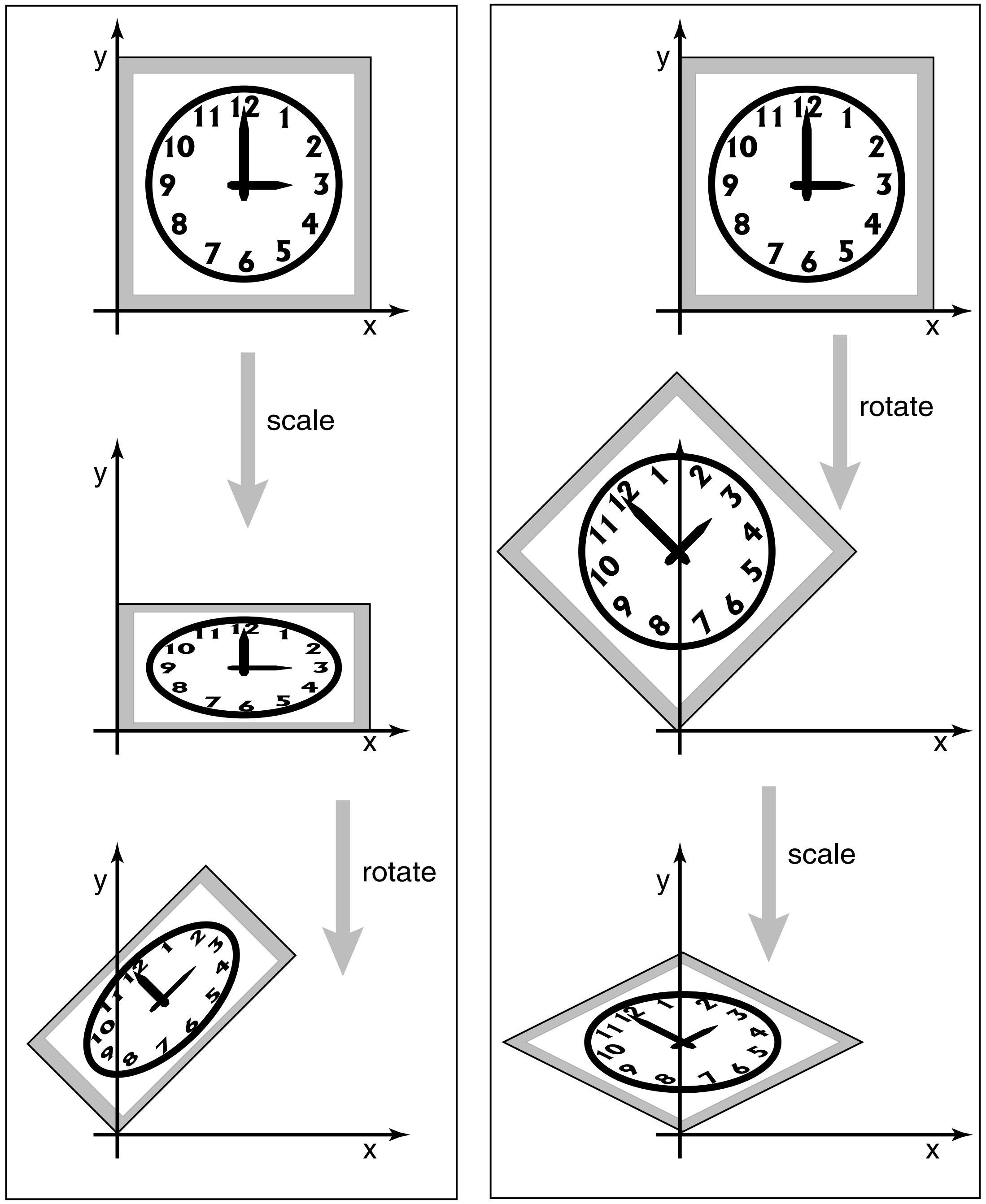
Transformations

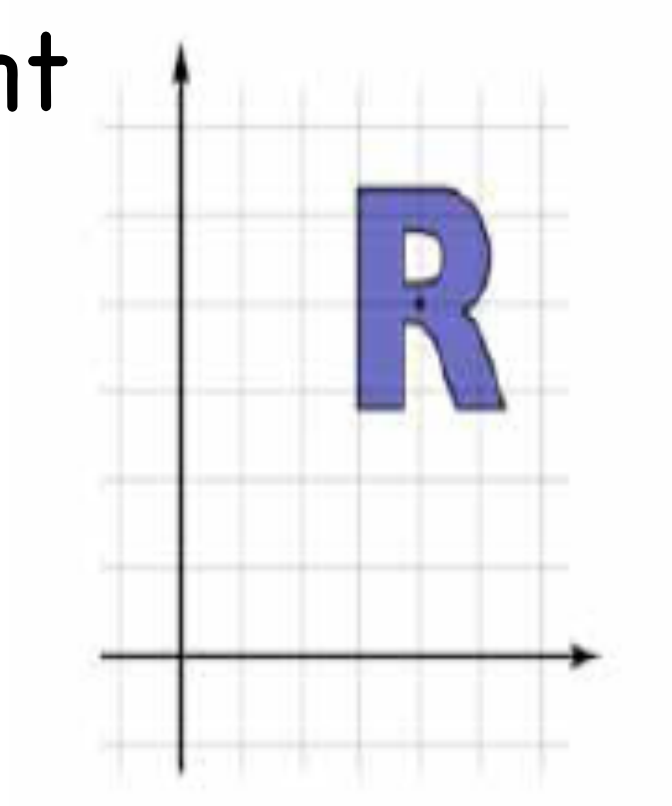
Composite transforms

- Generally **not** commutative!



Composing to change axis

- Want to rotate about a point
  - translate point to origin
