Visible Surface Reconstruction

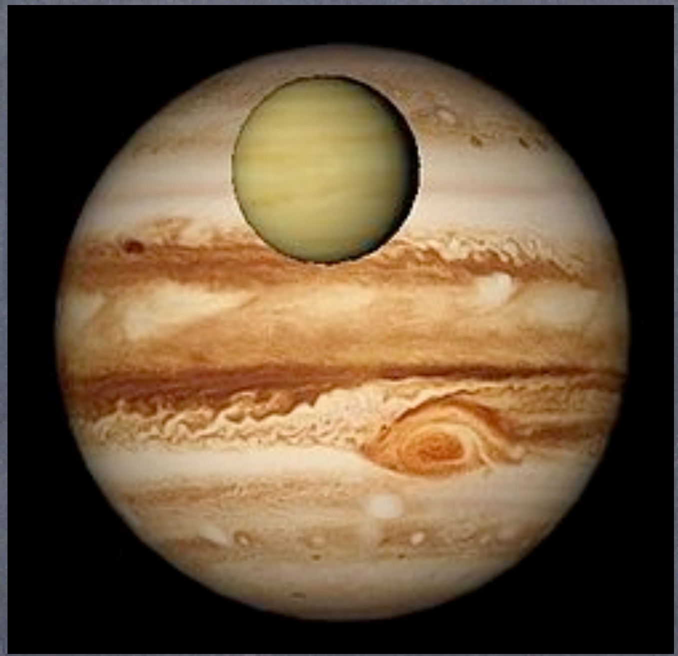
CAP5416 -- Vemuri

Problem Definition

- Given: Multiple Sources of Visual Information(Different Cues, eg., Surface Orientation, Surface Depth, Surface Curvature etc.)
- Goal: Visible Surface Reconstruction (Ref: Grimson'81, Terzopoulos'84, Boult'85, Vemuri'86)

Example Depth Cues

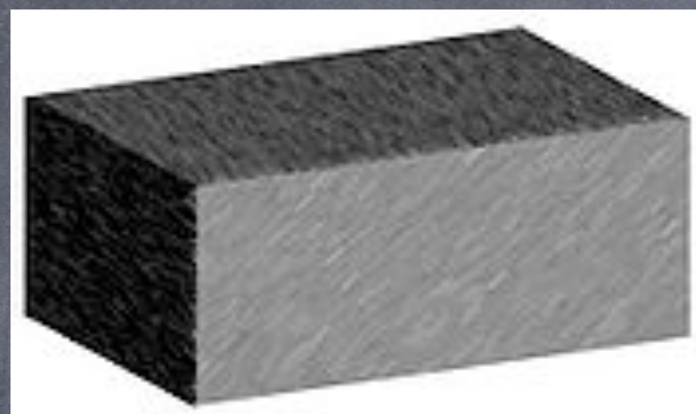
Objects appear smaller when they receed: linear perspective.

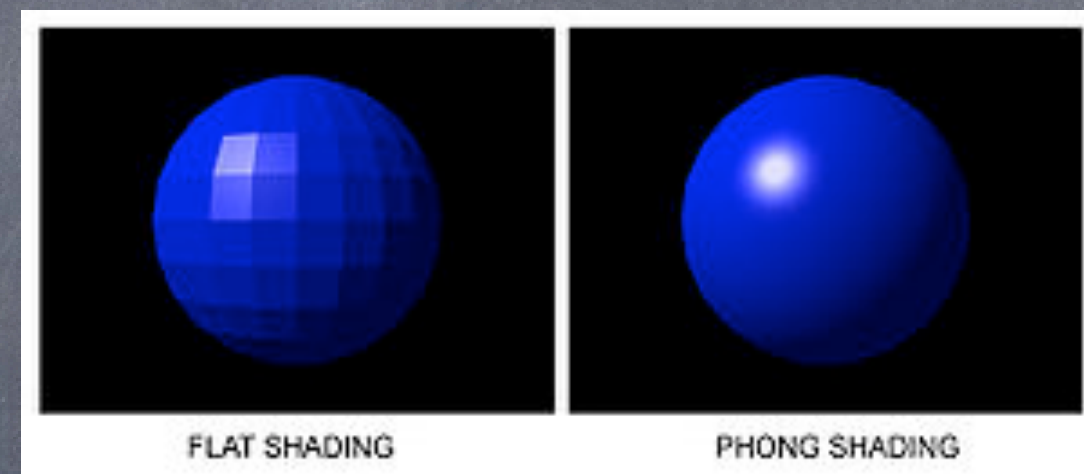


Transit: Smaller object in front of larger obj.

