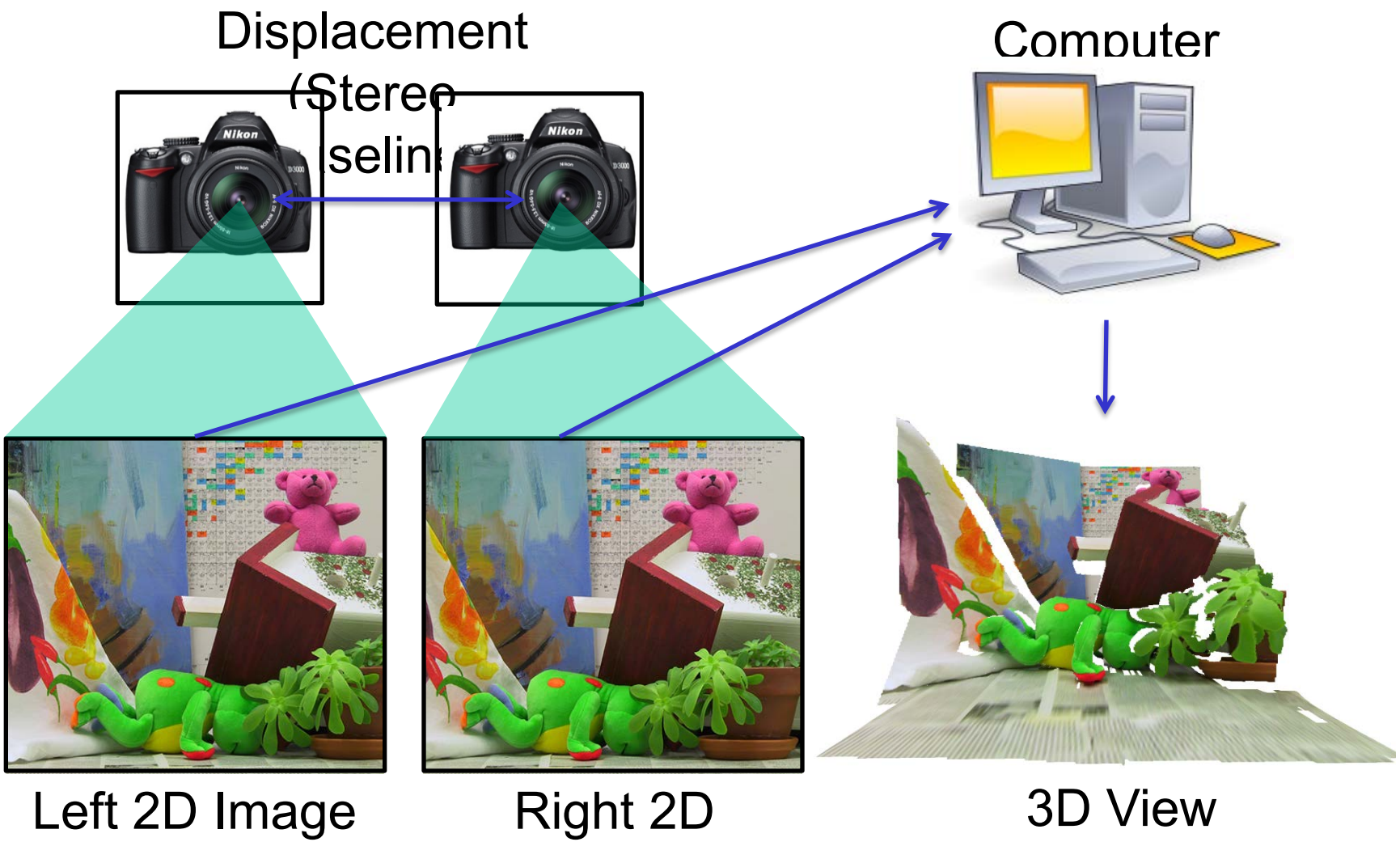
Left 2D Image

Right 2D Image

Computational Stereo



Computational Stereo



Computational Stereo

Displacement (Stereo (selin

Computer

How can we accomplish a fully automatic 2D to 3D conversion?

Left 2D Image

Right 2D Image

3D View

What is Disparity?

- The amount to which a single pixel is displaced in the two images is called disparity.
- A pixel's disparity is inversely proportional to its depth in the scene.

What is Disparity?

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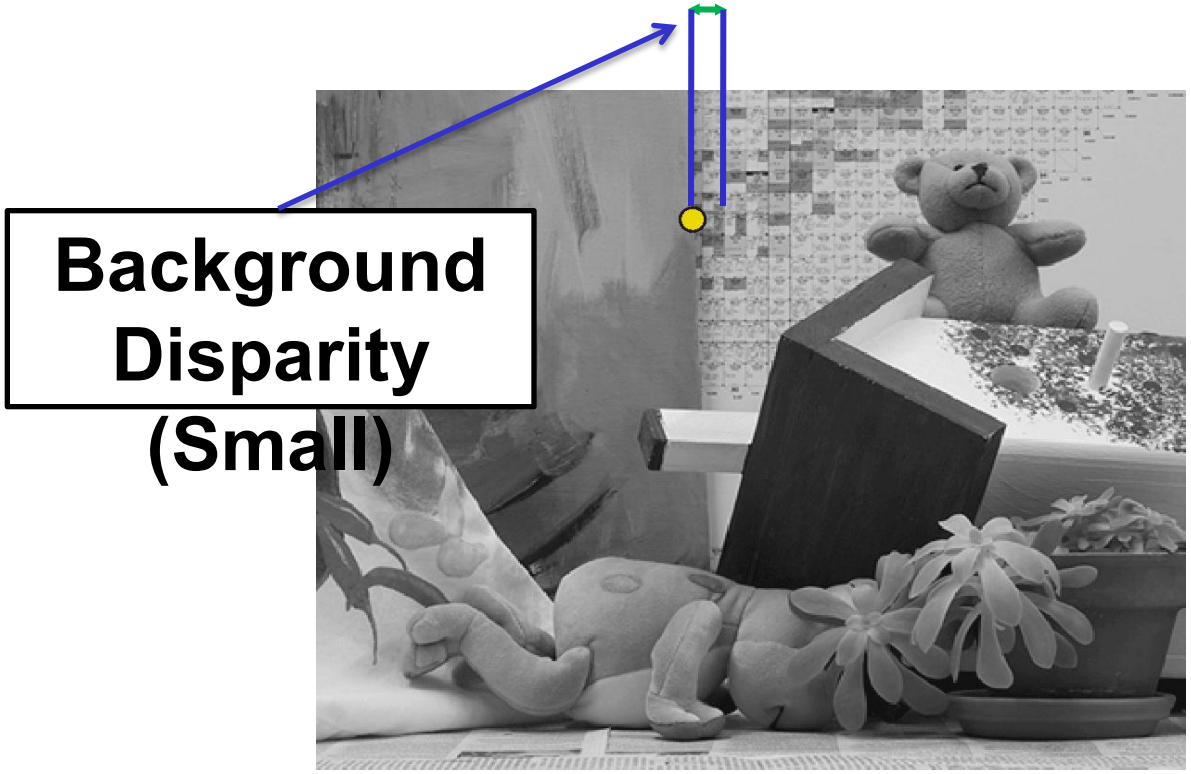
What is Disparity?



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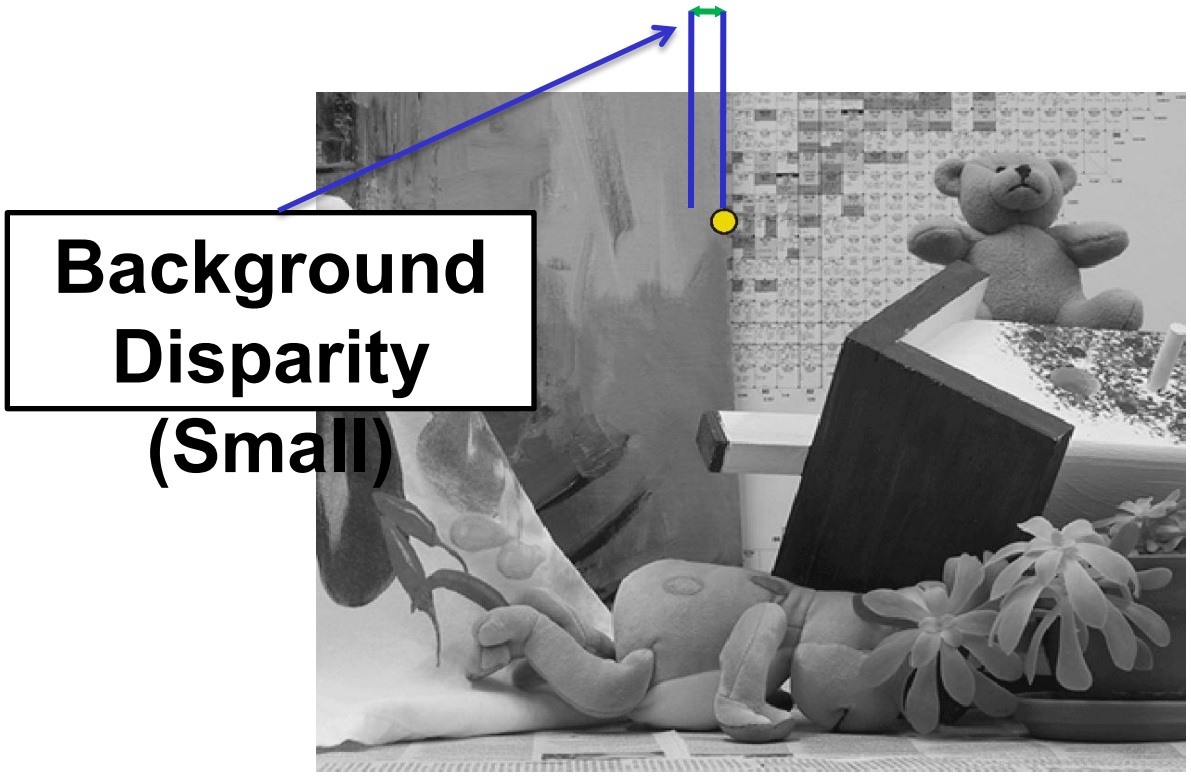

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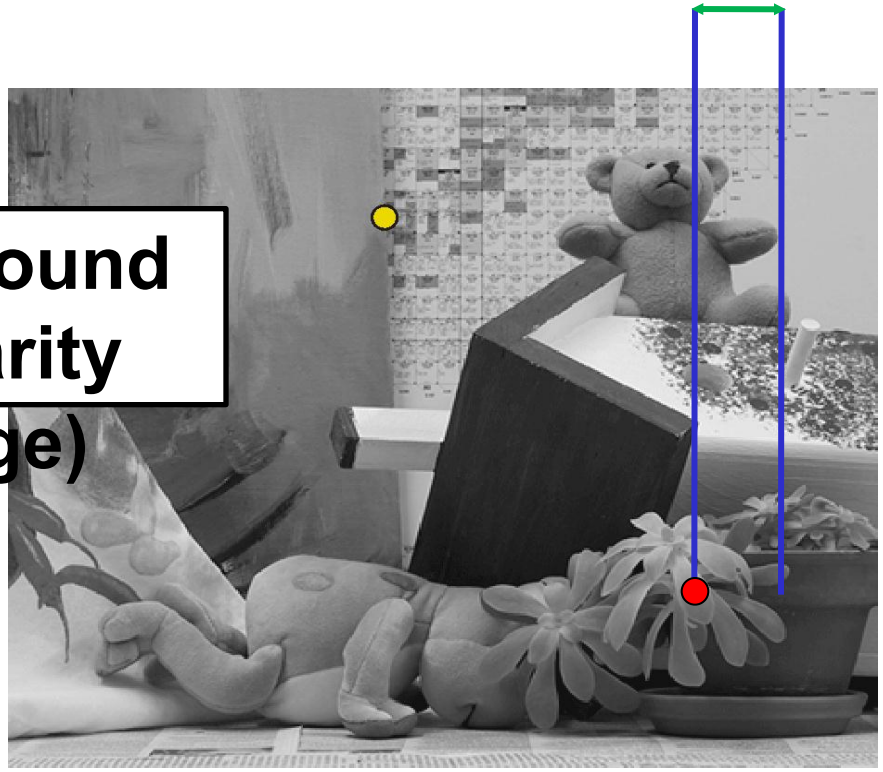

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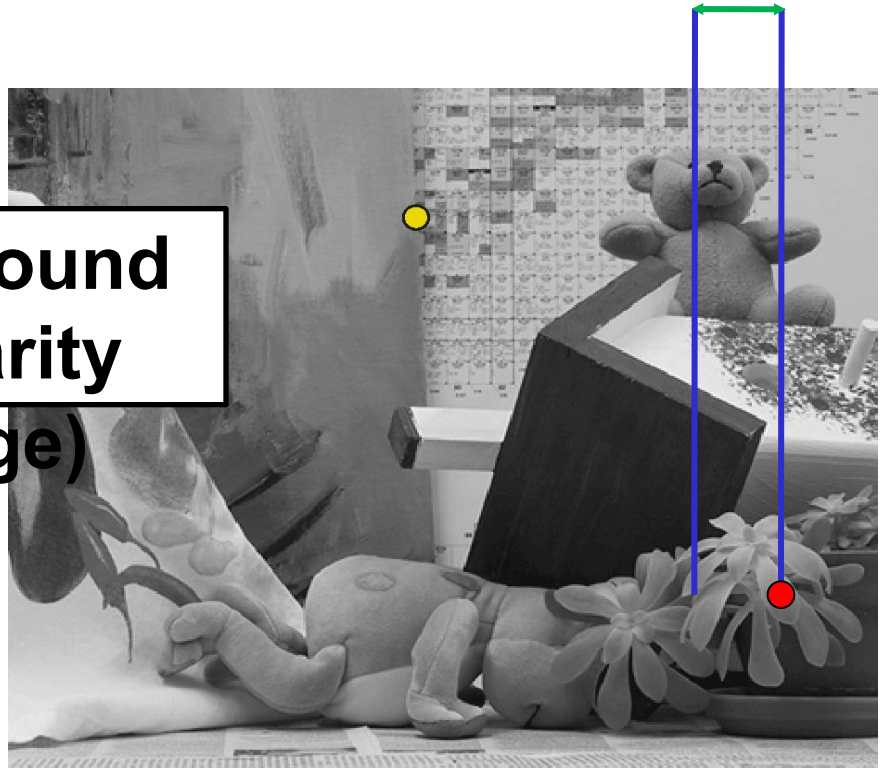

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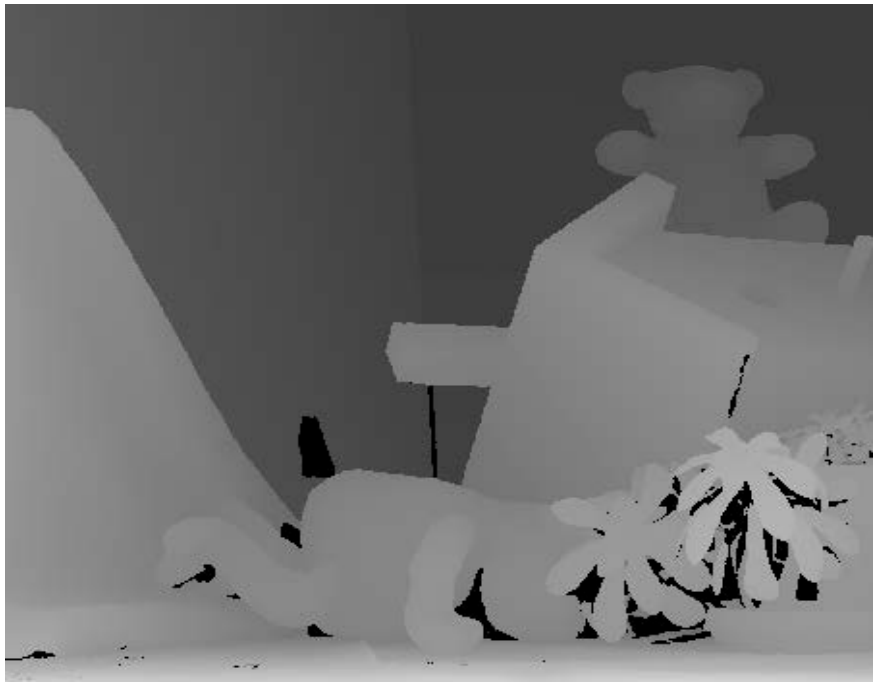
Disparity Encoding



- Disparity of each pixel is encoded by a gray value.
- High grey values represent high disparities (and low gray values small disparities).
- The resulting image is called disparity map.

Disparity and Depth

- The disparity map contains sufficient information for generating a 3D model.

Disparity and Depth

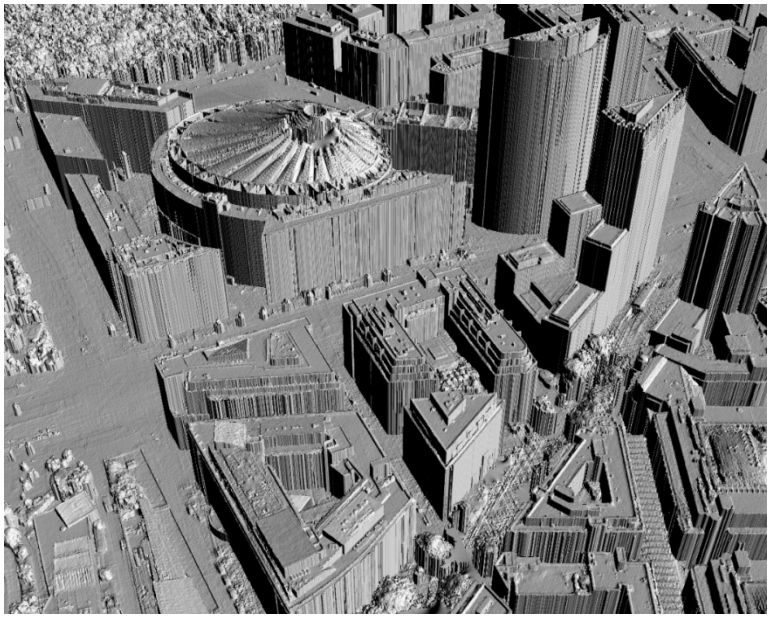
The challenging part is to compute the disparity map. This task is known as the stereo matching problem.

- The disparity map contains sufficient information for generating a 3D model.

Applications

(just a few examples)

3D Reconstruction from aerial images

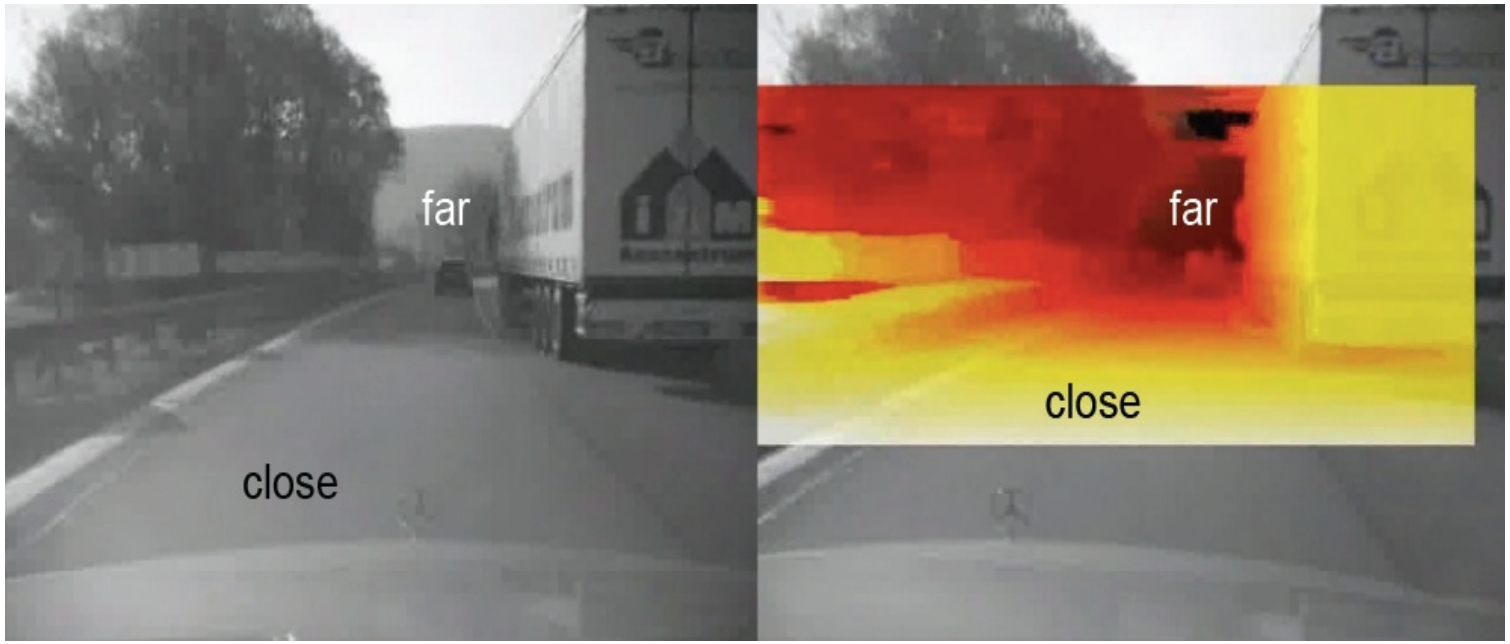


- Stereo cameras are mounted on an airplane to obtain a terrain map.
- Images taken from http://www.robotic.de/Heiko.Hirschmueller/

3D Reconstruction of Cities

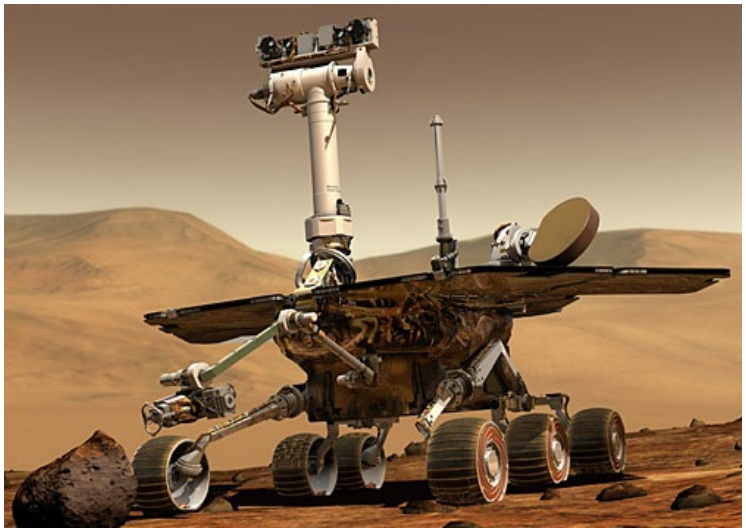
- City of Dubrovnik reconstructed from images taken from Flickr in a fully automatic way.
- [S. Agarwal, N. Snavely, I. Simon, S. Seitz and R. Szeliski “Building Rome in a Day”, ICCV, 2009]

Driver Assistance / Autonomous driving cars



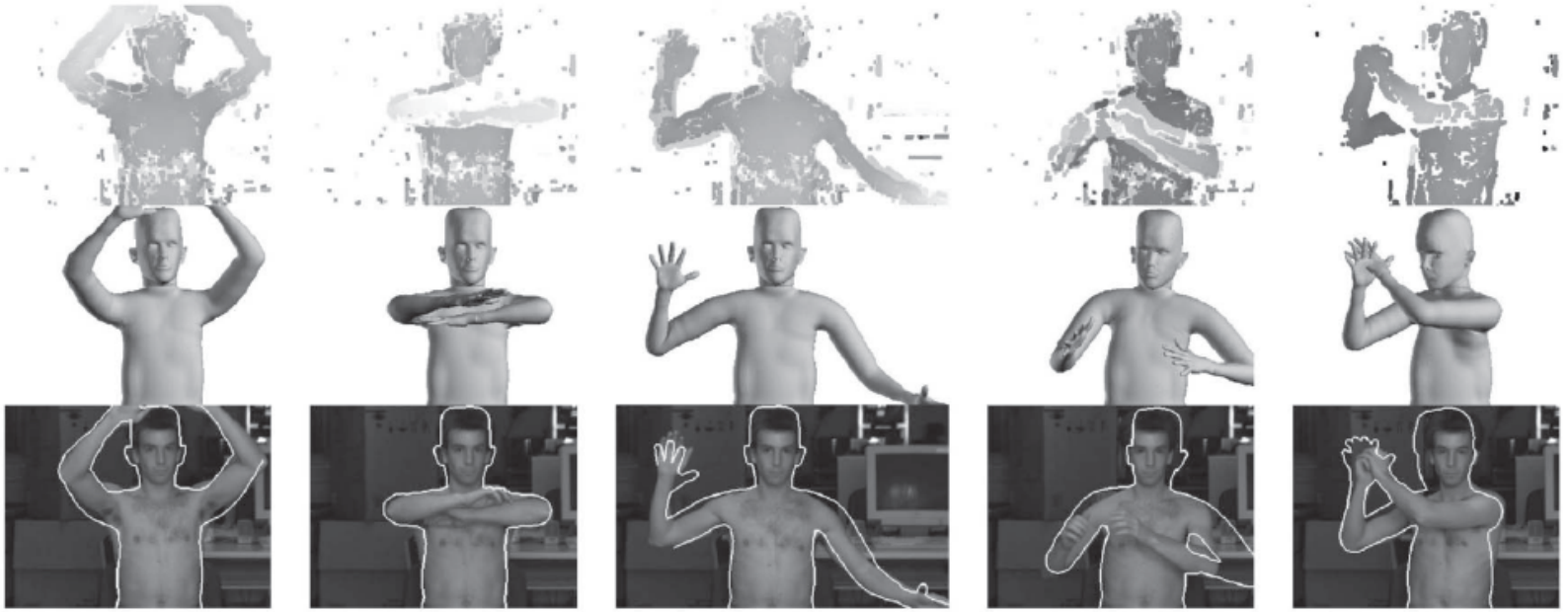
- For example, use stereo to measure distance to other cars.
- Image taken from http://www.cs.auckland.ac.nz/~rklette/talks/08_AI.pdf

The Mars Rover



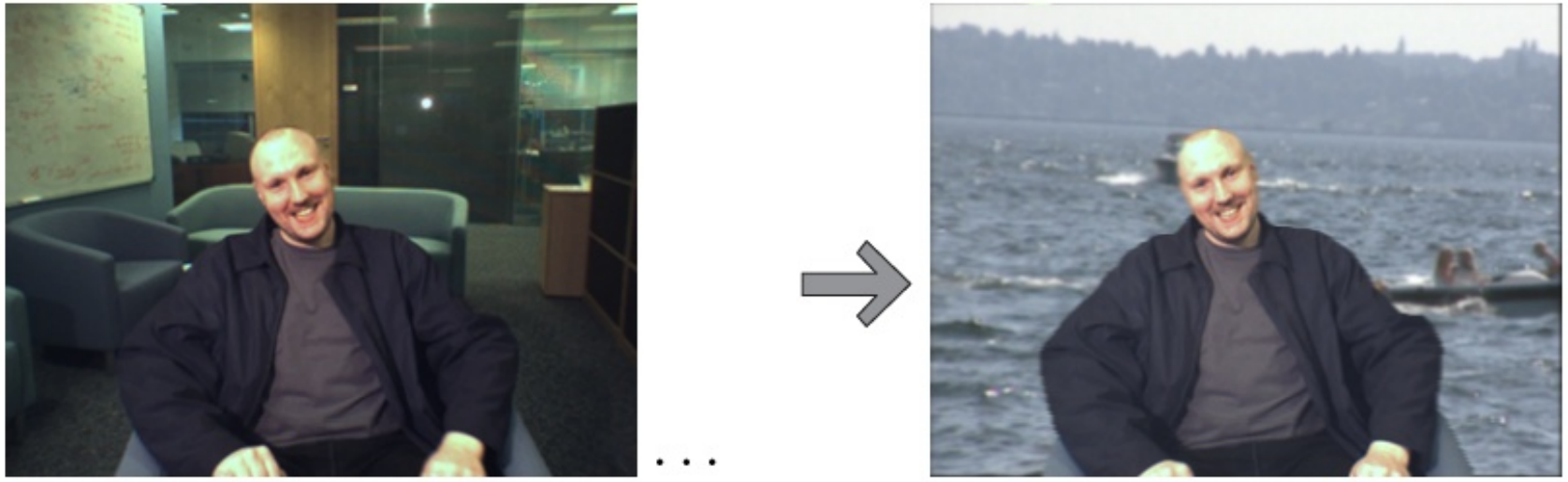
- Reconstruct the surface of Mars using stereo vision

Human Motion Capture



- Fit a 3D model of the human body to the computed point cloud.
- [R. Plänkers and P. Fua, “Articulated Soft Objects for Multiview Shape and Motion Capture”, PAMI, 2003]

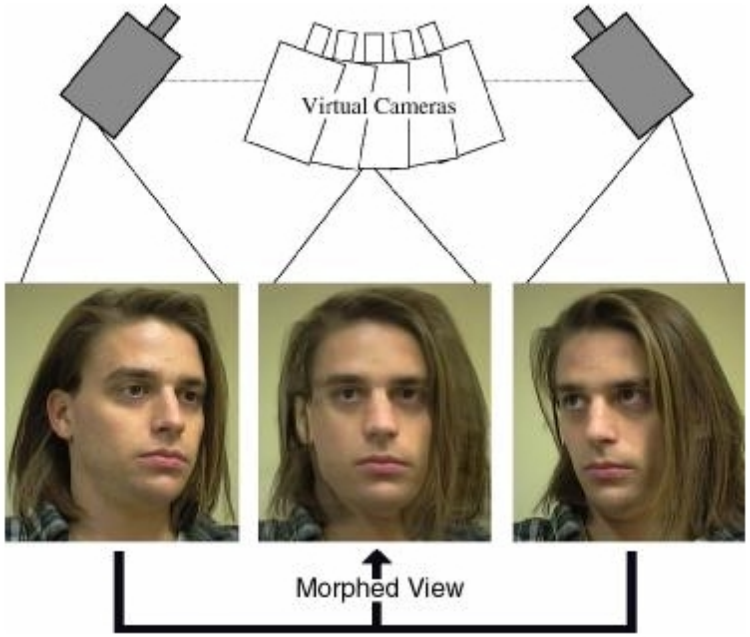
Bilayer Segmentation – Z-Keying

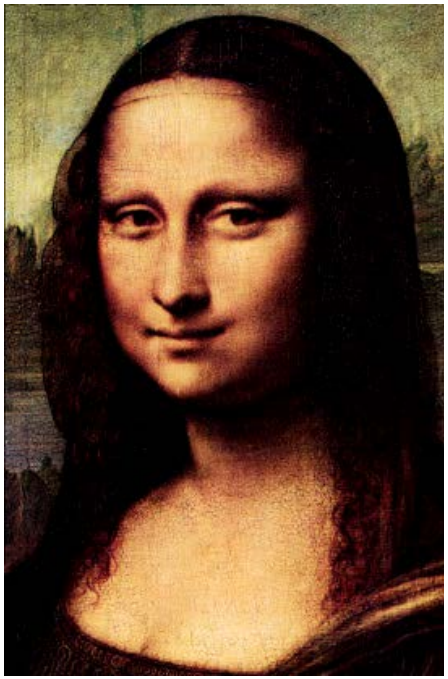


- Goal: Divide image into a foreground and a background region.
- Simple background subtraction will fail if there is motion in the background (see Jia's paper in ICCV 13).
- Solution:
  - Compute depth map
  - If the depth of a pixel is larger than a predefined threshold, pixels belongs to the foreground
- [A. Criminisi, G. Cross, A. Blake and V. Kolmogorov, "Bilayer Segmentation of Live Video", CVPR, 2006]

View Morphing

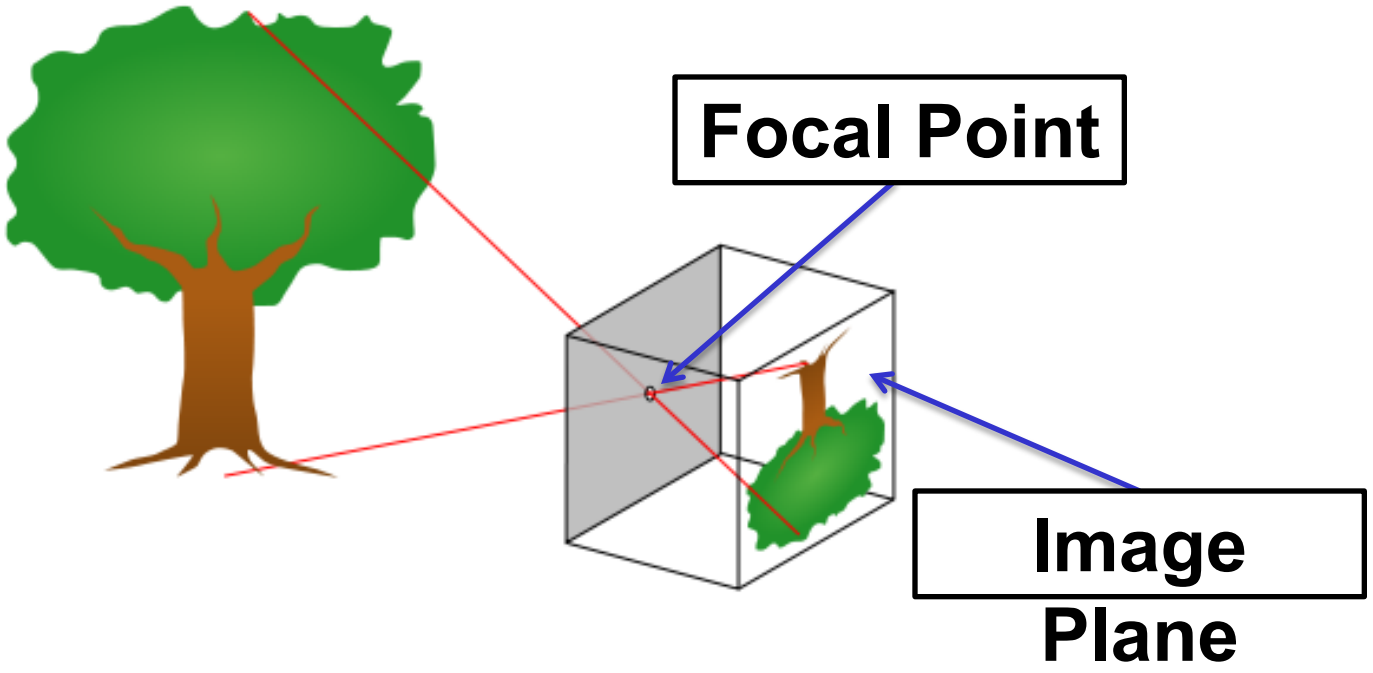
- Morph between pair of images using epipolar geometry [Seitz & Dyer, SIGGRAPH'96]





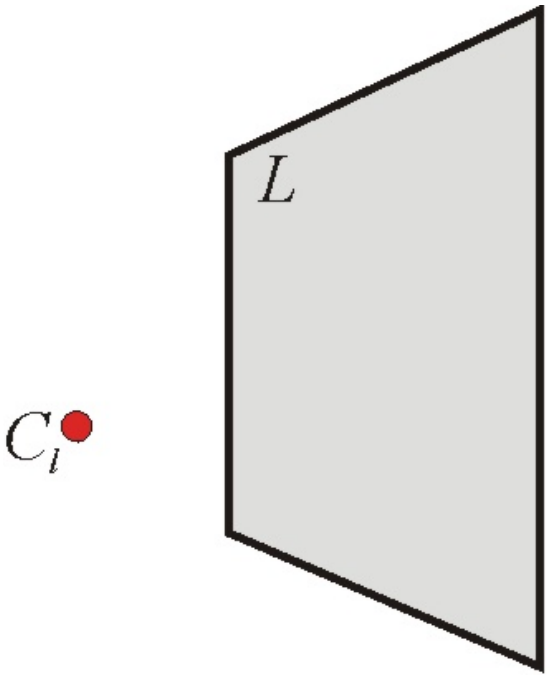
More technical version

Pinhole Camera



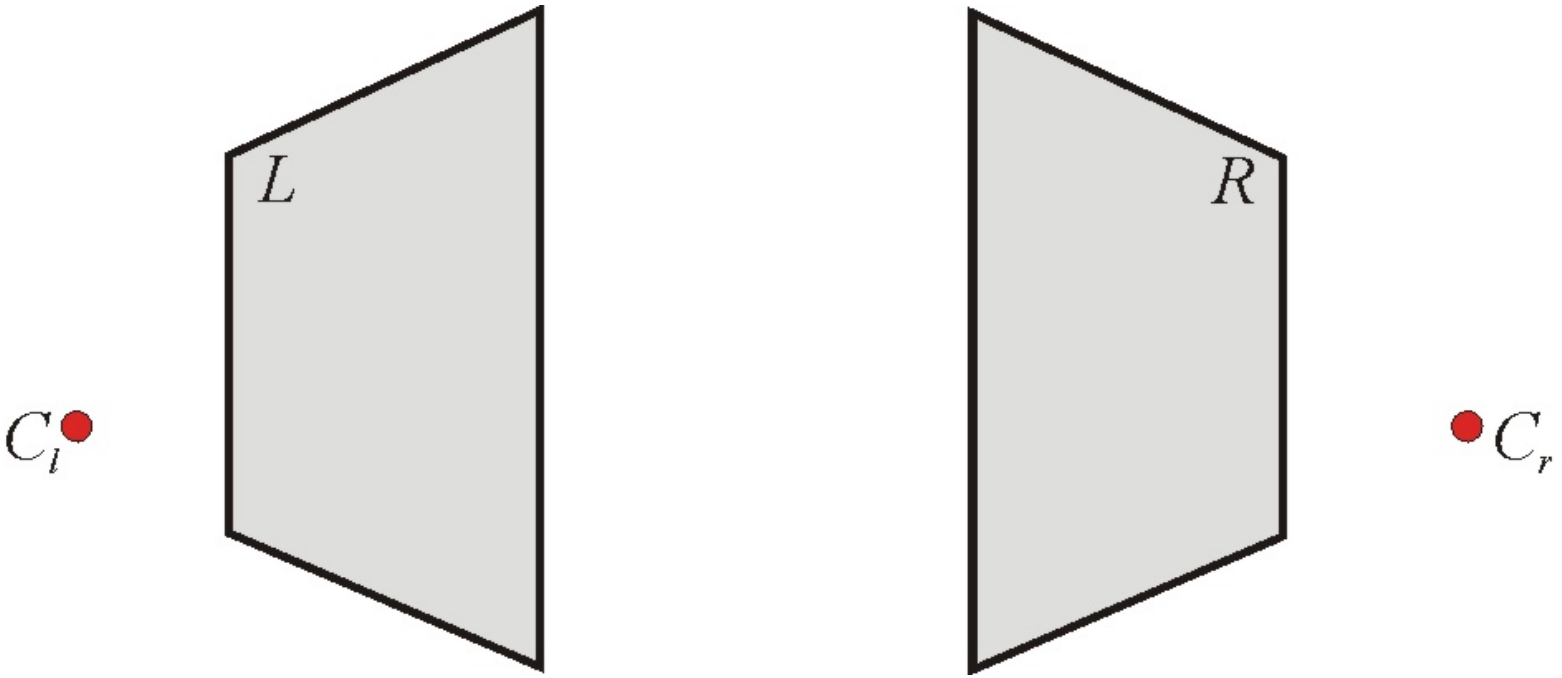
- Simplest model for describing the projection of a 3D scene onto a 2D image.
- Model is commonly used in computer vision.

Image Formation Process



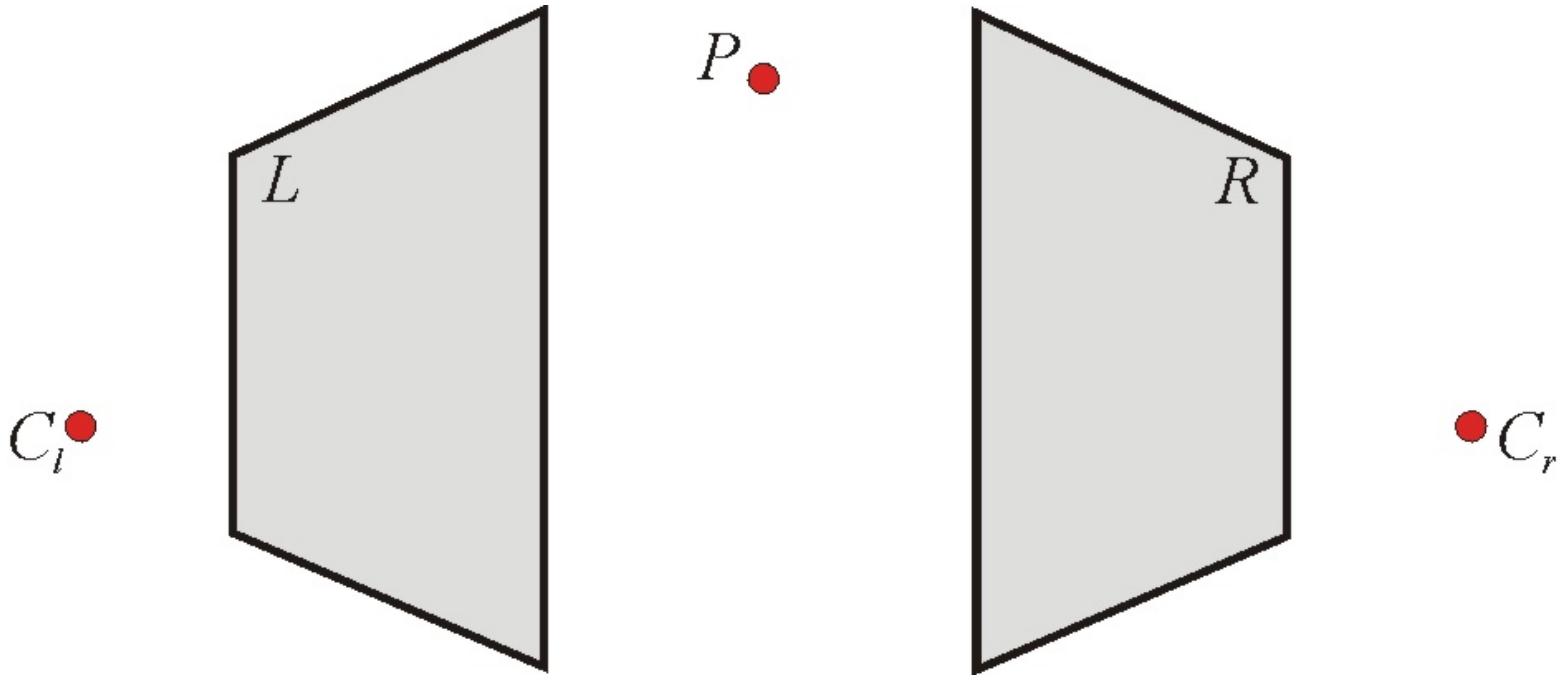
- Let us assume we have a pinhole camera.
- The pinhole camera is characterized by its focal point C_l and its image plane L.

