Probability Models Notes

Probability

Casework

Technical

$P[A \cap B] = P[A]P[B|A] = P[B]P[A|B]$

Independent: $P[A \cap B] = P[A]P[B]$

Pairwise Independent: $P[A_i \cap A_j] = P[A_i]P[A_j]$

Stronger Mutually Independent: every event is independent of any intersection of the other events. 0 information updating upon observation.

Complementary Counting: $P[!A] = 1 - P[A]$

Mutually Exclusive: $P[A \cap B] = 0$

Principle Of Inclusion Exclusion: calculable symmetric expressions. Bonferroni Inequalities that the partial sums alternate between being $\geq$ and $\leq$ the final true value.

$\mathrm{Sd}(X) = \sqrt{\mathrm{Var}(X)}$

$e_X(d) = \frac{\mathrm{E}[X]-\mathrm{E}[X \wedge d]}{S_X(d)}$

$\mathrm{E}[X] = e_X(d)S_X(d) + \mathrm{E}[X \wedge d]$

Law Of Large Numbers When Finite Exists

Multiplying Independent Through Odds Ratios Likelihood Functions

Generating Functions

The variance of a mixture is given simply by the formula $\mathrm{Var}(X) = \mathrm{E}[\mathrm{Var}(X|\Theta)] + \mathrm{Var}[\mathrm{E}(X|\Theta)]$. We compute Variance and Means in terms of the STAM tables provided $\mathrm{E}[X^k]$ formulae and $\mathrm{Var}(X) = \mathrm{E}[X^2] - (\mathrm{E}[X])^2$.

By definition the CV, the coefficient of variation is the variation divided by the mean i.e. $\frac{\sigma}{\mu}$.

So the Loss Elimination Ratio is given by LER $= \frac{E(X)-E(Y^L)}{E(X)} = \frac{E(X \wedge d)}{E(X)}$. Where X is the underlying loss and Y^L is the amount paid by the insurer i.e. 0 or $X - d$ when $X \geq d$.

The only distribution with the memoryless Markov property is the exponential/geometric.

The minimum of independent exponential variables is exponential. For example consider the Survival Function i.e. the fact that $P[\min(A, B) \geq c] = P[A \geq c] \cdot P[B \geq c] = e^{-\frac{c}{\tau_1}} \cdot e^{-\frac{c}{\tau_2}} = e^{-\frac{c}{\frac{\tau_1\tau_2}{\tau_1+\tau_2}}}$ and thus in particular this variable is isomorphic with an exponential of $\tau_3 = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$.

The sum of independent Poisson variables is Poisson. If S_1 and S_2 are independent Poisson distributions with parameters λ_1 and λ_2 then $S = S_1 + S_2$ is a Poisson distribution with the parameter $\lambda = l = \lambda_1 + \lambda_2$. This can be seen for example by directly multiplying the generating functions.

If $S_N = X_1 + \cdots + X_N$ are iid independent of N with μ, σ:
$\mathrm{Var}(S_N) =$
$E_N[\mathrm{Var}(S_N|N)] + \mathrm{Var}[E(S_N|N)]$
$= E_N[\sigma^2 N] + \mathrm{Var}_N[\mu N]$
$= \sigma^2 E[N] + \mu^2 \mathrm{Var}[N]$

Special Case: Poisson Distributed Frequency
If $N \sim \mathrm{Poi}(\lambda)$:
$E(N) = \mathrm{Var}(N) = \lambda$
$E(S_N) = \lambda E(X)$
$\mathrm{Var}(S_N) = \lambda(\sigma^2 + \mu^2) = \lambda E(X^2)$

$E[(S-d)_+]$ is the notation used to describe the expected value of the amount of the aggregate loss in excess of the deductible i.e. the net stop loss premium.

$(X \wedge x)$ is used to denote X thresholded upper bounded at x i.e. this variable takes on the value of x when $X > x$. And one can construct these thresholded deductible variables similarly. Or use $\vee$ notation.

$\mathrm{VaR}_q[X] = \inf\{x : F_X(x) \geq q\}$: Value At Risk measure of X with confidence level $q \in (0,1)$. This is simply the quantile. For example let X be the annual loss random variable of an insurance product, $\mathrm{VaR}_{0.95}[X] = 100000000$ means that there is no more than a 0.05 probability that the loss will exceed 100000000 over a given year.

$\mathrm{TVaR}_q[X] = E[X|X > \mathrm{VaR}_q[X]]$: Tail Value At Risk. Expected value of X given that X exceeds the Value At Risk and this expectation exists.

The Per Loss Random Variable

The Limited Loss Random Variable

Percentiles

Generating Functions

Raising To A Power

k-Point Mixtures

Continuous Mixing

Splicing

Ordinary Deductibles

Franchise Deductibles

The Loss Elimination Ratio

Upper Policy Limits

Coinsurance

The Poisson Distribution

The Poisson Thinning

The Negative Binomial Distribution

The $(a, b, 0)$ Class:
If an $\mathbb{N}_0$-Valued Distribution has a Partial Mass Function which satisfies the following recursion:
$p_k = p_{k-1}\left(a + \frac{b}{k}\right)$ for $k = 1, 2, \ldots$
Thus, we say that it is an $(a, b, 0)$ distribution. The Poisson, the Negative Binomial, and the Binomial are the only representations.

The Impact Of Deductibles On Claim Frequency:
On Compounding:
In general, for an $\mathbb{N}_0$-Valued Random Variable $\mathbb{N}$ with the Partial Generating Function $P_{\mathbb{N}}$ and a sequence of independent, identically distributed random variables $[M_1, M_2, \ldots]$ with a common Partial Generating Function P_M, we set $S = M_1 + M_2 + \cdots + M_N = \sum_{i=1}^{N} M_i$ [if $N = 0$, then $S = 0$]. What is the distribution of S? If N is independent from $[M_1, M_2, \ldots]$, then, $P_S(z) = P_N(P_M(z))$.

Approximation
$S \sim N(\text{mean} = \mu_s, \text{variance} = \sigma_s^2)$
$\mu_s = E[S] = E[N] \cdot E[X]$ Wald's Identity
$\sigma_s^2 = \text{Var}[S] = E[N] \cdot \text{Var}[X] + \text{Var}[N](E[X])^2$

Recall:

The Excess Loss Random Variable
$Y^P = X - d | X > d$
The Per Payment Random Variable

The Left Censored And Shifted Random Variable, usually denoted by Y^L, is defined by $Y^L = X - d$ if $X > d$ else 0 also known as the Per Loss Random Variable and can be denoted $Y^L = (X - d)_+$.

The Poisson Gamma Mixture

The Binomial Distribution

Poisson Thinning With Conditioning

The $(a, b, 0)$ Class

Aggregate Loss Models: Expectation And Variance

Aggregate Loss Models With A Normal Approximation

Aggregate Loss Models: The Partial Mass Function Of Aggregate Losses

Aggregate Loss Models: The Cumulative Distribution Function Of Aggregate Losses

Stop Loss Insurance

Interpolation Theorem

Compound Poisson With Stop Loss Insurance

Compound Poisson With A Probability Calculation

Aggregate Losses With An Ordinary Deductible Per Loss

Maximum Likelihood Estimation: First Principles

Maximum Likelihood Estimation: Individual Unmodified Data

Maximum Likelihood Estimation: Grouped Data

Maximum Likelihood Estimation: Truncation And Censoring

Maximum Likelihood Estimation: Discrete Distributions