  - rotate
  - translate back



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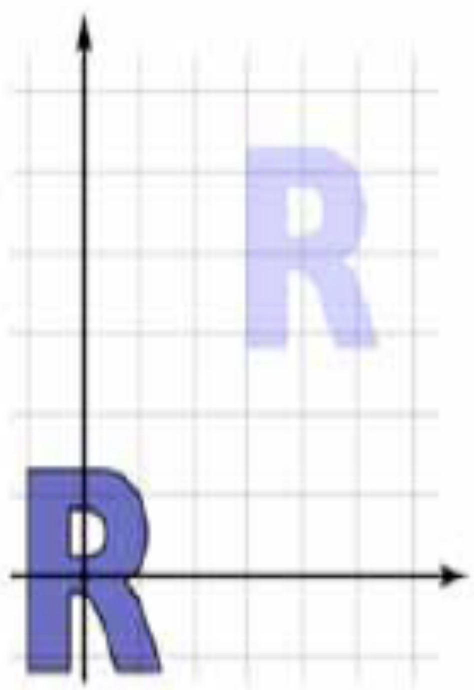

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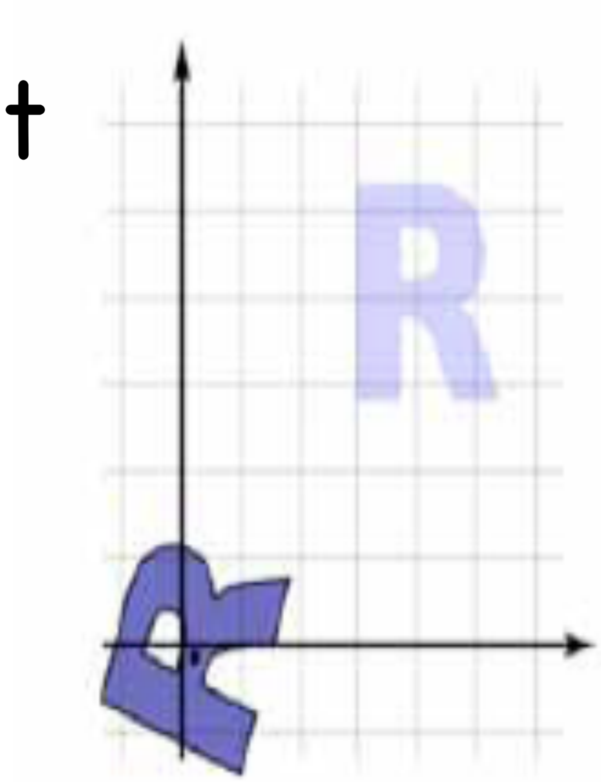