Image Formation Process



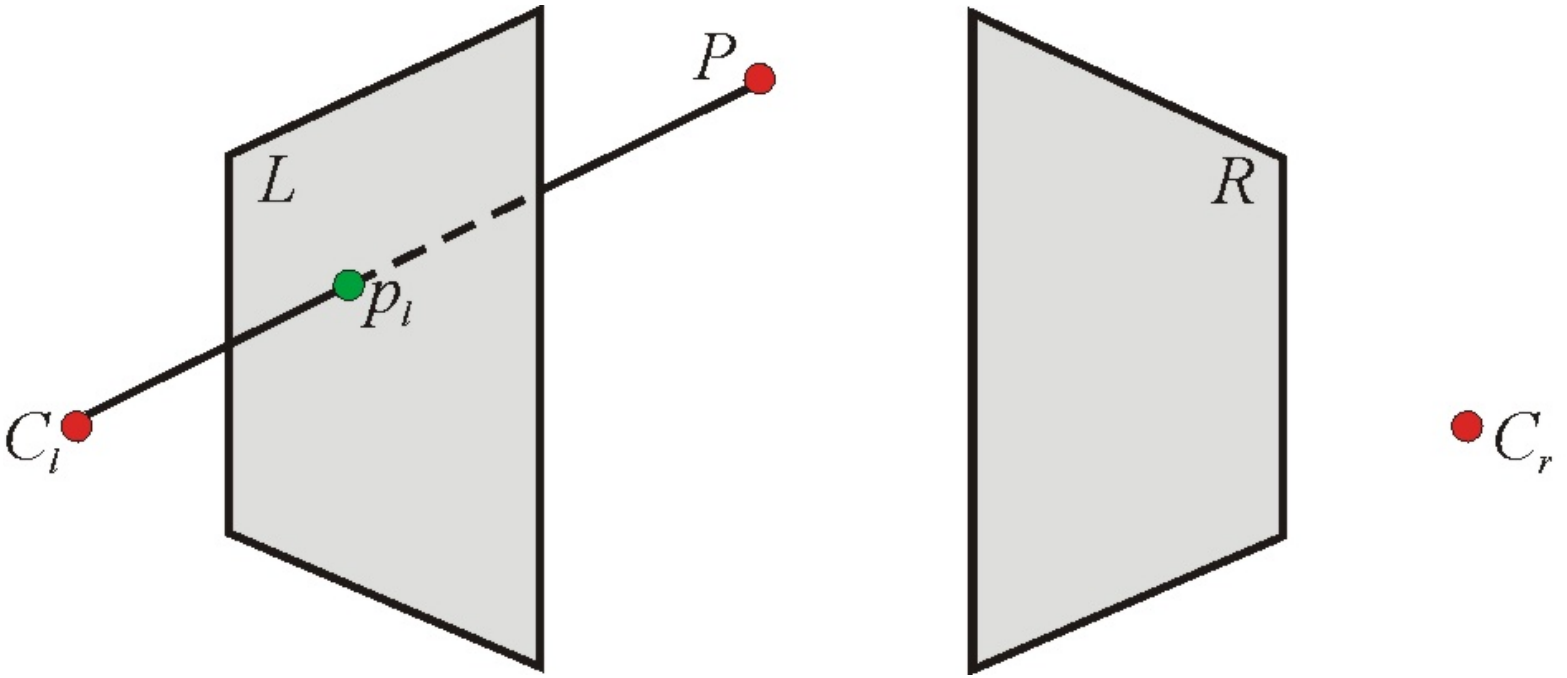
- We also have a second pinhole camera <C_r,R>.
- We assume that the camera system is fully calibrated, i.e. the 3D positions of <C_l, L> and <C_r,R> are known.

Image Formation Process



- We have a 3D point *P*.

Image Formation Process



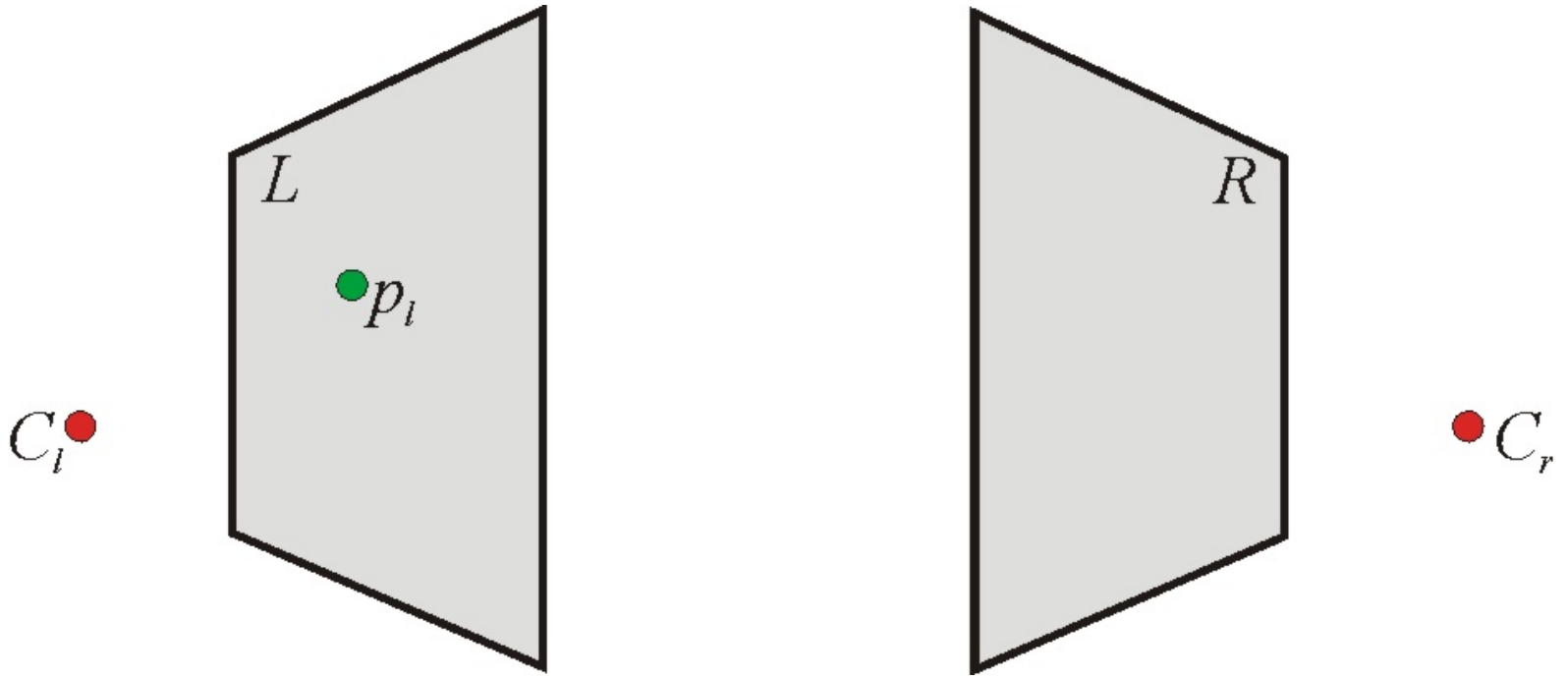
- We compute the 2D projection p_l of P onto the image plane of the left camera L by intersecting the ray from C_l to P with the plane L.
- This is what is happening when you take a 2D image of a 3D scene with your camera (image formation process).

Image Formation Process

Nice, but actually we want to do exactly the opposite: “We have a 2D image and want to make it 3D.”

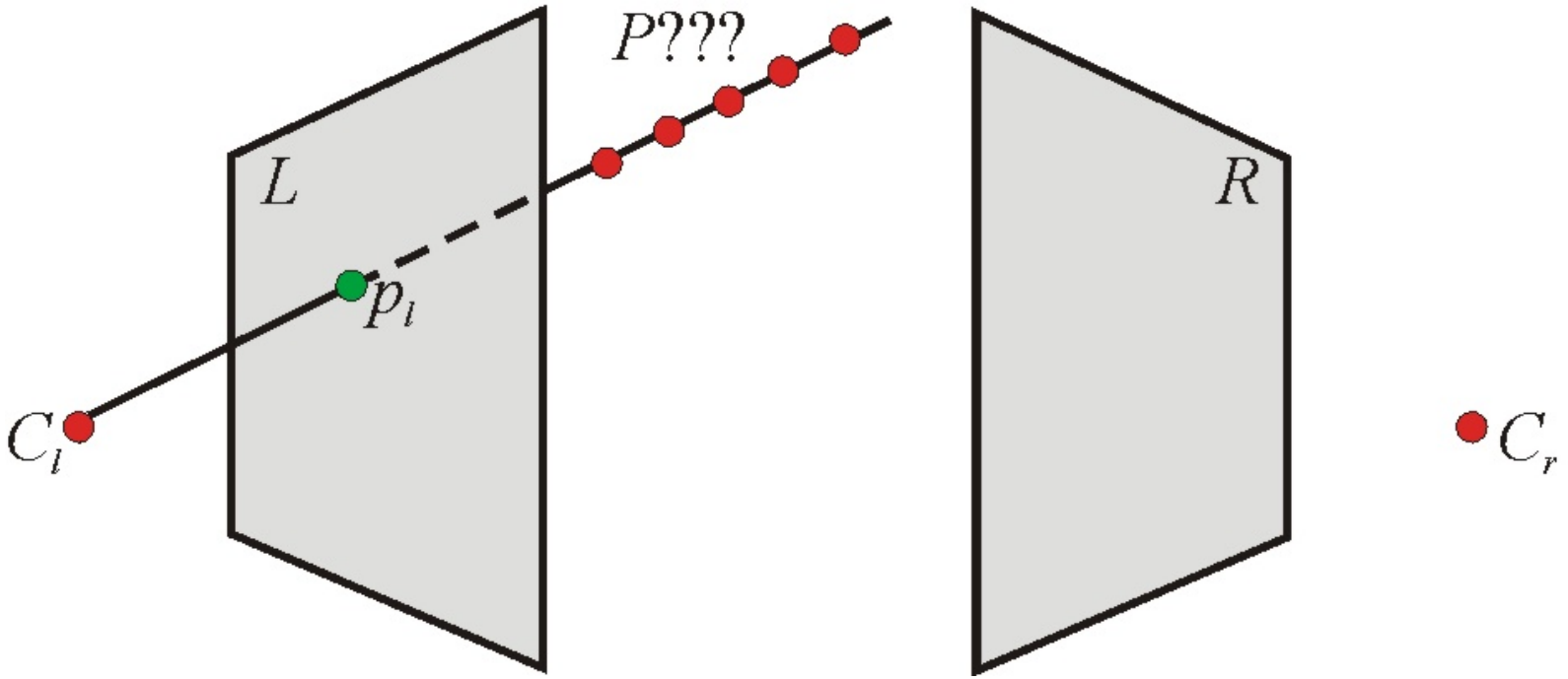
- We [illegible] the left camera *L* by intersecting the ray from C_l to *P* with the plane *L*.
- This is what is happening when you take a 2D image of a 3D scene with your camera (image formation process).

3D Reconstruction



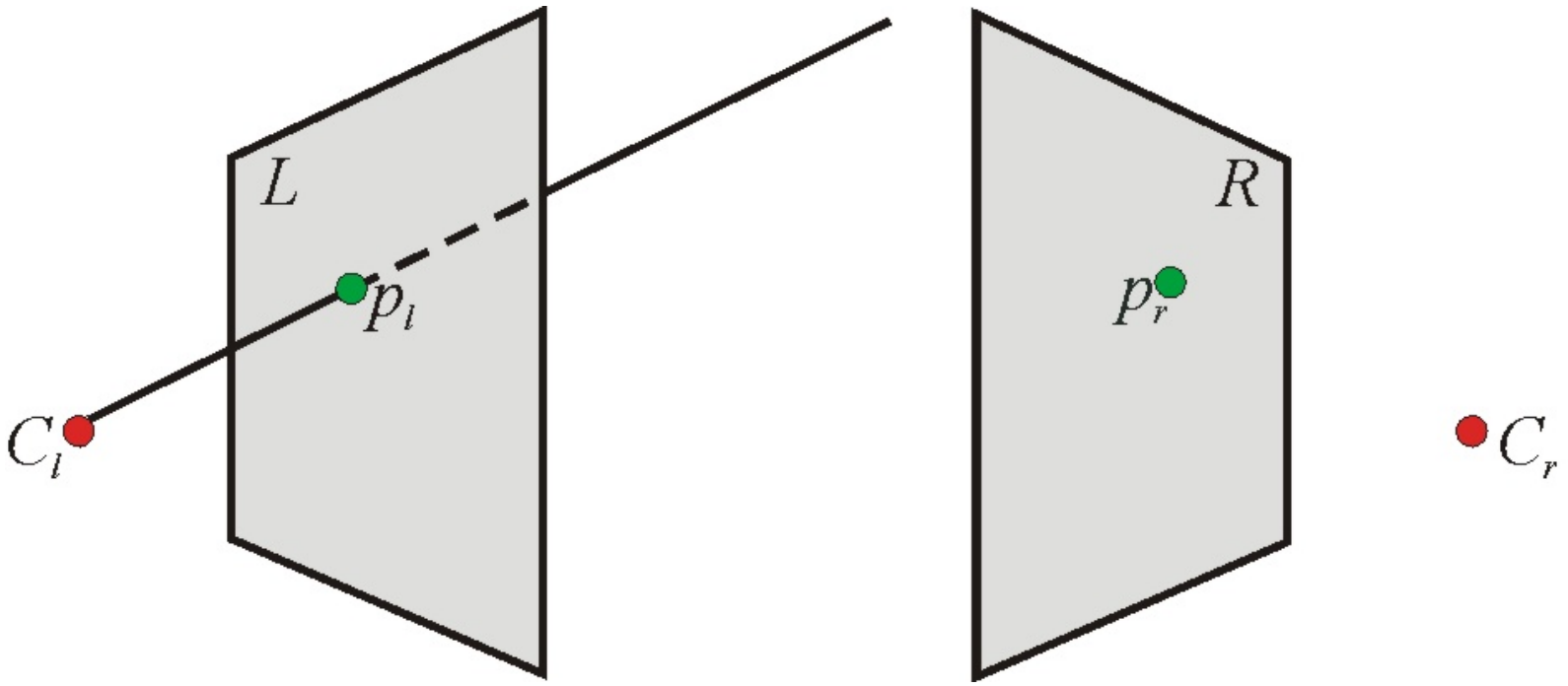
- Task: We have a 2D point p_l and want to compute its 3D position P.

3D Reconstruction



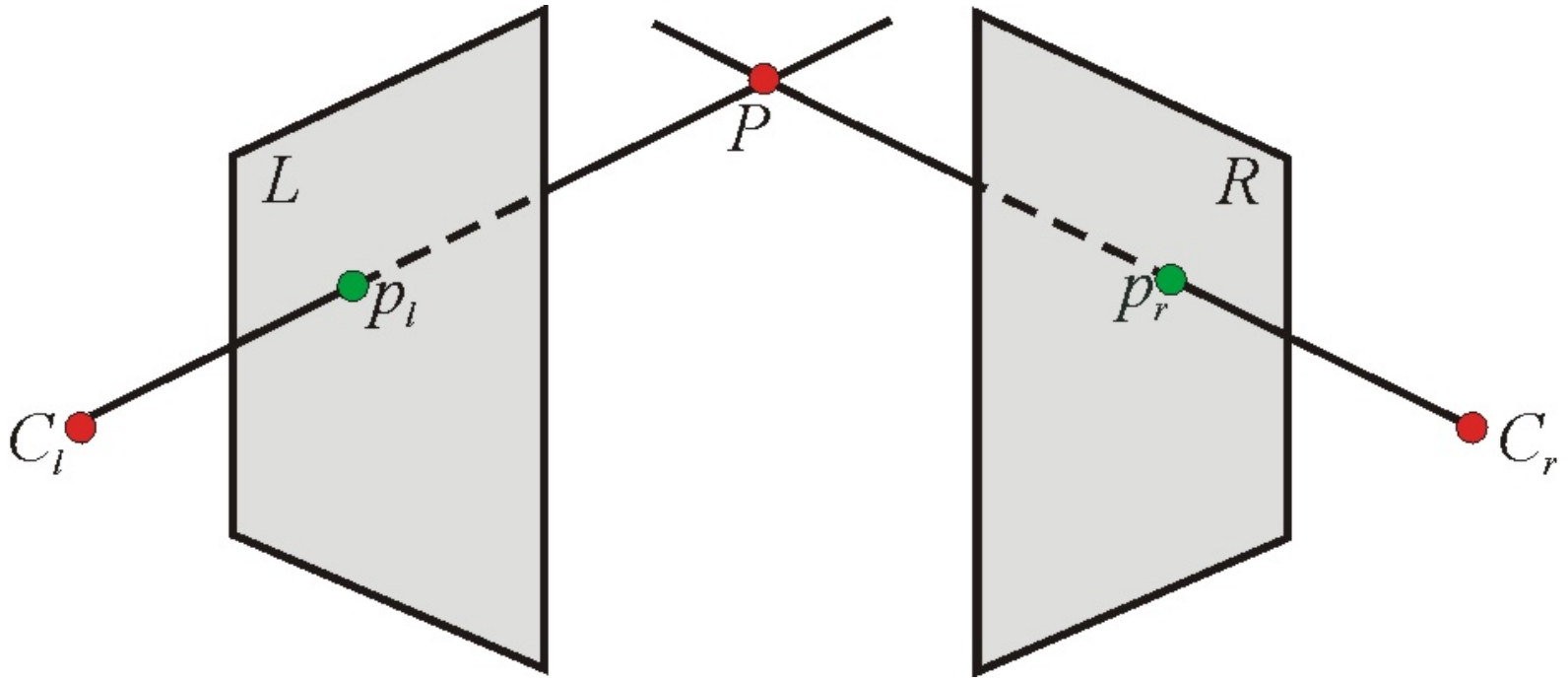
- *P* has to lie on the ray of C_l to p_l.
- Problem: It can lie anywhere on this ray.

3D Reconstruction



- Let us assume we also know the 2D projection p_r of P onto the right image plane R.

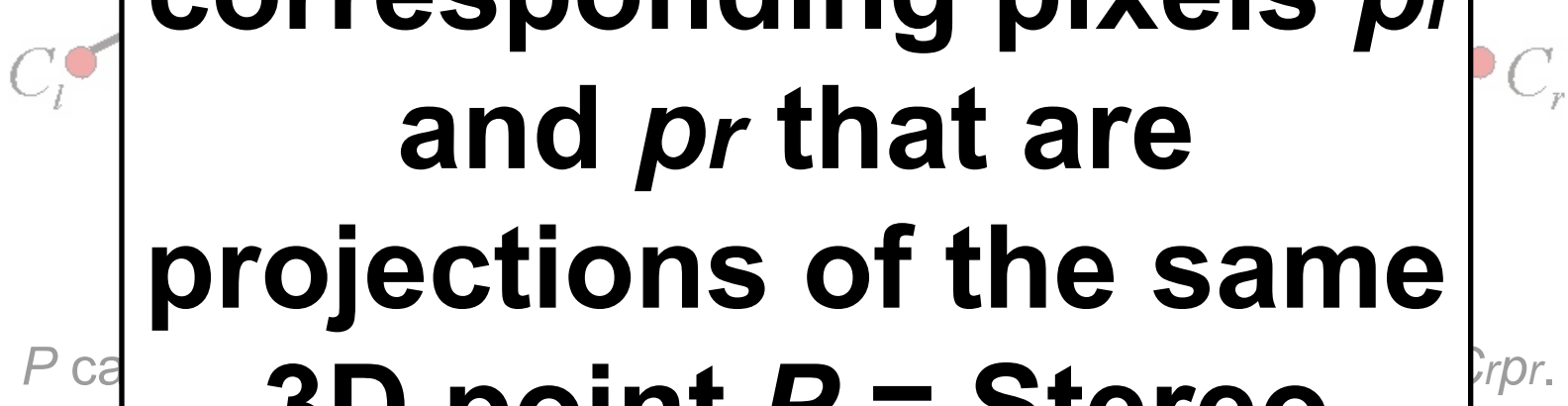
3D Reconstruction



- *P* can now be reconstructed by intersecting the rays *Clpl* and *Crpr*.

3D Reconstruction

The challenging part is to find the pair of corresponding pixels p_l and p_r that are projections of the same 3D point P = Stereo Matching Problem



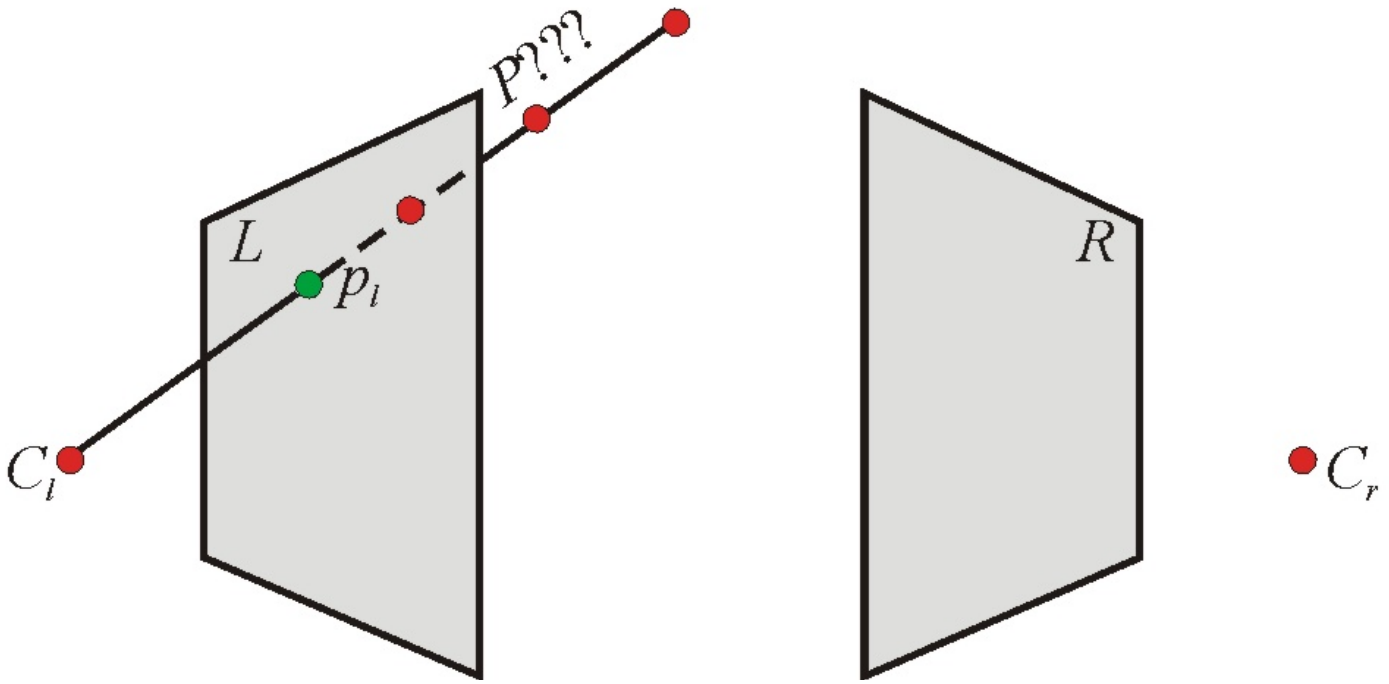
- *P ca* ... *Crpr*.

3D Reconstruction

Problem: Given p_l, the corresponding pixel p_r can lie at any x- and y-coordinate in the right image.

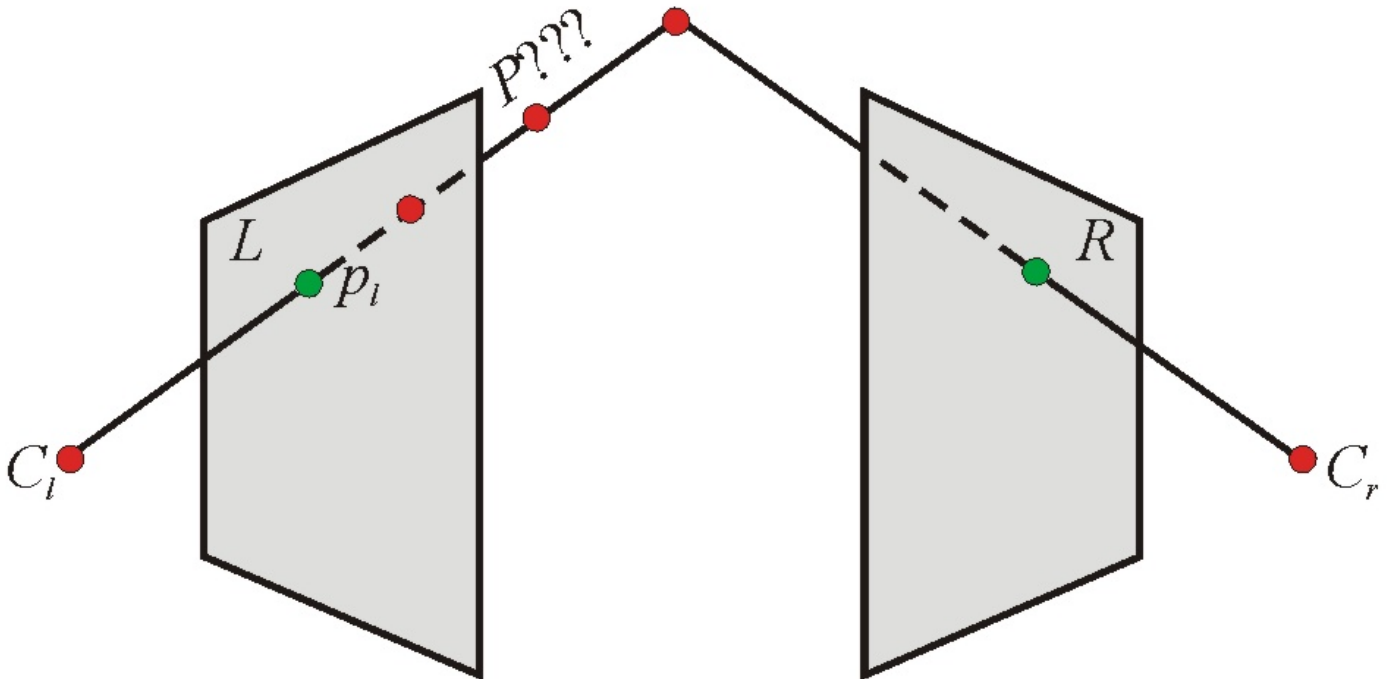
Can we make the search easier?

Epipolar Geometry



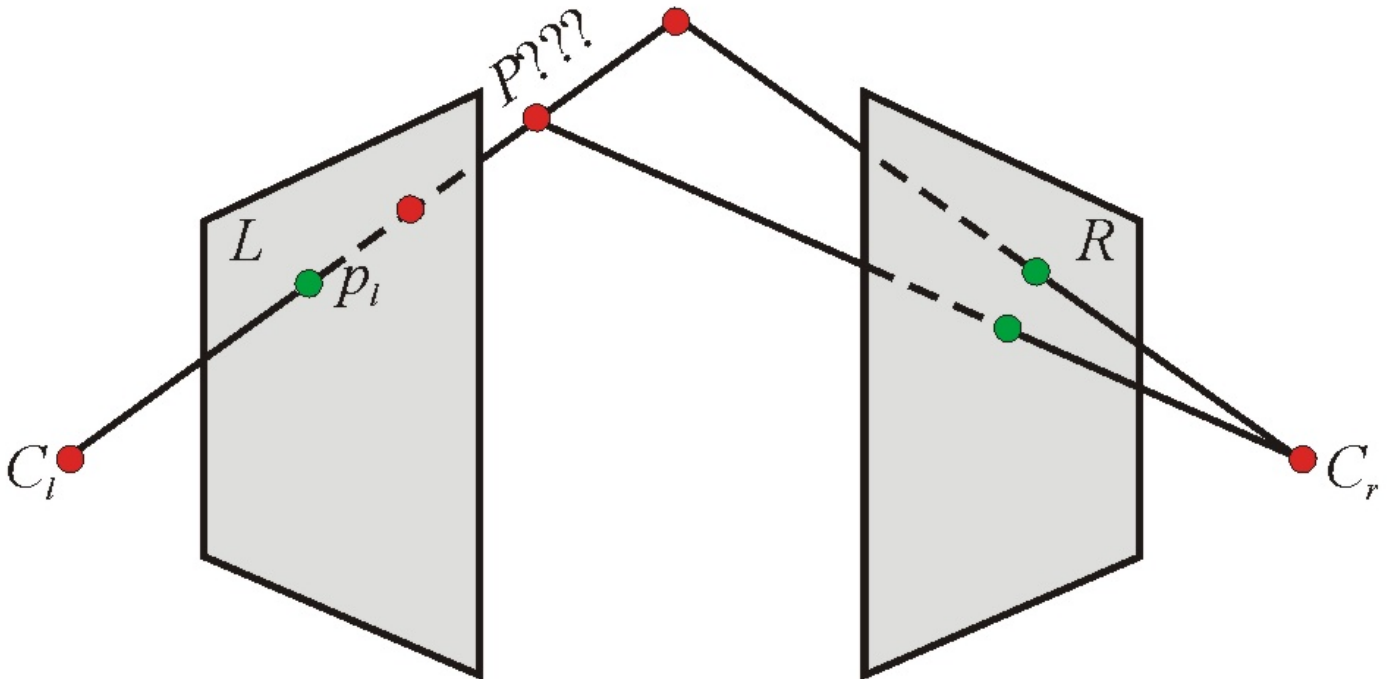
- We have stated that *P* has to lie on the ray $C_l p_l$.

Epipolar Geometry



- If we project each candidate 3D point onto the right image plane, we see that the all lie on a line in *R.*

Epipolar Geometry



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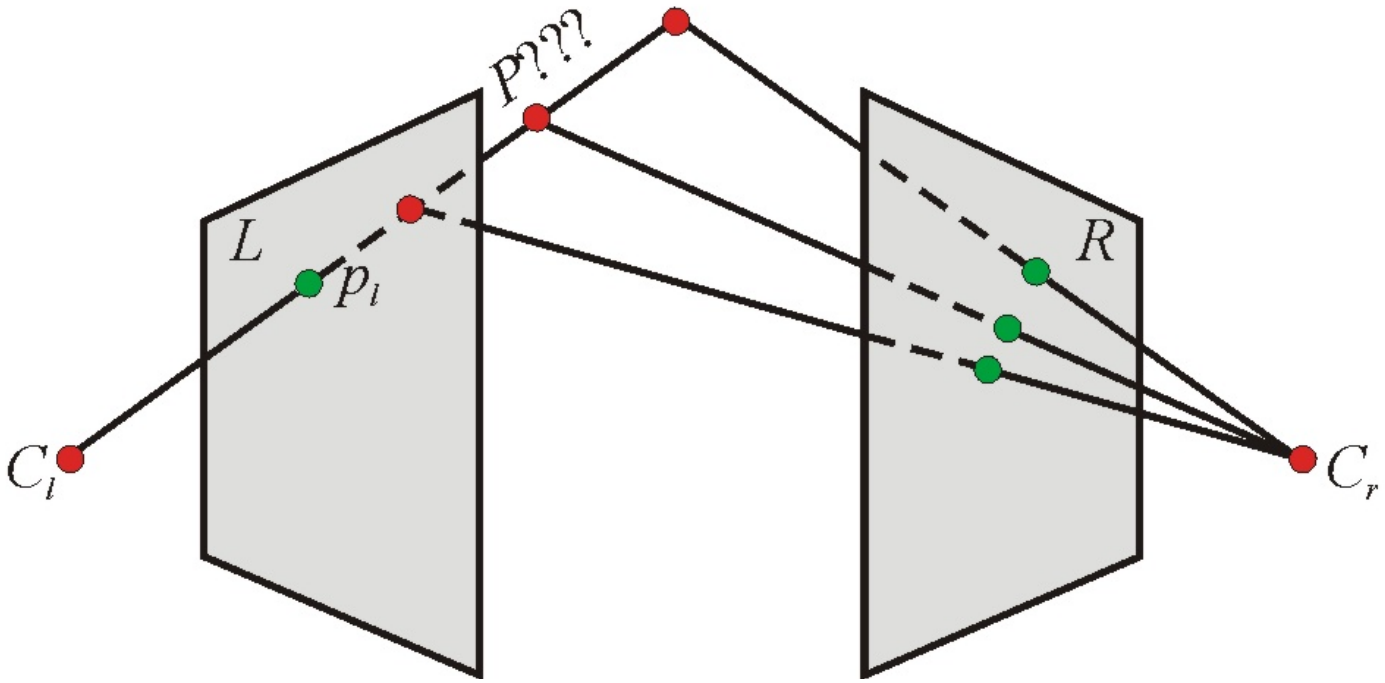

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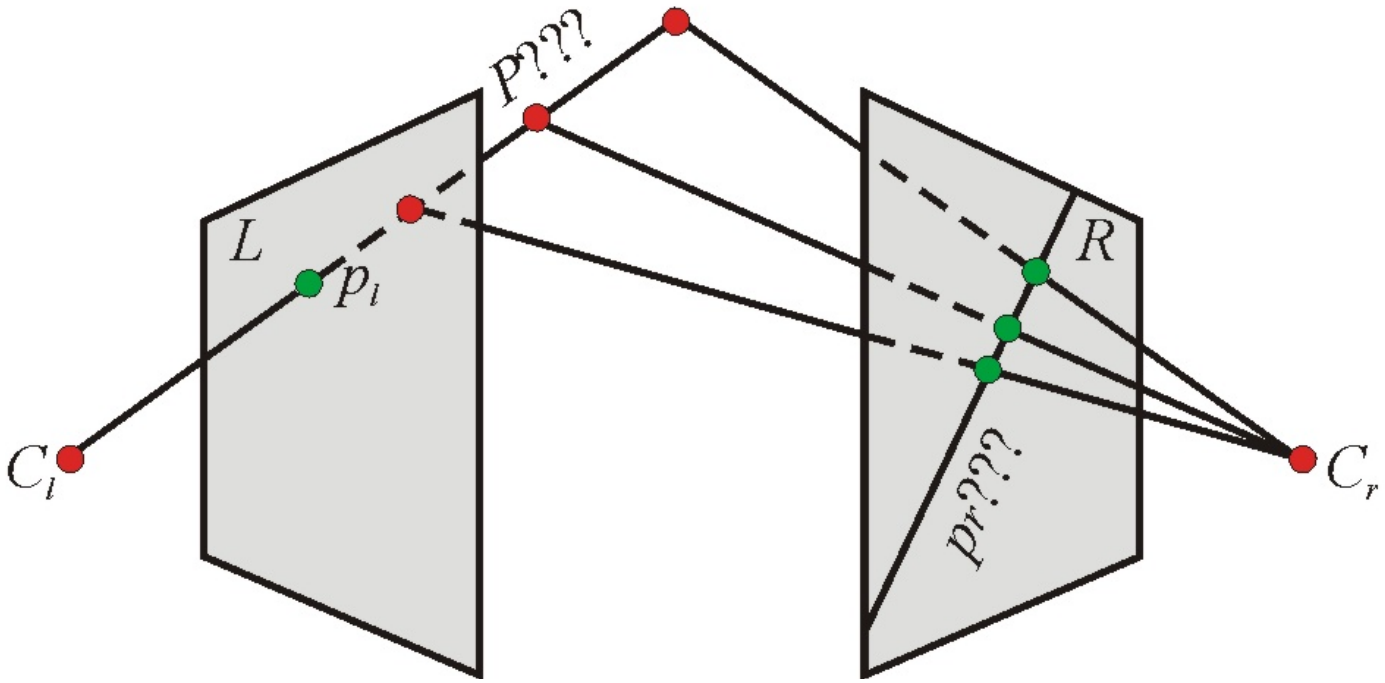

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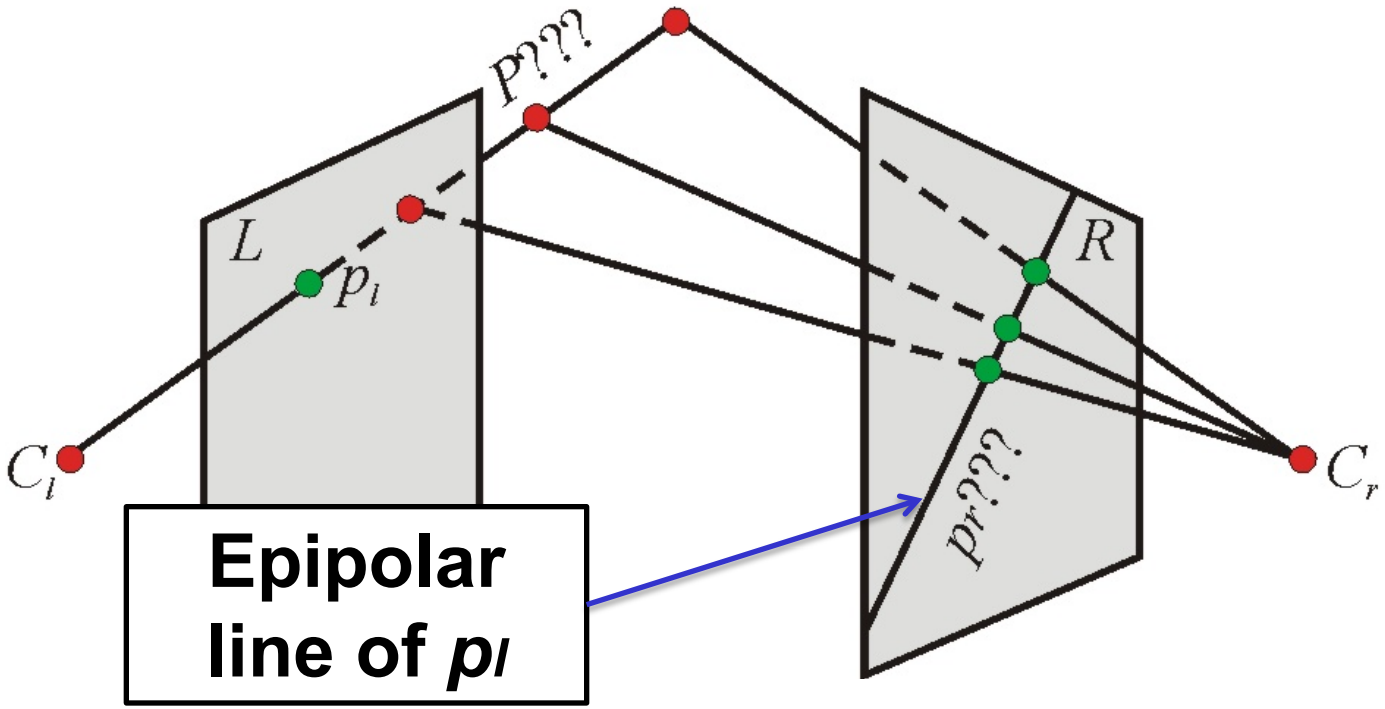
Epipolar Geometry



- This line is called epipolar line of p_l.
- This epipolar line is the projection of the ray $C_l p_l$ onto the right image plane R.
- The pixel p_r is forced to lie on p_l's epipolar line.

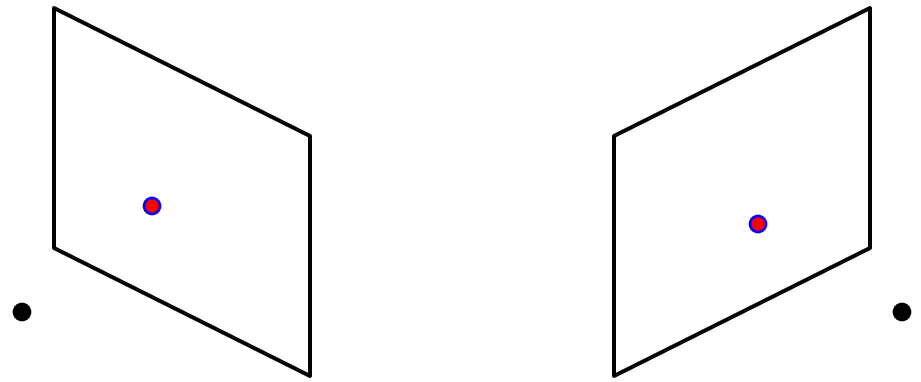
Epipolar Geometry

To find the corresponding pixel, we only have to search along the epipolar line (1D instead of 2D search).

This search space restriction is known as epipolar constraint.

