Support - set of outcomes with positive probability

Bernoulli Distribution: $[0,1]$ with $[1-p,p]$
E[Bernoulli] $= p$
Var[Bernoulli] $= p(1-p)$
$P_X(s) = ps + q$

Binomial Distribution (n,p): $[0,1,\ldots,n]$ with $[\binom{n}{0}p^0(1-p)^n,\ldots]$
E[Binomial(n,p)] $= np$
Var[Binomial(n,p)] $= np(1-p)$
$P_X(s) = (ps+q)^n$

Geometric Distribution: $[0,1,2,\ldots]$ with $[p, p(1-p), p(1-p)^2, \ldots]$
E[Geometric] $= \frac{1-p}{p}$
Var[Geometric] $= \frac{1-p}{p^2}$
$P_X(s) = \frac{p}{1-qs}$

Poisson Distribution: $[0,1,2,\ldots]$ with $[e^{-\lambda}\frac{\lambda^k}{k!}]$
E[Poisson] $= \lambda$
Var[Poisson] $= \lambda$
$P_X(s) = e^{\lambda(s-1)}$

Gambler's Ruin: start with x and ++ with p and $--$ with $1-p$ and halt at a or 0.

Generating Functions - special case of Fourier/Laplace harmonic analysis way to capture probability distribution into function

$P_X(s) = \sum_{k=0}^{\infty} p_k s^k$

$P_{X+Y}(s) = P_X(s)P_Y(s)$ convolution by definition

$Y = \sum_{k=1}^{N} g_k$ on N independent distribution then $P_Y(s) = P_N(P_g(s))$

$\mathrm{E}[X], \mathrm{E}[X(X-1)], \mathrm{E}[X(X-1)(X-2)], \ldots$ factorial moments given by kth derivative of $P_X(s)$ evaluated at $s=1$ whence we may deduce moments $\mathrm{E}[X^k]$ linearly. And note $\mathrm{Var}[X] = P''(1) + P'(1) - (P'(1))^2$

$\mathrm{E}[X] = \mathrm{E}[X]$
$\mathrm{E}[X^2] = \mathrm{E}[X(X-1)] + \mathrm{E}[X]$
$\mathrm{E}[X^3] = \mathrm{E}[X(X-1)(X-2)] + 3\mathrm{E}[X(X-1)] + \mathrm{E}[X]$

Branching process has offspring distribution p_n then if $\mathrm{E}[p_n] = u$ with finite variance then by definition $\mathrm{E}[Z_n] = u^n$ and $\mathrm{Var}[Z_n] = \mathrm{Var}[p_n]u^n\frac{1-u^{n+1}}{1-u}$ if $u \neq 1$ and $\mathrm{Var}[p_n](n+1)$ if $u = 1$

Extinction probability p_E is smallest non-negative solution of the equation $x = P(x)$ where P is the generating function of the offspring distribution.

$i \to j$, i communicates with j, j is a consequent of/accesible from/follows i - nonzero P of arriving at j from i

$i \leftrightarrow j$, i and j intercommunicate if $i \to j$ and $j \to i$ a set B of states is irreducible if so classes of a chain, maximal irreducible sets, are strongly connected components in this digraph.

A set of states is closed if once there never escapes i.e. sink component.

A closed singleton is an absorbing state.

Recurrent if return with P $= 1$, positive recurrent if the Expected Value of time between 2 visits is finite, null if the EV is infinite, transient if positive P of not returning.

Recurrence and transience are class properties i.e. belong to all members of the same connected component.

Class on a finte state space is recurrent if and only if it is closed.

Write transition matrix P in the canonical form:
$\begin{bmatrix} PC & 0 \\ R & Q \end{bmatrix}$

So Q is T to T edges and PC is C to C edges and R is T to C edges where T is transient states i.e. union of all transient classes and PC is recurrent/closed states i.e. union of all sink connected components whence:

$U = (1-Q)^{-1}R = (1 + Q + Q^2 + \ldots)R = FR$ is transition probabilities

F captures expected number of times hitting each state in T prior to transitioning into C