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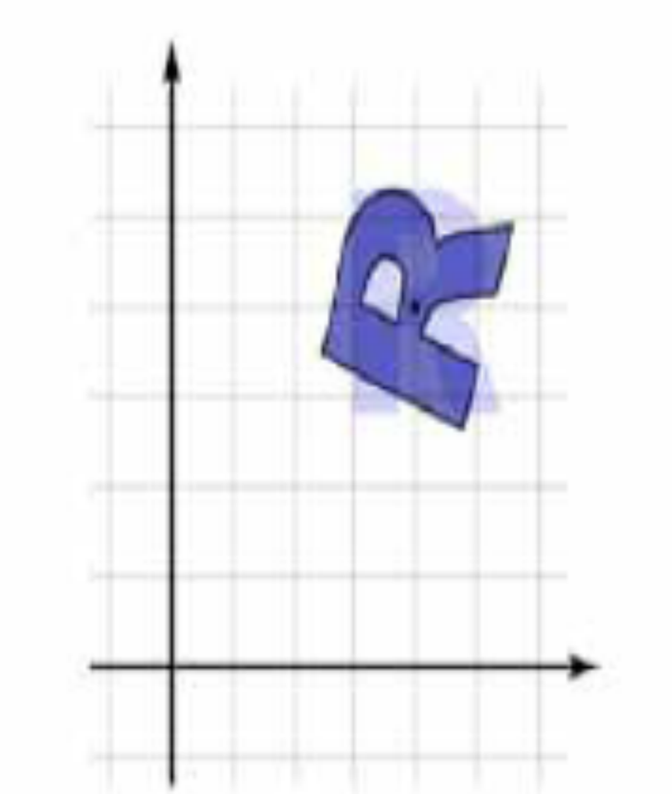
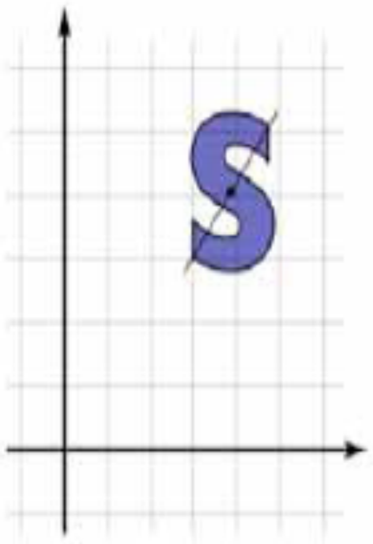


$$M = T^{-1} R T$$

Composing transforms

- Scaling along a particular axis
  - translate to origin
  - rotate to align the x-axis
  - scale along x
  - un-rotate to original orientation
  - translate back



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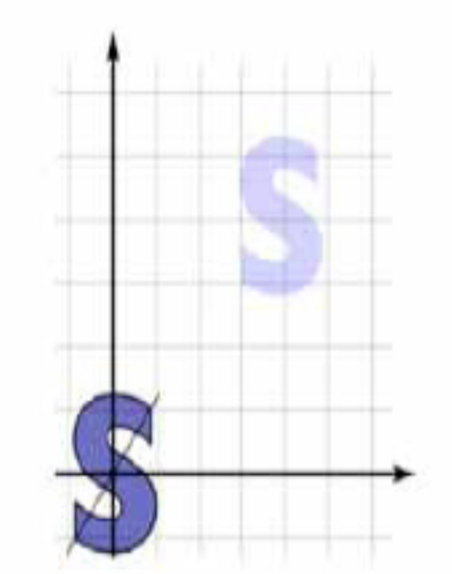

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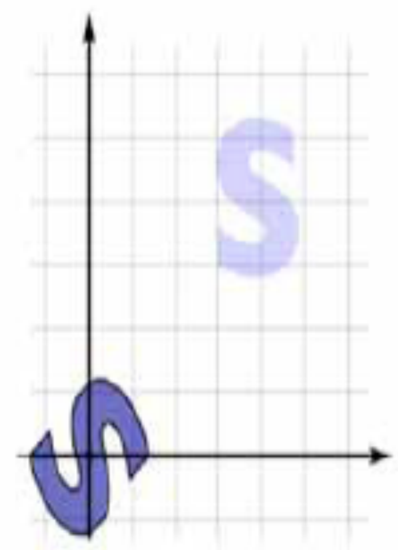