Stereo correspondence

- Determine Pixel Correspondence
  - Pairs of points that correspond to same scene point



Epipolar Constraint

- Reduces correspondence problem to 1D search along *epipolar lines*
- Java demo: http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

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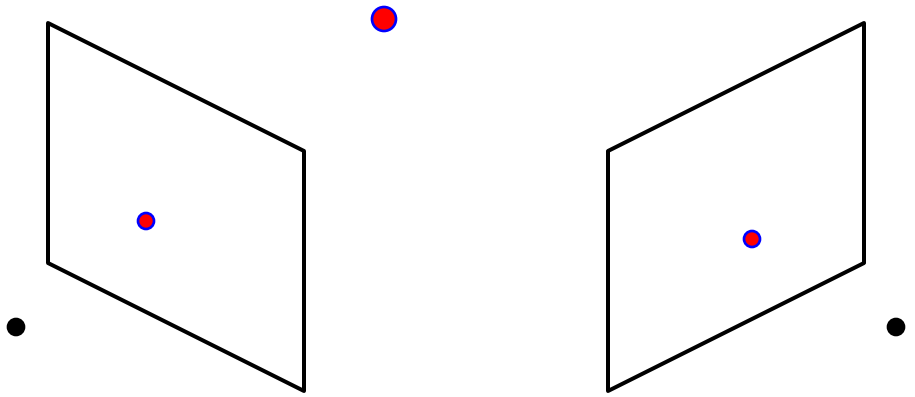

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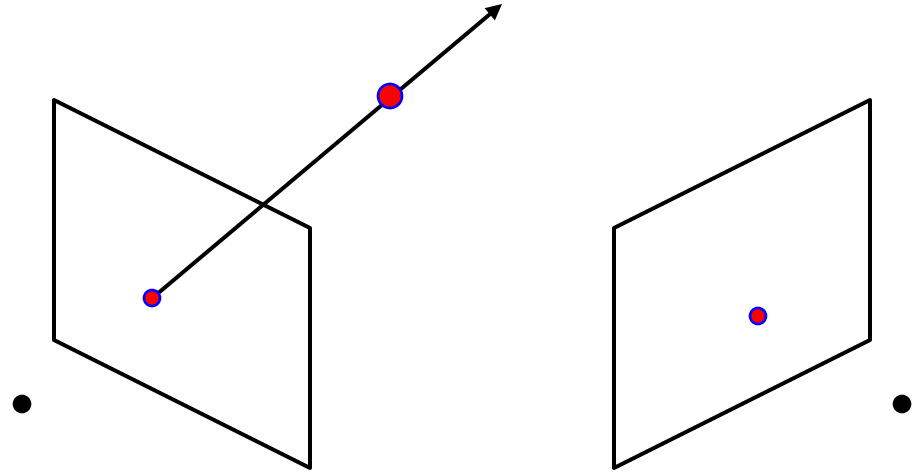

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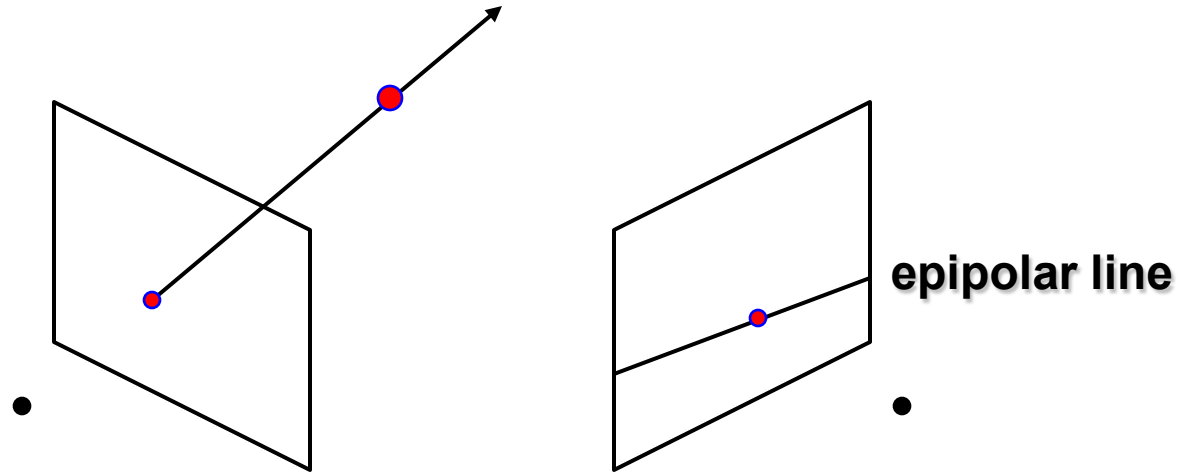

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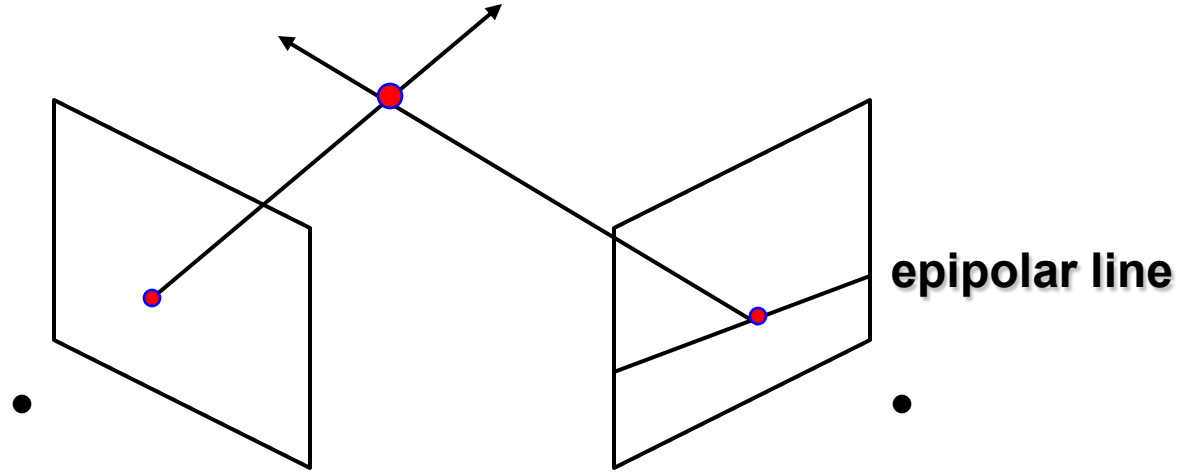

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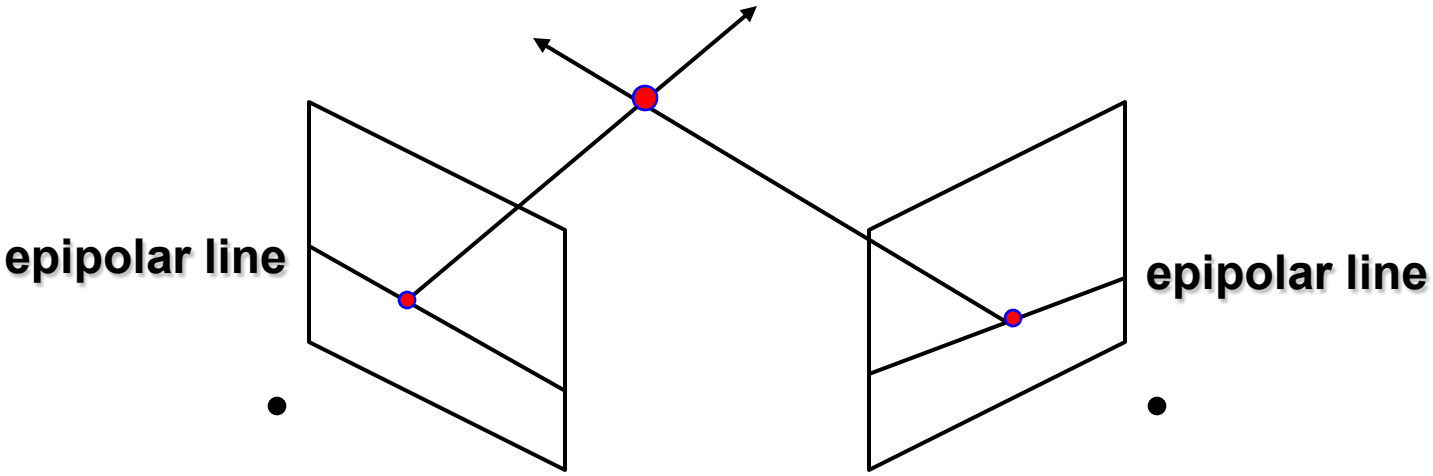

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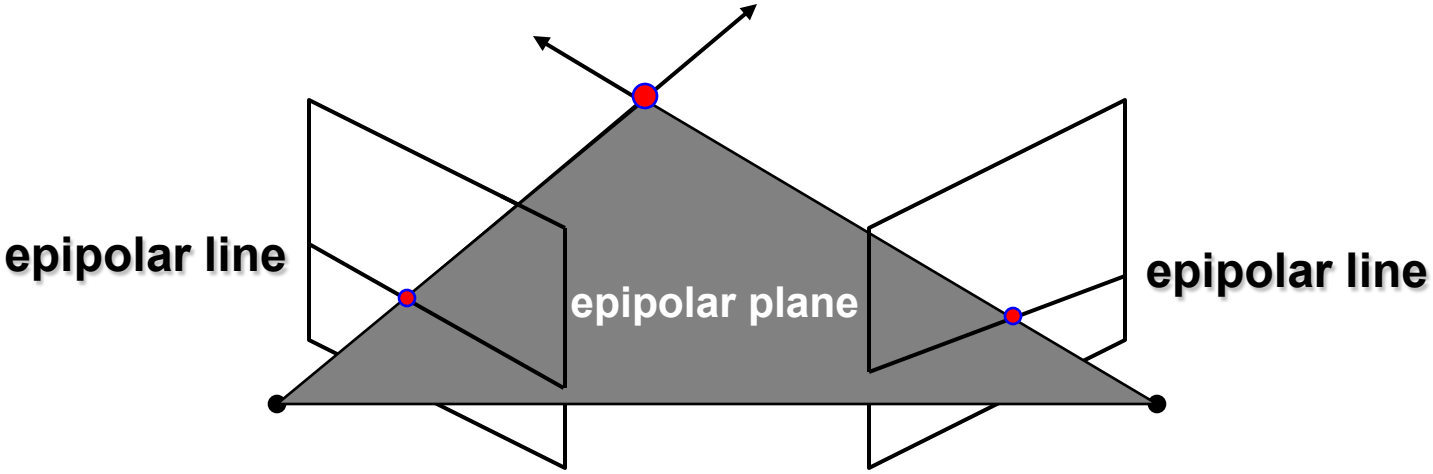

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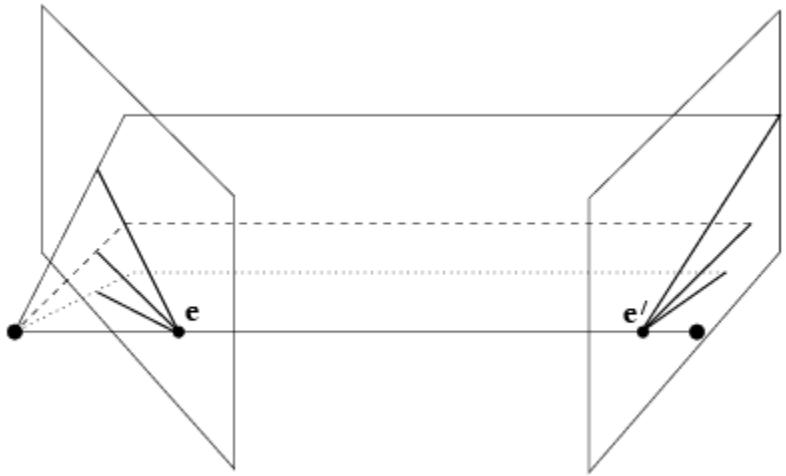
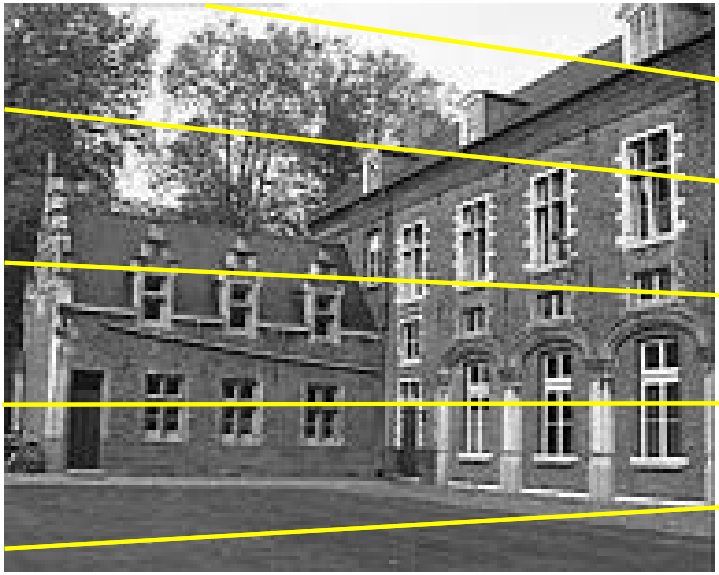
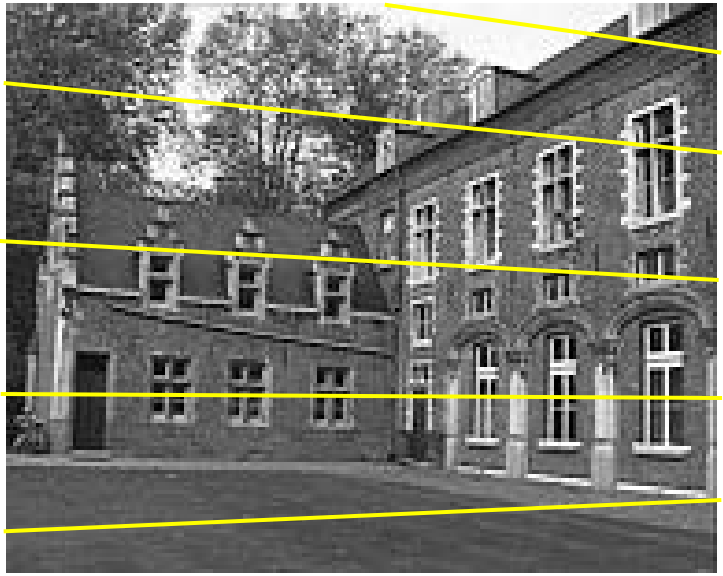
Epipolar Line Example

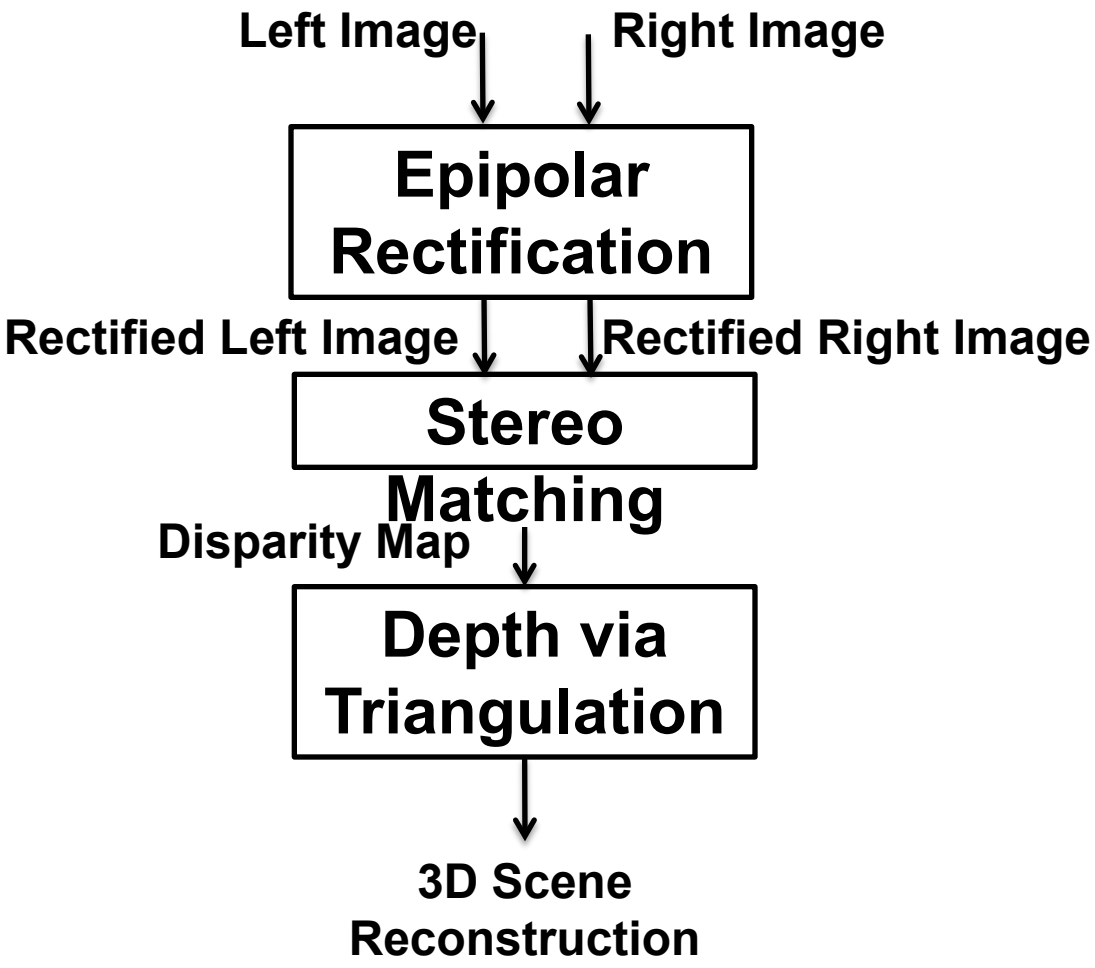




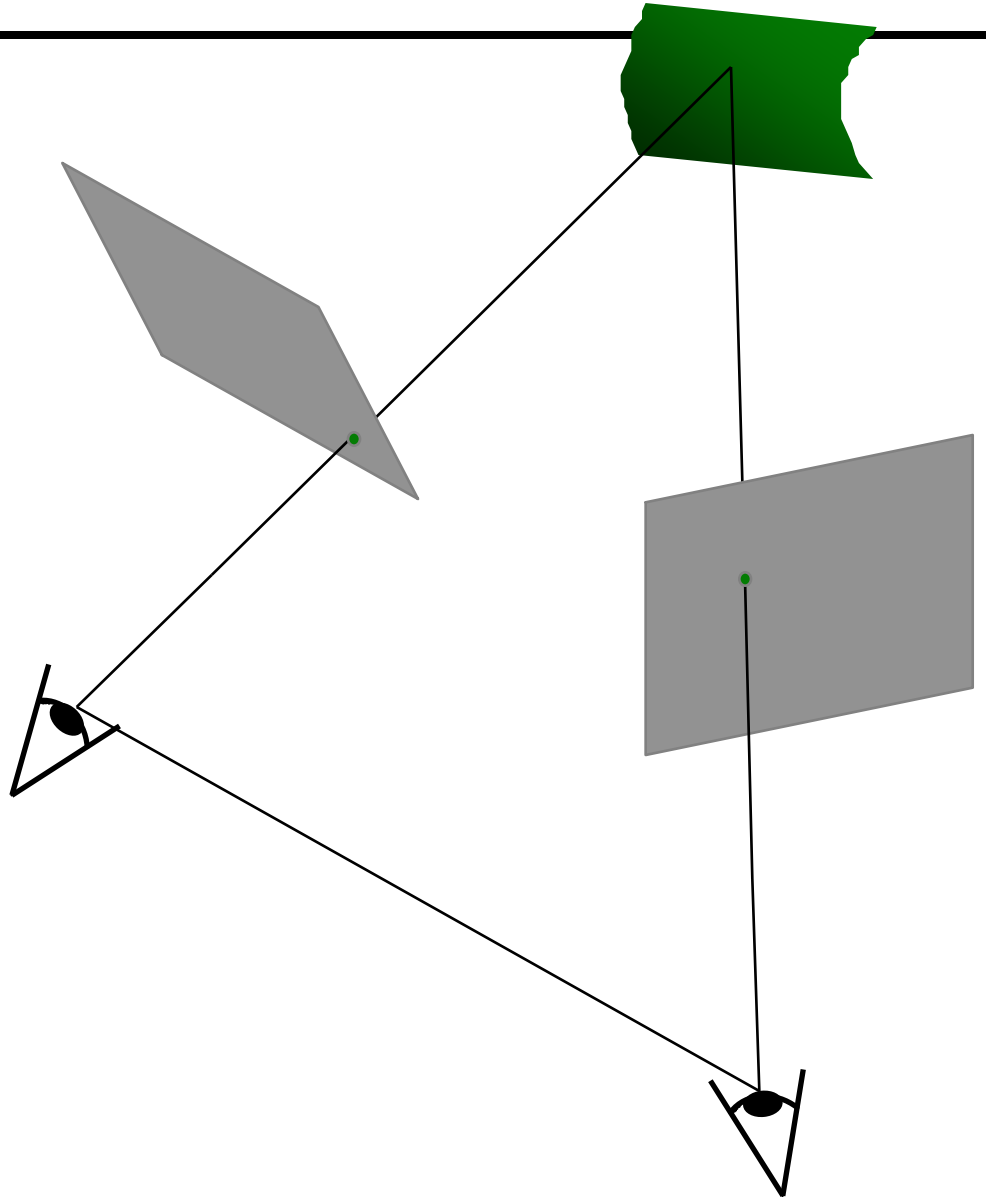


courtesy of Marc Pollefeys

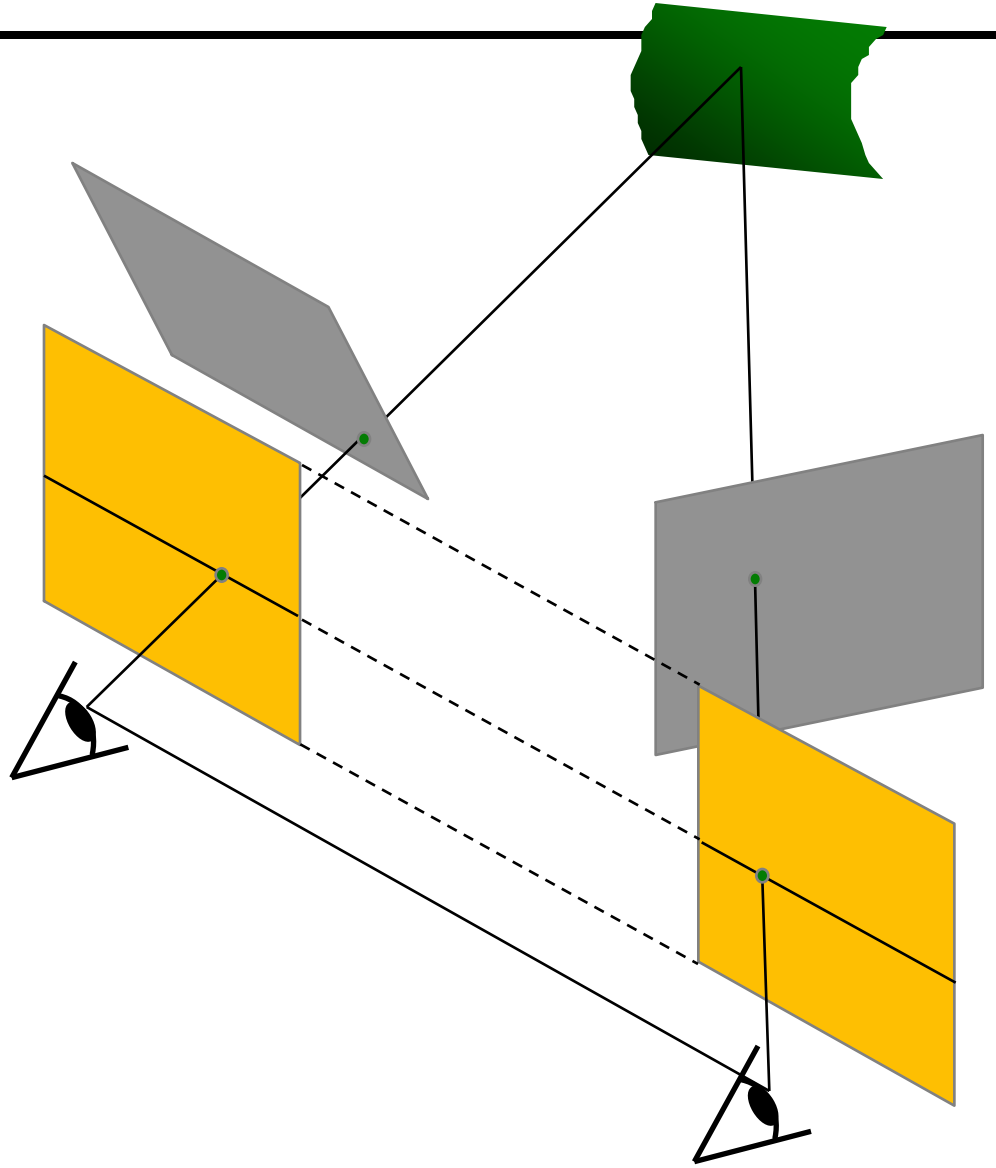
Stereo Pipeline



Stereo image rectification

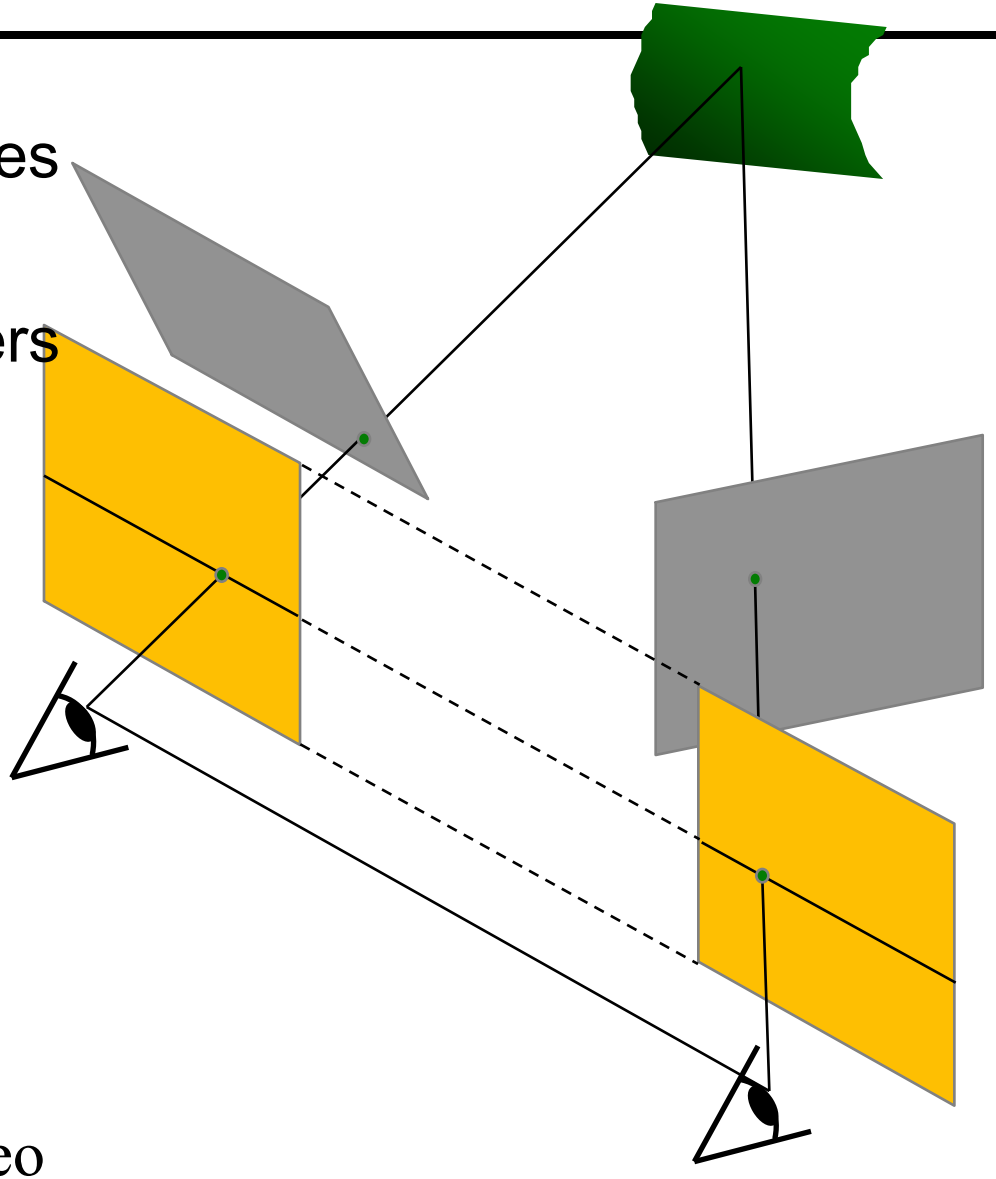


Stereo image rectification



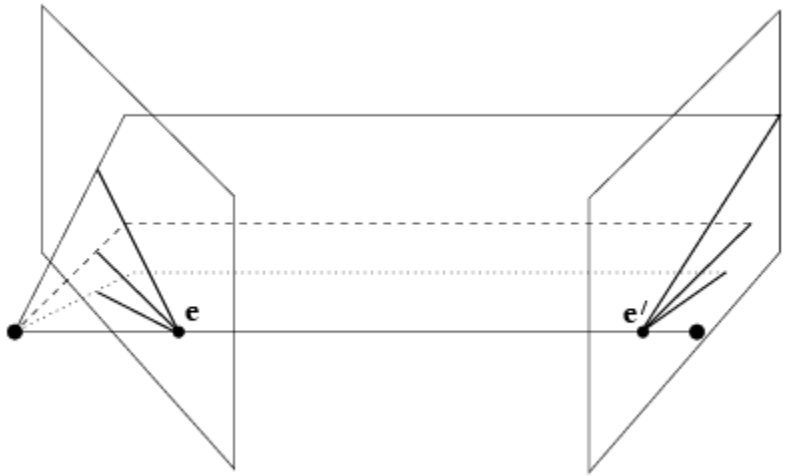
Stereo image rectification

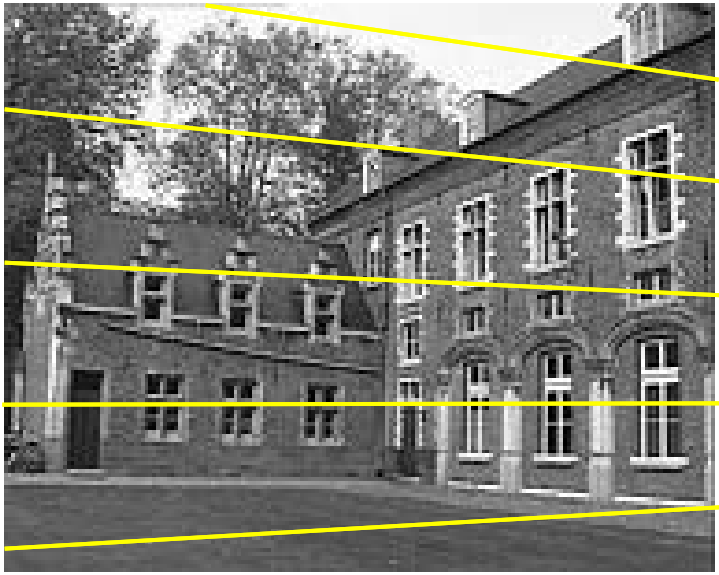
- Reproject image planes onto a common plane parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- Two homographies, one for each input image reprojection

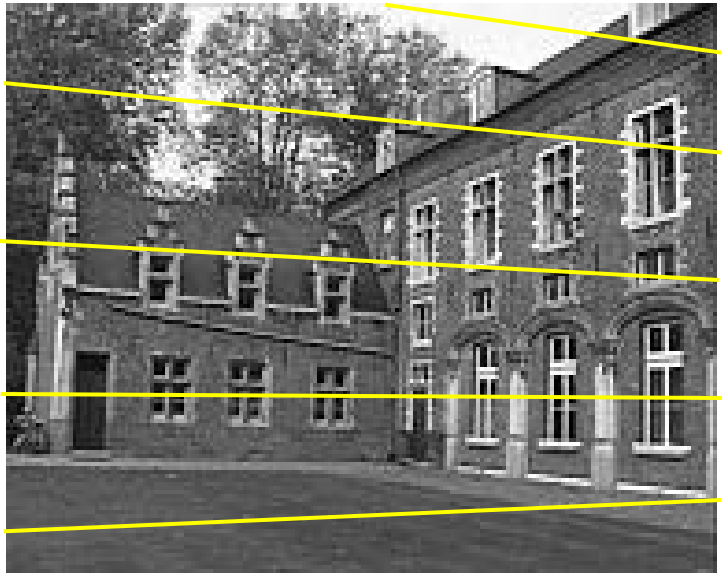


Computing Rectifying Homographies for Stereo Vision. (CVPR 1999)

Epipolar Line Example

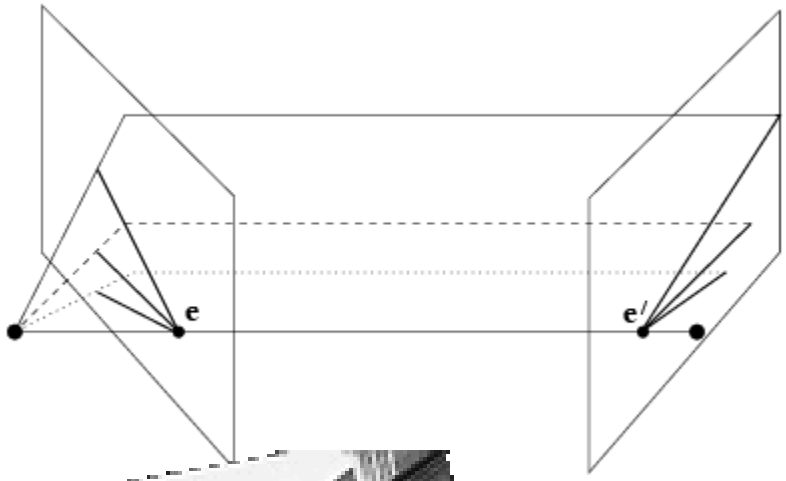


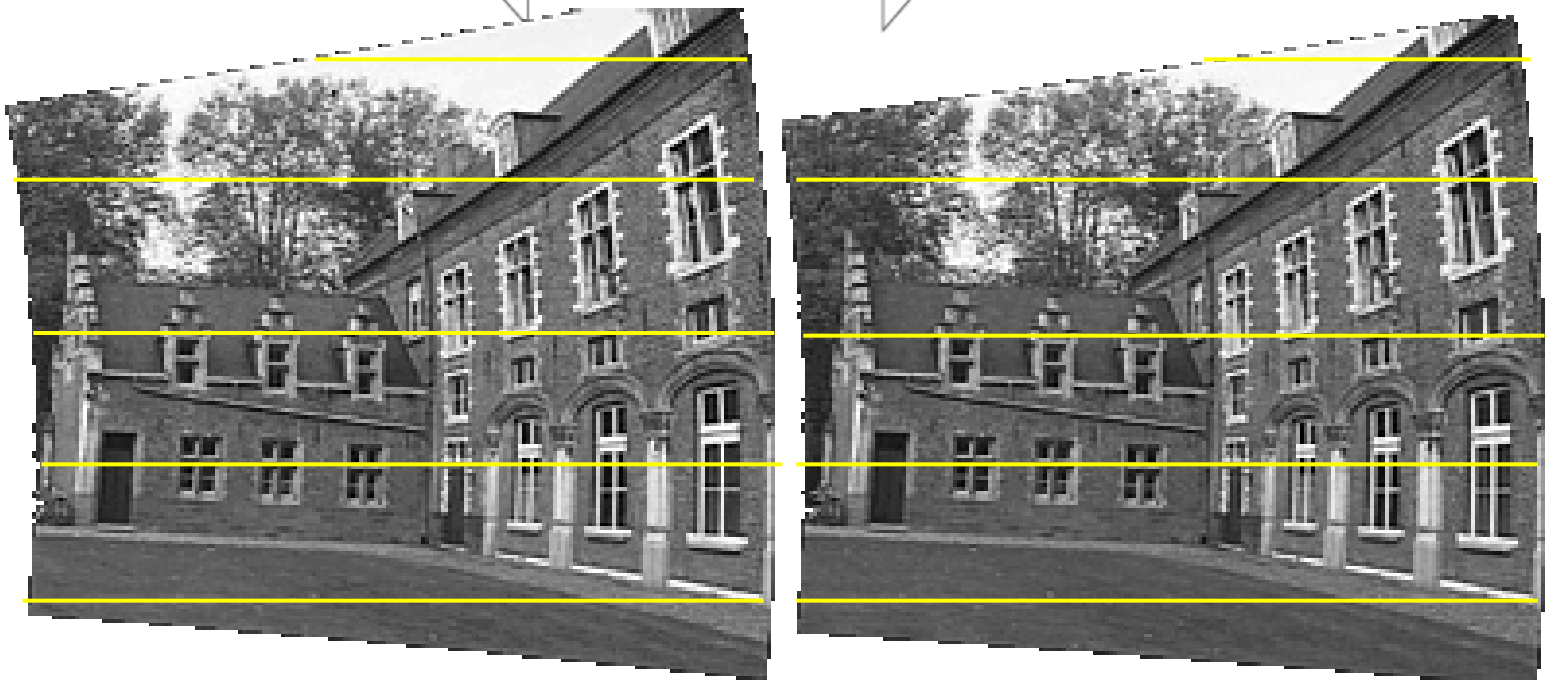




courtesy of Marc Pollefeys

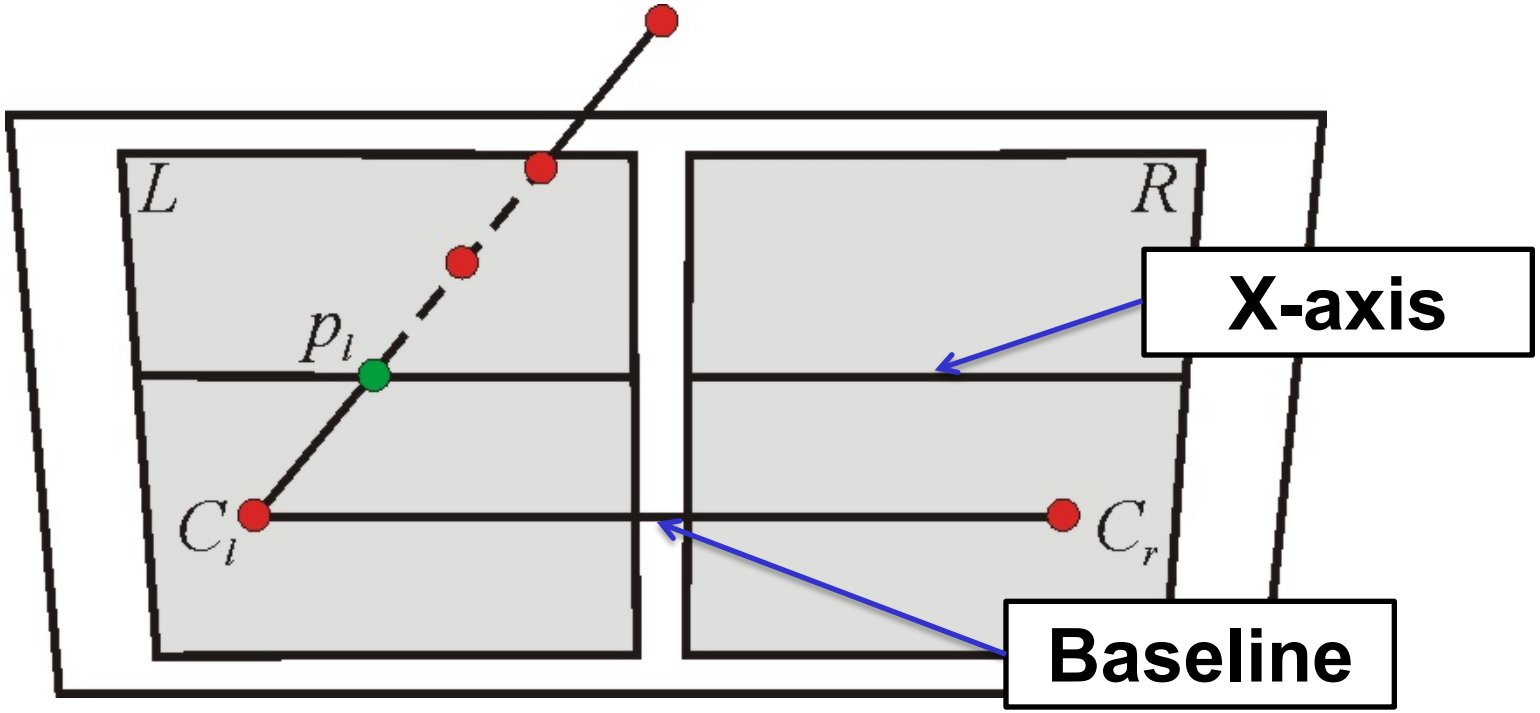
Epipolar Line Example





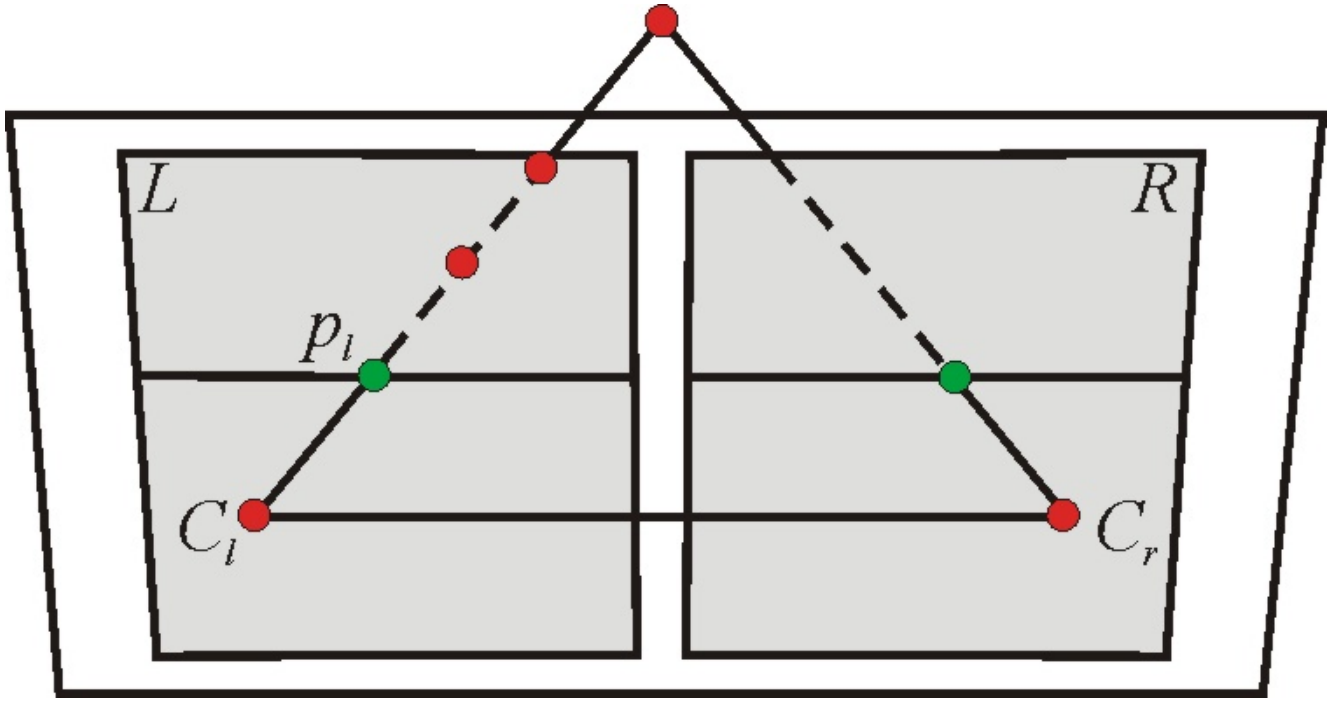
courtesy of Marc Pollefeys

Epipolar Rectification



- Specifically interesting case:
  - Image plane *L* and *R* lie in a common plane.
  - X-axes are parallel to the baseline
- Epipolar lines coincide with horizontal scanlines => corresponding pixels have the same y-coordinate

Epipolar Rectification



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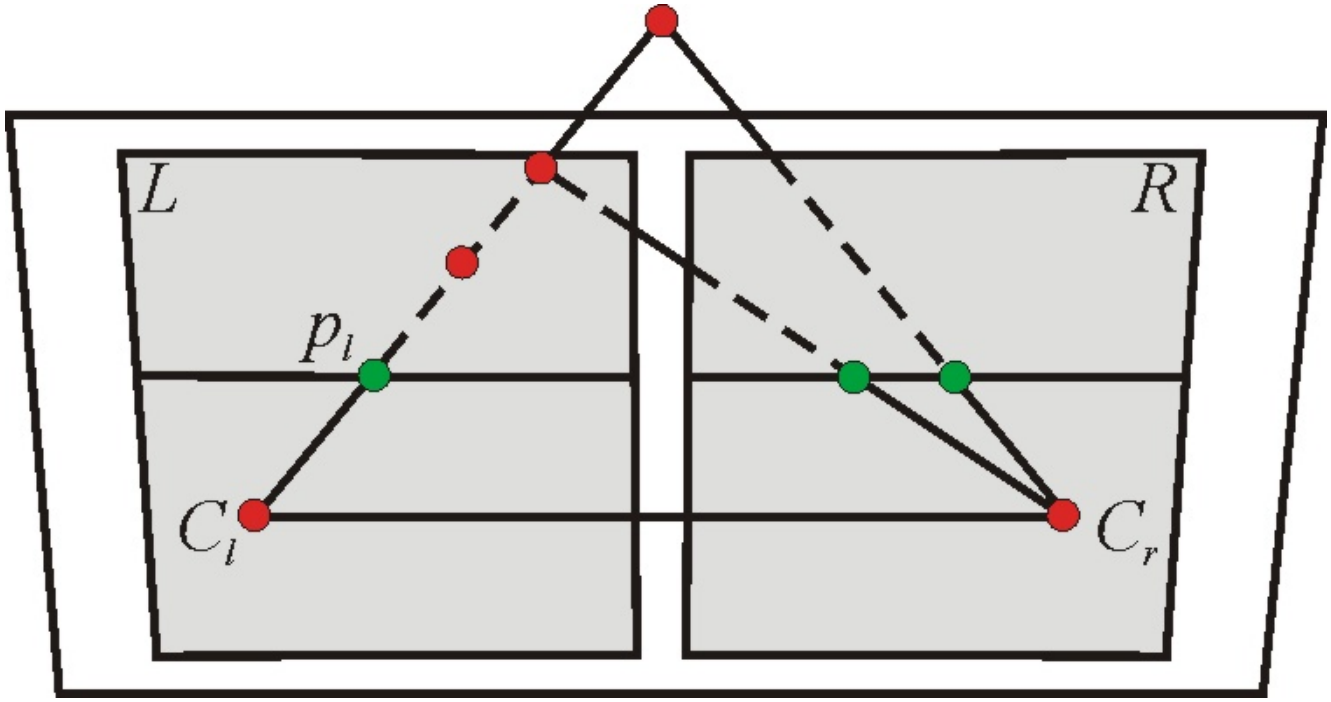

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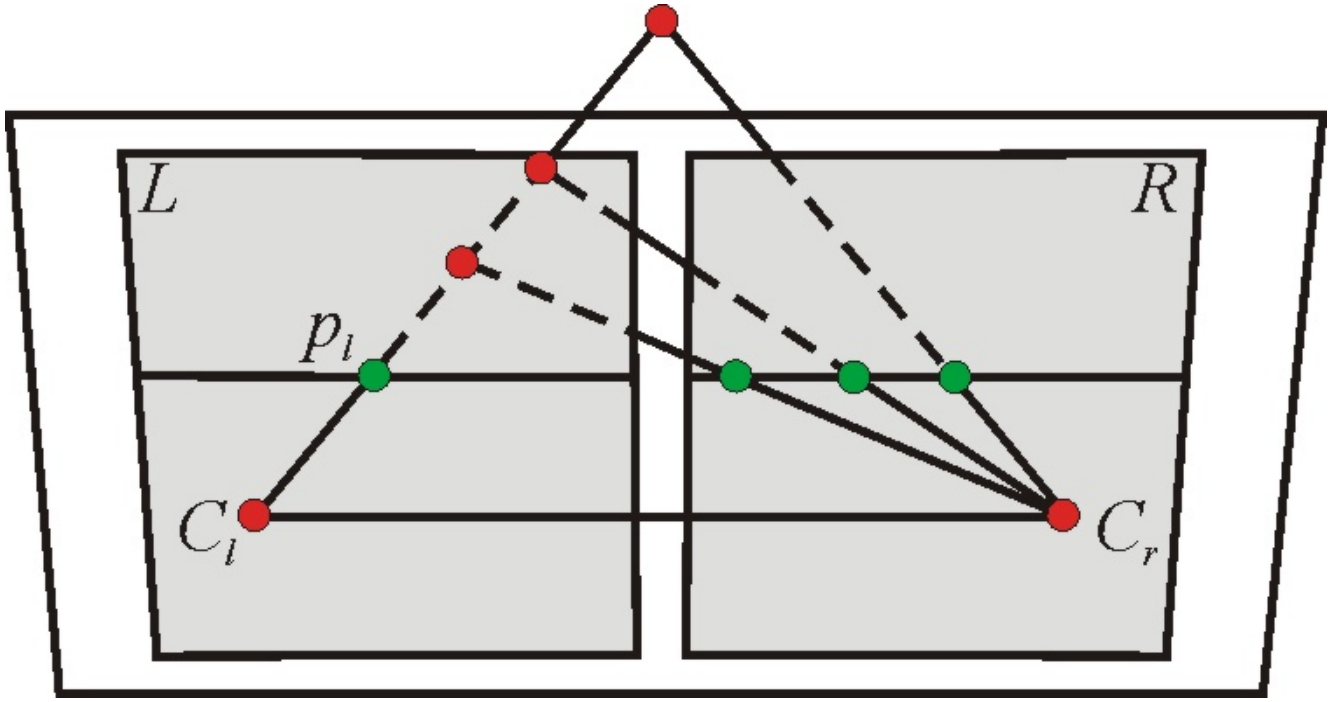

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Epipolar Rectification

To find the corresponding pixel, we only have to search along the horizontal scanline.