Texture gradient

Depth Cues (Contd.)





- Depth from shading

- Depth from motion

A Computational Model that Unifies

- Integration of Constraints From Different Visual Cues
- Interpolation (\Filling in Gaps")
- Discontinuity Map
- Computational Efficiency

Mathematical Basis

- z = Z(x; y) : Distance from the Viewer to the Visible Surface
- Noise Corrupted Shape Estimates (Constraints) from Low-level Visual Processes:

$$\{C_i\} = \mathcal{L}_i\{Z(x,y)\} + \epsilon_i$$

$\mathcal{L}_i$: Measurement functionals

ϵ_i : Associated errors

Goal

- Reconstruct from $\{C_i\}$ (Available Constraints) the Depth Function $Z(x,y)$ Along with Explicit Representation of Discontinuities Over the Visual Field
- Note: This is an Inverse Problem and is Ill-Posed in the Hadamard Sense

- Definition: A Mathematical Problem is Said to be Well-Posed if its Solution

1. Exists
2. is Unique
3. Depends continuously on initial data (solution is robust to noise).

Contd.

- Early Vision is Inverse Optics
- Most Inverse Problems are Ill-Posed e.g.,eg., Obtain x, Given y From $y = Ax$, Where A is a Known Operator.

Visible Surface Reconstruction is Ill-Posed

- Coincident, Slightly Inconsistent Shape Estimates From Different Visual Processes Over determine the Shape.
- Sparse, Scattered Constraints Restrict Surface Shape Locally but Do not Determine it Uniquely Everywhere.
- Erroneous Shape Estimates Locally Perturb the Surface Radically (Stability of the Solution).

Solution: Regularization

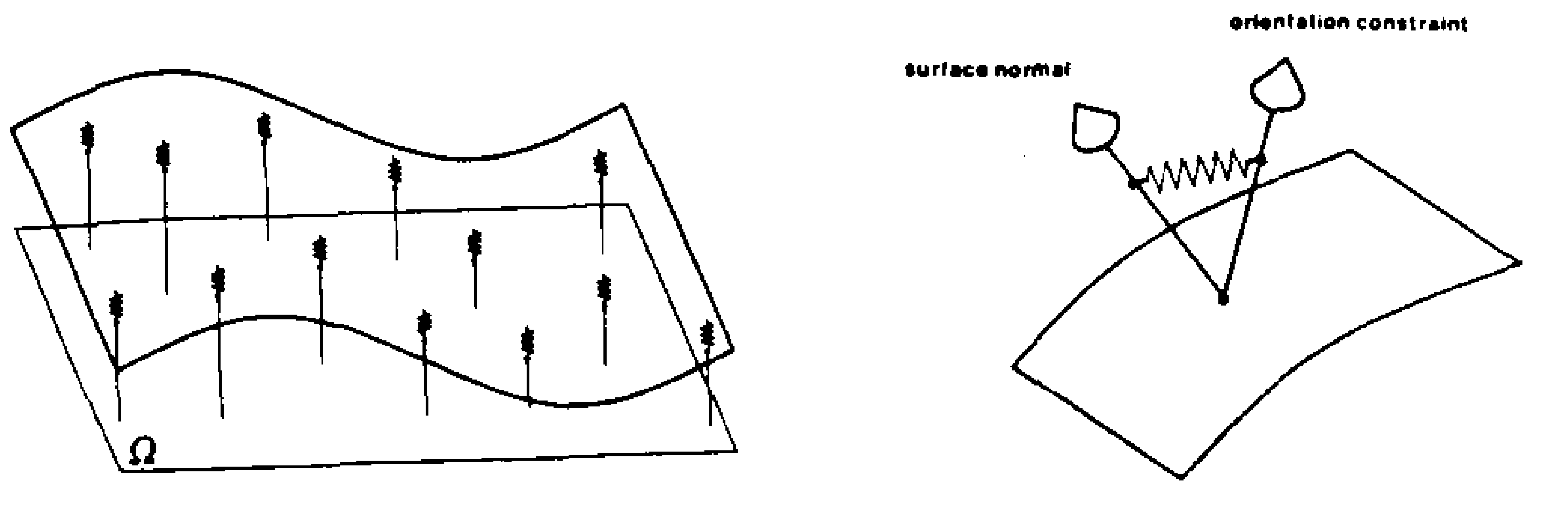
- $\mathcal{H}$: Linear Space of Admissible Functions on $\Re^d$
- $S(\mathbf{v})$: A Stabilizing Functional that Measures the (Lack of) Smoothness of a Function

