热统习题的部分答案 (野生版)

2022 年 10 月 19 日

目录

1 第一章 1

1.1 习题 1.6 1

1.2 习题 1.8 1

1.3 习题 1.11 2

1.4 习题 1.13 3

1.5 习题 1.22 3

1.6 习题 1.23 3

1 第一章

1.1 习题 1.6

$$
\begin{aligned}
\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{\nu R}{pV} \quad &\Rightarrow \quad \left(\frac{\partial V}{\partial T}\right)_p = \frac{\nu R}{p} \\
&\Rightarrow \quad V = \frac{\nu R T}{p} + g(p) \quad \text{代入 } \kappa = \frac{1}{p} + \frac{a}{V} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \\
&\Rightarrow \quad \mathrm{d}(pg(p)) = -\frac{a}{2}\mathrm{d}(p^2) \\
&\Rightarrow \quad pg(p) = -\frac{a}{2}p^2 + \mathrm{const} \quad \text{代入第二行式子并整理} \\
&\Rightarrow \quad pV = \nu R T - \frac{a}{2}p^2 + \mathrm{const}\,.
\end{aligned}
$$

1.2 习题 1.8

1) 利用广延量假设,

$$
\begin{aligned}
S = Ns, V = Nv, U = Nu \quad &\Rightarrow \quad s = A(vu)^{1/3} \\
&\Rightarrow \quad T\mathrm{d}s = TA\frac{v\mathrm{d}u + u\mathrm{d}v}{3(vu)^{2/3}} = \mathrm{d}u + p\mathrm{d}v \\
\mathrm{d}u, \mathrm{d}v \text{ 前的系数对应相等并恢复广延量} \quad &\Rightarrow \quad \begin{cases} T = \frac{3u^{2/3}}{Av^{1/3}} = \frac{3U^{2/3}}{A(NV)^{1/3}} \quad \text{易得: } U \sim T^{3/2} \\ p = \left(\frac{N}{V}\right)^{1/2}\left(\frac{AT}{3}\right)^{3/2} \end{cases}
\end{aligned}
$$

$$
C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{1}{2}\sqrt{\frac{A^3NVT}{3}} \quad \text{利用了前面的温度表达式.}
$$

2) 对于每个热源而言, 我们有:

$$
\text{đ}Q = \mathrm{d}U \sim T^{1/2}\mathrm{d}T,
$$

由于高温热源放出热量 ($\Delta Q_h < 0$) 必定大于等于低温热源吸热 ($\Delta Q_c > 0$), 不失一般性, 我们假设 $T_1 > T_2$,

$$
\begin{aligned}
\Delta Q_h + \Delta Q_c \leq 0 \quad &\Rightarrow \quad \left(\int_{T_1}^{T_f} + \int_{T_2}^{T_f}\right) T^{1/2}\mathrm{d}T \leq 0 \\
&\Rightarrow \quad T_f \leq \left(\frac{T_1^{3/2} + T_2^{3/2}}{2}\right)^{2/3}\,;
\end{aligned}
$$

另一方面, 由热力学第二定律: $\Delta S \geq 0$,

$$
\begin{aligned}
\Delta S = \left(\int_{T_1}^{T_f} + \int_{T_2}^{T_f}\right)\frac{\text{đ}Q}{T} &\sim \left(\int_{T_1}^{T_f} + \int_{T_2}^{T_f}\right)\frac{1}{T^{1/2}}\mathrm{d}T \geq 0 \\
&\Rightarrow \quad T_f \geq \left(\frac{T_1^{1/2} + T_2^{1/2}}{2}\right)^2\,.
\end{aligned}
$$

由第一题的温度表达式, 我们得到:

$$
U = \left(\frac{AT(NV)^{1/3}}{3}\right)^{3/2} = CT^{3/2}.
$$

对于整个系统而言,

$$
W = -\Delta U_{\text{总}} = C\left(T_1^{3/2} + T_2^{3/2} - 2T_f^{3/2}\right) \leq \sqrt{\frac{A^3NV}{27}}\left[T_1^{3/2} + T_2^{3/2} - 2\left(\frac{T_1^{1/2} + T_2^{1/2}}{2}\right)^3\right]\,.
$$