More convenient than tracing arbitrary epipolar lines.

The difference in x-coordinates of corresponding pixels is called *disparity*

Epipolar Rectification

- This special case can be achieved by reprojecting left and right images onto virtual cameras.
- This process is known as epipolar rectification.
- After this, we can assume that images have been rectified.

Original images – white lines represent epipolar lines

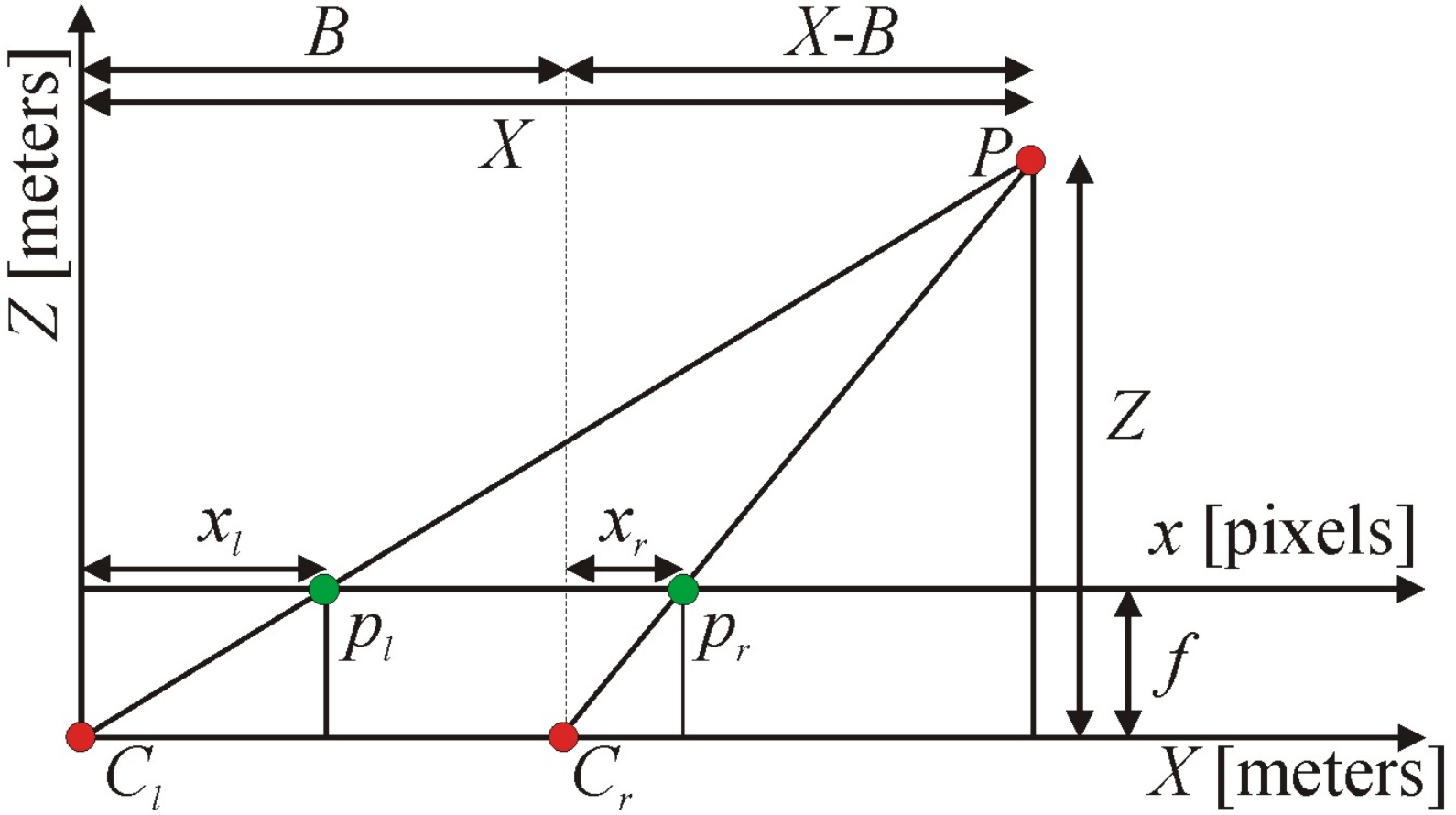
Rectified images – epipolar lines coincide with horizontal scanlines

Images taken from http://profs.sci.univr.it/~fusiello/rectif_cvol/node6.html

Summary

- 1D search is computationally faster than 2D search.
- Reduced search range lowers chance of finding a wrong match (Quality of depth maps).
- More or less the only constraint that will always be valid in stereo matching (unless there are calibration errors).

Depth via Triangulation



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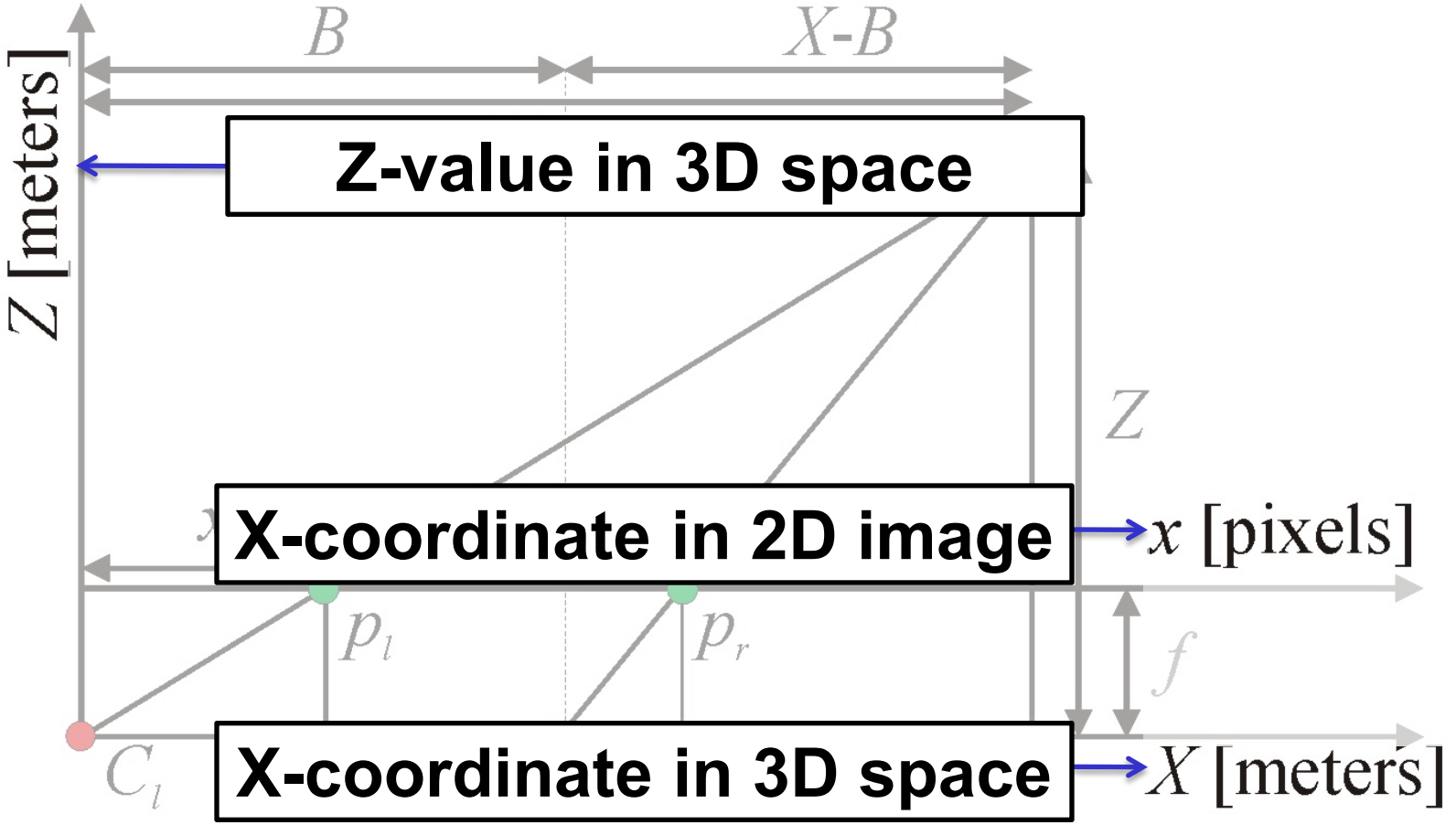

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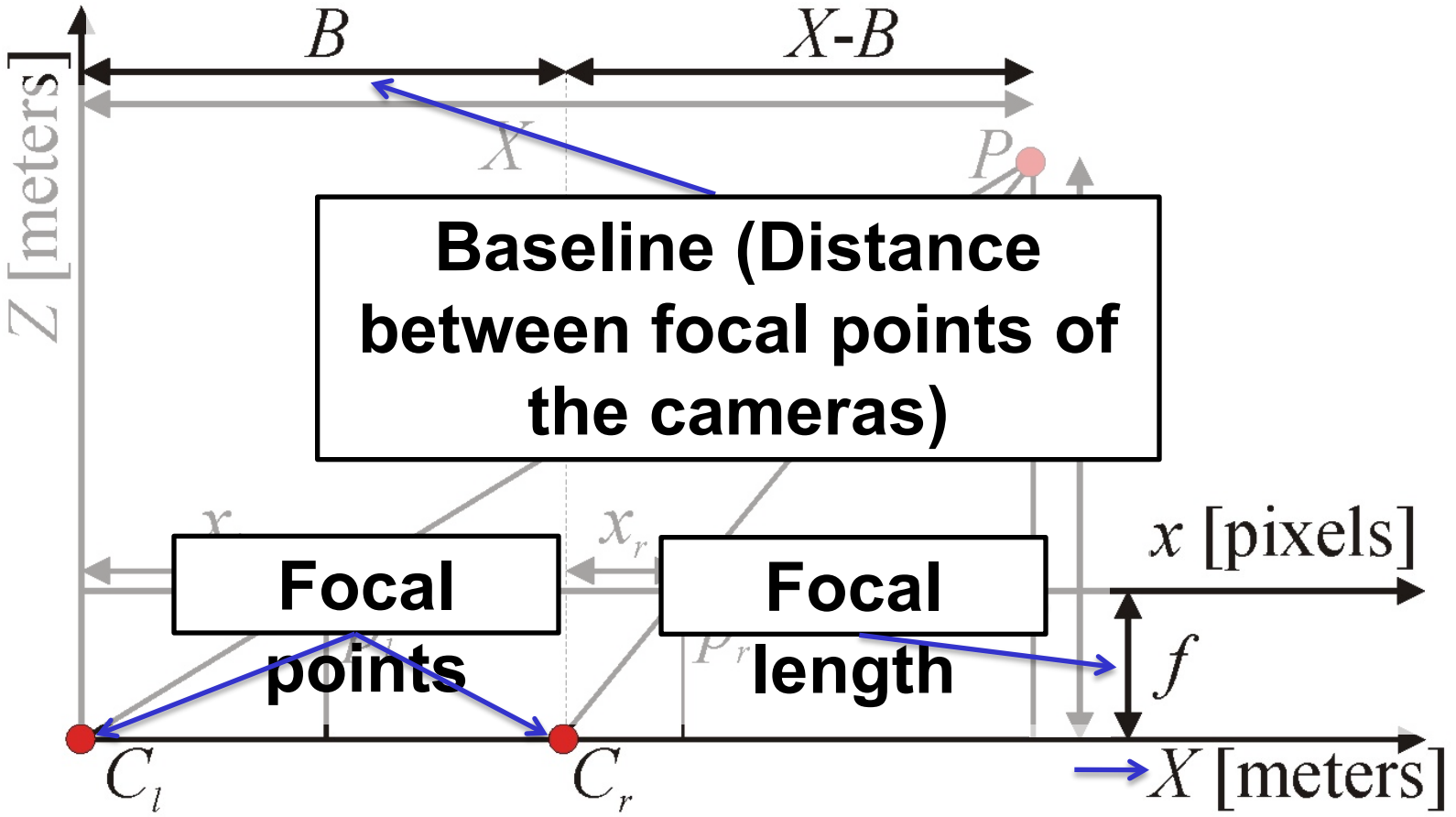
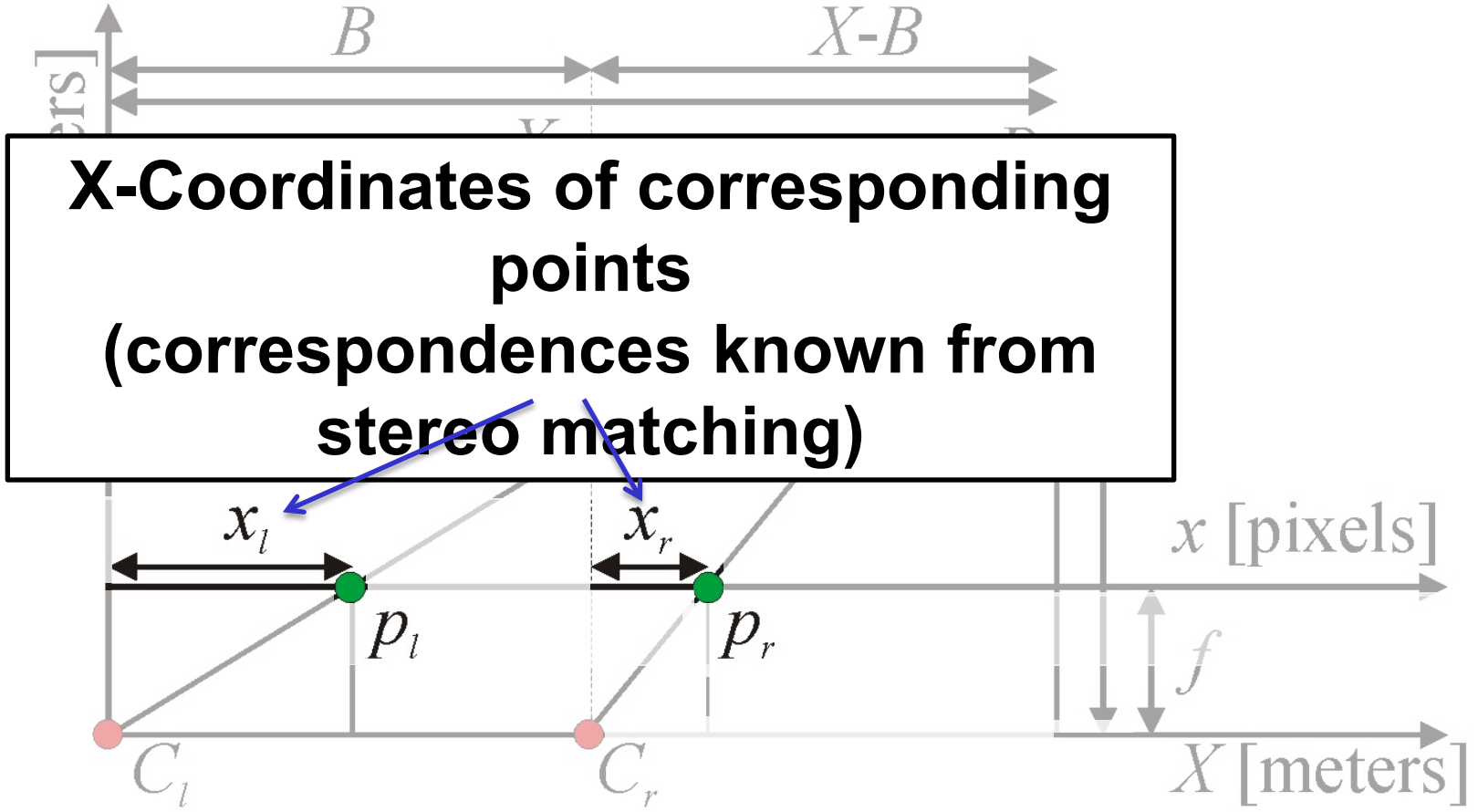


All of these values are known from camera calibration

Depth via Triangulation



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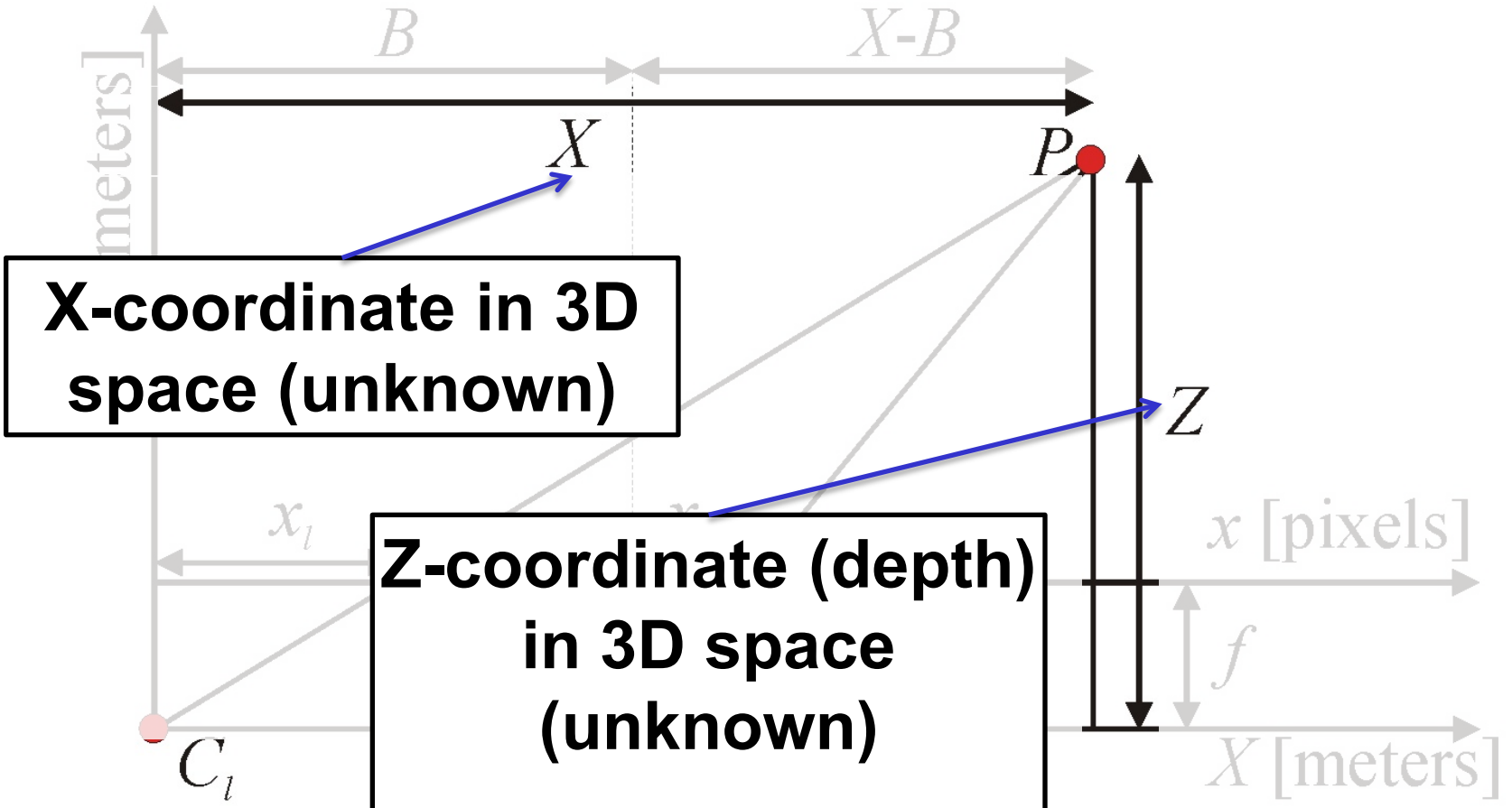

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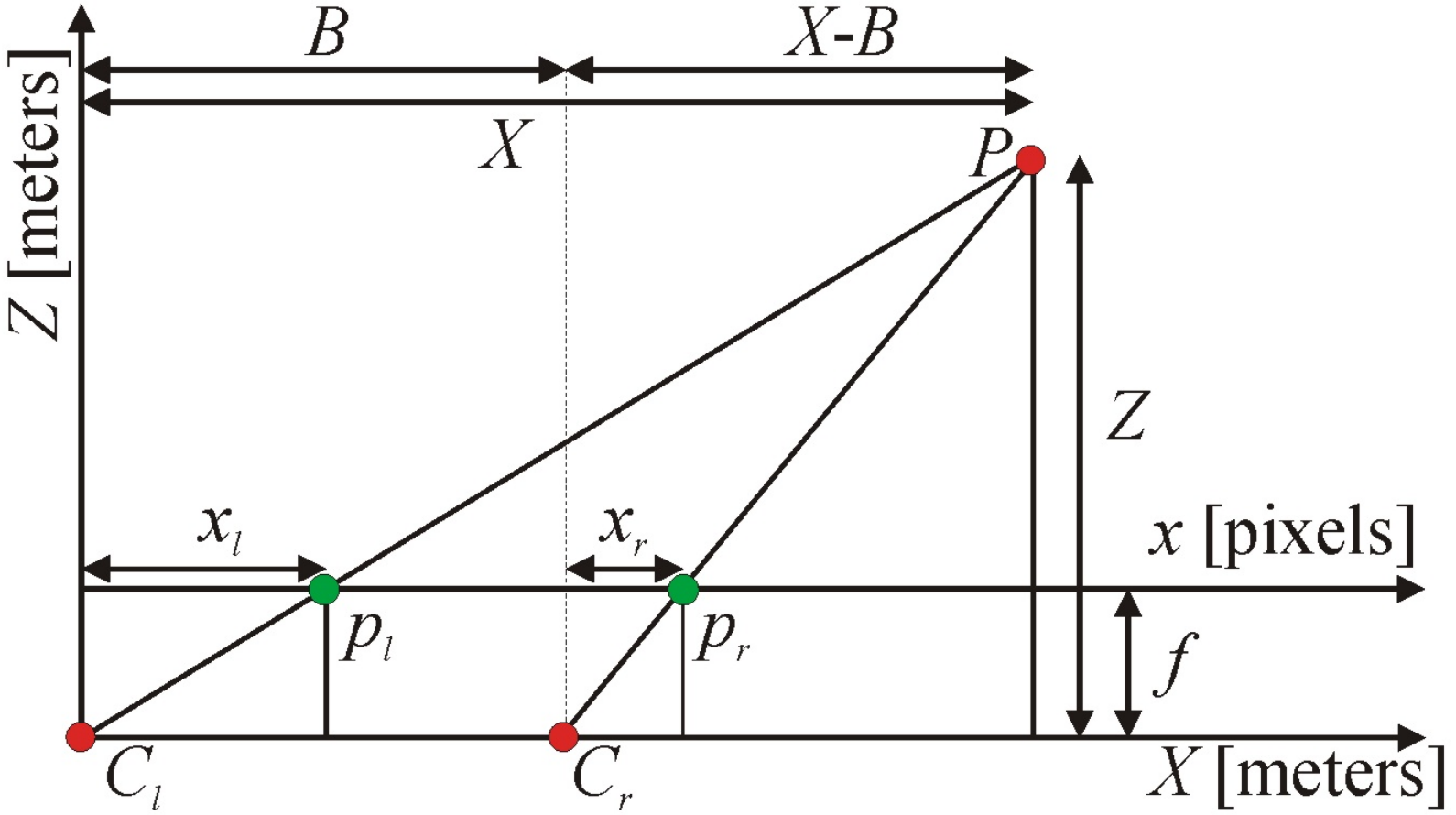

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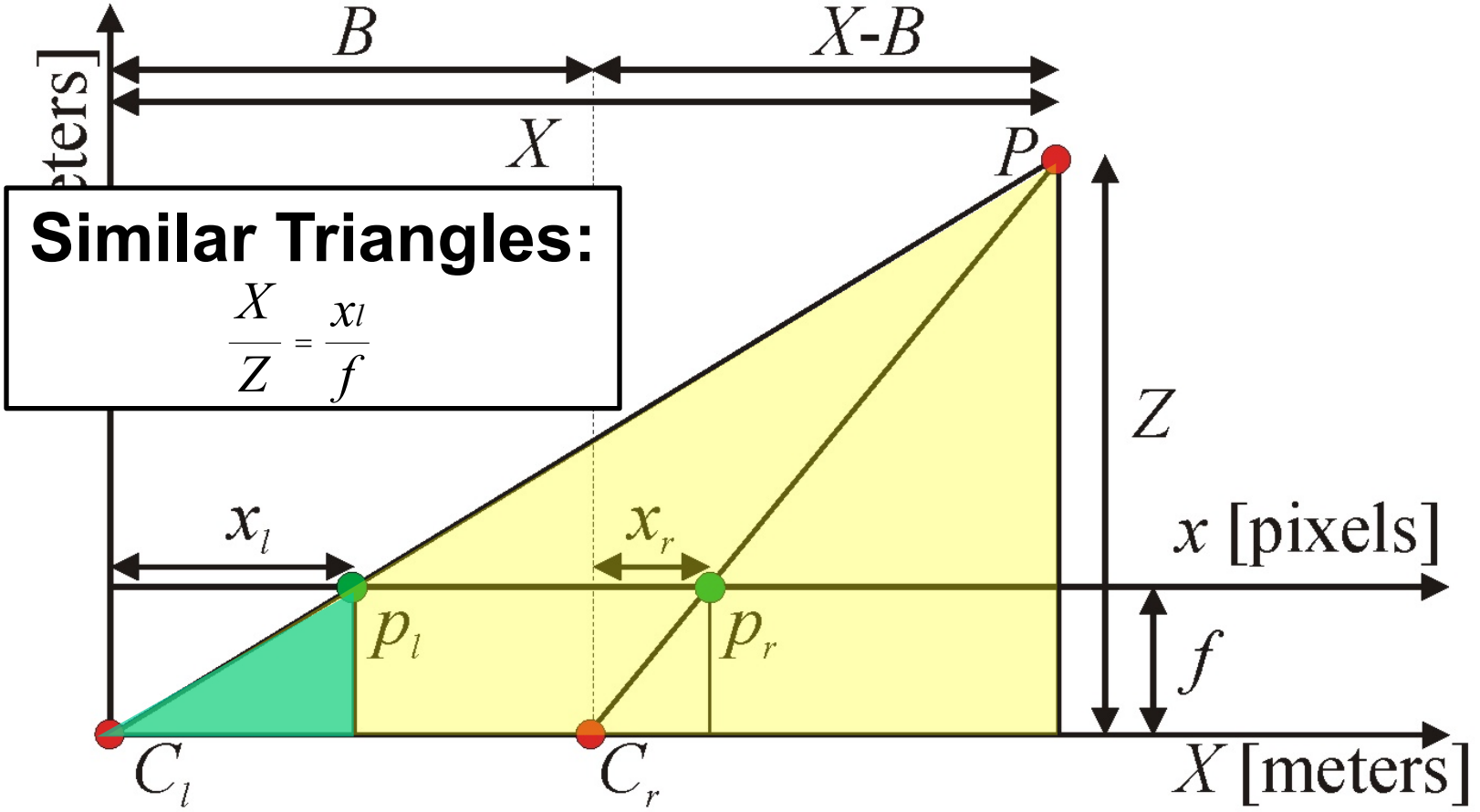

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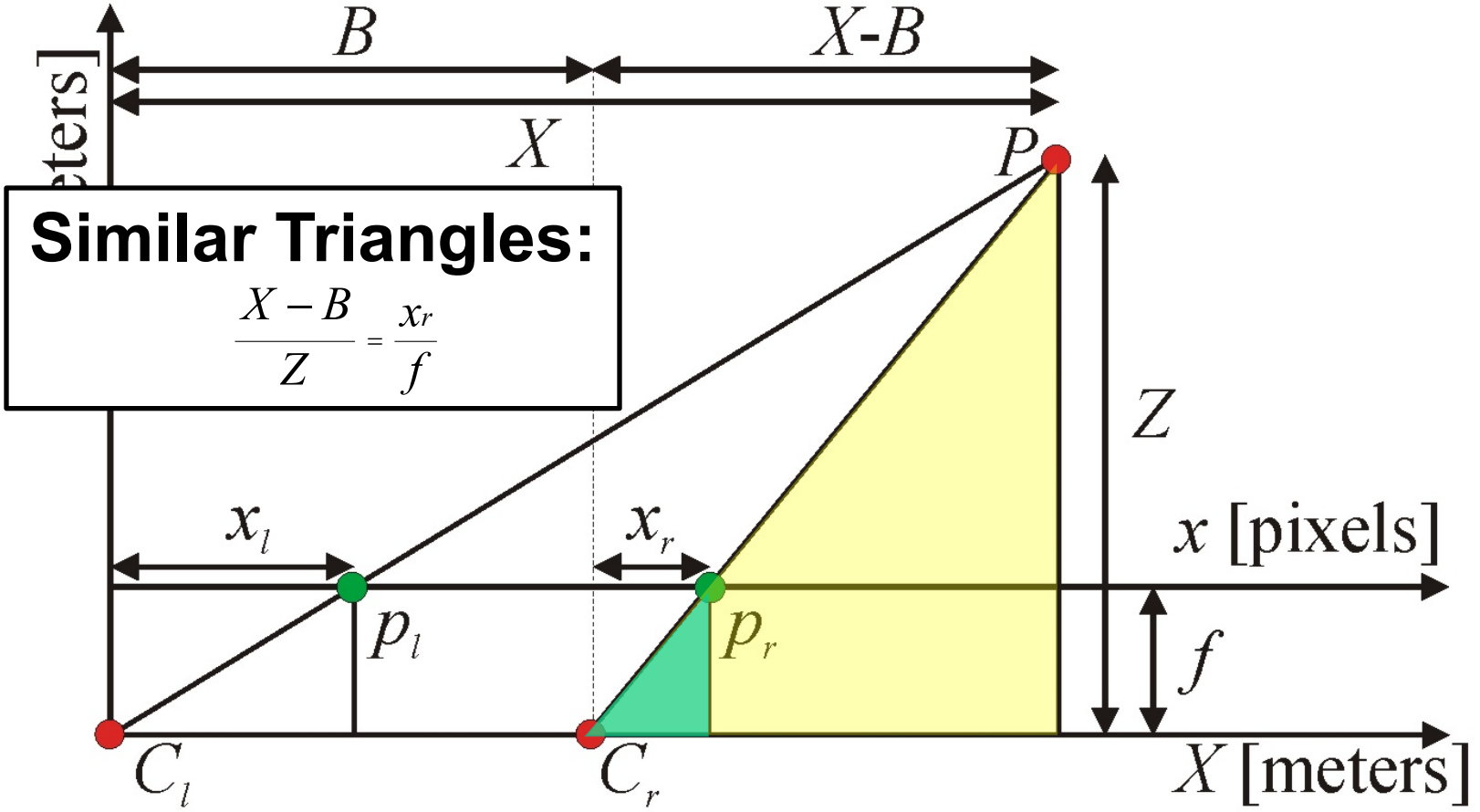

Depth via Triangulation

- From similar triangles:

$$\frac{X}{Z} = \frac{x_l}{f} \qquad \frac{X - B}{Z} = \frac{x_r}{f}$$

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$$X = \frac{Z.x_l}{f} \qquad X = \frac{Z.x_r}{f} + B$$

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- Combine both equations:

$$\frac{Z.x_l}{f} = \frac{Z.x_r}{f} + B$$

$$Z.x_l = Z.x_r + B.f$$

$$Z.(x_l - x_r) = B.f$$

- Write Z in explicit form:

$$Z = \frac{B.f}{x_l - x_r} = \frac{B.f}{d}$$

This is disparity

Depth via Triangulation

- From similar triangles:

Disparity and depth are inversely proportional!

- Write X i

Therefore, disparity is commonly used synonymously with depth.

- Combine

$$Z.x_l = Z.x_r + B.f$$

$$Z.(x_l - x_r) = B.f$$

- Write Z in explicit form:

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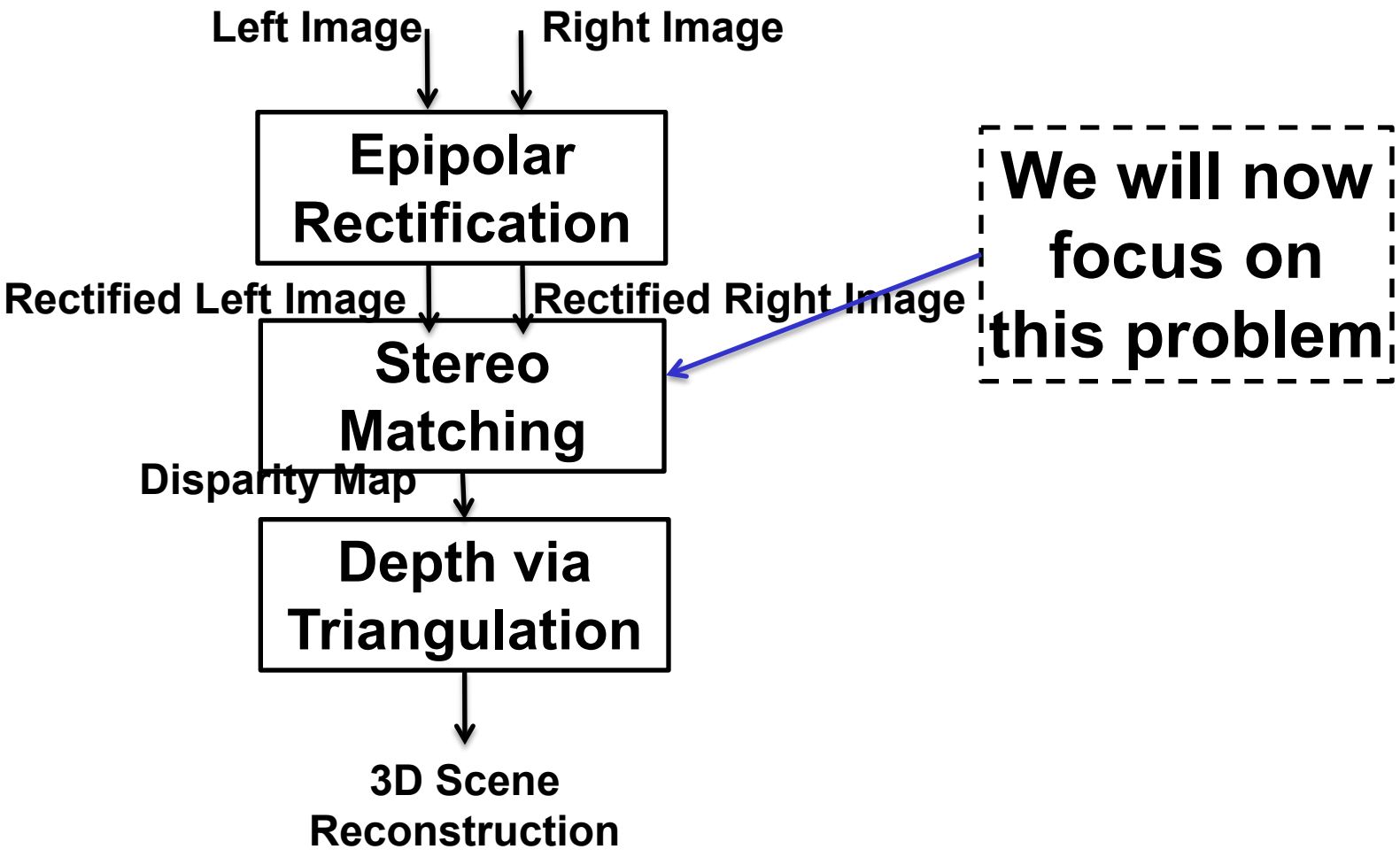
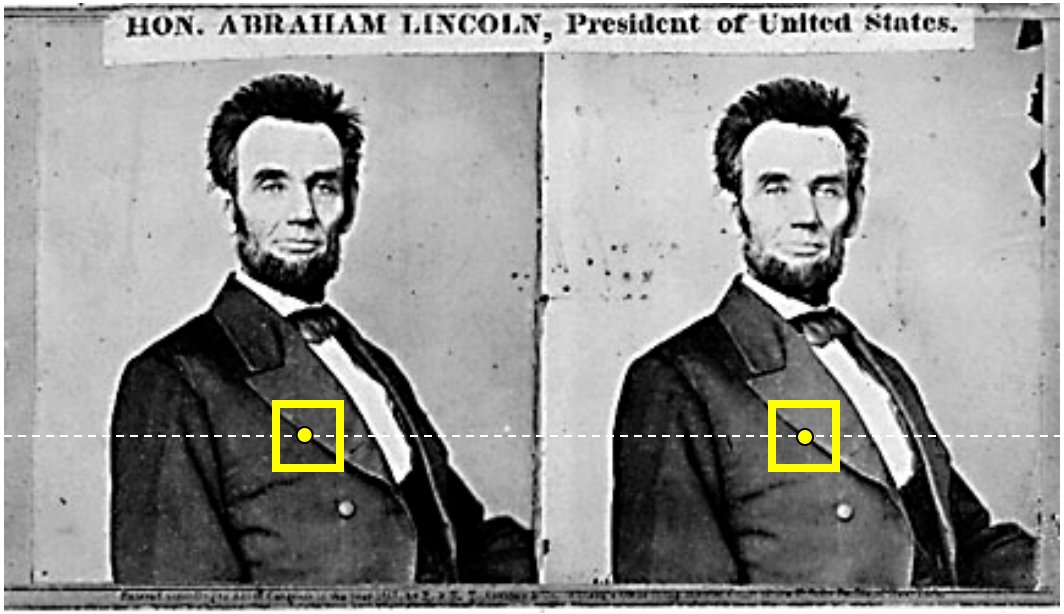
This is disparity

Stereo Pipeline



Basic stereo algorithm



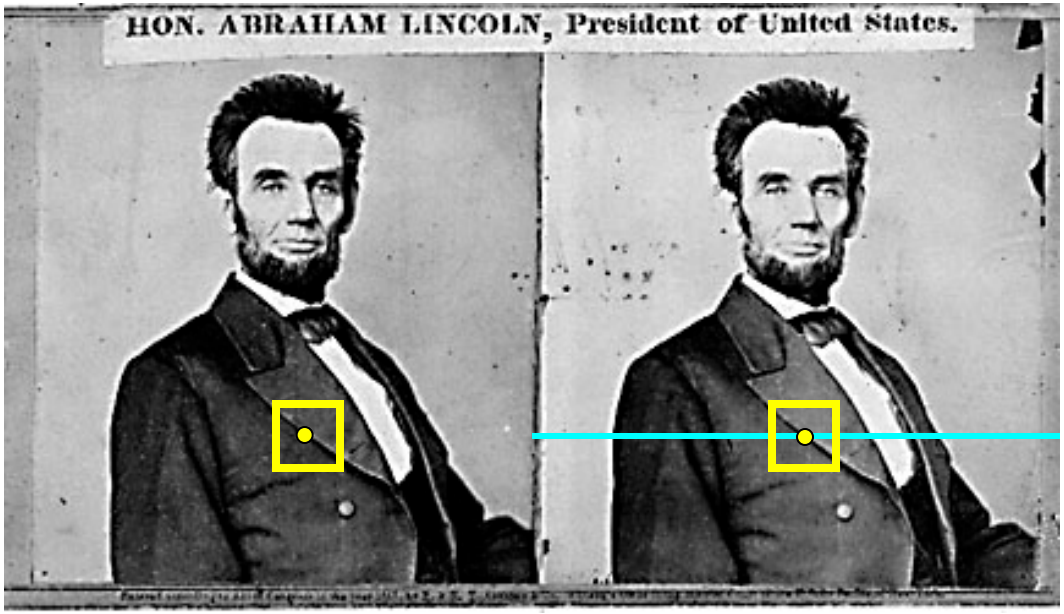
For each epipolar line (raster scan top to bottom)

For each pixel in the left image (left to right)

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

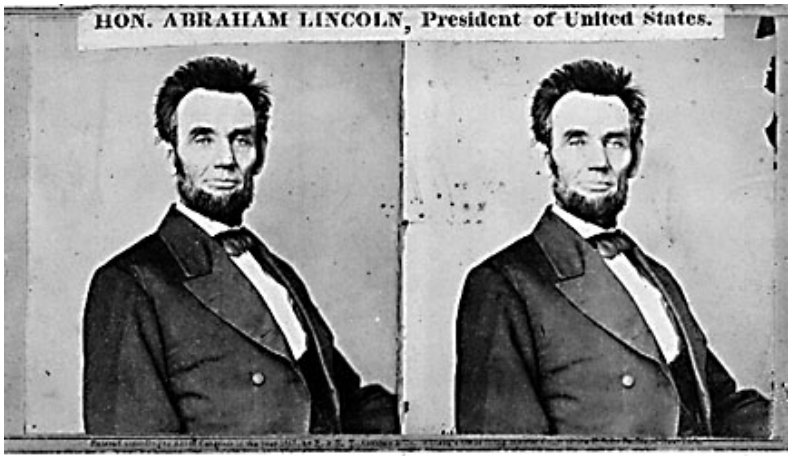
Improvement: match *windows*

The similarity constraint



- Corresponding regions in two images should be similar in appearance
- and non-corresponding regions should be different
- When will the similarity constraint fail?

