1.1 $\bar{x} = [2, 8, 1]$

$\bar{y} = [3, 12, -1]$

СП: $(\bar{x}, \bar{y}) = 2 \cdot 3 + 8 \cdot 12 + 1 \cdot (-1) = 101$

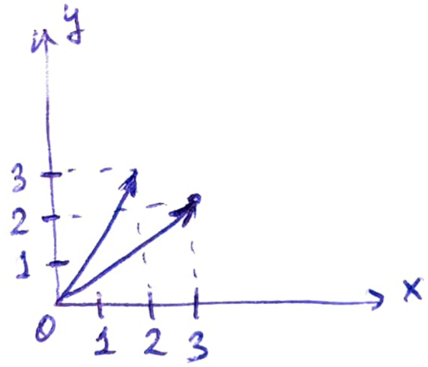
Угол: $\cos(\widehat{\bar{x}\bar{y}}) = \dfrac{(\bar{x}, \bar{y})}{|\bar{x}| \cdot |\bar{y}|} = \dfrac{101}{\sqrt{2^2+8^2+1^2} \cdot \sqrt{3^2+12^2+(-1)^2}} =$

$= \dfrac{101}{8{,}306 \cdot 12{,}41} \approx \dfrac{101}{103} = 0{,}98$

$\widehat{\bar{x}, \bar{y}} = \arccos(0{,}98) \approx 11^\circ$

1.2 $\bar{t} = [2, 3]$

$\bar{z} = [3, 2]$



$\bar{i} = [1, 0]$

$\bar{j} = [0, 1]$

$(\bar{i}, \bar{t}) = 2 \cdot 1 + 3 \cdot 0 = 2 \Rightarrow (\bar{i}, \bar{t}) = (\bar{j}, \bar{z})$.

$(\bar{j}, \bar{z}) = 3 \cdot 0 + 2 \cdot 1 = 2$

2.1 1) $f(x) = x^2 + 2x$

$f'(x_0) = 2x_0 + 2 = 0$ - экстремум ф-ии

$x_0 = -1$ - точка экстремума.

$f''(x_0) = 2 \Big|_{x_0=-1} > 0$

2) $f(x) = 2\sin x + 1$

$f'(x_0) = 2\cos x_0 = 0 \Rightarrow x_0 = \frac{\pi}{2} + \pi n, \ n \in \mathbb{Z}$

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3) $f(x) = \log_2 x + 3$

$f'(x_0) = \frac{1}{x_0 \ln 2} = 0$ - нет решения.

3

$$\begin{pmatrix} 1 & -3 & 2 \\ 3 & -4 & 1 \\ 2 & -5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 & 6 \\ 1 & 2 & 5 \\ 1 & 3 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2-3+2 & 5-6+6 & 6-15+4 \\ 6-4+1 & 15-8+3 & 18-20+2 \\ 4-5+3 & 10-10+9 & 12-25+6 \end{pmatrix} = \begin{pmatrix} 1 & 5 & -5 \\ 3 & 10 & 0 \\ 2 & 9 & -7 \end{pmatrix}$$

$$\overline{x} = [6, 8]$$

$$|\overline{x}| = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$\overline{y} = [2, 2, 3]$$

$$|\overline{y}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{25} = 5$$

4

$z = x - 2y$

$x^2 + y^2 \le 2y \Rightarrow x^2 + y^2 - 2y + 1 - 1 \le 0$

$x^2 + (y-1)^2 \le 1$

Условия на экстремум:

$$\begin{cases} \dfrac{\partial z}{\partial x} = 0 \Rightarrow 1 = 0 \\ \dfrac{\partial z}{\partial y} = 0 \Rightarrow -2 = 0 \end{cases}$$

Точек экстремума нет, в том числе в области

Условный экстремум на границе:

$L = z(x,y) + \lambda(x^2 + (y-1)^2 - 1)$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 1 + 2\lambda x = 0 \\ \dfrac{\partial L}{\partial y} = -2 + 2\lambda(y-1) = 0 \\ x^2 + (y-1)^2 - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = -\dfrac{1}{2\lambda} & (1) \\ y = \dfrac{\lambda+1}{\lambda} & (2) \\ x^2 + (y-1)^2 - 1 = 0 & (3) \end{cases}$$

Подставим (1) и (2) в (3)

$$\frac{1}{4\lambda^2} + \left(1 + \frac{1}{\lambda} - 1\right)^2 - 1 = 0.$$

$$\frac{1}{4\lambda^2} + \frac{1}{\lambda^2} - 1 = 0 \Rightarrow \frac{5}{4} = \lambda^2 \Rightarrow \lambda_{1,2} = \pm\frac{\sqrt{5}}{2}$$

$$x_{1,2} = \mp\frac{1}{\sqrt{5}};\ y_{1,2} = 1 \pm \frac{2}{\sqrt{5}}.$$

$$x_{1,2} = \mp \frac{1}{\sqrt{5}}$$

$$y_{1,2} = 1 \pm \frac{2}{\sqrt{5}}$$

$$z(x_1, y_1) = -\frac{1}{\sqrt{5}} - 2\cdot\left(1 + \frac{2}{\sqrt{5}}\right) = -2 - \sqrt{5}$$

$$z(x_2, y_2) = \frac{1}{\sqrt{5}} - 2\cdot\left(1 - \frac{2}{\sqrt{5}}\right) = -2 - \frac{3}{5}\sqrt{5}.$$

$\Rightarrow$ Минимум достигается в т. $\left(-\frac{1}{\sqrt{5}}; 1 + \frac{2}{\sqrt{5}}\right)$

$$z_{min} = -2 - \sqrt{5}$$