Students Id:

Total Number of Pages: 2

B.TECH
MA102

Second Semester Examination – 2015

Mathematics-II

BRANCH: ALL
Time: 3 Hours

Max marks: 50

Answer Question No.1 which is compulsory and any four from the rest.

The figures in the right hand margin indicate marks.

1. (2×5)

a) Find the unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1,2,-1)$.

b) Verify whether the vector field $(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is conservative.

c) Find the inverse Laplace of $F(s) = \frac{s}{(s^2+1)^2}$ using convolution theorem.

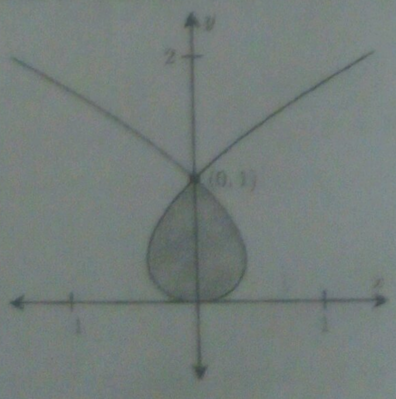
d) Define Dirac-delta function and find its Fourier transform.

e) If $e^{-kx} = \frac{2}{\pi}\int_0^\infty B\sin(\omega x)d\omega$, find the value of B.

2. (5+5)

a) Let R be the region determined by the inequalities $x^2 + y^2 \le 4$ and $y^2 \le x^2$. Evaluate the following integral $\iint_R \sin(x^2 + y^2)\, dA$.

b) The following picture shows the parametric curve $(x, y) = (t - t^3, t^2)$. Use Green's theorem to find the area of the shaded region.



(2+3+5)

3.

a) Find the parametric representation of elliptic cone $z = \sqrt{x^2 + 4y^2}$.

b) Evaluate $\int_C ydx + 2xdy$ where C is the boundary of the rectangle enclosed by the lines $x = 0, x = 1, y = 0, y = 2$.

c) Let C be the rectangle in space with vertices $(0,0,0), (1,0,0), (1,1,1)$, and $(0,1,1)$, oriented in the given order. Use stoke's theorem to evaluate the line integral

$$\int_C \sin(x^2)dx + xy^2dy + xz^2dz$$

4. State Gauss divergence theorem and Green's identities. (2+2+6)

Let S be the surface given by the parametric equation

$$x = 2 + \cos u + \cos v,\ y = u,\quad z = \sin v,\quad 0 \le u \le 2\pi,\ 0 \le v \le 2\pi$$

Use the divergence theorem to find the volume of the region inside S.

5. (a) Find the Fourier series of the periodic function (5+5)

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1 - x, & 1 < x < 2 \end{cases} \qquad p = 2L = 2.$$

(b) Solve the following differential equations using Laplace transform:

$$t\frac{d^2y}{dt^2} - \frac{dy}{dt} = -1; \quad y(0) = 0.$$

6. (a) Determine the Fourier transform for $f(x) = e^{-2x^2}, x \ge 0$. (5+5)

(b) Find the Laplace transform of the periodic function defined by the triangular wave $f(t) = t, 0 \le t \le a$, and $f(t + 2a) = f(t)$.

7. (a) State the Parsevall's identity for Fourier transform. (2+ 8)

(b) Determine the Fourier cosine transform for $f(t) = e^{-t}\cos t, t \ge 0$ and show that

$$e^{-t}\cos t = \frac{2}{\pi}\int_0^{\infty} \frac{(\alpha^2 + 2)\cos \alpha t}{\alpha^4 + 4} d\alpha, \quad t \ge 0.$$

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