1.3 习题 1.11

首先，我们需要求出范氏气体的内能表达式,

$$
\begin{aligned}
\text{由范氏气体的状态方程} \quad &\Rightarrow \quad \left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{V-b} \\
\text{代入书上式子 (1.9.16)} \quad &\Rightarrow \quad \left(\frac{\partial U}{\partial V}\right)_T = -p + T\left(\frac{\partial p}{\partial T}\right)_V = \frac{a}{V^2} \\
&\Rightarrow \quad \mathrm{d}U = C_V \mathrm{d}T + \left(\frac{\partial U}{\partial V}\right)_T \mathrm{d}V = C_V \mathrm{d}T + \frac{a}{V^2}\mathrm{d}V \\
&\Rightarrow \quad U = C_V T - \frac{a}{V} + U_0 \,.
\end{aligned}
$$

1) 对于等温过程,

$$
\begin{aligned}
&\Delta U = -\frac{a}{V_f} + \frac{a}{V_i} \\
&W = \int_{V_i}^{V_f} p\mathrm{d}V = \int_{V_i}^{V_f} \left(\frac{RT_i}{V-b} - \frac{a}{V^2}\right)\mathrm{d}V = RT_i \ln\left(\frac{V_f - b}{V_i - b}\right) + \frac{a}{V_f} - \frac{a}{V_i} \\
&\Delta S = \frac{\Delta U + W}{T_i} = R\ln\frac{V_f - b}{V_i - b}\,;
\end{aligned}
$$

2) 对于等压过程,

$$
W = p_i(V_f - V_i) = \left(\frac{RT_i}{V_i - b} - \frac{a}{V_i^2}\right)(V_f - V_i)
$$

$$
\begin{aligned}
\text{利用状态方程} \quad &\Rightarrow \quad T_f = \left(p_i + \frac{a}{V_f^2}\right)(V_f - b)/R = \left(\frac{RT_i}{V_i - b} - \frac{a}{V_i^2} + \frac{a}{V_f^2}\right)(V_f - b)/R \\
&\Rightarrow \quad \Delta U = C_V(T_f - T_i) - \frac{a}{V_f} + \frac{a}{V_i} = C_V\left[\left(\frac{RT_i}{V_i - b} - \frac{a}{V_i^2} + \frac{a}{V_f^2}\right)(V_f - b)/R - T_i\right] - \frac{a}{V_f} + \frac{a}{V_i}
\end{aligned}
$$

$$
\begin{aligned}
\text{由等压条件} \quad &\Rightarrow \quad T\mathrm{d}S = đQ = C_p \mathrm{d}T \\
&\Rightarrow \quad \mathrm{d}S = C_p \mathrm{d}(\ln T) \\
&\Rightarrow \quad \Delta S = C_p \ln\frac{T_f}{T_i} = C_p \ln\left[\frac{V_f - b}{V_i - b} - a\left(\frac{1}{V_i^2} - \frac{1}{V_f^2}\right)\frac{V_f - b}{RT_i}\right]\,;
\end{aligned}
$$

3) 对于绝热过程,

$$
đQ = \mathrm{d}U + p\mathrm{d}V = \left(\frac{\partial U}{\partial T}\right)_V \mathrm{d}T + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\mathrm{d}V = C_V \mathrm{d}T + \left(\frac{a}{V^2} + p\right)\mathrm{d}V = 0\,,
$$

$$
\begin{aligned}
\text{结合状态方程} \quad &\Rightarrow \quad C_V \mathrm{d}T + \frac{RT}{V-b}\mathrm{d}V = 0 \\
&\Rightarrow \quad C_V \ln T + R\ln(V-b) = 0 \\
&\Rightarrow \quad T^{C_V}(V-b)^R = \mathrm{const} \\
&\Rightarrow \quad T_f = T_i\left(\frac{V_i - b}{V_f - b}\right)^{R/C_V}
\end{aligned}
$$