Limitations of similarity constraint

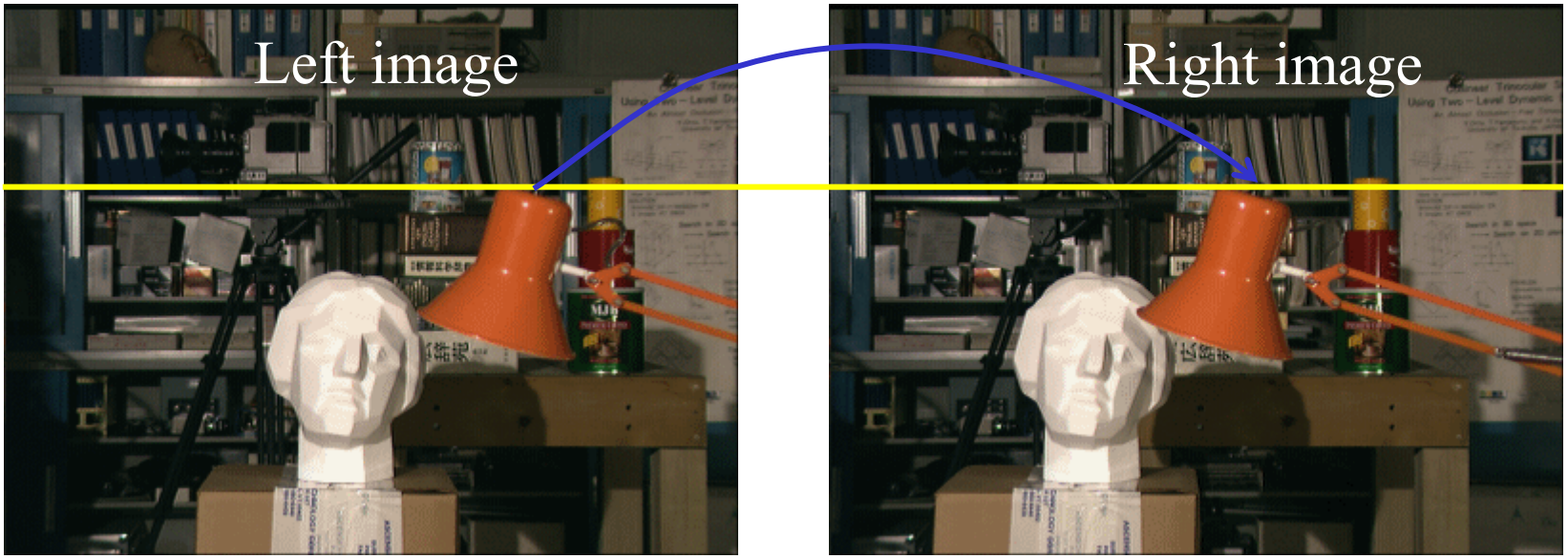


Textureless surfaces

Occlusions, repetition

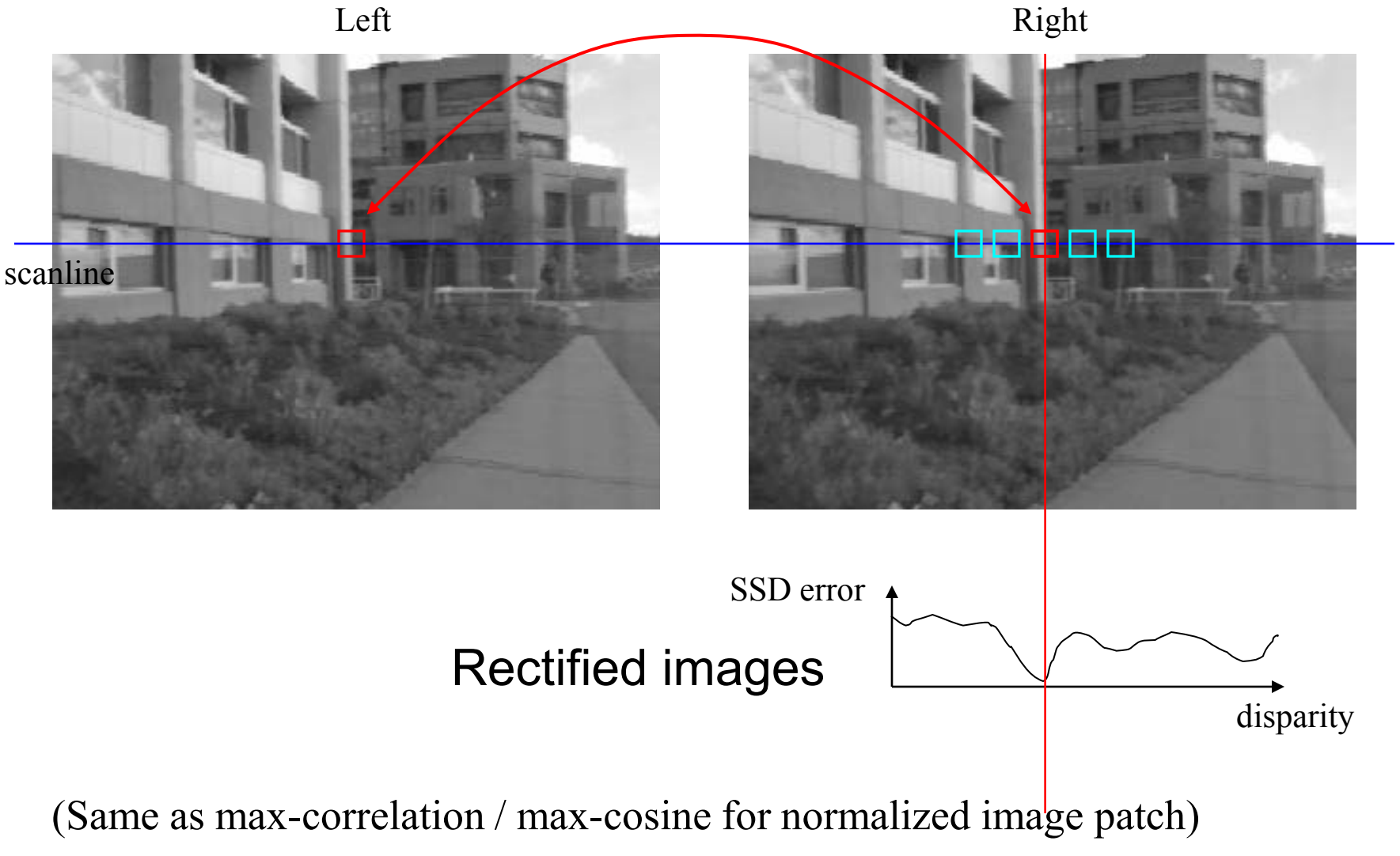
Non-Lambertian surfaces

Correspondence



(After calibration) all correspondences are along the same horizontal scan lines

Correspondence via Correlation



Rectified images

(Same as max-correlation / max-cosine for normalized image patch)

Image Normalization

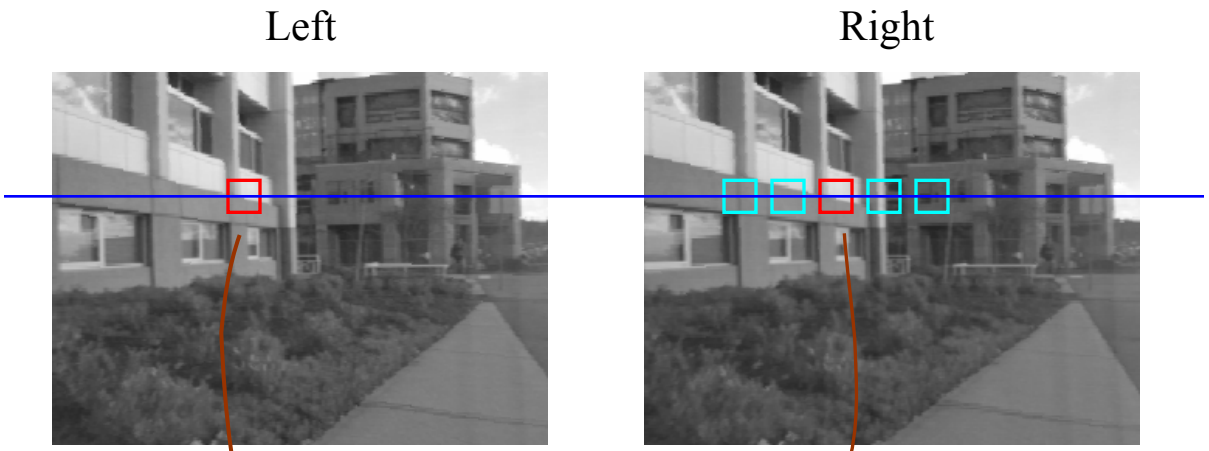
- The cameras do not see exactly the same surfaces, so their overall light levels will differ: good idea to normalize the pixels in each window (accounts for changes in gain and sensitivity)

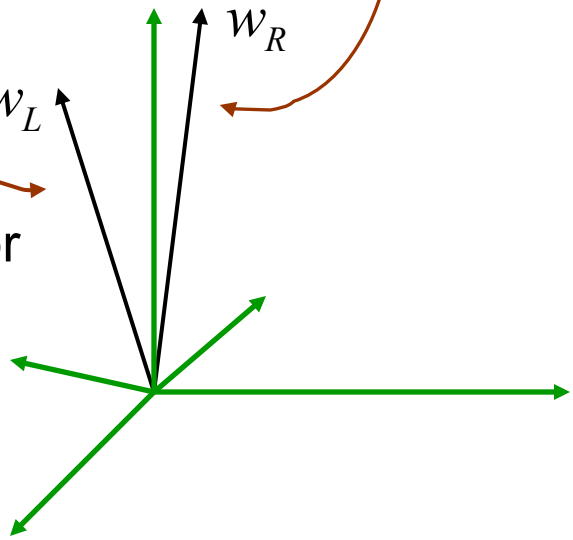
$$\bar{I} = \frac{1}{|W_m(x,y)|} \sum_{(u,v)\in W_m(x,y)} I(u,v) \qquad \text{Average pixel}$$

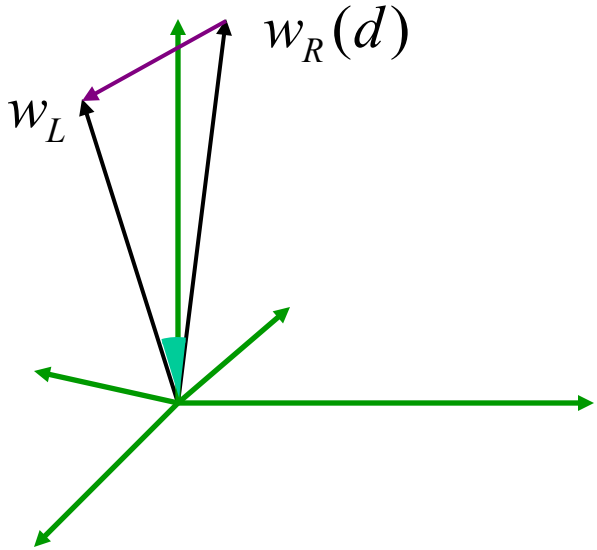
$$\|I\|_{W_m(x,y)} = \sqrt{\sum_{(u,v)\in W_m(x,y)} [I(u,v)]^2} \qquad \text{Window magnitude}$$

$$\hat{I}(x,y) = \frac{I(x,y) - \bar{I}}{\|I - \bar{I}\|_{W_m(x,y)}} \qquad \text{Normalized pixel}$$

Images as Vectors







Each window is a vector in an m^2 dimensional vector space.
Normalization makes them unit length.

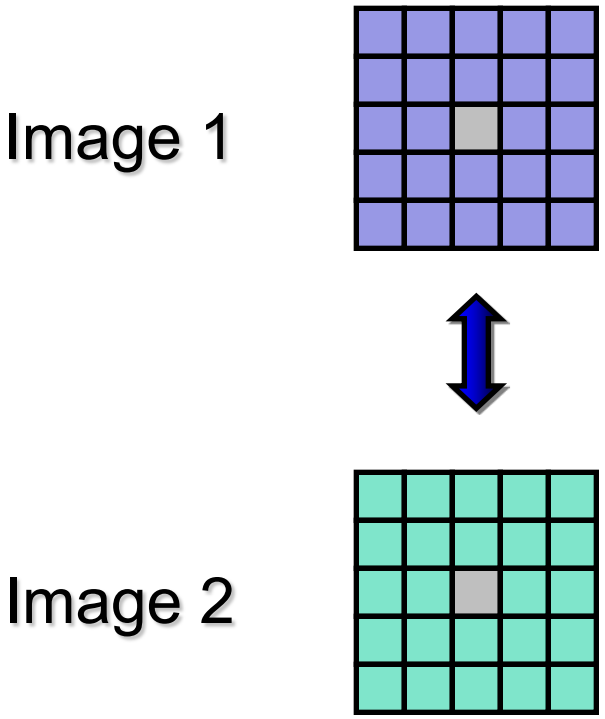
Basic stereo algorithm

- For each disparity
  - For each pixel
    - For each pixel in window

      Compute difference
- Find disparity with minimum SSD at each pixel

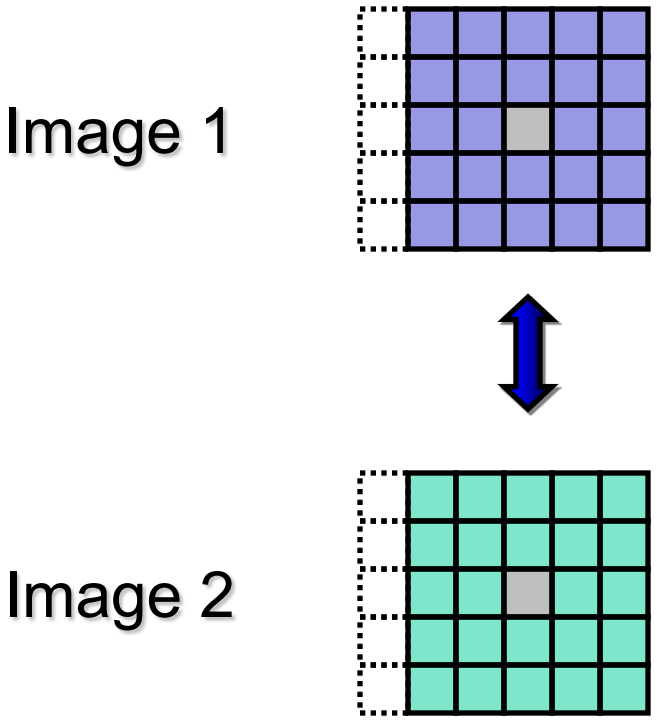
Incremental computation

- Given SSD of a window, at some disparity



Incremental computation

- Want: SSD at next location



Incremental computation

- Subtract contributions from leftmost column, add contributions from rightmost column

Image 1

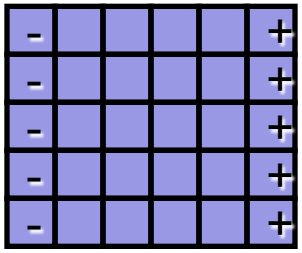
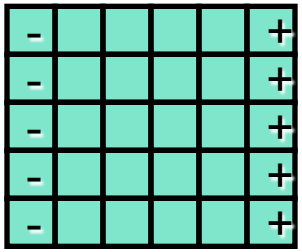


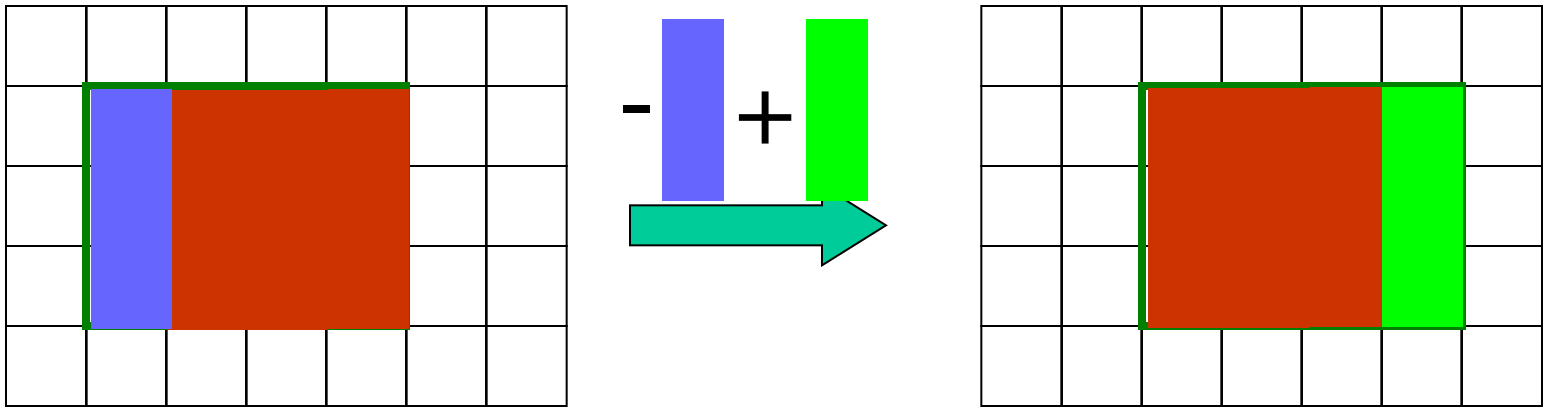
Image 2



Incremental computation

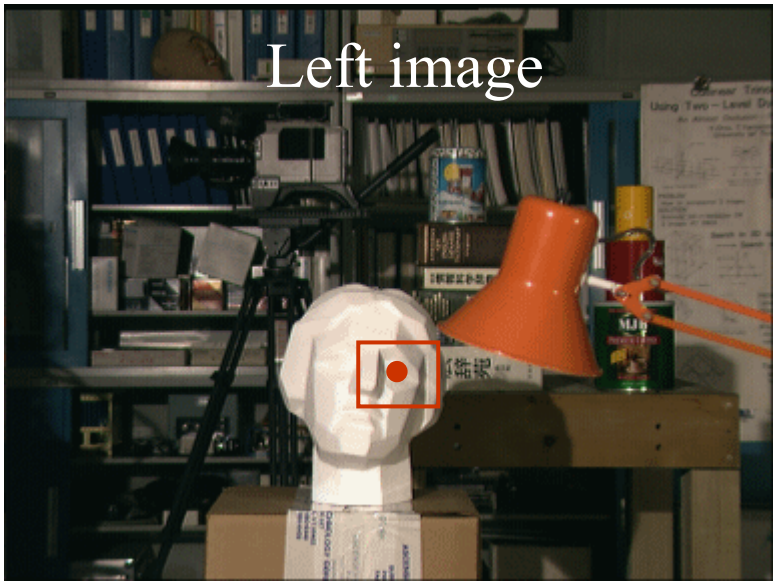
- Efficient implementation with “sliding column”, Faugeras et. al.’1993 for window cost function

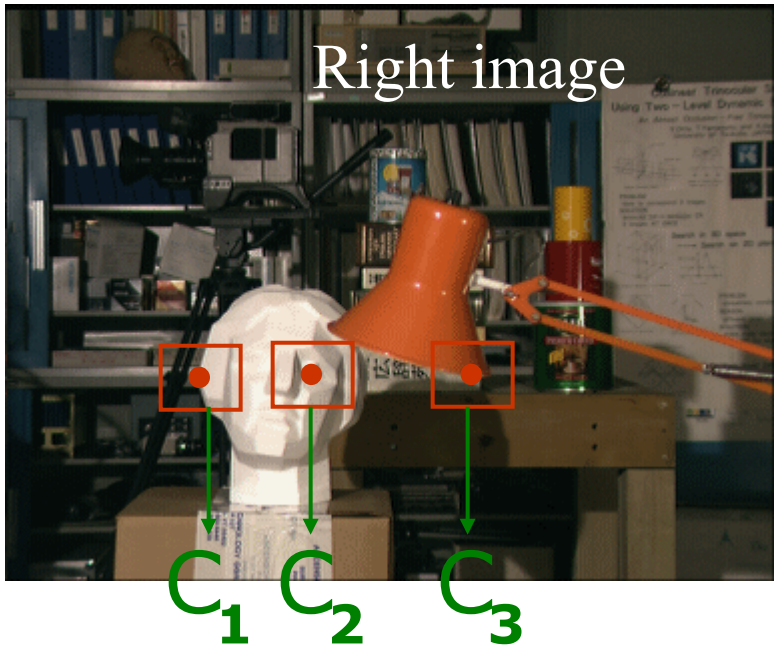
$$\sum_{x,y \in \square} f(x,y)$$



- Running time is independent of window size

Window based approach





- Typical window cost function

$$\sum_{x,y \in \square} f(x,y)$$

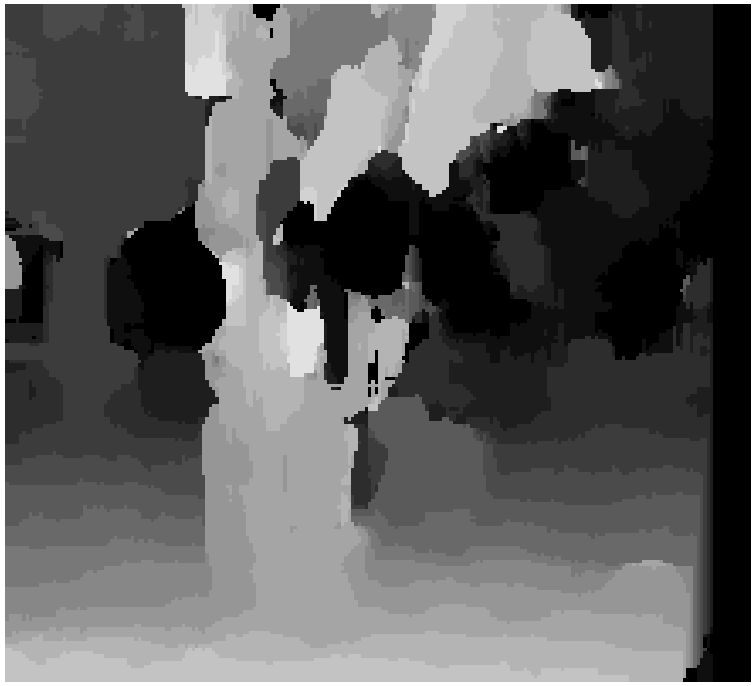
- Location with the best cost wins

Selecting window size

- Small window: more detail, but more noise
- Large window: more robustness, less detail
- Example:

Selecting window size

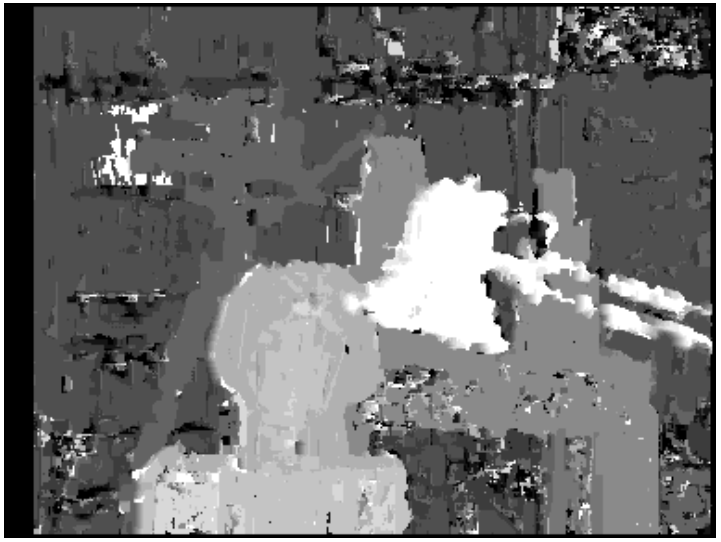
3 pixel window



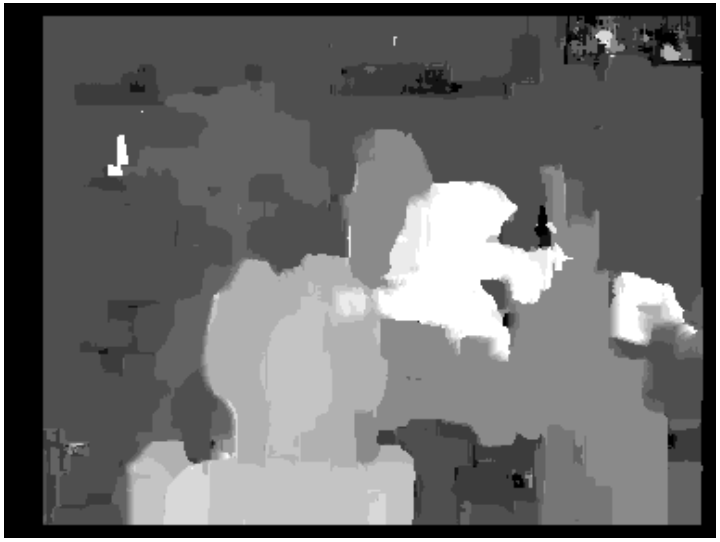
20 pixel window

Problems with Fixed Windows

small window



large window



better at boundaries

noisy in low texture areas

better in low texture areas

blurred boundaries

Non-square windows

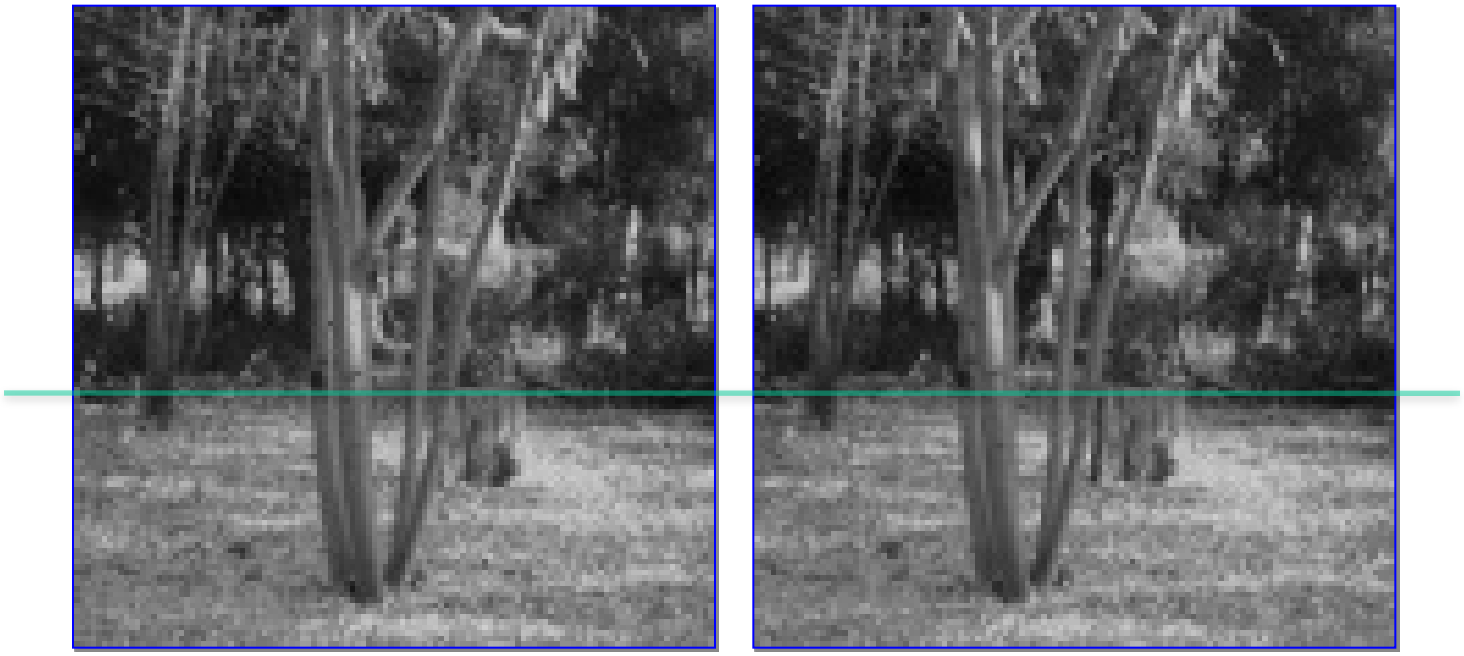
- Compromise: have a large window, but higher weight near the center
- Example: Gaussian

Problems with window matching

- No guarantee that the matching is one-to-one
- Hard to balance window size and smoothness

A global approach

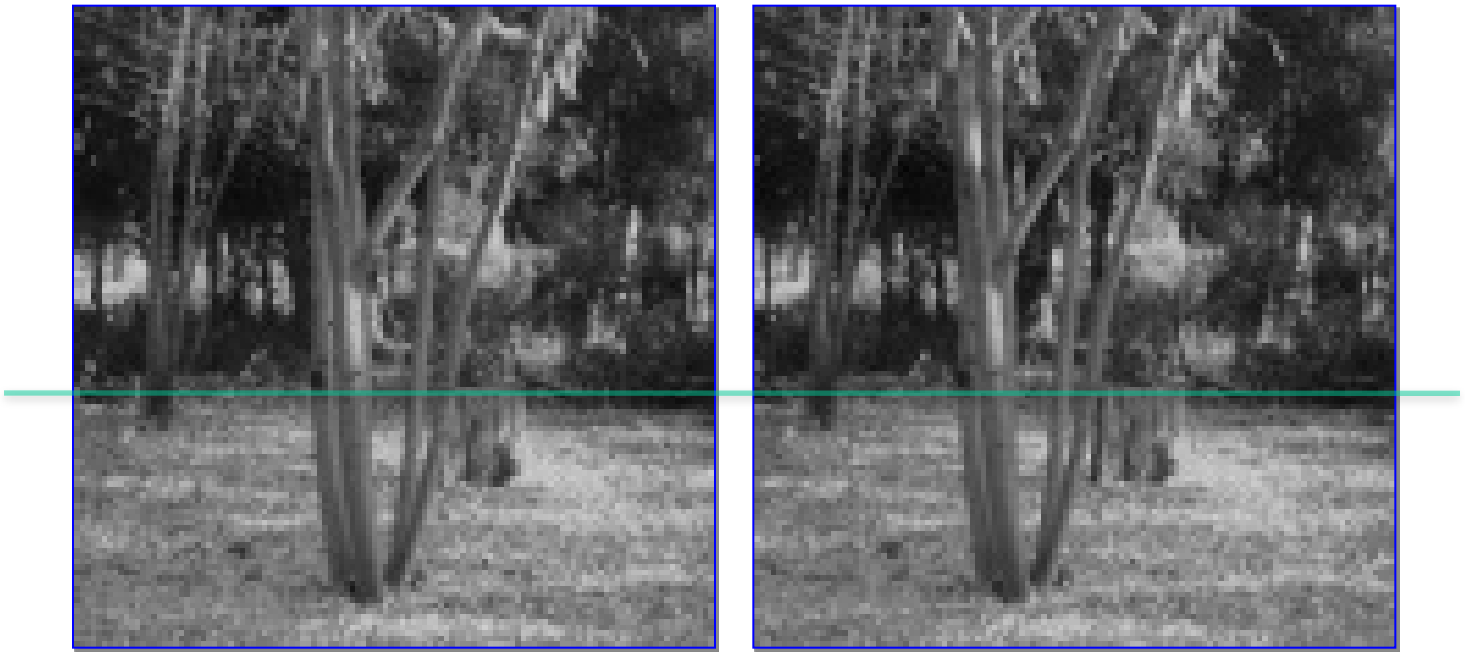
- Finding correspondence between a pair of epipolar lines for all pixels simultaneously



A global approach

- Finding correspondence between a pair of epipolar lines for all pixels simultaneously

Still challenging .



Why is Stereo Matching Challenging? (1)

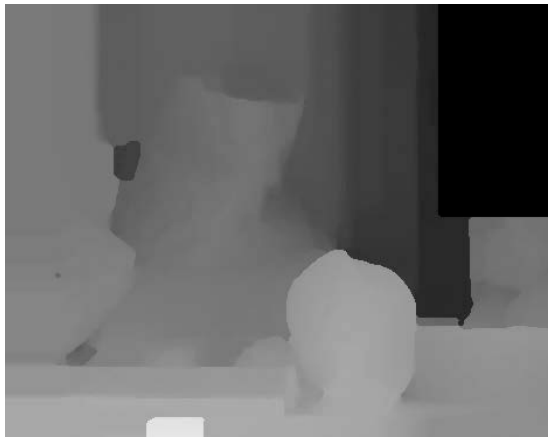
- Color inconsistencies:
  - When solving the stereo matching problem, we typically assume that corresponding pixels have the same intensity/color (= Photo consistency assumption)
  - That does not need to be true due to:
    - Image noise
    - Different illumination conditions in left and right images
    - Specular reflections

Why is Stereo Matching Challenging? (2)

- Untextured regions (Matching ambiguities)
  - There needs to be a certain amount of intensity/color variation (i.e. texture) so that a pixel can be uniquely matched in the other view.
  - Can you (as a human) depict depth if you are standing in front of a wall that is completely white?

Left image (no texture in the background)

Right image

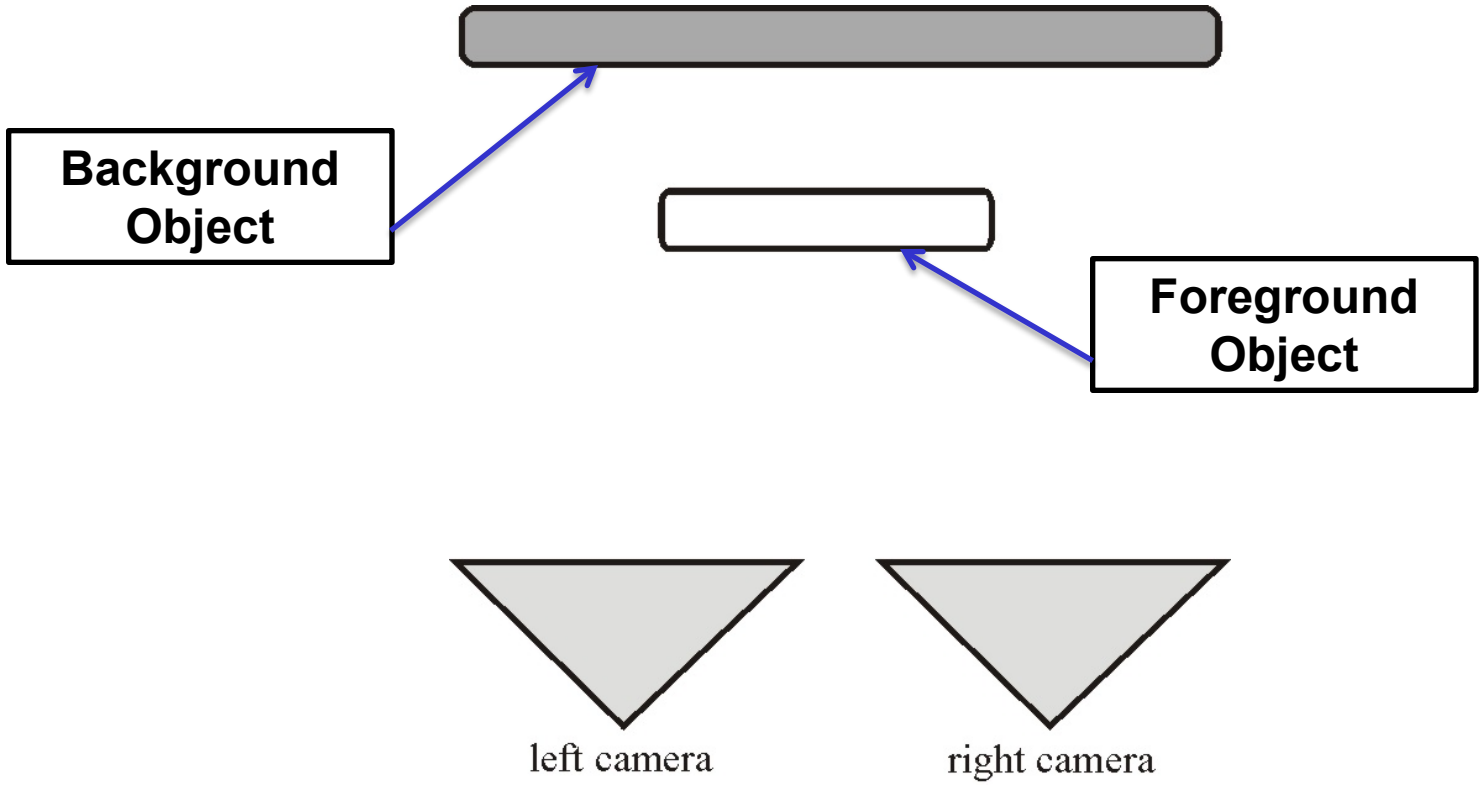


Computed disparity map (errors in background)

Why is Stereo Matching Challenging? (3)

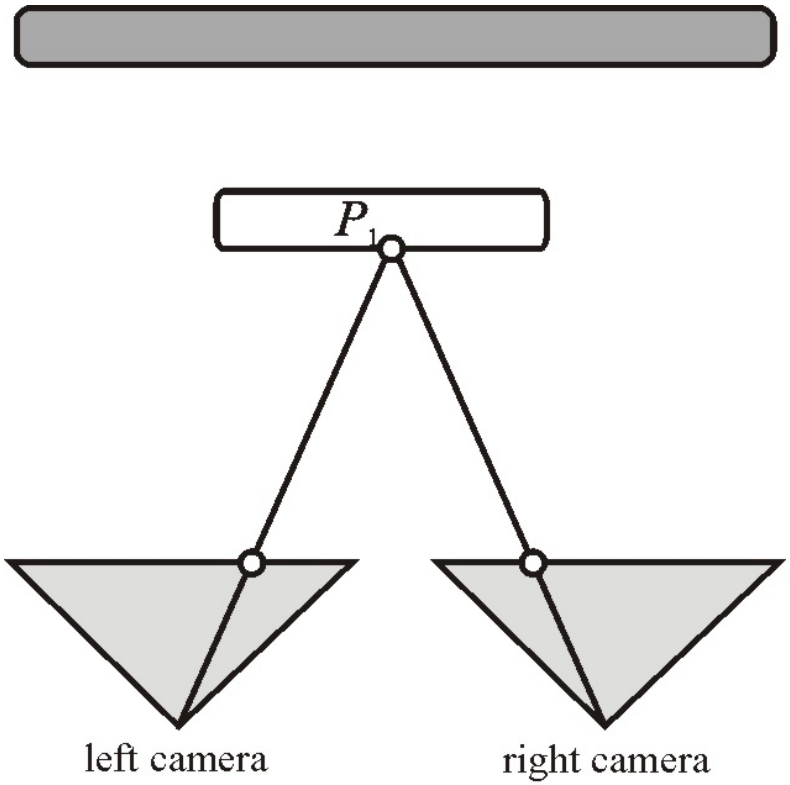
- Occlusion Problem
  - There are pixels that are only visible in exactly one view.
  - We call this pixels occluded (or half-occluded)
  - It is difficult to estimate depth for these pixels.

