Differential Equations

$y' + p(t)y = g(t)$
$u(t)y' + u(t)p(t)y = u(t)g(t)$
Integrating Factor $u(t)$ Satisfies
$u(t)p(t) = u'(t)$
$\frac{u'(t)}{u(t)} = p(t)$
$(\ln(u(t)))' = p(t)$
$\ln(u(t)) = \int p(t)dt + c$
$u(t) = e^{\int p(t)dt+c} = e^c e^{\int p(t)dt} = ce^{\int p(t)dt}$
$u(t)y' + u'(t)y = (u(t)y(t))' = u(t)g(t)$
$u(t)y(t) + c = \int u(t)g(t)dt$
$y(t) = \frac{\int u(t)g(t)dt+c}{u(t)}$

$N(y)\frac{dy}{dx} = M(x)$
$\int N(y)dy = \int M(x)dx$

$M(x,y) + N(x,y)\frac{dy}{dx} = 0$
If $F_x + F_y\frac{dy}{dx} = 0$
$\frac{d}{dx}(F(x,y(x))) = 0$
$F(x,y) = c$
Check $F_{xy} = F_{yx}, M_y = N_x$
Perhaps Solve

Bernoulli Equations
$y' + p(x)y = q(x)y^n$
$y^{-n}y' + p(x)y^{1-n} = q(x)$
$v = y^{1-n}$
$v' = (1-n)y^{-n}y'$
$\frac{1}{1-n}v' + p(x)v = q(x)$
$v' + (1-n)p(x)v = (1-n)q(x)$
Solve Linear Equation And Solve For y

Substitutions

$y' = f\left(\frac{y}{x}\right)$
Homogeneous Equations
$v(x) = \frac{y}{x}$
$y = xv$
$y' = v + xv'$
$v + xv' = f(v)$
$xv' = f(v) - v$
$\frac{1}{f(v)-v}dv = \frac{1}{x}dx$
$x = ce^{\int \frac{1}{f(v)-v}dv}$

$y' = g(ax+by)$
$v = ax + by$
$v' = a + by'$
$\frac{1}{b}(v' - a) = g(v)$
$v' = a + bg(v)$
$\frac{1}{a+bg(v)}dv = dx$
$x = \int \frac{1}{a+bg(v)}dv + c$

Logistic Growth
$P' =$
$(\text{Growth Rate})\left(1 - \frac{P}{\text{Saturation Level}}\right)P$
$P(t) = 0, P(t) = \text{Saturation Level}$
Unstable/Stable Equilibria

Euler, Runge-Kutta Methods, Assert Maximum Error Bound Obtained, Error $\Theta(h)$ Of Step Size

$ay'' + by' + cy = 0$
$ar^2 + br + c = 0$
Distinct Real
$y(t) = c_1e^{r_1t} + c_2e^{r_2t}$
Complex (Roots $a \pm ib$) (Same As Real)
$y(t) = c_1e^{at}\cos(bt) + c_2e^{at}\sin(bt)$
Repeated Roots
$y(t) = c_1e^{rt} + c_2te^{rt}$

Reduction Of Order
Given $y_1(t)$ Solution Solve
$y_2(t) = v(t)y_1(t)$

For particular solutions ad hoc inspect functions related to the right hand side functions. For computing mutual antiderivative inspect terms which would produce such a term, and note degrees of terms which can match e.g.

Undetermined Coefficients
Variation Of Parameters
Solve $y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$
From Homogeneous Solutions

$a_n\frac{d^n y}{dx^n} + \cdots + a_2\frac{d^2 y}{dx^2} + a_1\frac{dy}{dx} + a_0 = f(x)$
Characteristic Equation:
$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0 = 0$
General Solution To Homogeneous Differential Equation:
$y(x) =$
$P_1(x)e^{\lambda_1 x} + P_2(x)e^{\lambda_2 x} + \cdots + P_r(x)e^{\lambda_r x}$
$P_i(x)$ is a polynomial of degree 1 less than the multiplicity of λ_i