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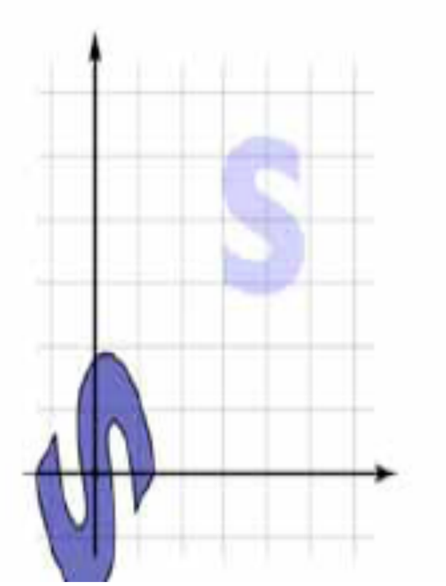

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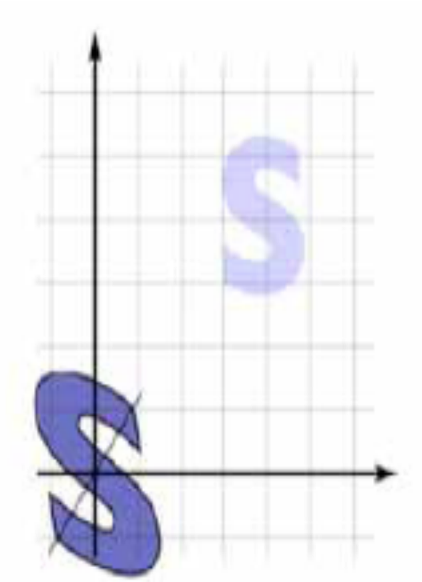

Composing transforms

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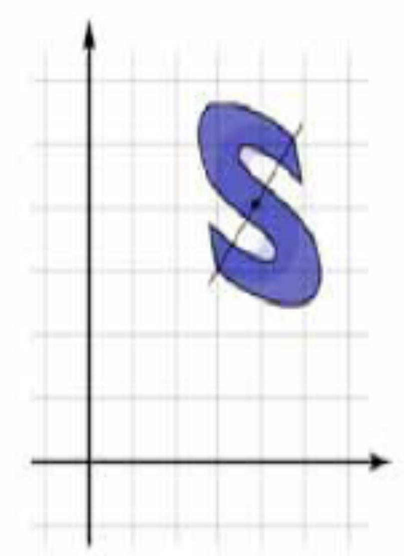

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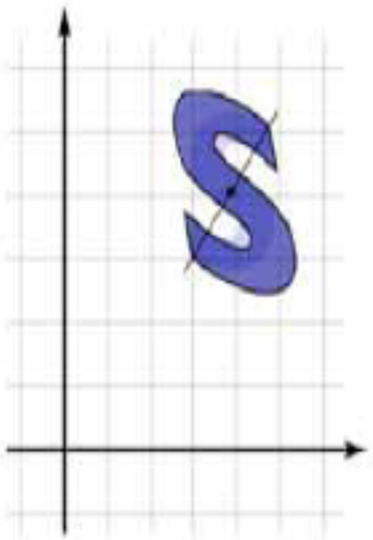
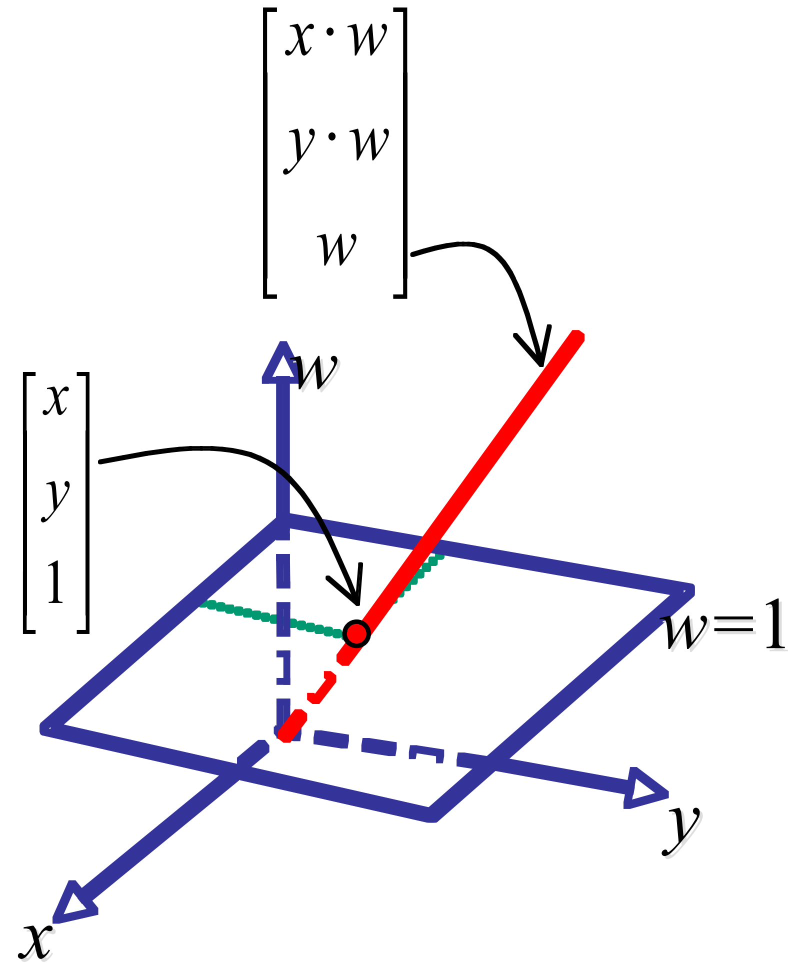