  $\mathbf{v} \in \mathcal{H}.\ S(\mathbf{v}) : \mathcal{H} \rightarrow \Re$

- $\mathcal{P}(\mathbf{v})$:Penalty Functional, A Measure of Discrepancy Between $\mathbf{v}$ and Given Constraints.

$\mathcal{P}(\mathbf{v}) : \mathcal{H} \rightarrow \Re$

A Controlled Continuity Surface Model



(a) (b)

Fig. 2. The physical model. (a) Thin-plate surface under tension and depth constraints. (b) Local influence of an orientation constraint.

Controlled Continuity Splines

- Convex Combination of the Deformation Energies (Linearized) in a Thin Plate and a Membrane are

$$S_{\rho\tau}(\mathbf{v}) = \int\int_{\Omega} \rho(x,y)\{\tau(x,y)(\mathbf{v}_{xx}^2 + 2\mathbf{v}_{xy}^2 + \mathbf{v}_{yy}^2) + [1-\tau(x,y)](\mathbf{v}_x^2 + \mathbf{v}_y^2)\}\,\mathrm{d}x\mathrm{d}y$$

- $\rho(x,y), \tau(x,y)$:Continuity Control Functions, Range Over [0,1].

- Model Orientation Discontinuities with $\tau(x,y) = 0$ and $\rho(x,y) = 1$.

- Model depth discontinuities with $\rho(x,y) = 0$.

Data Constraints

- Penalty Functional $\mathcal{P}(\mathbf{v})$: Synthesize from data.

$$\mathcal{P}(\mathbf{v}) = \frac{1}{2}\sum_i \alpha_i \left(\mathcal{L}_i(\mathbf{v}) - C_i\right)^2$$