The Occlusion Problem



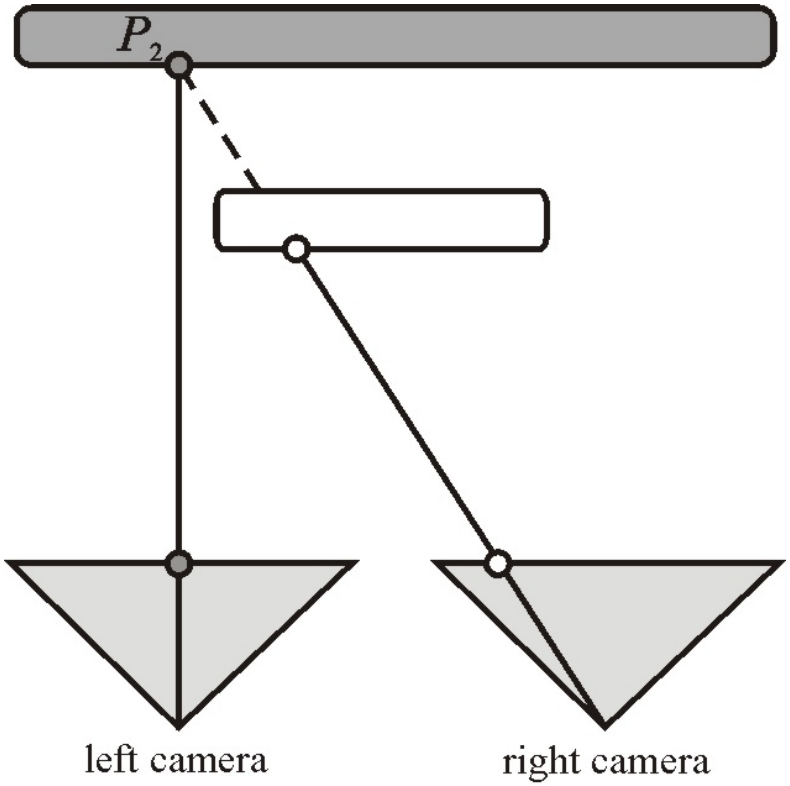
- Let's consider a simple scene composed of a foreground and a background object

The Occlusion Problem



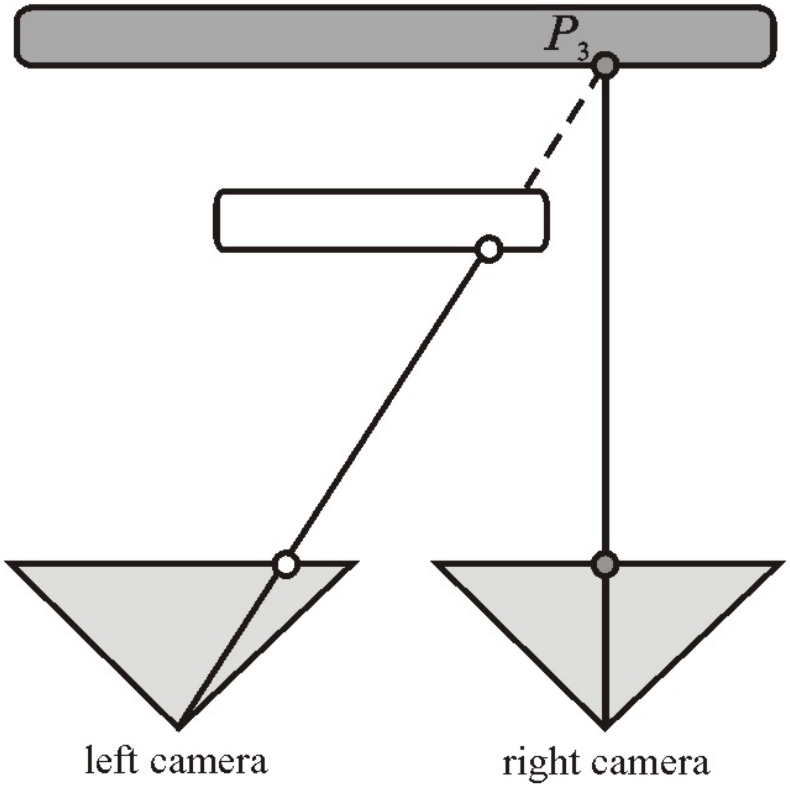
- Regular case:
  - The white pixel P_1 can be seen by both camera.

The Occlusion Problem



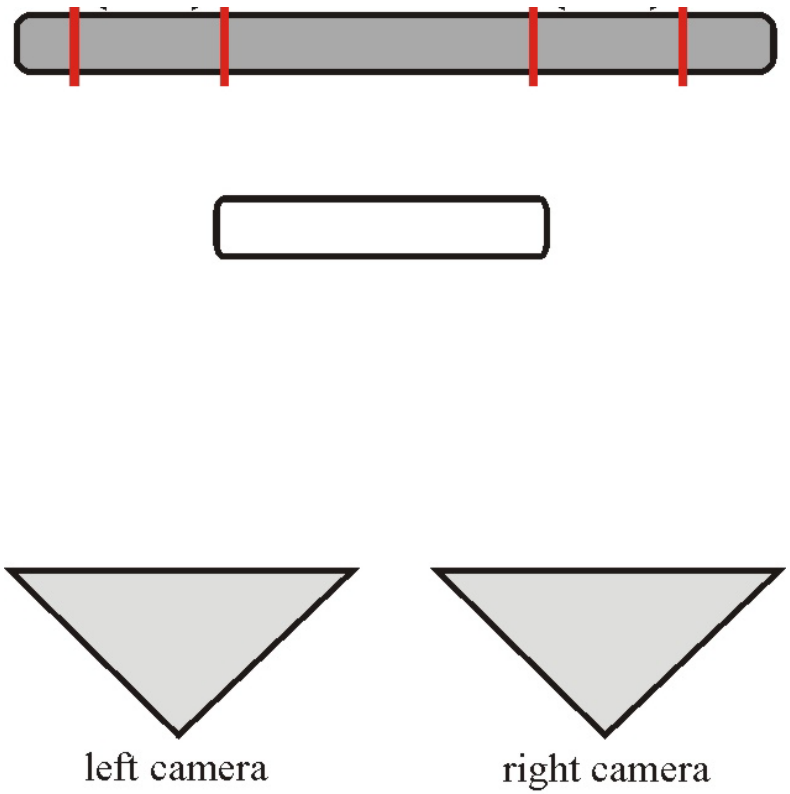
- Occlusion in the right camera:
  - The left camera sees the grey pixel *P*2.
  - The ray from the right camera to *P*2 hits the white foreground object => *P*2 cannot be seen by right camera.

The Occlusion Problem



- Occlusion in the left camera:
  - The right camera sees the grey pixel *P*3.
  - The ray from the left camera to *P*3 hits the white foreground object => *P*3 cannot be seen by left camera.

The Occlusion Problem



- Occlusions occur in the proximity of disparity discontinuities.

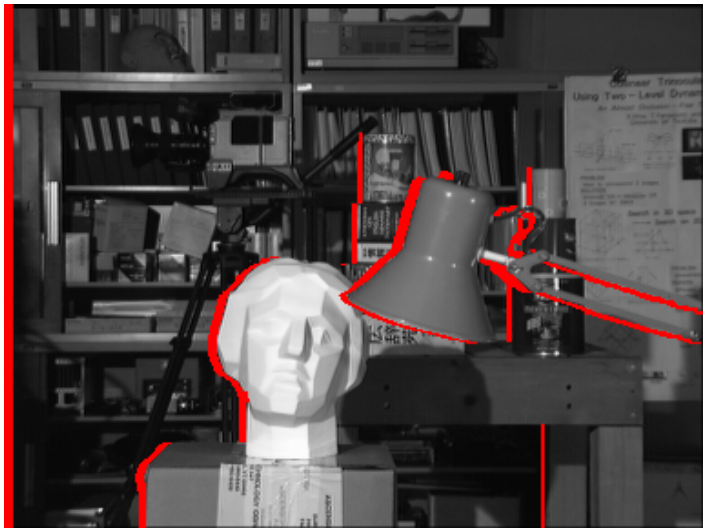
The Occlusion Problem

Occlusions occur as a consequence of discontinuities in depth.

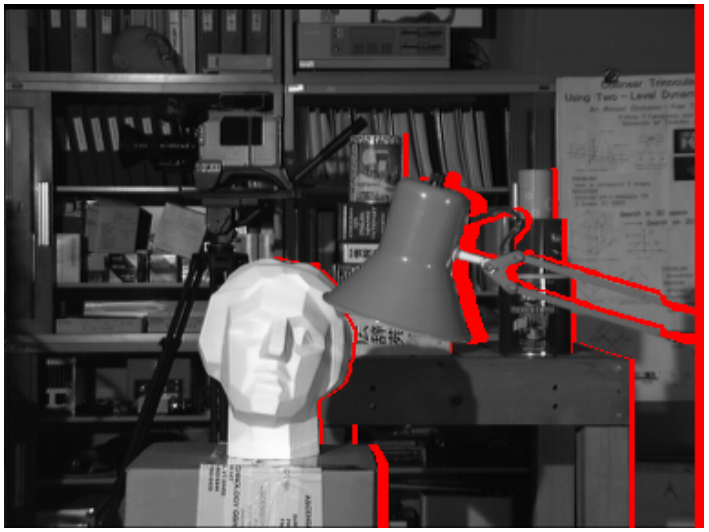
They occur close object/depth boundaries.

They occur in both frames

The Occlusion Problem



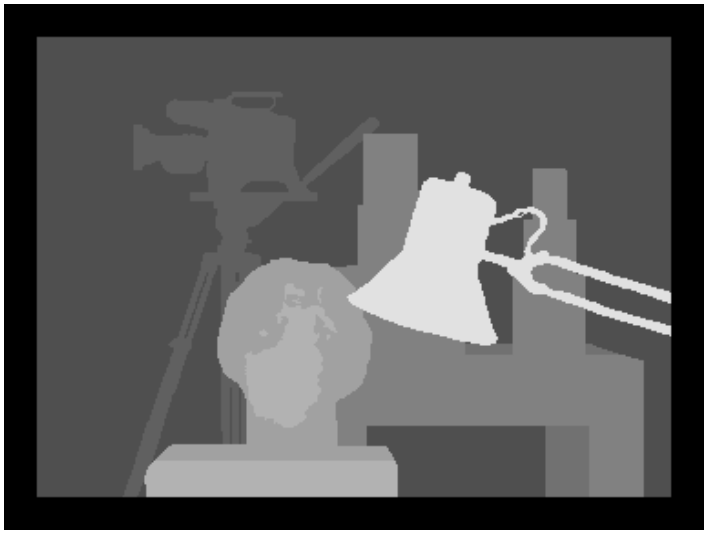
Left Image (Occlusions in red color)



Right Image (Occlusions in red color)

- In the left image, occlusions are located to the left of a disparity boundary.
- In the right image, occlusions are located to the right of a disparity boundary.

The Occlusion Problem



Correct Disparity Map (Geometry of Left Image)

Computed Disparity Map (Occlusions Ignored)

- It is difficult to find disparity if the matching point does not exist!
- Ignoring the occlusion problem leads to disparity artefacts near disparity borders.

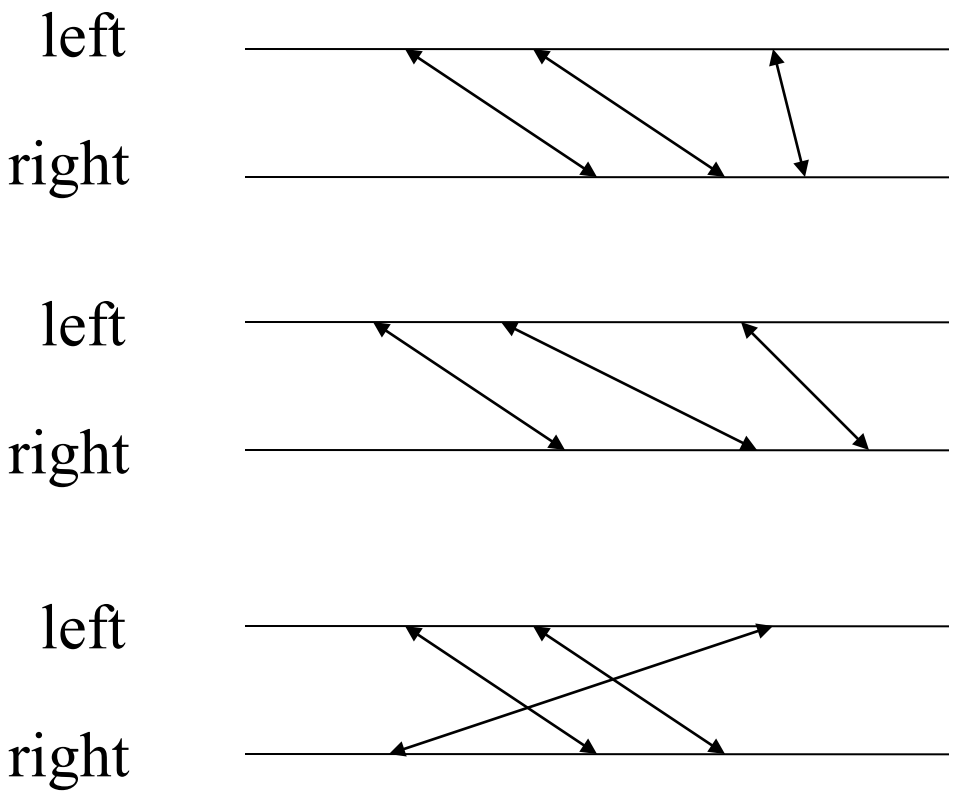
Stereo

If x-shifts (disparities) are known for all pixels in the left (or right) image then we can visualize them as a disparity map: the image above makes sense globally

Variable window size algorithms

- Correspondences are still found independently at each pixel
- All Window-based solution can be thought of as “local” solutions
  - Fast
- How to introduce spatial coherence
  - Introduce Energy function
  - Searching for “global” solutions

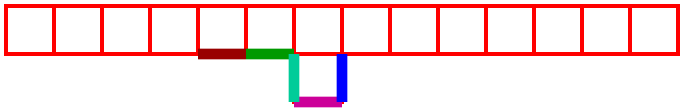
A global approach: Dynamic Programming



Define a global evaluation score for each configuration, choose the best matching configuration

Evaluation score: the sum of corresponding pixel difference?

Correspondences

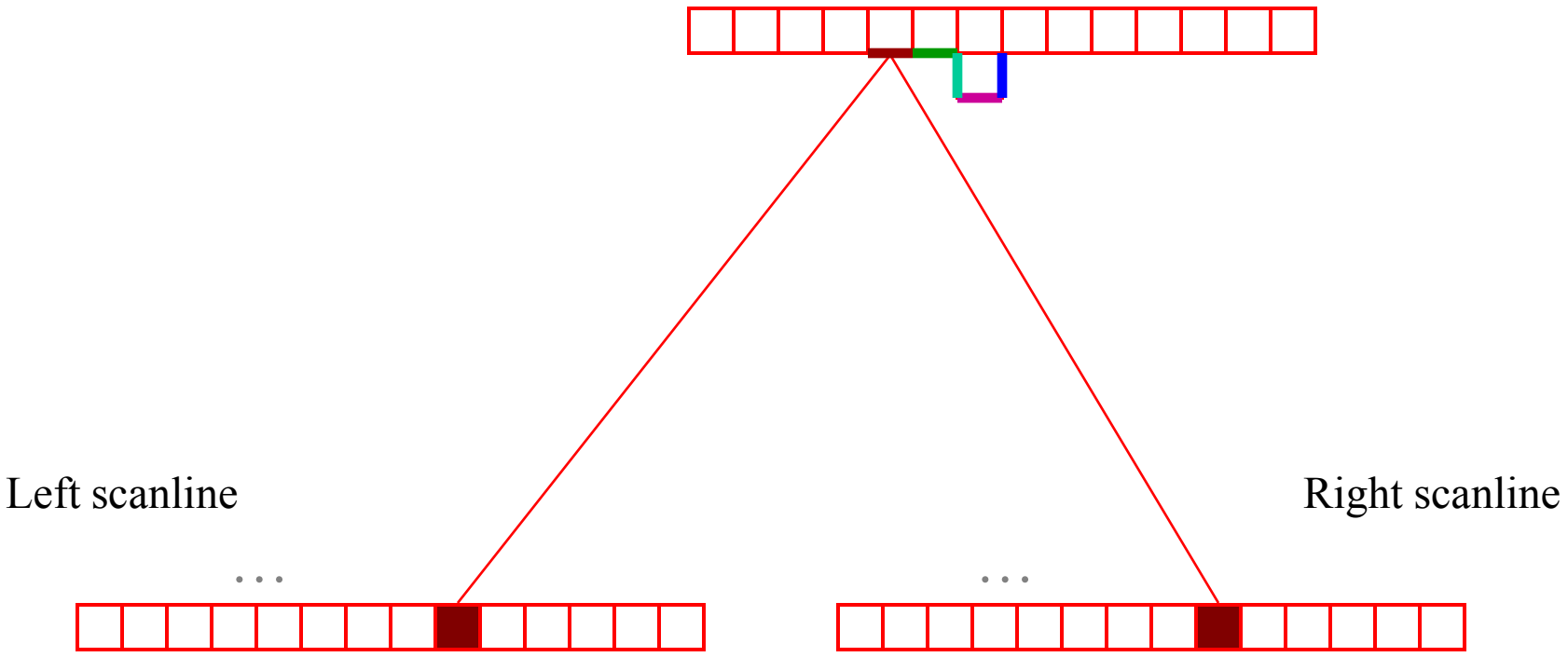


Left scanline

Right scanline

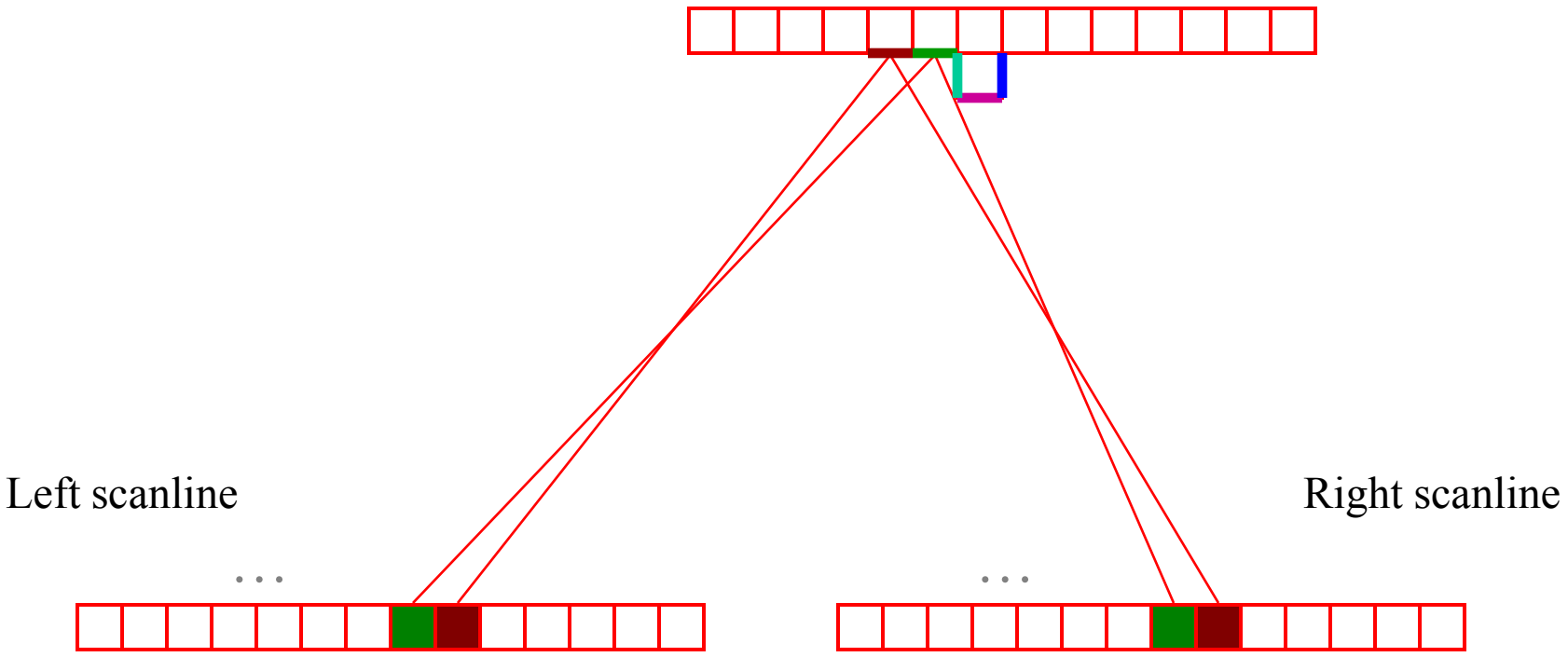
Match intensities sequentially between two scanlines

Correspondences



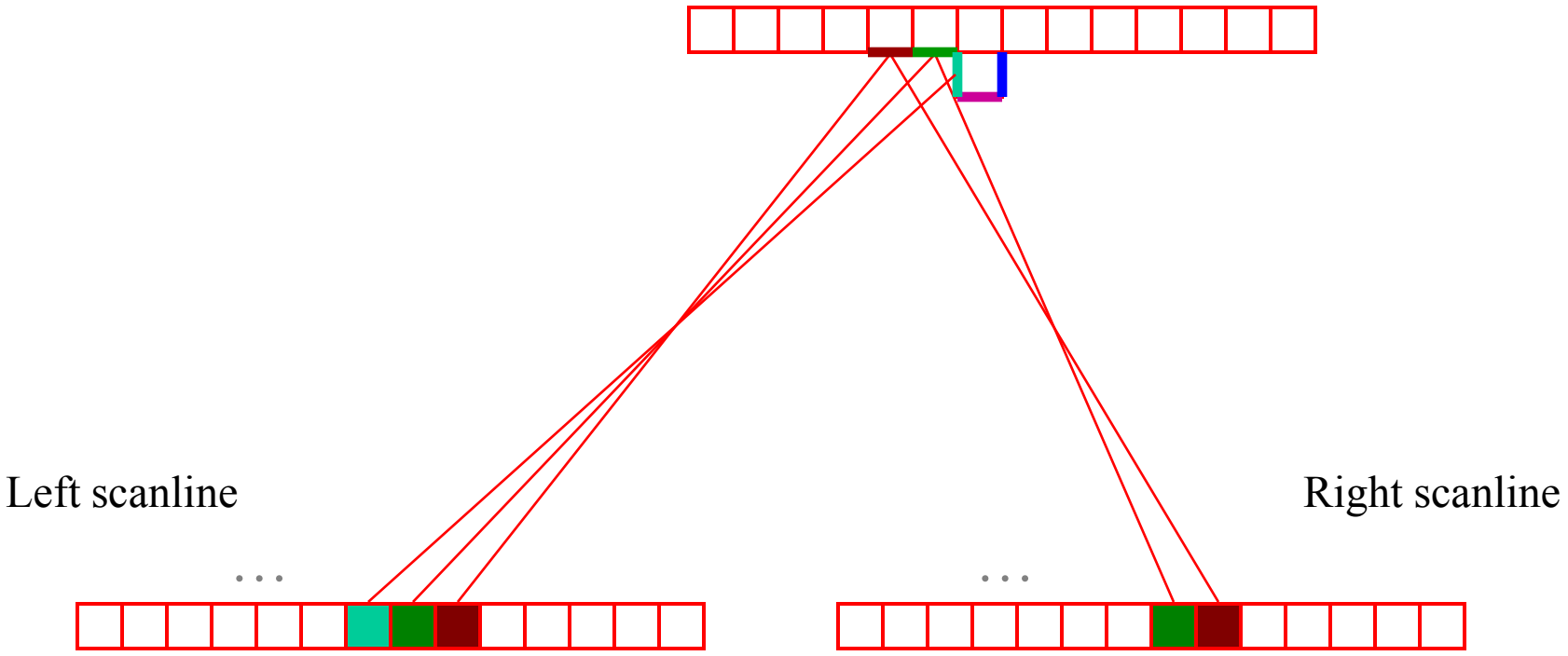
Match intensities sequentially between two scanlines

Correspondences



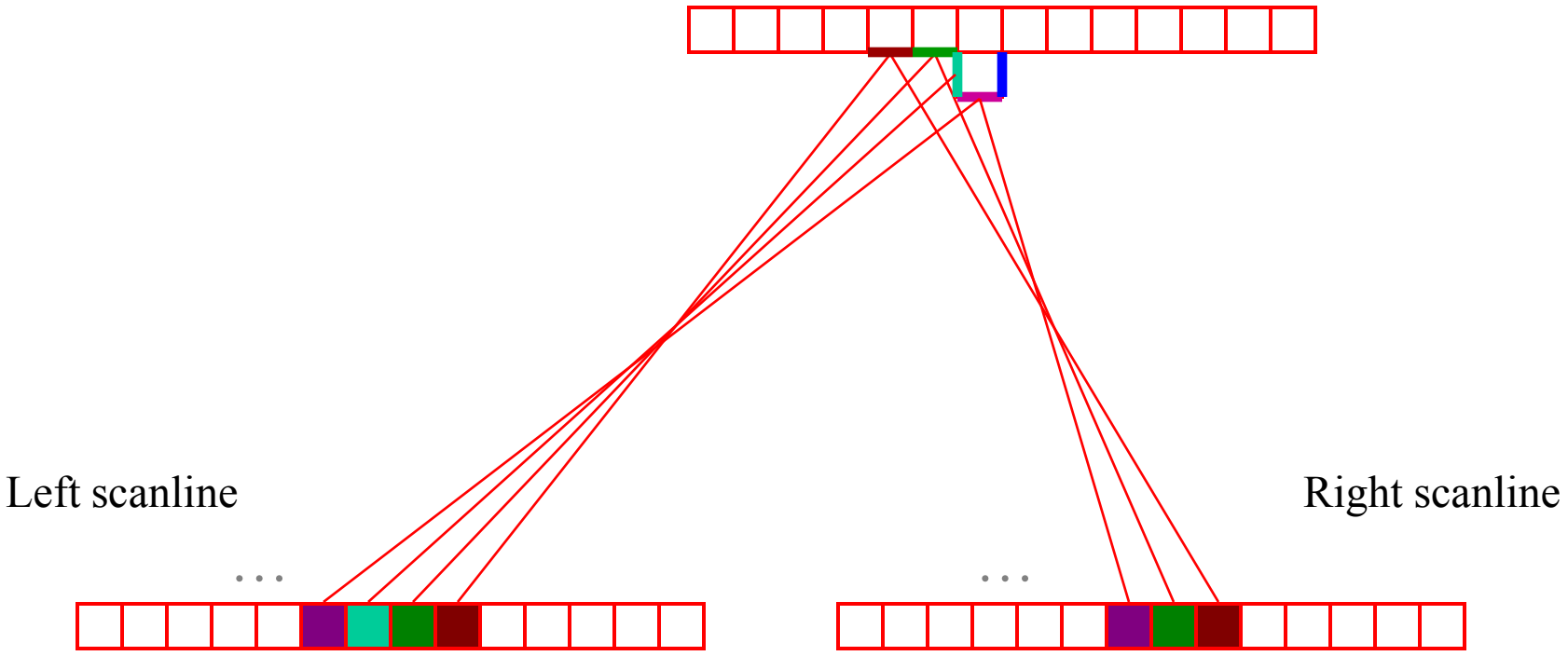
Match intensities sequentially between two scanlines

Correspondences



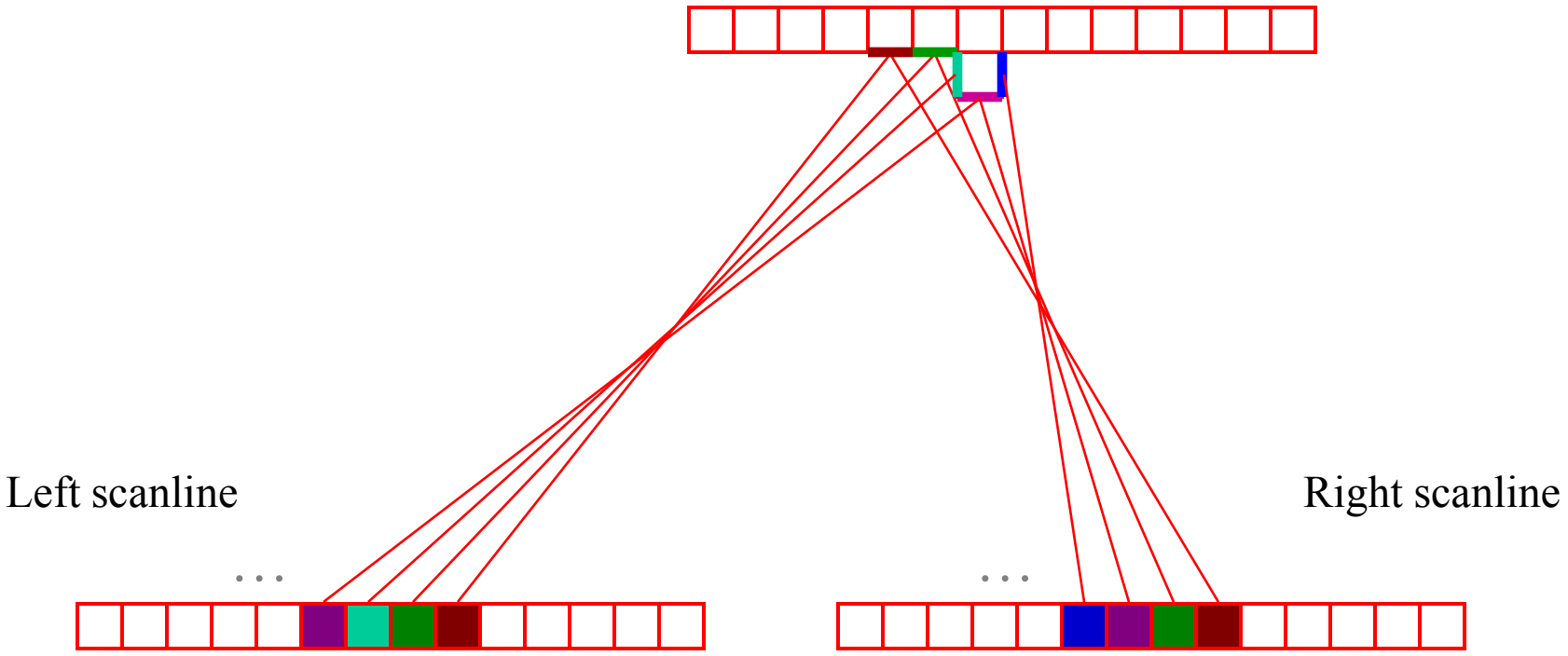
Match intensities sequentially between two scanlines

Correspondences



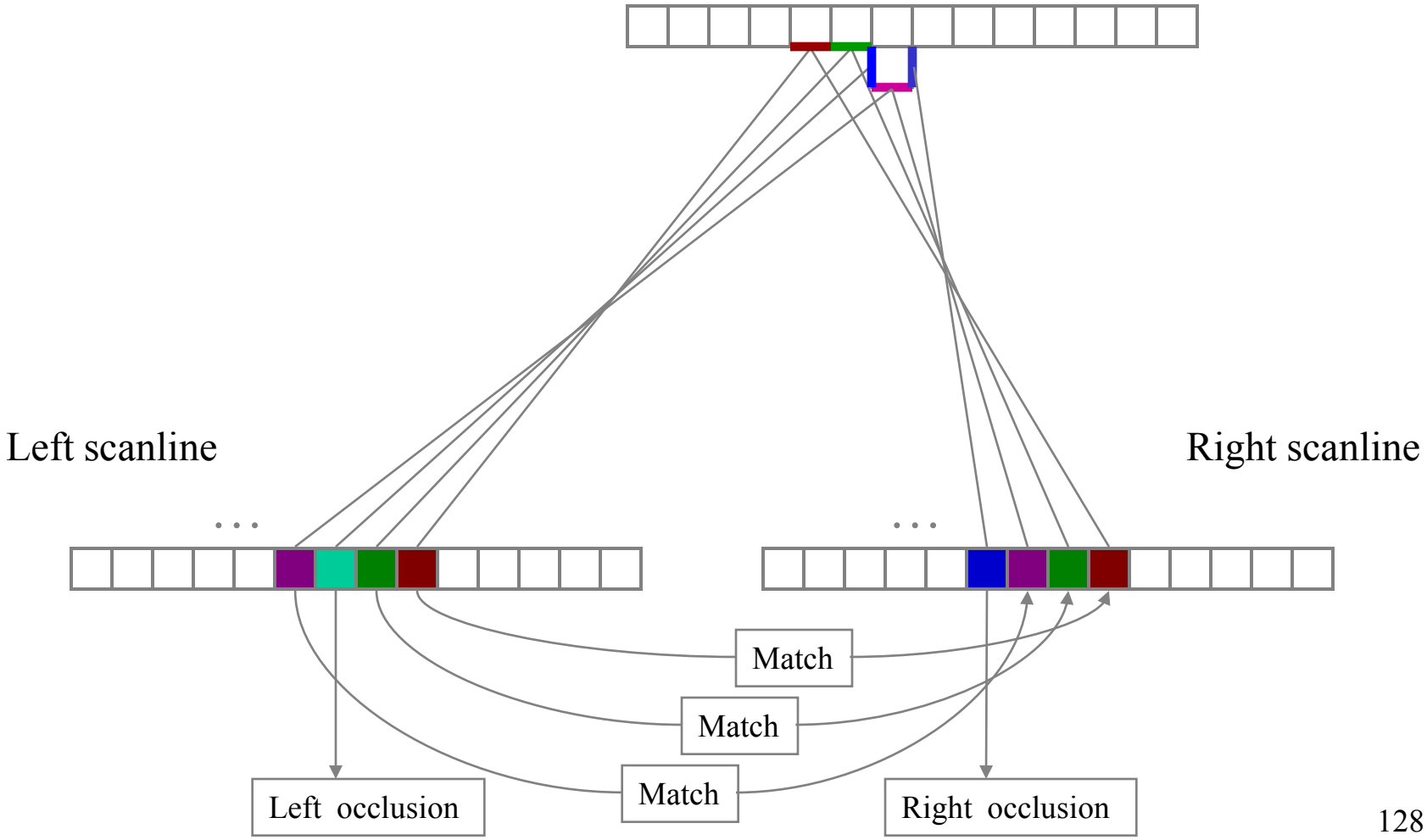
Match intensities sequentially between two scanlines

Correspondences

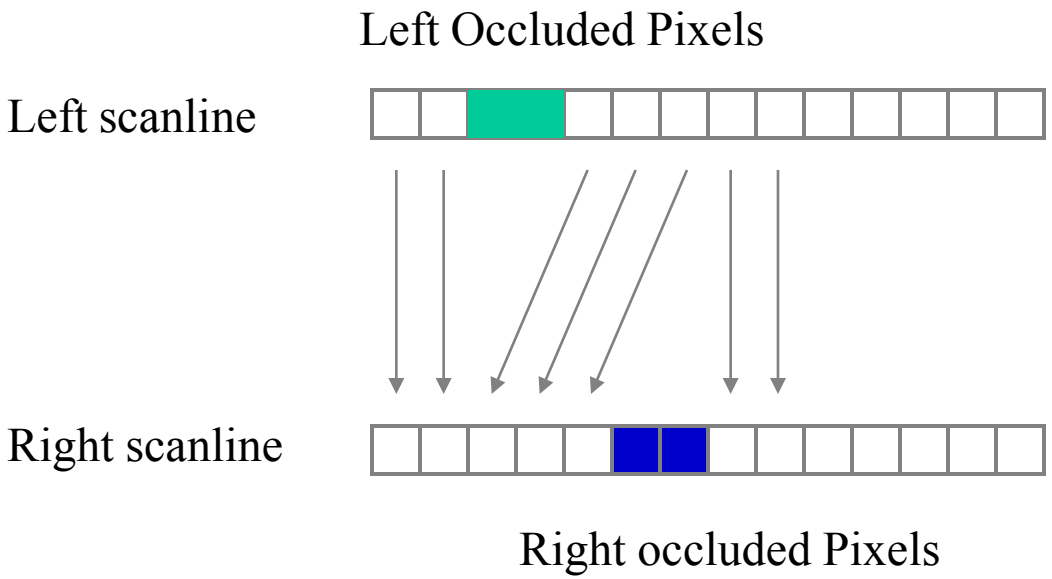


Match intensities sequentially between two scanlines

Correspondences



Search Over Correspondences



Three cases:

- Sequential – cost of match
- Left occluded – cost of no match
- Right occluded – cost of no match

Digression….

Dynamic programming example

Consider the matrix multiplication example: A1: 5x4, A2: 4x6, A3: 6x2

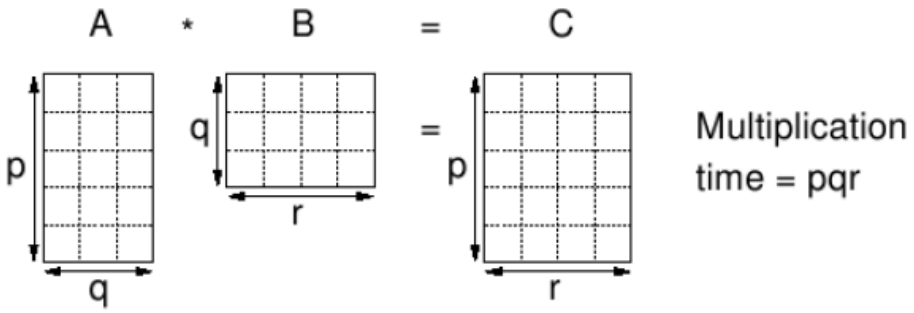


Fig. 7: Matrix Multiplication.