$$M = T^{-1}R^{-1}SRT$$

Change of Basis?

Homogeneous coords

- Homogenize: to convert 3D homog. coord to 2D Cartesian point:
  - Divide by w: (x/w, y/w, 1)
  - When w=0, consider it as a direction: points at infinity
  - (0,0,0) undefined

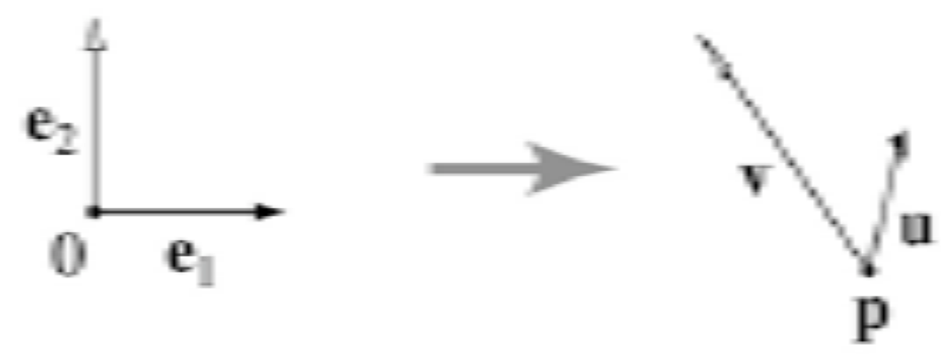


homogeneous **cartesian**

$$(x, y, w) \xrightarrow{/\,w} \left(\frac{x}{w}, \frac{y}{w}\right)$$

Affine change of coordinates

- Six degrees of freedom in 2D: **Coordinate frame**
- Frame:
  - **Basis + Point** $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 1 \end{bmatrix}$ Or $\begin{bmatrix} \boldsymbol{u} & \boldsymbol{v} & \boldsymbol{p} \\ 0 & 0 & 1 \end{bmatrix}$



Coordinate frame

- Frame = point + basis
- Frame matrix (frame-to-canonical)

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{u} & \boldsymbol{v} & \boldsymbol{p} \\ 0 & 0 & 1 \end{bmatrix}$$

- Move points to and forth by multiplying F

$$p_e = \boldsymbol{F} p_F \text{ and } p_F = \boldsymbol{F}^{-1} p_e$$

- Move transformations using similarity trans

$$\boldsymbol{T}_e = \boldsymbol{F}\boldsymbol{T}_F\boldsymbol{F}^{-1} \qquad \boldsymbol{T}_F = \boldsymbol{F}^{-1}\boldsymbol{T}_e\boldsymbol{F}$$

Affine change of coordinates

- When we move an object to the origin to apply a transformation, we are "changing coordinates": $\boldsymbol{T}_e = \boldsymbol{F}\boldsymbol{T}_F\boldsymbol{F}^{-1}$
  - $\boldsymbol{T}_e$ is the transformation in global frame
  - $\boldsymbol{T}_F$ is the transformation in local frame
  - F is the frame-to-canonical matrix
  - This is similarity transformation