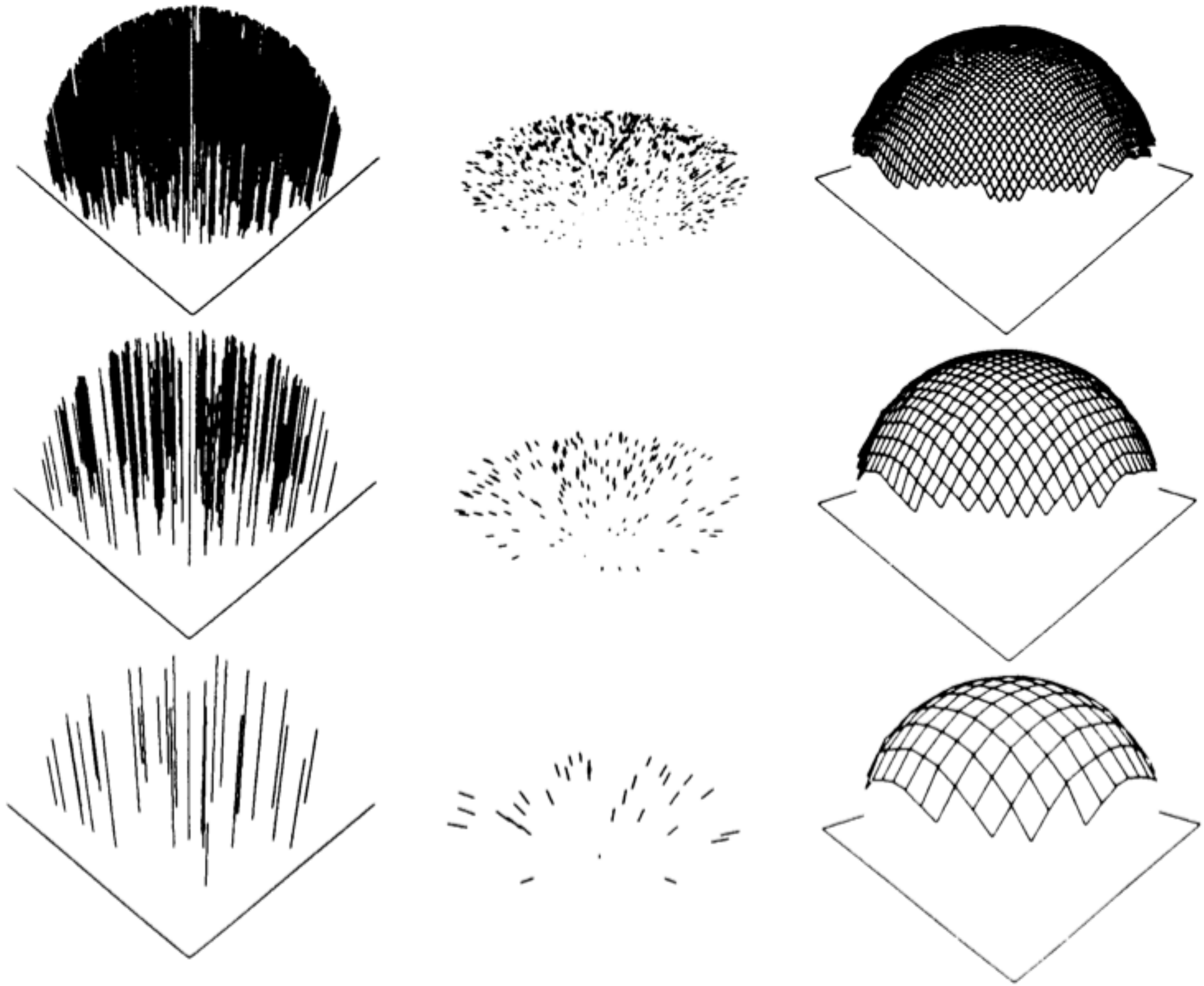
- α_i : Non-negative real valued parameter.
- Can synthesize $\mathcal{L}_i$ from the k^{th} order derivatives. $\mathcal{L}_i(\mathbf{v}) = \left(\frac{\partial^k \mathbf{v}}{\partial x^j \partial y^{k-j}}\right)\Big|_{(x_i,y_j)\in\Omega \ and \ j=0\ldots k}$
- k=0 gives the depth constraints $\mathcal{L}_i[\mathbf{v}(x,y)] = \mathbf{v}(x_i,y_j)$
- $C_i = \mathbf{v}(x_i,y_j) + \epsilon_i = \mathrm{d}(x_i,y_j) \ldots for \ i \in D_\Omega$. Its straight forward to get constraints of higher order.

Finding the Approximating Spline

- Total potential energy: $\mathcal{E}(\mathbf{v}) = \lambda \mathcal{S}_{\rho\tau}(\mathbf{v}) + \mathcal{P}(\mathbf{v})$, where λ is the regularization parameter.
- Variational principle, $VP1: \ \mathcal{E}(\mathbf{u}) = \overset{inf}{\mathbf{v} \in \mathcal{H}} \quad \mathcal{E}(\mathbf{v})$
- Necessary condition for a minimum is the vanishing of the first variation of VP1.

$$\delta_{\mathbf{u}} \mathcal{E}(\mathbf{u}) = \delta_{\mathbf{u}} S_{\rho\tau}(\mathbf{u}) + \delta_{\mathbf{u}} \mathcal{P}(\mathbf{u}) = 0$$

Expts.



