Random Sample

$X_1 \dots X_n$ are called Random Sample of size n from a population $f(x)$

if
* $X_1 \dots X_n$ are mutually independent
* marginal PDF or PMF of each X_i is $f(x)$

Also,

$X_1 \dots X_n \rightarrow$ independent and identically distributed random samples (iid)

$$f(x_1, \dots x_n) = f(x_1) f(x_2) \dots f(x_n) = \prod_{i=1}^{n} f(x_i) \quad \text{— (I)}$$

$$f(x_1 \dots x_n | \theta) = \prod_{i=1}^{n} f(x_i | \theta) \quad \text{— (II)}$$

The Likelihood Function

$f(x|\theta) \longrightarrow$ joint PDF or PMF of $X = (X_1 \dots X_n)$

Given $X = x$ is observed,

the function of θ defined by

$$L(\theta|x) = f(x|\theta)$$ is

the likelihood function.

$$P_{\theta_1}(X=x) = L(\theta_1|x) > L(\theta_2|x) = P_{\theta_2}(X=x)$$

means

θ_1 is more plausible than θ_2

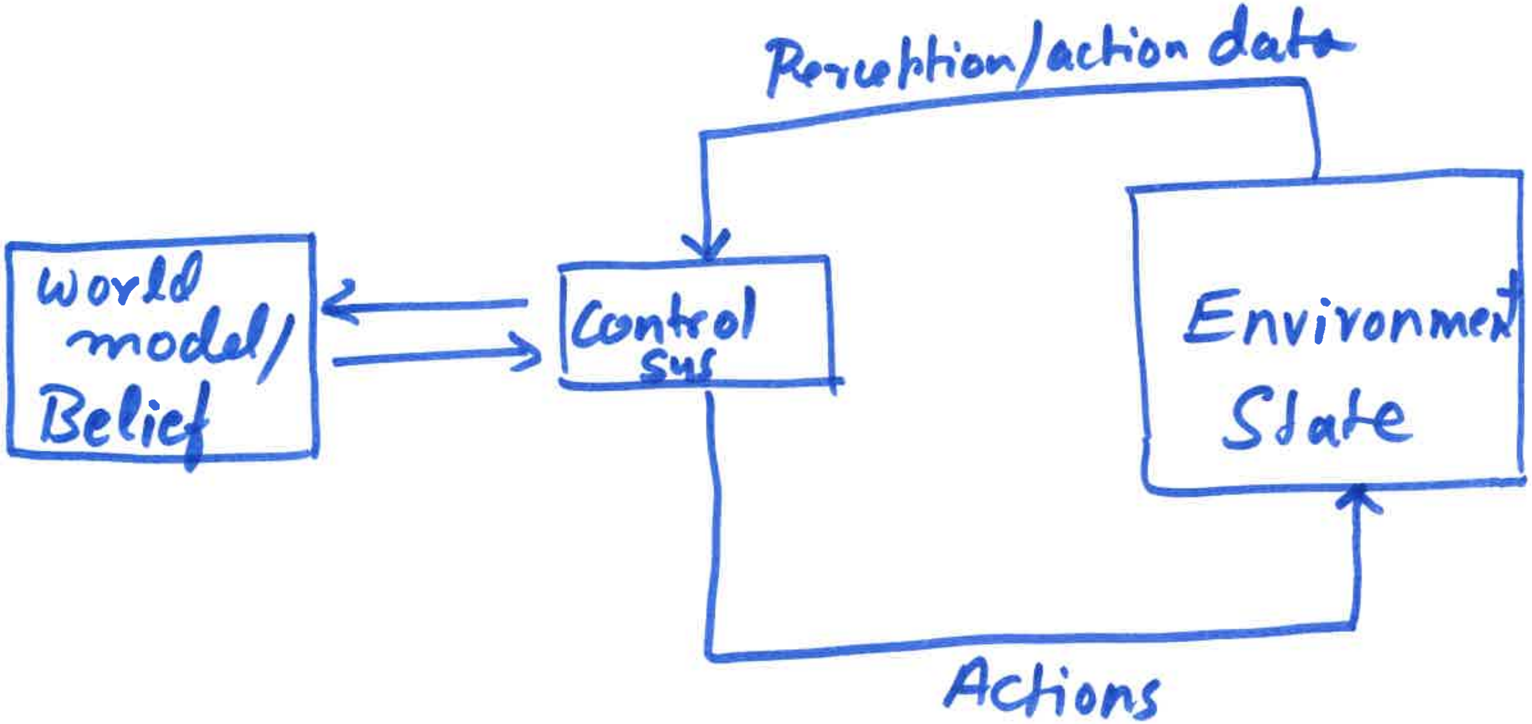
Estimation

Classic Estimation	Bayesian Estimation
$\theta \rightarrow$ fixed, unknown	$\theta \rightarrow$ variable, unknown
* Maximum likelihood estimator (MLE)	* Normal Bayes estimator
* $\hat{\theta}(x)$ is the parameter at which $L(\theta \mid x)$ attains maximum as a function of θ, x is held fixed	* $\pi(\theta \mid x) = \dfrac{f(x \mid \theta)\, \pi(\theta)}{m(x)}$ $m(x) = \int f(x \mid \theta)\, \pi(\theta)\, d\theta$

Bayesian Estimation

Content: mobile robot localization / tracking of targets

[Thrun et al "Probabilistic Robotics" 2005]



Robot Environment Interaction.

State

State — Environment
State — Dynamic
State — Static

x_t

* Pose — Position
* Pose — Velocity
* Pose — Attitude

* Landmarks — features of the environment.

Environment Interaction

* Measurement (using sensors)

  - $z_{t_1:t_2} = z_{t_1}, z_{t_1+1} \cdots z_{t_2}$

    $t_1 < t_2$

  - Obtain information about the state

* Control (using actuators)

  - $u_{t_1:t_2} = u_{t_1}, u_{t_1+1} \cdots u_{t_2}$

  - actions that change the state of the world

Probabilistic generative laws

Evolution of State and Measurements

State:

x_t is stochastically generated from x_{t-1}

$$p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Measurement:

model the process by which measurements are generated.

$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

Belief Distributions

belief with regard to the state

$$\boxed{bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})}$$

Algorithm Bayes Filter.

Alg Bayes-fiter($bel(x_{t-1}, u_t, z_t)$)

for all x_t do

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$$

$$bel(x_t) = \eta(z_t \mid x_t)\, \overline{bel}(x_t)$$

end for

return $bel(x_t)$

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