Note that although any legal parenthesization will lead to a valid result, not all involve the same number of operations. Consider the case of 3 matrices: A_1 be 5×4, A_2 be 4×6 and A_3 be 6×2.

$$\begin{aligned} \text{multCost}[((A_1A_2)A_3)] &= (5 \cdot 4 \cdot 6) + (5 \cdot 6 \cdot 2) = 180, \\ \text{multCost}[(A_1(A_2A_3))] &= (4 \cdot 6 \cdot 2) + (5 \cdot 4 \cdot 2) = 88. \end{aligned}$$

Look for

- Optimal substructure
- Overlapping subproblems

Matrix Chain-Products

- **Matrix Chain-Product:**
  - Compute $A=A_0*A_1* \quad *A_{n-1}$
  - Problem: How to parenthesize?
- Example
  - B is 3 × 100
  - C is 100 × 5
  - D is 5 × 5
  - (B*C)*D takes 1500 + 75 = 1575 ops
  - B*(C*D) takes 1500 + 2500 = 4000 ops

Dynamic Programming Algorithm

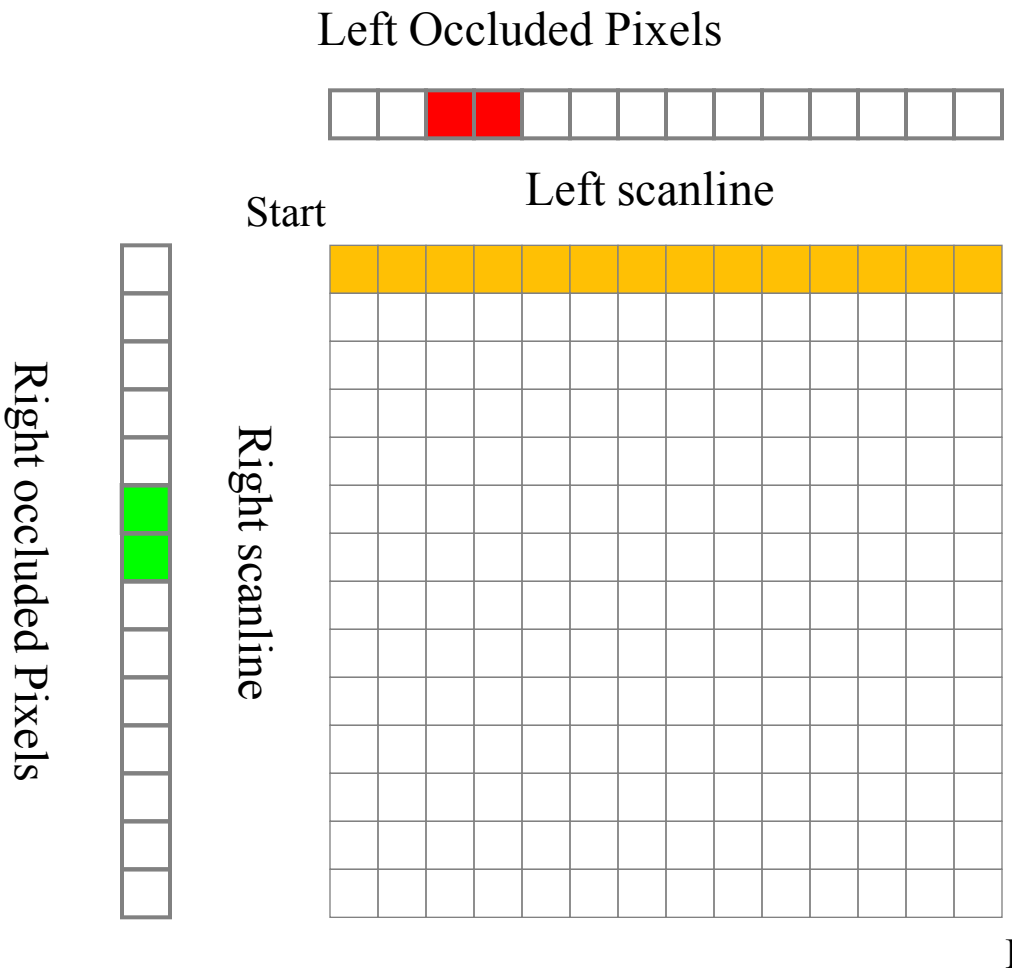
- A_0: 30 X 35; A_1: 35 X15; A_2: 15X5;
  A_3: 5X10; A_4: 10X20; A_5: 20 X 25

0	1	2	3	4	5	
0	15,750	7,875	9,375	11,875	15,125	0
	0	2,625	4,375	7,125	10,500	1
		0	750	2,500	5,375	2
			0	1,000	3,500	3
				0	5,000	4
					0	5

$$N_{i,j} = \min_{i \le k < j} \{ N_{i,k} + N_{k+1,j} + d_i d_{k+1} d_{j+1} \}$$

$$N_{1,4} = \min \{$$
$$N_{1,1} + N_{2,4} + d_1 d_2 d_5 = 0 + 2500 + 35*15*20 = 13000,$$
$$N_{1,2} + N_{3,4} + d_1 d_3 d_5 = 2625 + 1000 + 35*5*20 = 7125,$$
$$N_{1,3} + N_{4,4} + d_1 d_4 d_5 = 4375 + 0 + 35*10*20 = 11375$$
$$\}$$
$$= 7125$$

Standard 3-move Dynamic Programming for Stereo

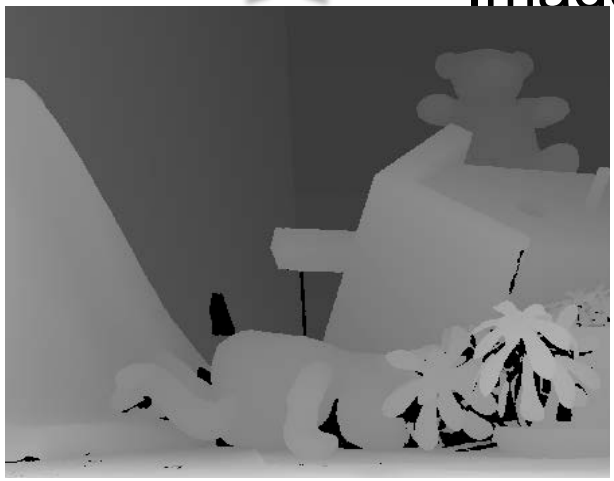


Dynamic programming yields the optimal path through grid. This is the best set of matches that satisfy the ordering constraint

Dynamic Programming Results

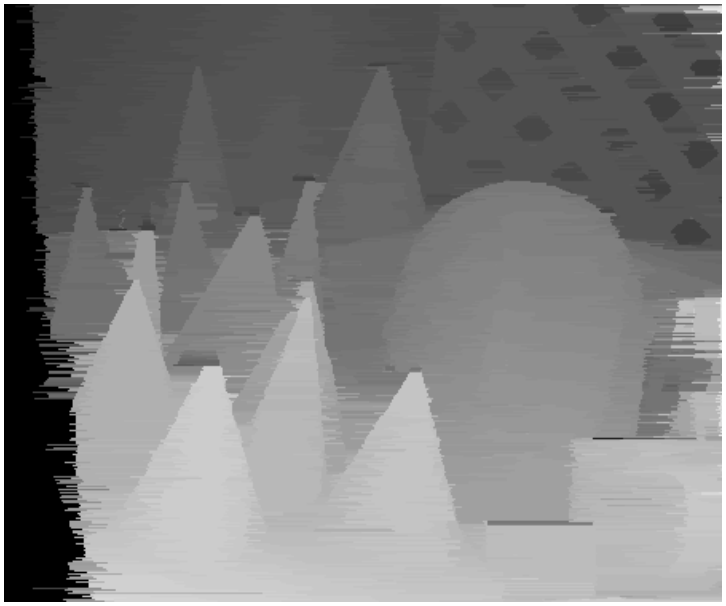
Left 2D Image

Right 2D Image



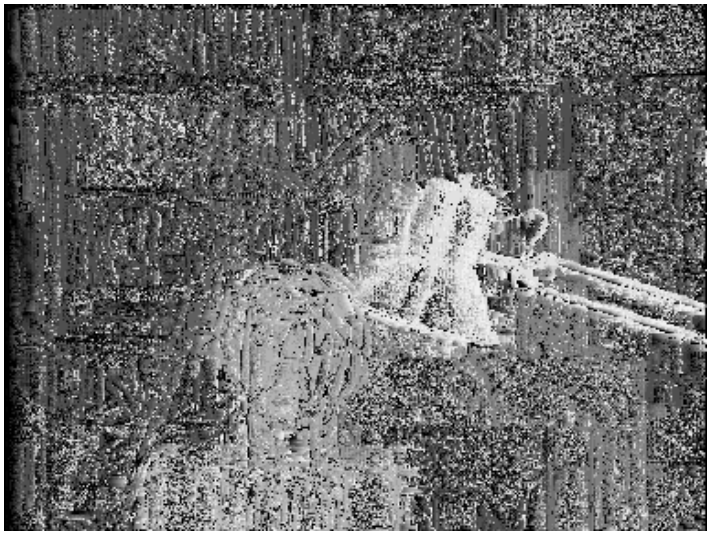
Disparity Map

Dynamic Programming Results



No global optimization

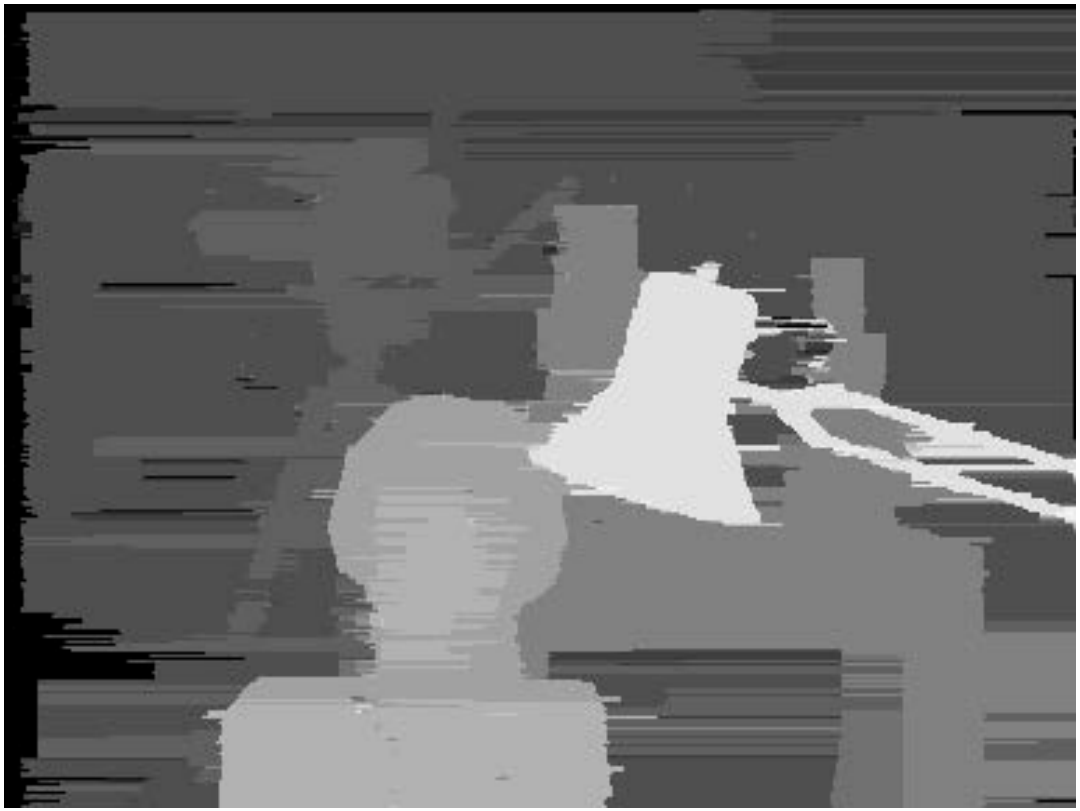
Left Image



Computed Disparity Map

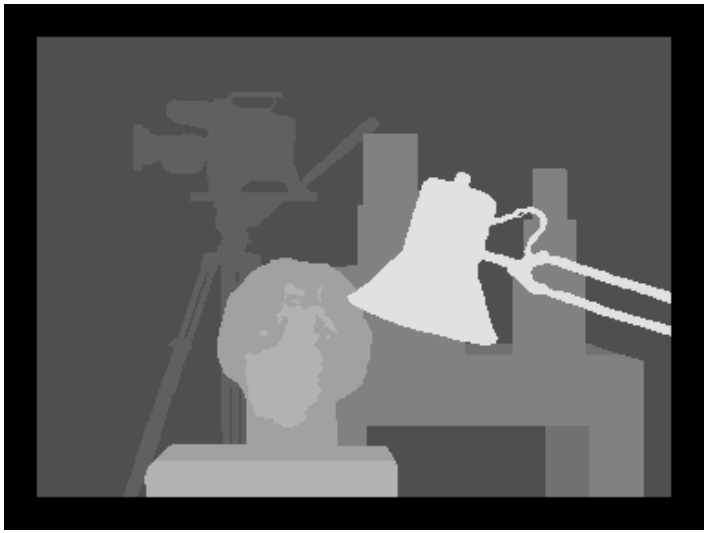
- Quite disappointing, why?
- We have posed the following task:
  - I have a red pixel. Find me a red pixel in the other image.
- Problem:
  - There are usually many red pixels in the other image (ambiguity)
- We need additional assumptions.

Dynamic Programming Result

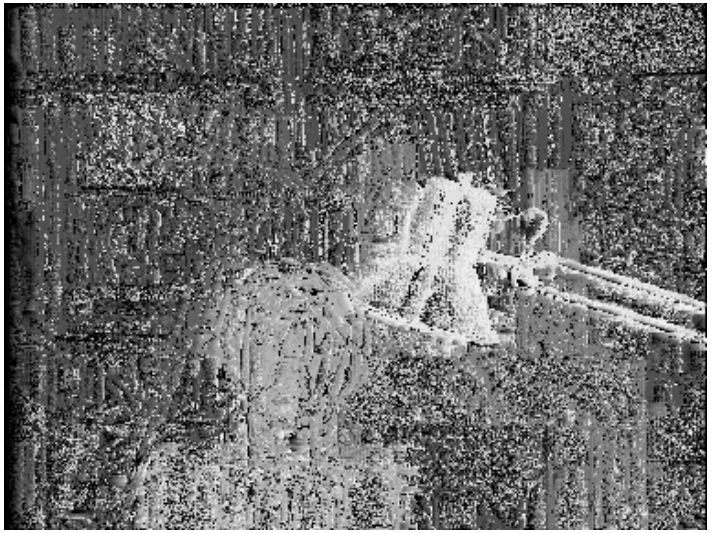


Note the horizontal streaks!
It doesn't want to change its mind!

What is the most obvious problem?



Correct Disparity Map



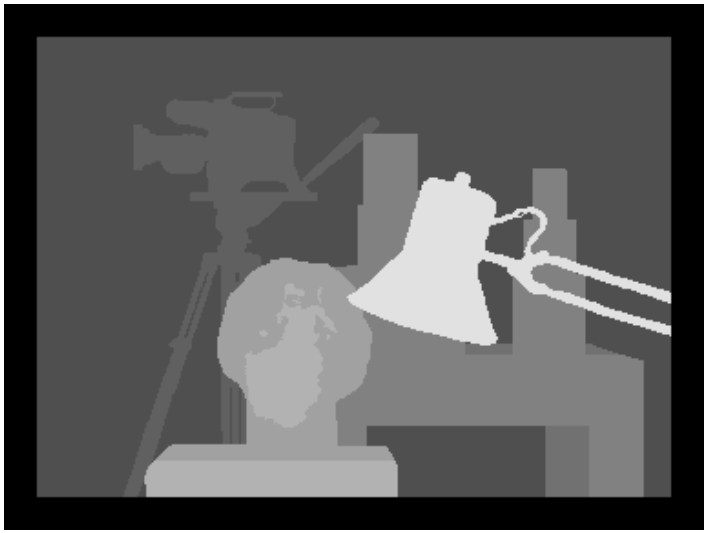
Computed Disparity Map

- What is the most obvious difference between the correct and the computed disparity maps?

Smoothness Assumption (1)

- Observation:
  - A correct disparity map typically consists of regions of constant (or very similar) disparity. For example, lamp, head, table,
- We can give this apriori knowledge to a stereo algorithm in the form of a smoothness assumption

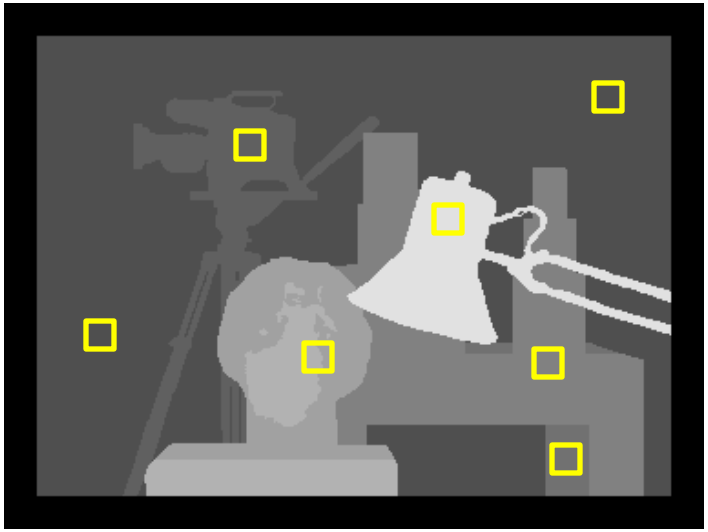
Left Image



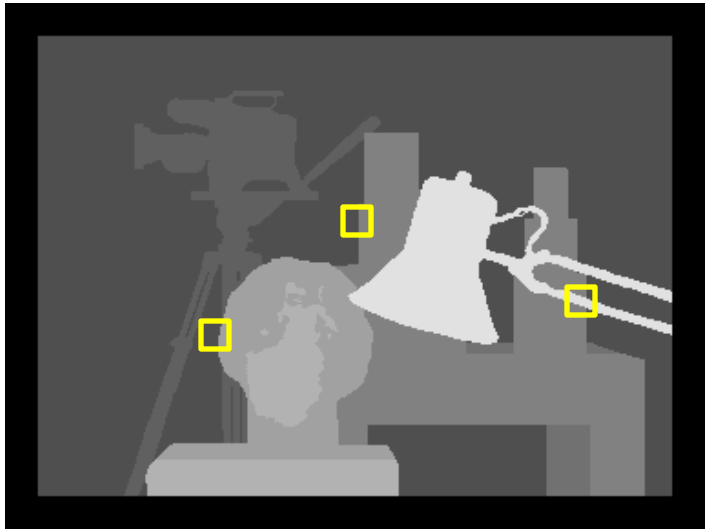
Correct Disparity Map

Smoothness Assumption

- Smoothness assumption:
  - Spatially close pixels have the same (or similar) disparity.
  - (By spatially close I mean pixels of similar image coordinates.)
- Smoothness assumption typically holds true almost everywhere, except at disparity borders.

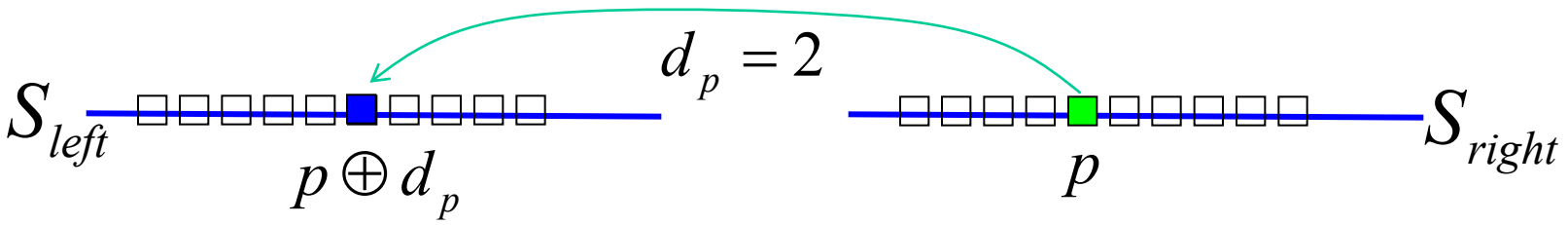


Regions where smoothness assumption is valid

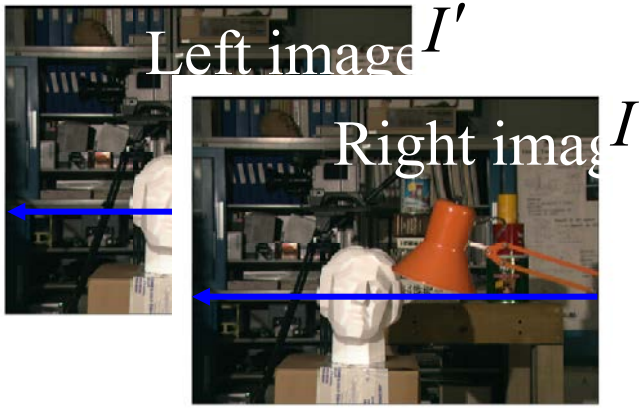


Regions where smoothness assumption is *not* valid

Revisit DP for scan-line stereo



Viterbi algorithm can be used to optimize the following energy of *disparities* $\mathbf{d} = \{d_p \mid p \in S\}$ of pixels p on a fixed scan-line S_{right}



$$E(\mathbf{d}) = \sum_{p \in S} D_p(d_p) + \sum_{p \in S} V(d_p, d_{p+1}) = \sum_{\{p,q\} \in N} E(d_p, d_q)$$

$D_p(d_p) = |I_p - I'_{p \oplus d_p}|$ — photo consistency

$V(d_p, d_{p+1}) = |d_p - d_{p+1}|$ — spatial coherence

Viterbi can handle this on non-loopy graphs (e.g., scan-lines)

Other Global Methods: graph based

- Define a cost function to measure the quality of a disparity map:
  - High costs mean that the disparity map is bad.
  - Low costs mean it is good.
- Costs function is typically in the form of:

where $$E = E_{data} + E_{smooth}$$

  - E_{data} measures photo consistency
  - E_{smooth} measures smoothness
- Global methods express smoothness assumption in an explicit form

Coherent stereo on 2D grid?

- Scan-line stereo generates streaking artifacts
- Can’t use DP (or Dijkstra) to find spatially coherent disparities/correspondances on a 2D grid

- Now lets talk about **graph cuts** algorithm

Graph cuts for spatially coherence on grids

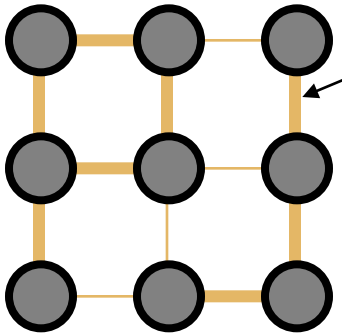
We will globally minimize the same energy of disparities $\mathbf{d} = \{d_p \mid p \in G\}$ for pixels *p* on a grid *G*

$$E(\mathbf{d}) = \sum_{p \in G} D_p(d_p) + \sum_{\{p,q\} \in N} V(d_p, d_q)$$

$$D_p(d_p) = |I_p - I'_{p \oplus d_p}| \qquad V(d_p, d_q) = w_{pq} \cdot |d_p - d_q|$$

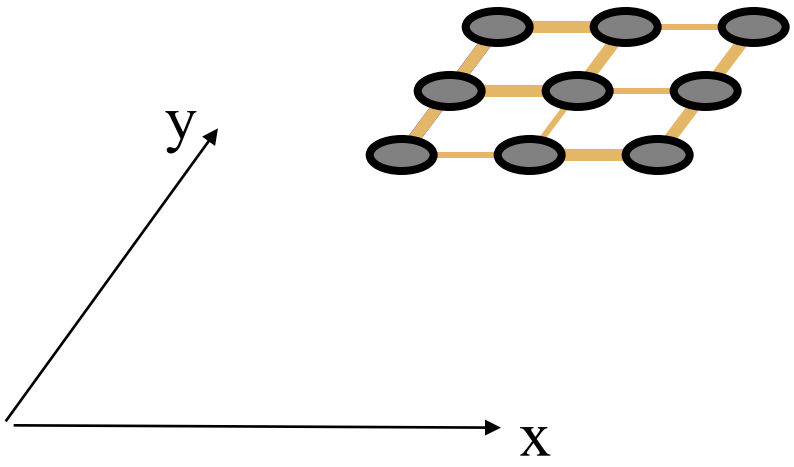
photo consistency spatial coherence

Consider a 2D grid *G* corresponding to (right) image pixels

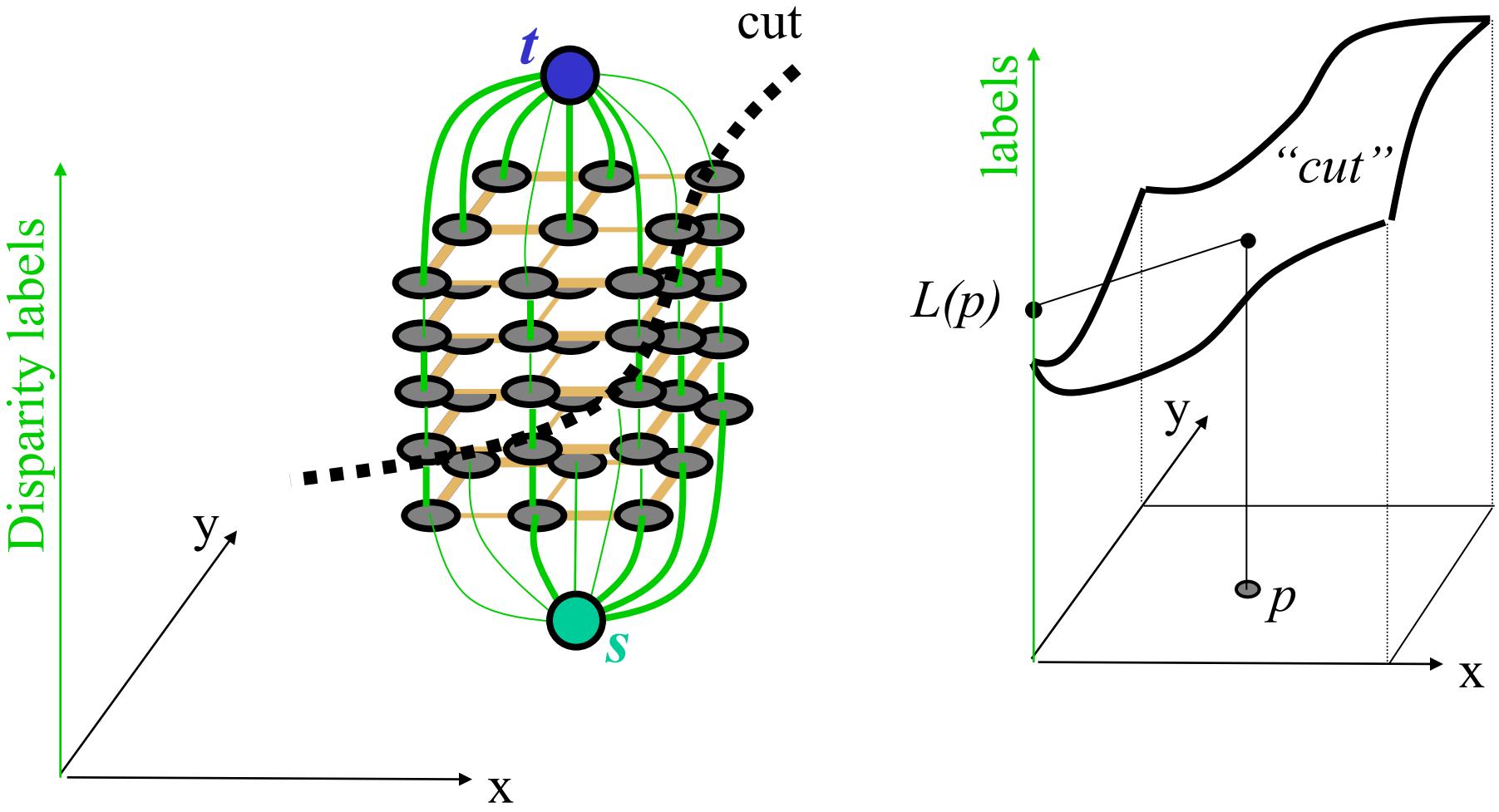


w_{pq} weights of neighborhood edges, **n-links**, may reflect (right) image intensity gradients (**static cues**)

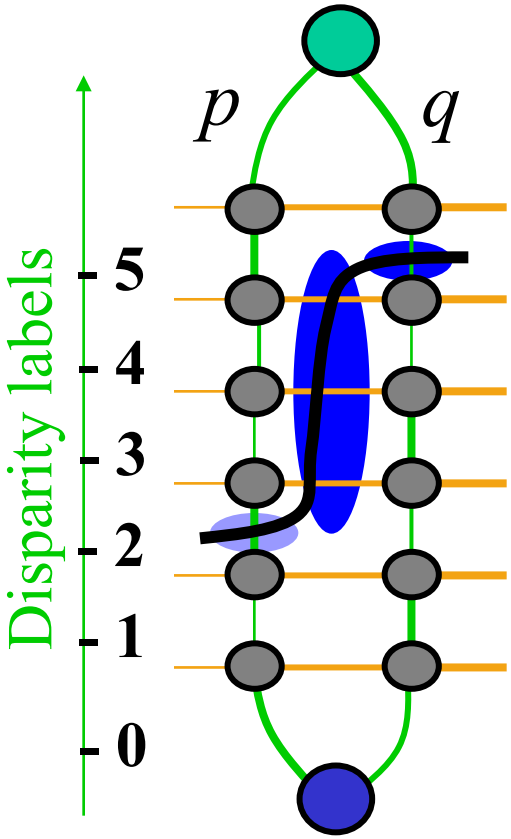
Multi-scan-line stereo with graph cuts (Roy&Cox'98)



Multi-scan-line stereo with graph cuts (Roy&Cox'98)



Concentrate on one pair of neighboring pixels $\{p,q\} \in N$



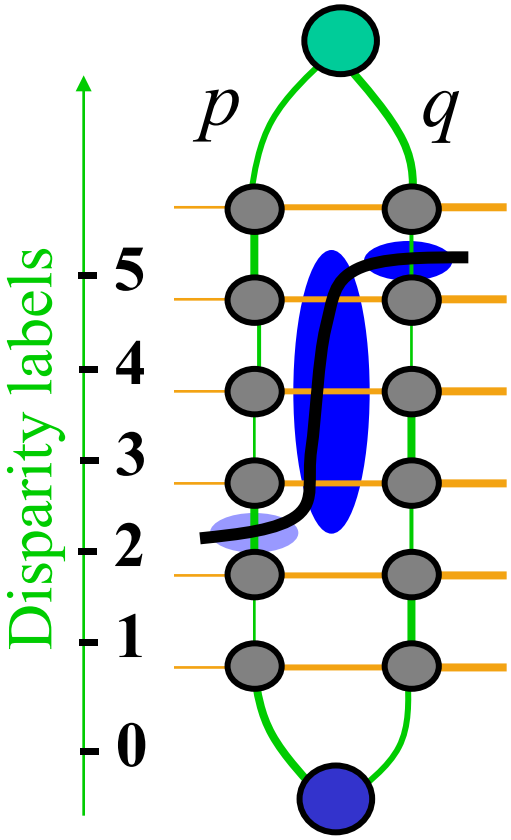
$$E(d_p, d_q) = D_p(2) + D_q(5) + \ldots$$

cost of vertical edges

$$+ \quad w_{pq} \cdot |3| \quad + \ldots$$

cost of horizontal edges

Concentrate on one pair of neighboring pixels $\{p,q\} \in N$



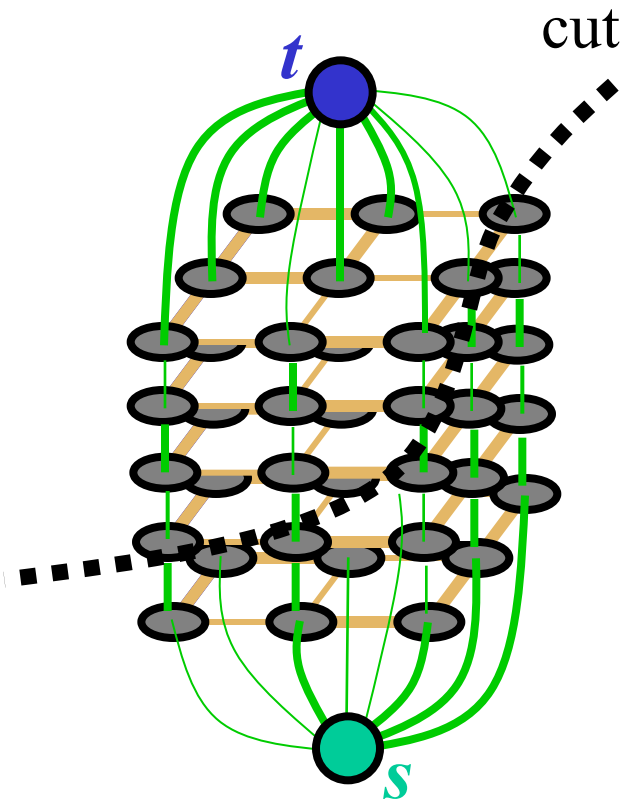
cost of vertical edges

$$E(d_p, d_q) = D_p(d_p) + D_q(d_q) + \ldots$$

$$+ \quad w_{pq} \cdot |d_p - d_q| + \ldots$$

cost of horizontal edges

The combined energy over the entire grid G is

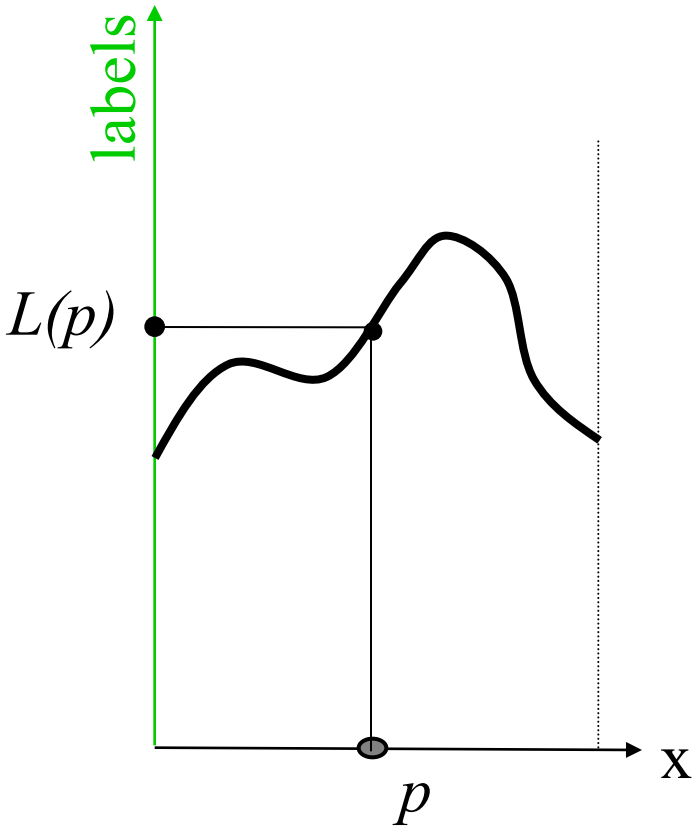


$$E(\mathbf{d}) = \sum_{p \in G} D_p(d_p) + \sum_{\{p,q\} \in N} w_{pq} \cdot |d_p - d_q|$$

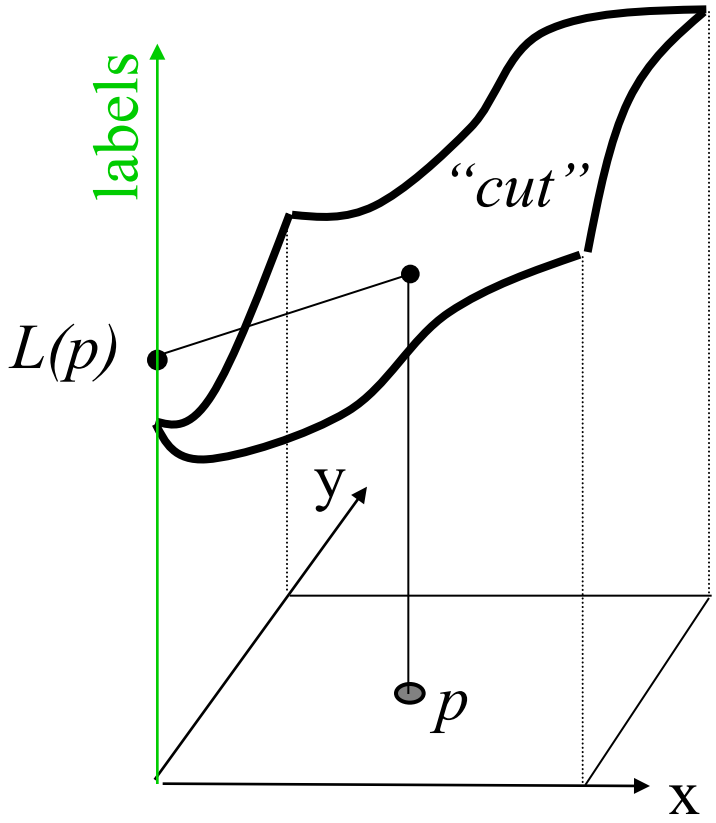
(**photo consistency**)
cost of vertical edges

cost of horizontal edges
(**spatial consistency**)

Multi-scan-line stereo with graph cuts (Roy&Cox'98)



Dynamic Programming
(single scan line optimization)

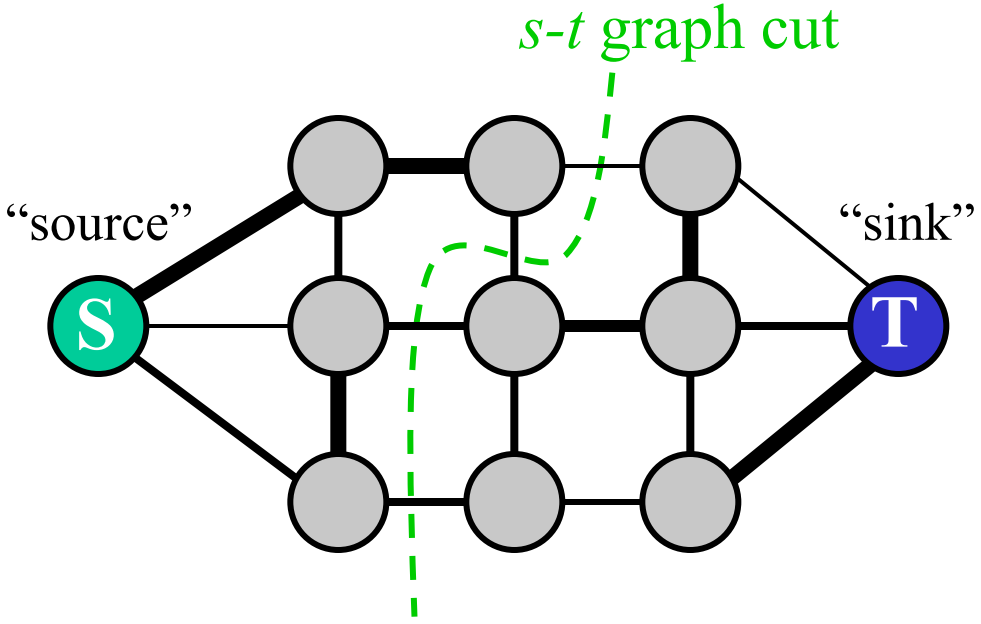


s-t Graph Cuts
(multi-scan-line optimization)

Graph Cuts Basics (see CLRS)

Simple 2D example

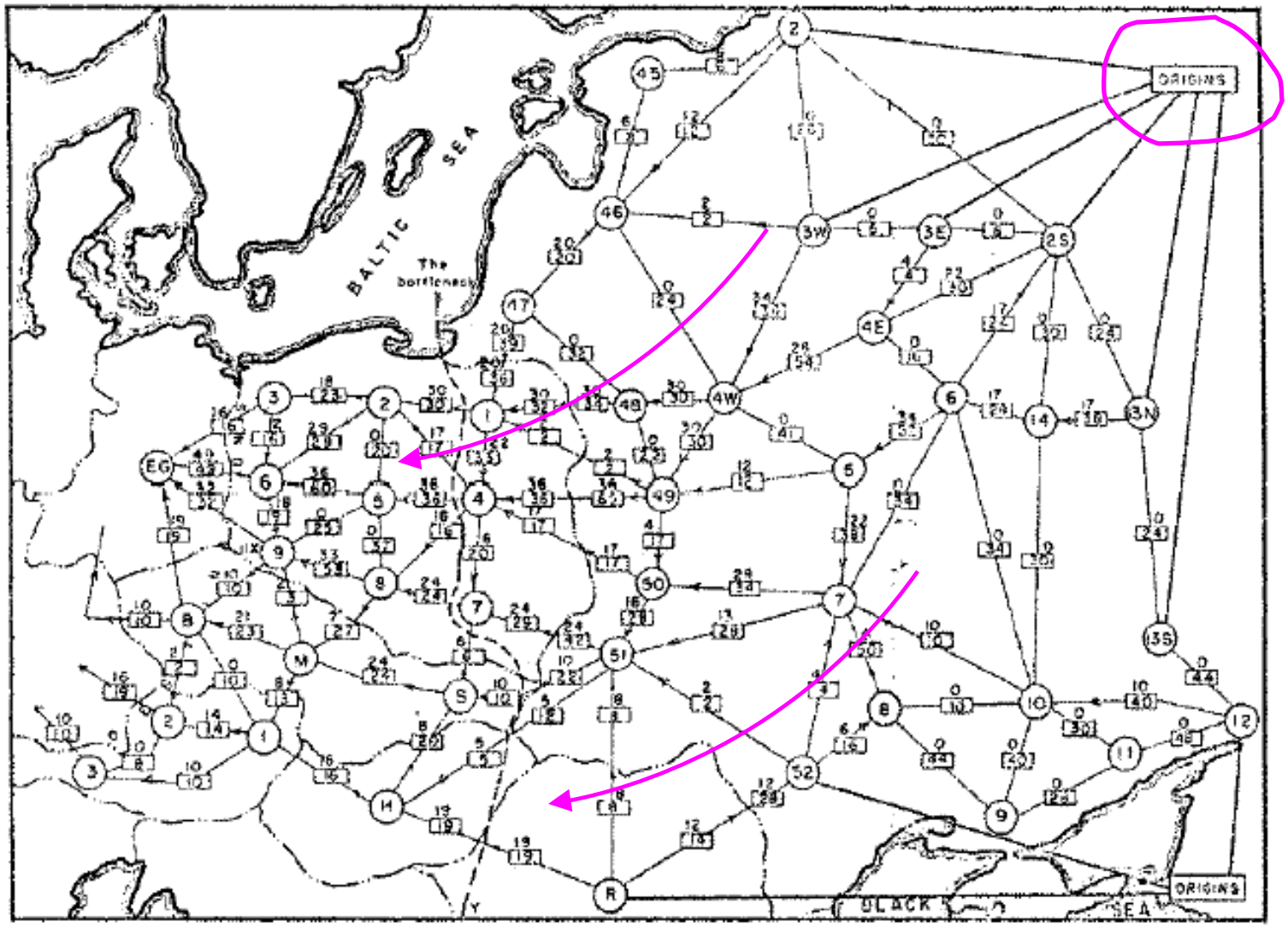
Goal: divide the graph into two parts separating red and blue nodes



A graph with two terminals *S* and *T*

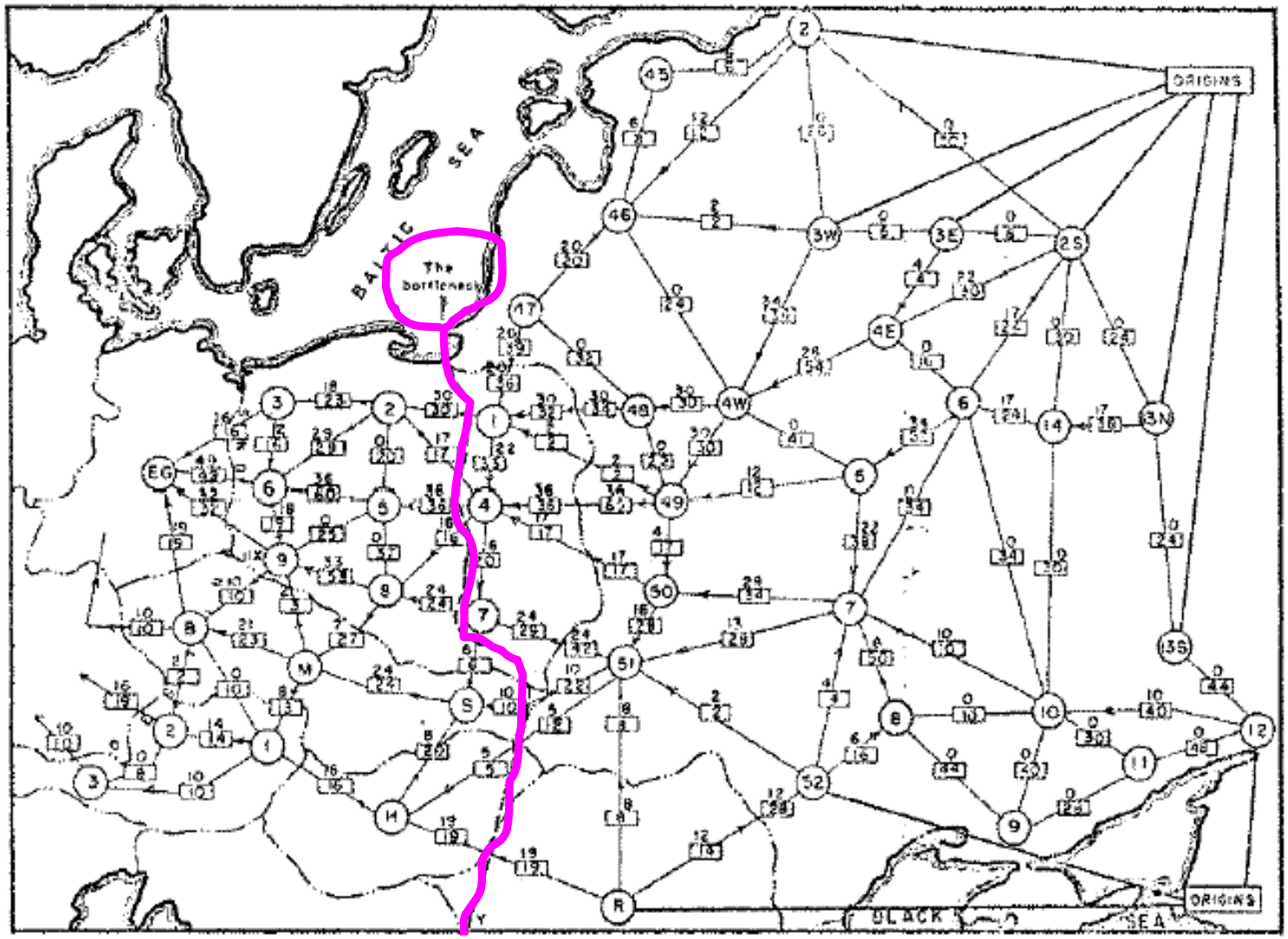
- Cut cost is a sum of severed edge weights
- Minimum cost *s-t* cut can be found in polynomial time

Graph cut is an old problem



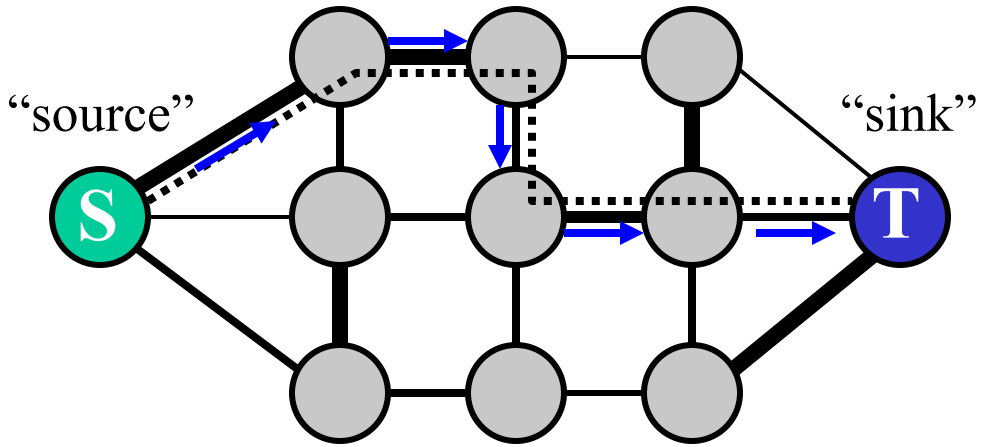
From Harris & Ross [1955]