(a) (b) (c)

Fig. 13. Multiresolution reconstruction of the hemisphere from depth and orientation constraints. (a) Depth constraints and (b) orientation constraints consistent with a hemisphere at three resolutions. (c) Recon-

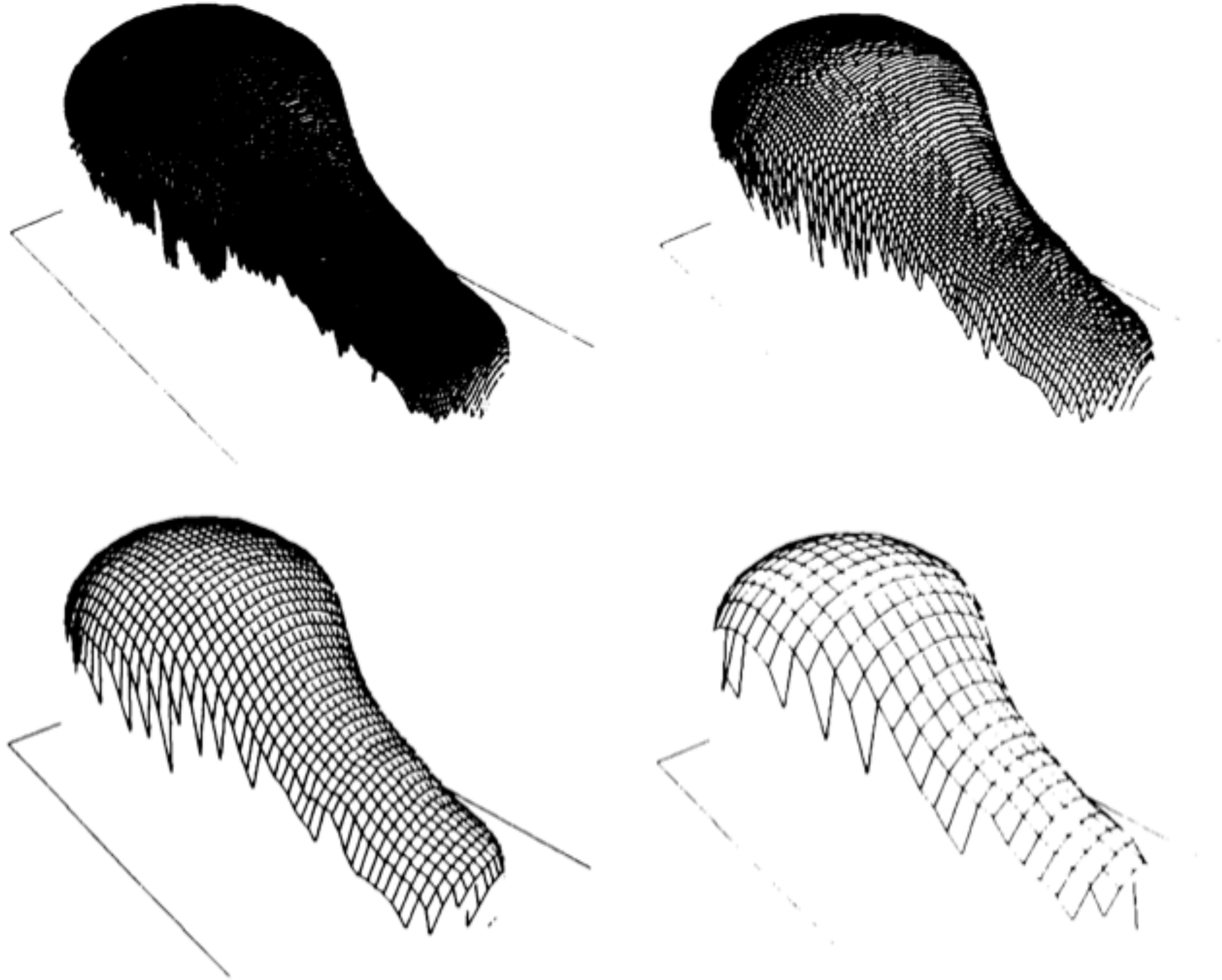


Fig. 14. Reconstruction of a light bulb from range data. (Finest grid dimensions: $N_x^{h_4} \times N_y^{h_4} = 257 \times 281$. Grid spacings: $h_1 = 0.8$, $h_2 = 0.4$, $h_3 = 0.2$, and $h_4 = 0.1$. Constraint parameters: $\alpha_d^{h_j} = 0.2/h_j$. Computation: 9.78 work units.)

Surface Representation

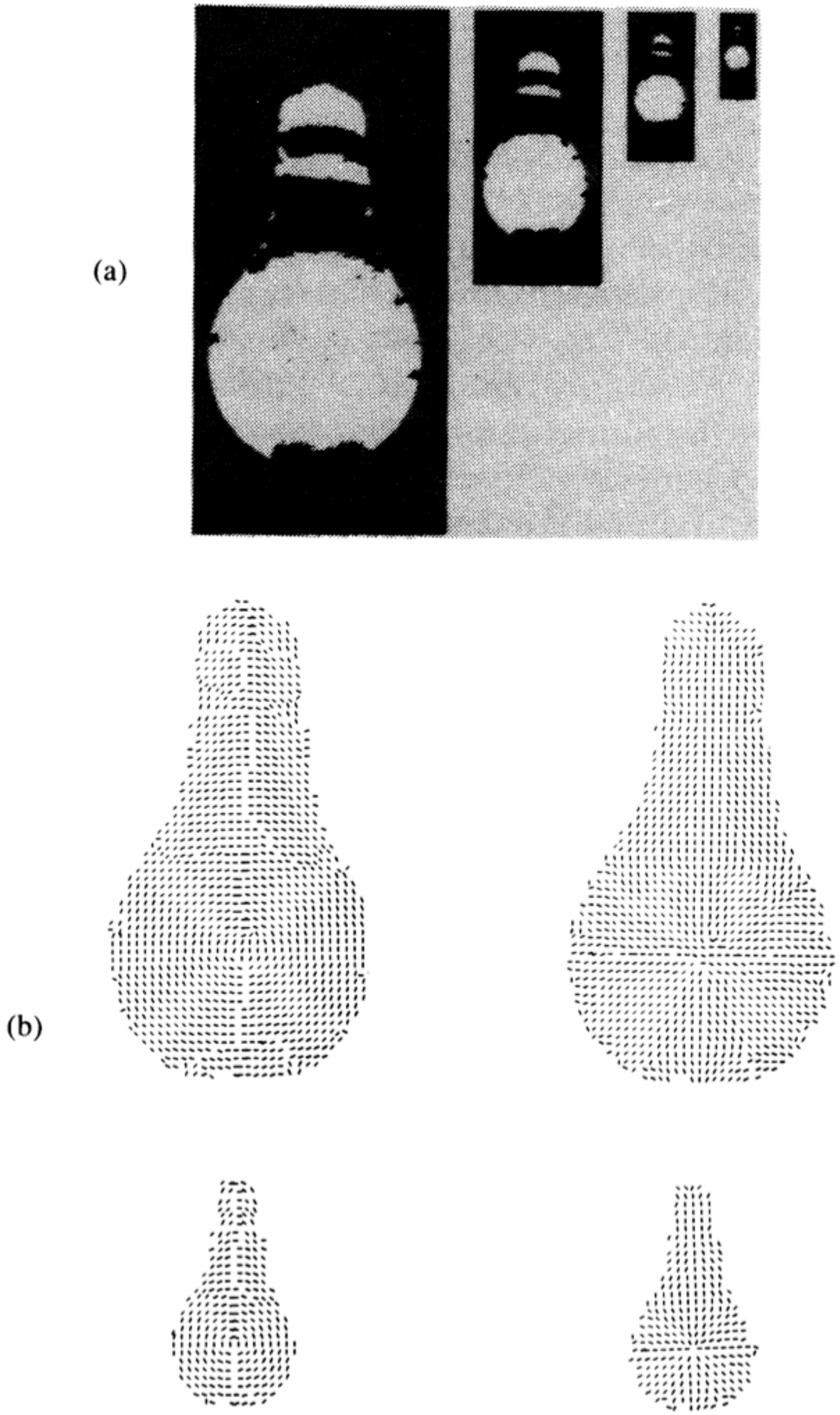


Fig. 24. Computing intrinsic and extrinsic surface properties in the visible-surface representation. (a) Computed Gaussian curvature $K_{i,j}$ of reconstructed lightbulb at four scales. Elliptic points ($K > 0$) are white, hyperbolic points ($K < 0$) are black, and parabolic points ($K = 0$) separate regions. (b) Computed principal direction field for reconstructed lightbulb at the two coarsest scales showing directions of greatest curvature (left) and least curvature (right).