Graph cut is an old problem



From Harris & Ross [1955]

"Augmenting Paths"

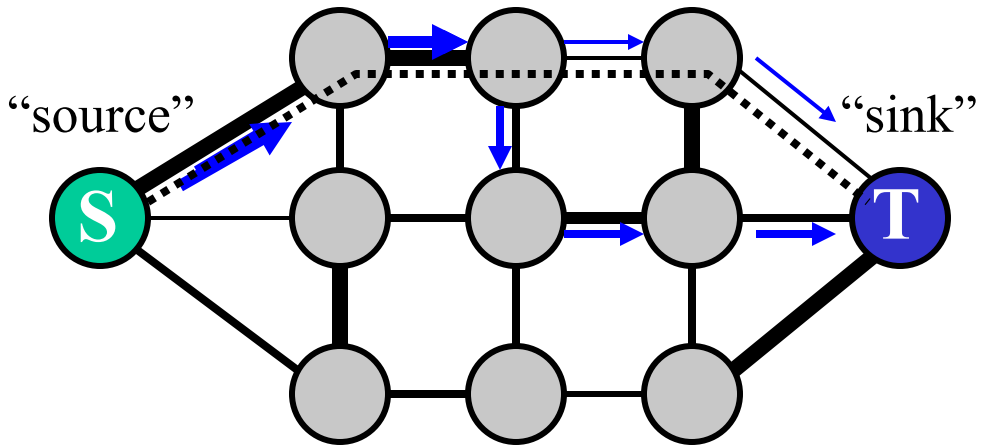


A graph with two terminals

- Find a path from S to T along non-saturated edges

Increase flow along this path until some edge saturates

"Augmenting Paths"



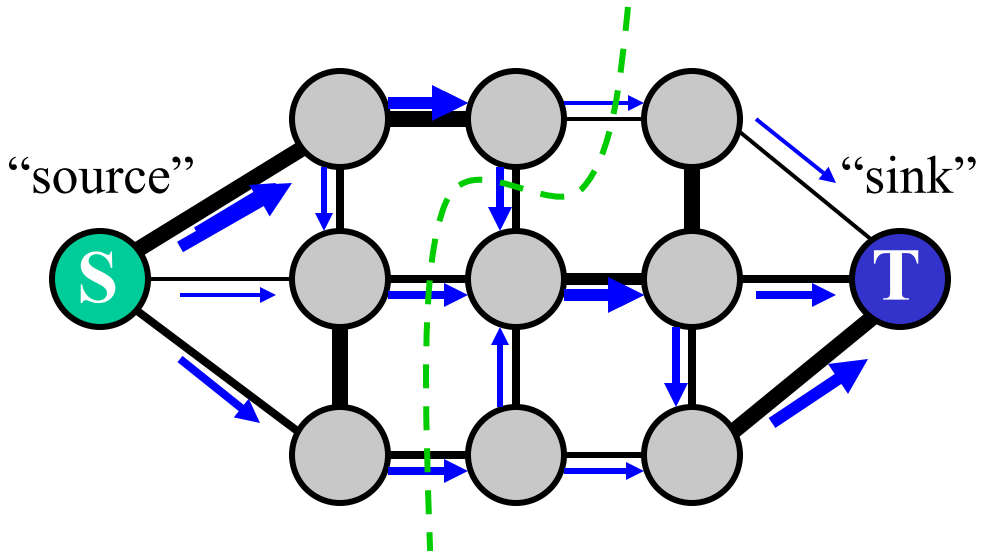
A graph with two terminals

- Find a path from S to T along non-saturated edges

Increase flow along this path until some edge saturates

- Find next path
- Increase flow

"Augmenting Paths"



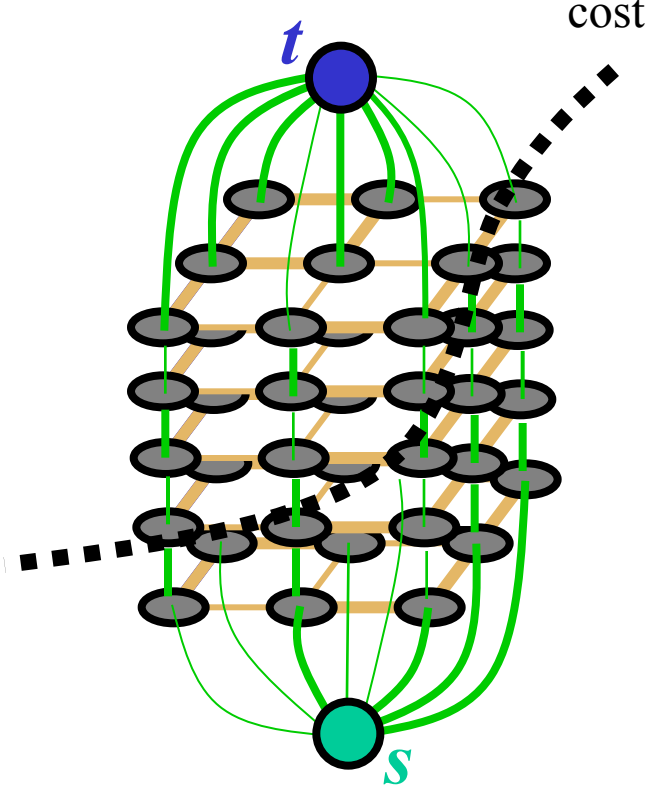
A graph with two terminals

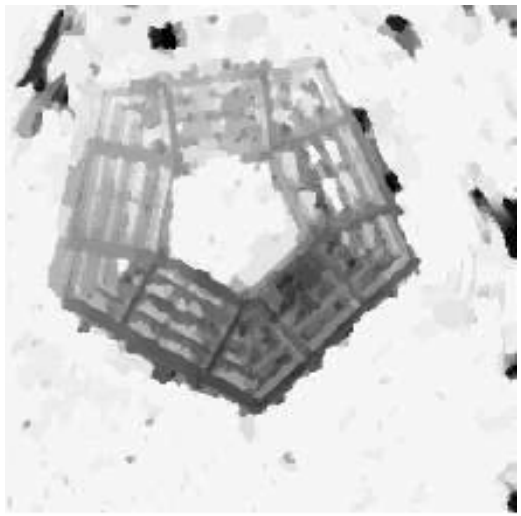
- Find a path from S to T along non-saturated edges
- Increase flow along this path until some edge saturates

Iterate until … all paths from S to T have at least one saturated edge

MAX FLOW ⇔ MIN CUT

Some results from Roy&Cox

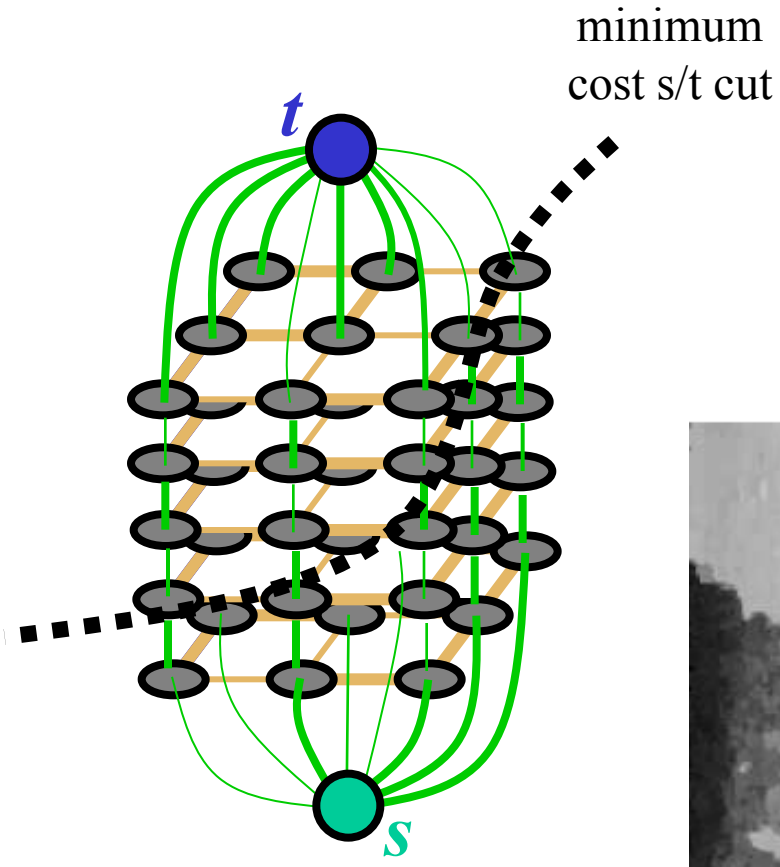




multi scan line stereo (graph cuts)

single scan-line stereo (DP)

Some results from Roy&Cox



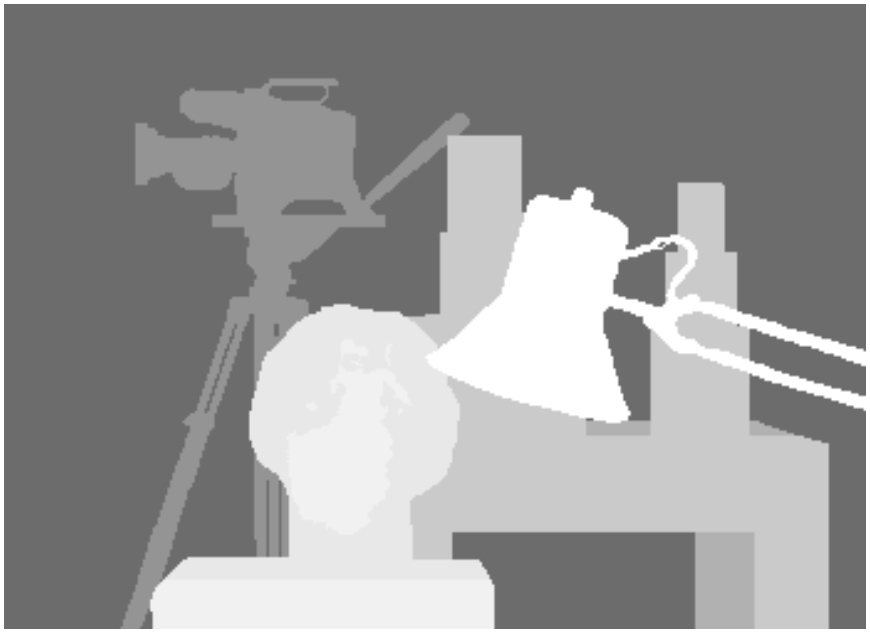
multi scan line stereo (graph cuts)

single scan-line stereo (DP)

Stereo results

- Data from University of Tsukuba

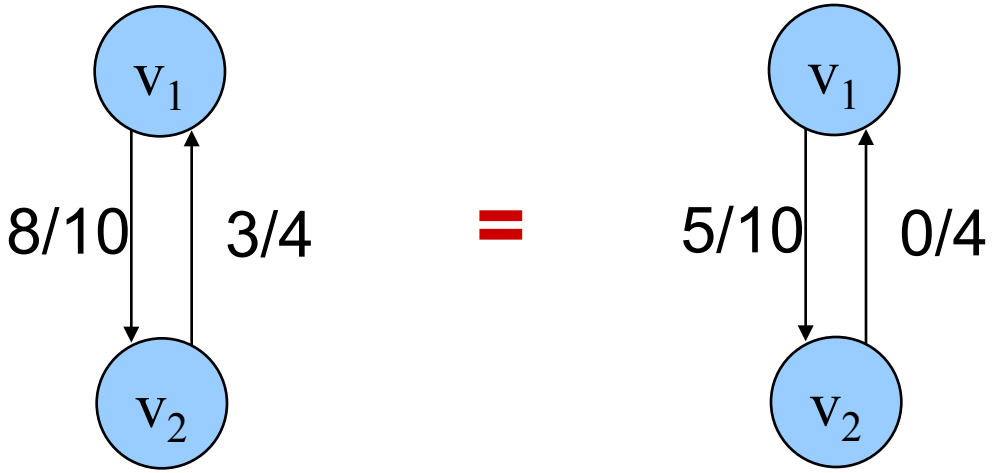
scene



ground truth

http://cat.middlebury.edu/stereo/

Cancellation of Flow



Definition of Network Flow

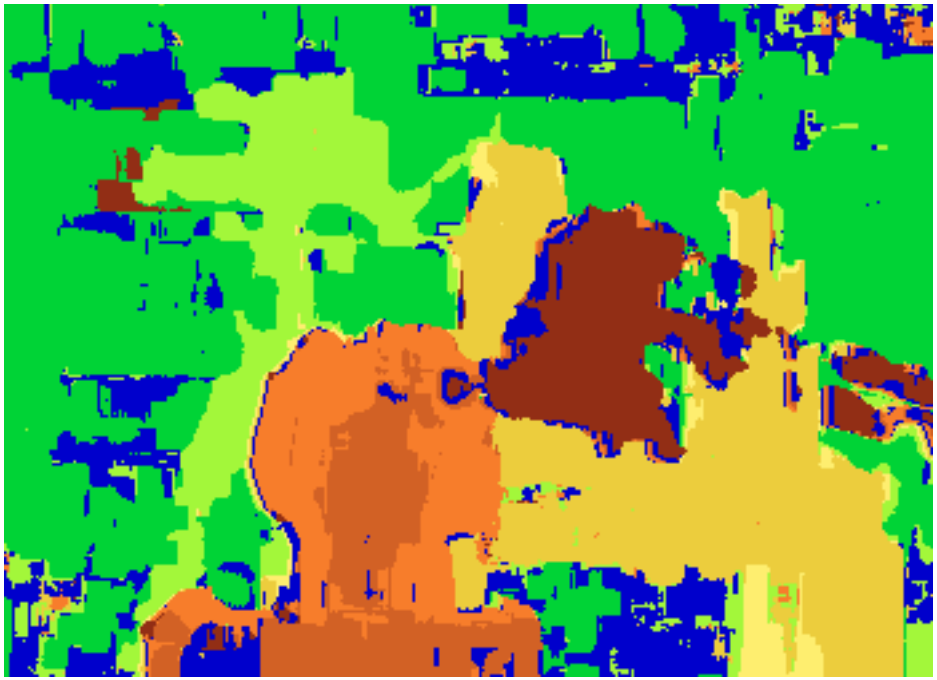
flow $f: (E \cup E^R) \rightarrow R^+$

(1) $f(e) = -f(e^R)$ for all $e \in E$

(2) $f(e) \leq c(e)$

(3) $\sum_{(v,u) \in E} f(v,u) = 0$ for all $v \in V-\{s,t\}$

Results with window correlation



normalized correlation
(best window size)

ground truth

Results with data + smooth

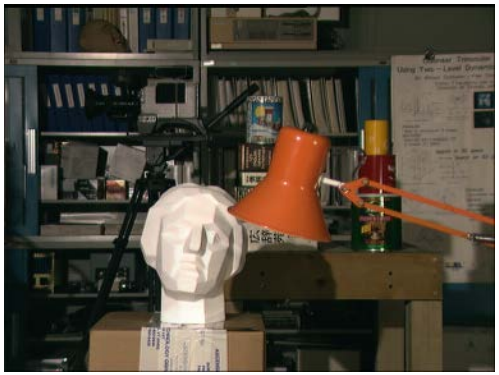
so-called “graph cuts”

ground truth

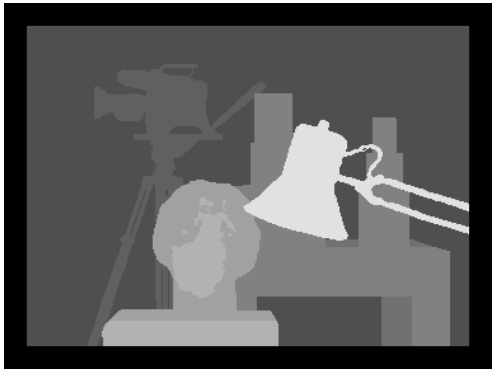
Middlebury Stereo Benchmark

Ground Truth Data

Left image



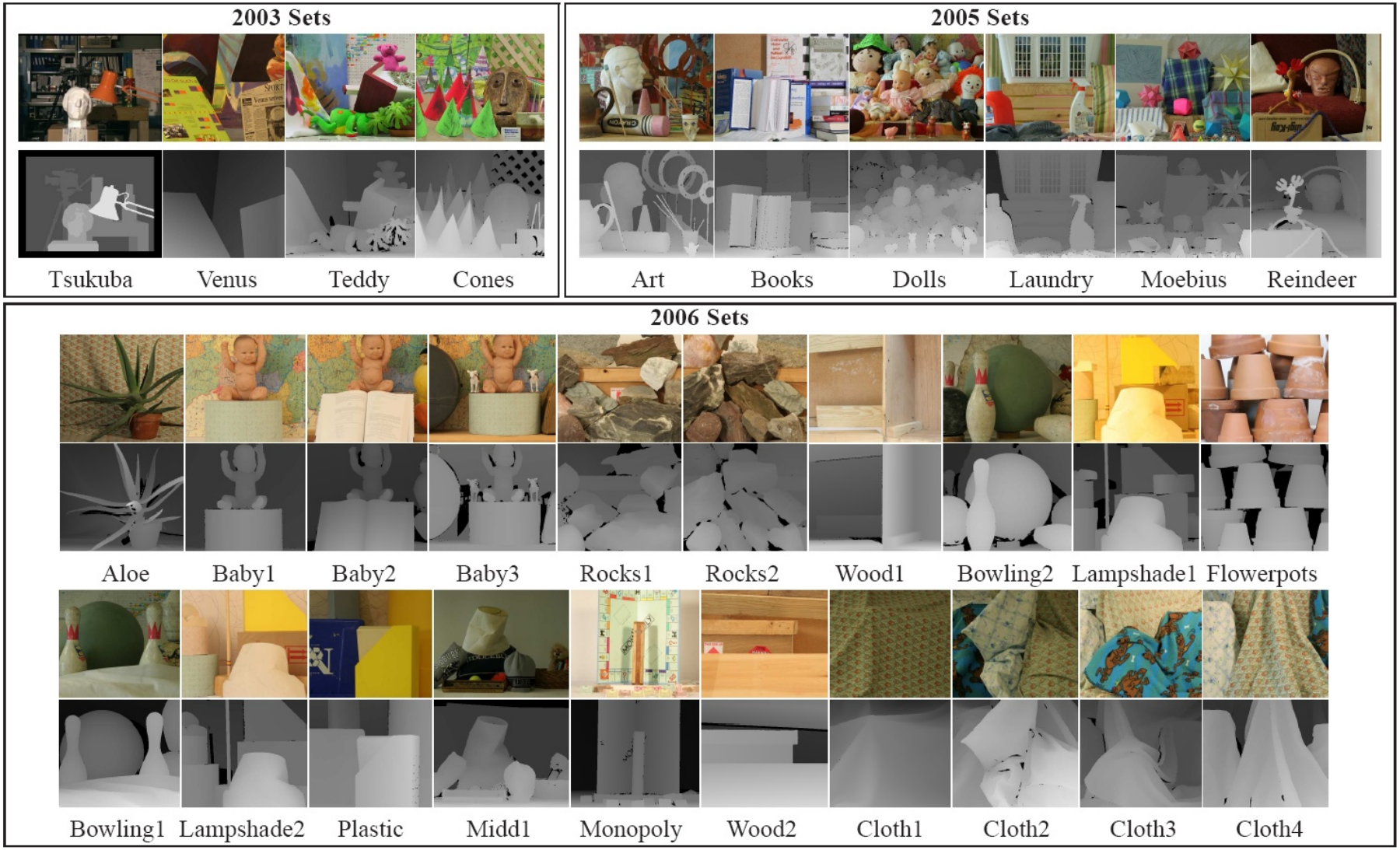
Right image



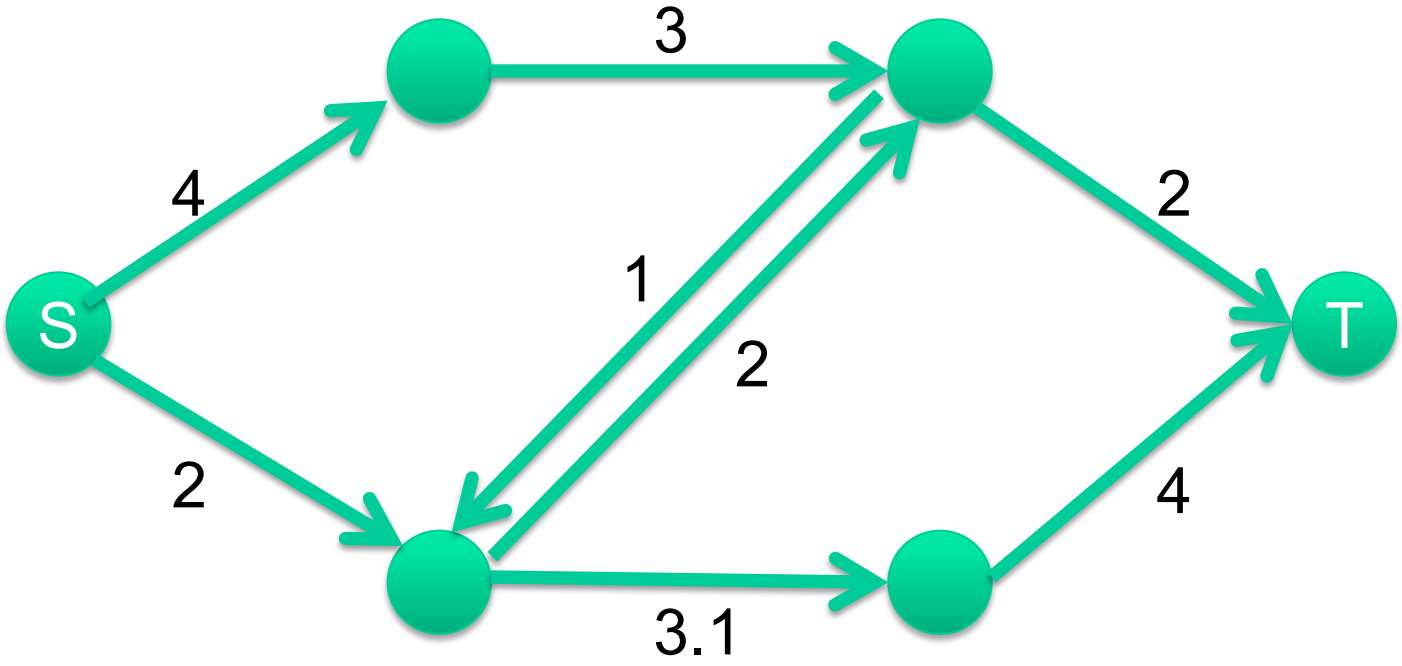
Ground truth disparities

- For stereo, ground truth data is available on the Middlebury Stereo Evaluation website http://vision.middlebury.edu/stereo/
- The Middlebury set is widely adopted in the stereo community.

The Middlebury Set

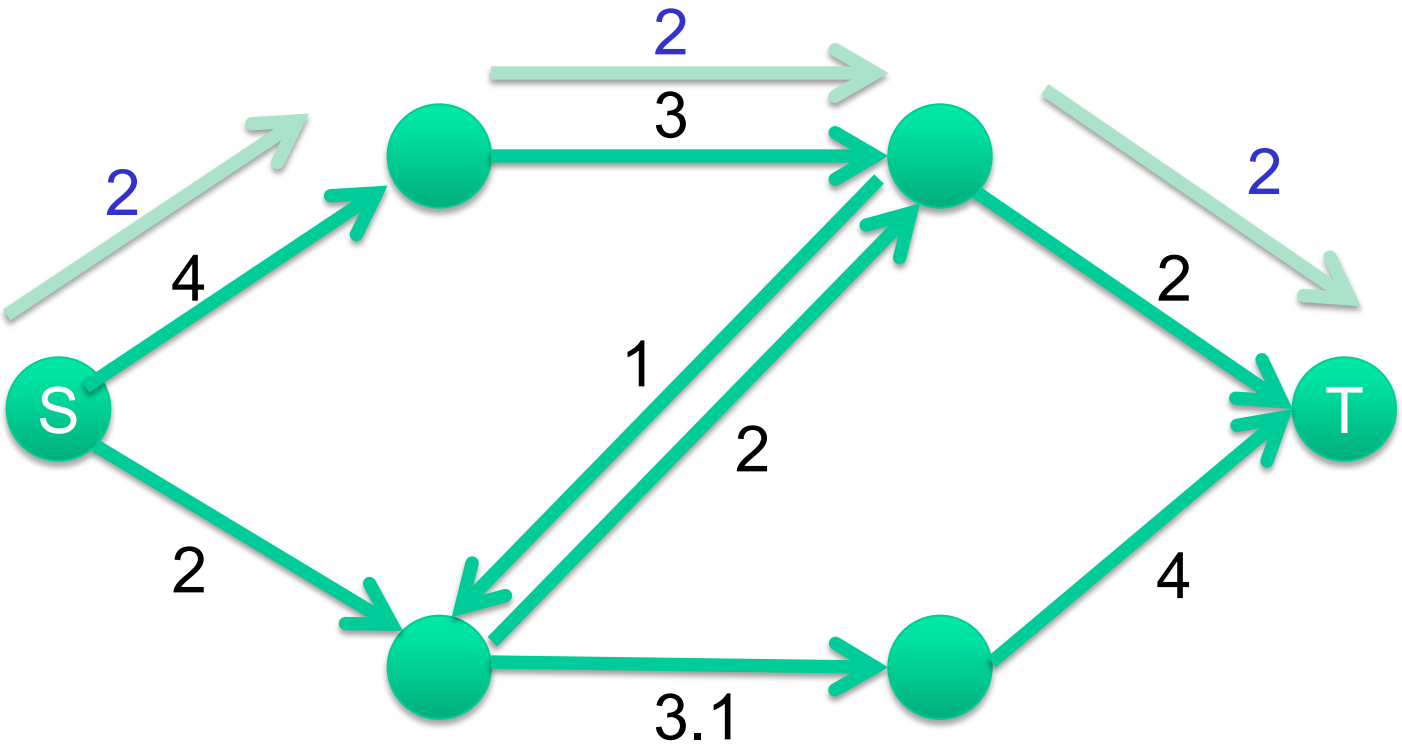


Max Flow – Ford Fulkerson



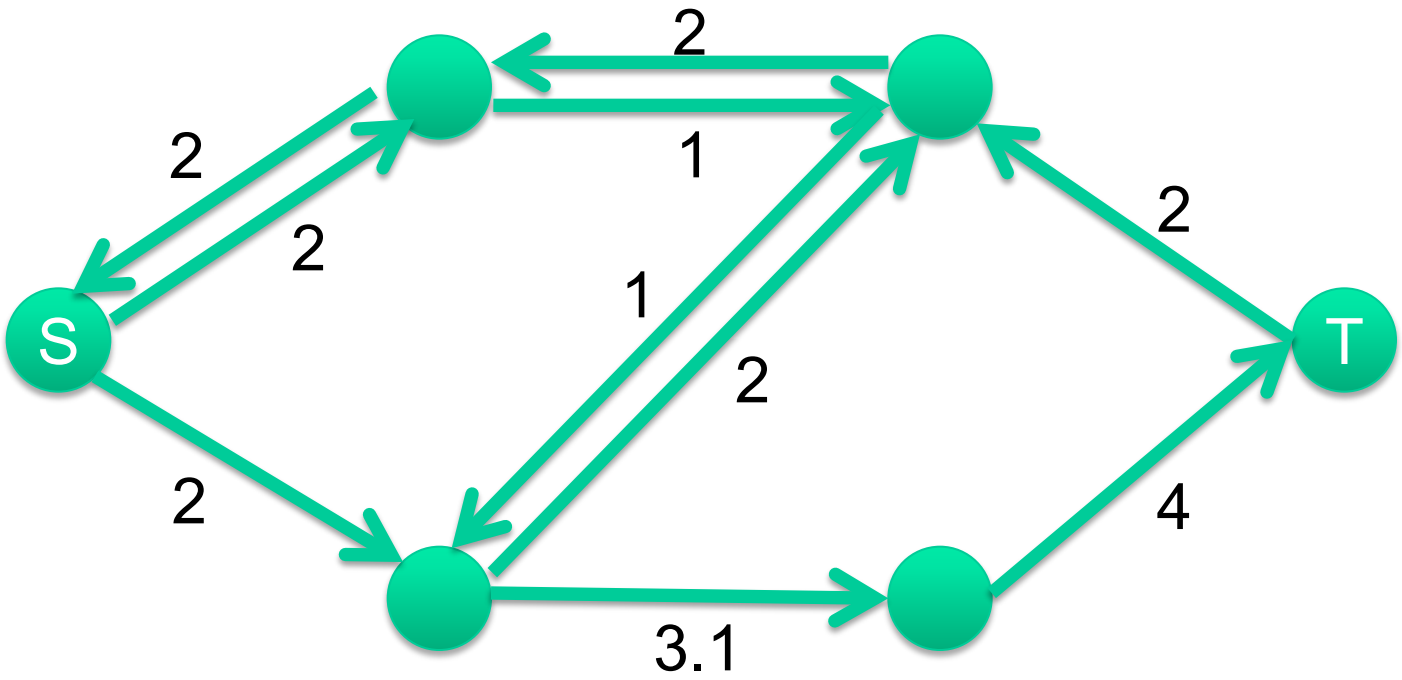
Push Flow of 2

Total Flow pushed: 2



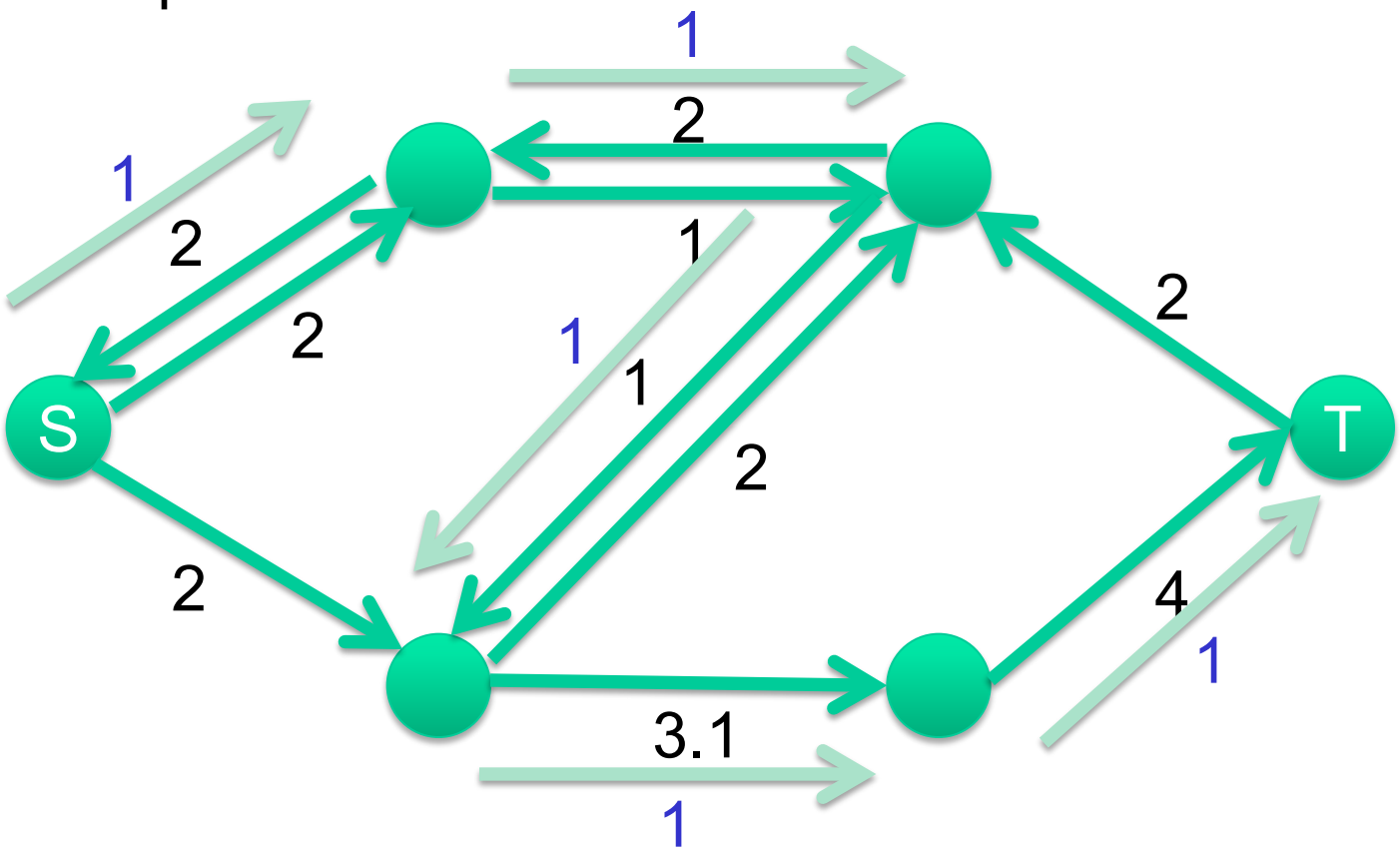
Residual Graph

Total Flow pushed: 2



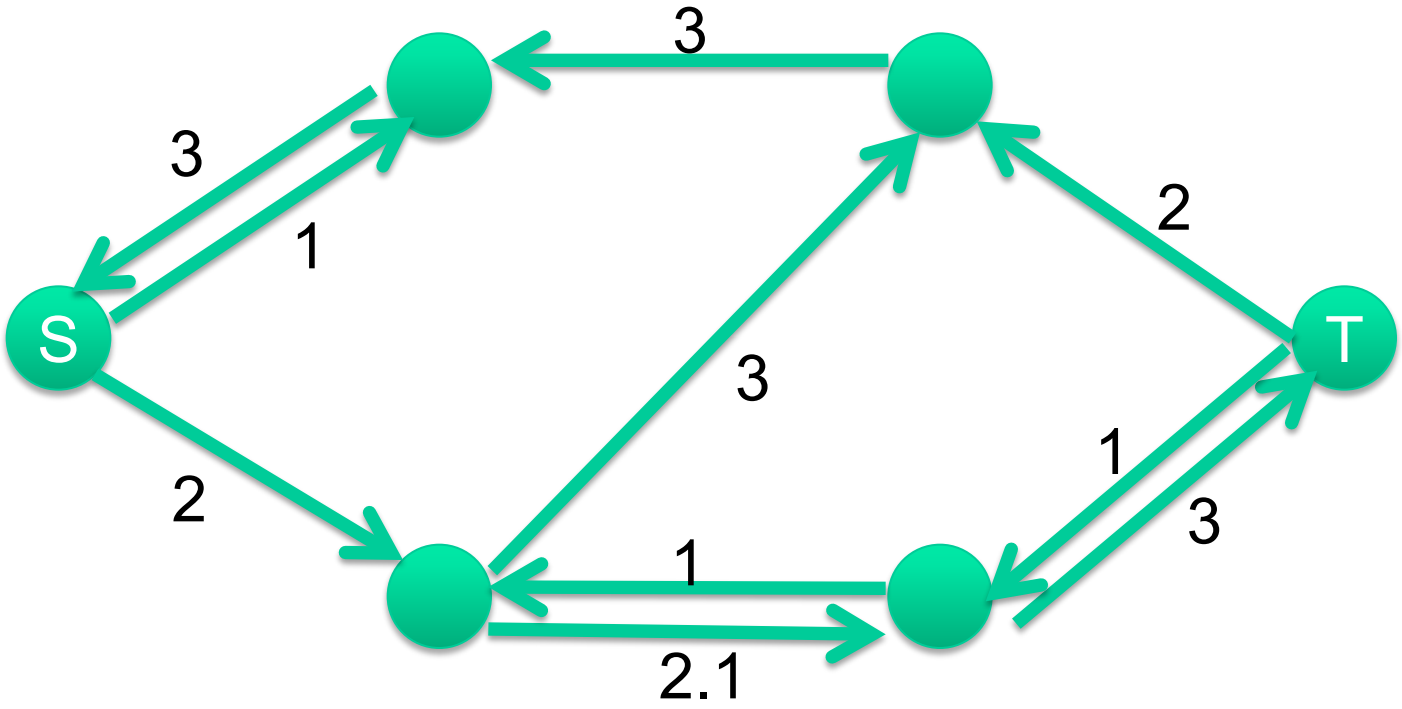
Residual Graph

Total Flow pushed: 3



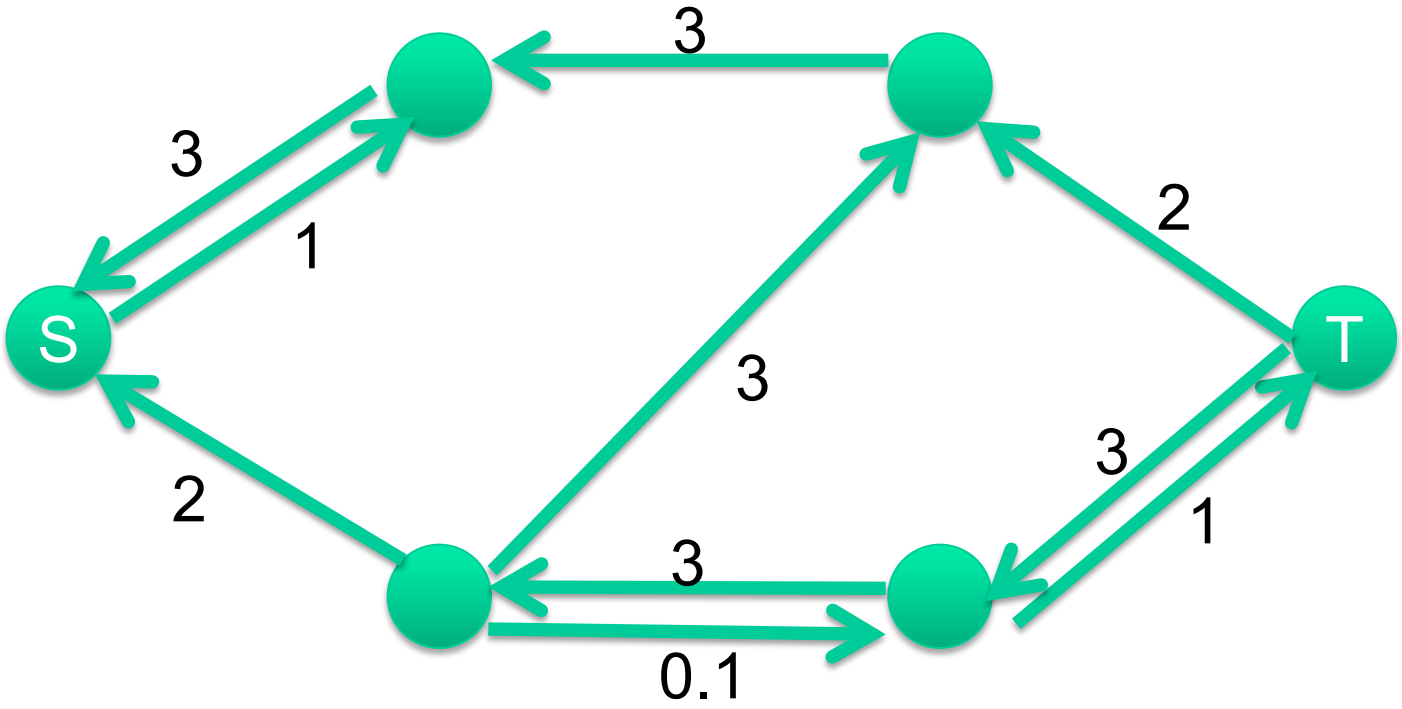
Residual Graph

Total Flow pushed: 3



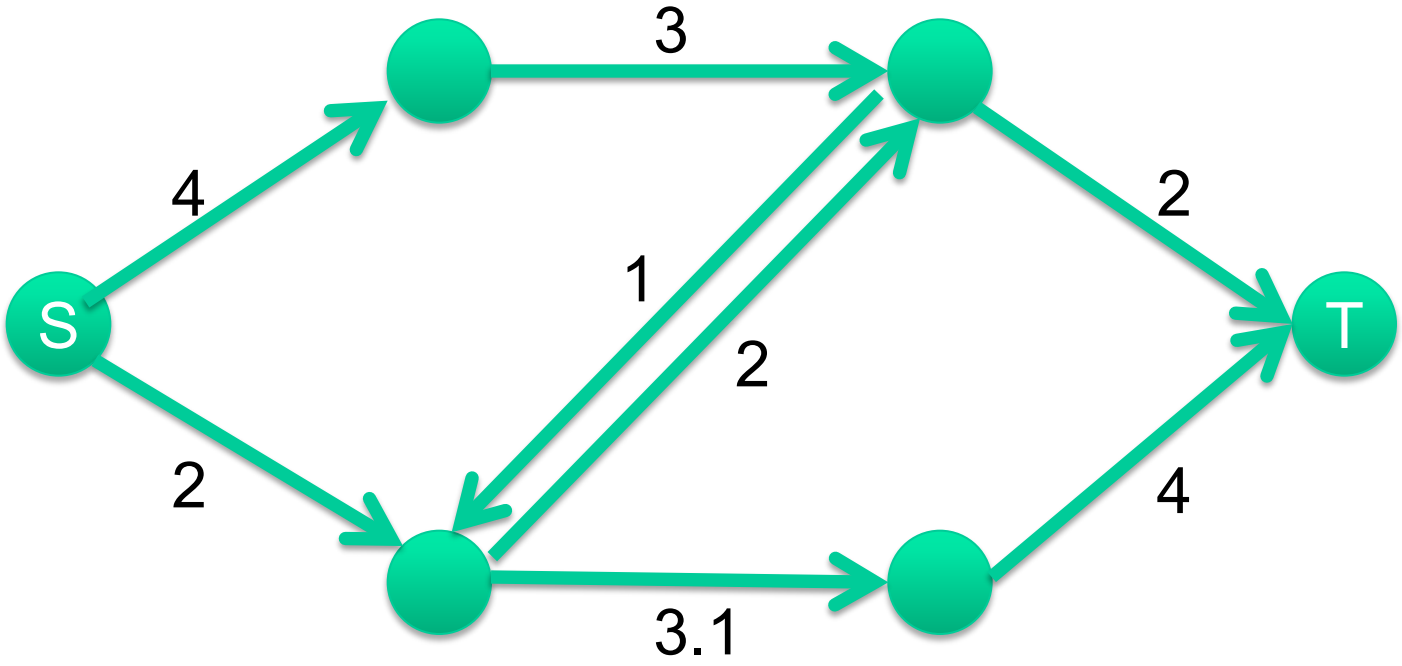
Residual Graph

Total Flow pushed: 5



Max flow Min Cut

Total Flow pushed: 5



Min Cut

Total Flow pushed: 5