因此, 我们容易得到:

$$
\begin{aligned}
&\Delta U = C_V T_i\left[\left(\frac{V_i - b}{V_f - b}\right)^{R/C_V} - 1\right] - \frac{a}{V_f} + \frac{a}{V_i} \\
&W = -\Delta U \\
&\Delta S = 0\,.
\end{aligned}
$$

1.4　习题 1.13

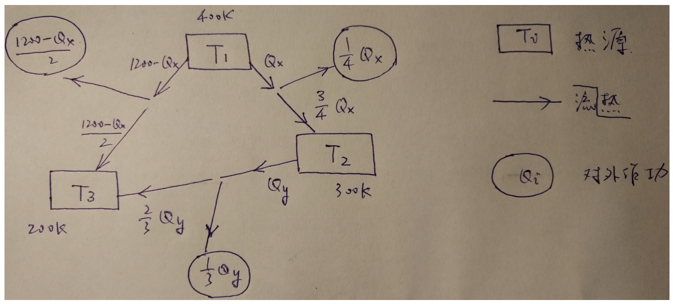
其中一个物体降温 (放热): $T_1 \to T_2$, 另一个物体升温 (吸热): $T_1 \to T_x$. 考虑整个体系, 由热力学第二定律,

$$\Delta S = \left(\int_{T_1}^{T_2} + \int_{T_1}^{T_x} \right) \frac{C_p \mathrm{d}T}{T} \geq 0 \quad \Rightarrow \quad T_x \geq \frac{T_1^2}{T_2} .$$

因此,

$$W \geq \Delta Q = C_p(T_x + T_2 - 2T_1) \geq C_p \left(\frac{T_1^2}{T_2} + T_2 - 2T_1 \right) = C_p \frac{(T_1 - T_2)^2}{T_2} .$$

1.5　习题 1.22



由图, 我们有以下关系:

$$\begin{aligned} \frac{1200 - Q_x}{2} + \frac{Q_y}{3} + \frac{Q_x}{4} = 200 \quad & \Rightarrow \quad \frac{3}{4} Q_x - Q_y = 1200 \\ & \Rightarrow \quad \begin{cases} Q_2 = \frac{3}{4} Q_x - Q_y = 1200\mathrm{J} \\ Q_3 = \frac{1200 - Q_x}{2} + \frac{2Q_y}{3} = -200\mathrm{J} \end{cases} \\ & \Rightarrow \quad \begin{cases} \Delta S_1 = \frac{Q_1}{T_1} = \frac{-1200}{400} = -3\mathrm{J/K} \\ \Delta S_2 = \frac{Q_2}{T_2} = \frac{1200}{300} = 4\mathrm{J/K} \\ \Delta S_3 = \frac{}{Q_3} T_3 = \frac{-200}{200} = -1\mathrm{J/K} \\ \Delta S_{\text{总}} = \sum_{i=1}^{3} \Delta S_i = 0\mathrm{J/K} . \end{cases} \end{aligned}$$

1.6　习题 1.23

考虑等熵过程 $(S = S_0)$,

$$\begin{aligned} W_{S_0} = \int_{V_0}^{V} p(S_0, V') \mathrm{d}V' = R S_0 \ln \frac{V}{V_0} \quad & \Rightarrow \quad p(S_0, V) = \frac{\mathrm{d}W_{S_0}}{\mathrm{d}V} = \frac{R S_0}{V} \\ U(S_0, V) - U(S_0, V_0) = -W_{S_0} \quad & \Rightarrow \quad U(S_0, V) = U_0 - R S_0 \ln \frac{V}{V_0} . \end{aligned}$$

现在考虑一个等体过程 $(V, S_0 \to V, S)$,

$$\begin{aligned} U(S, V) - U(S_0, V) &= \int_{T(S_0, V)}^{T(S, V)} T(S', V) \mathrm{d}S' \\ &= \int_{S_0}^{S} \frac{R V_0}{V} \left(\frac{S'}{S_0} \right)^a \mathrm{d}S' \\ &= \frac{R V_0 S_0}{(a+1) V} \left[\left(\frac{S}{S_0} \right)^{a+1} - 1 \right] \end{aligned}$$

因此,

$$U(S,V) = \frac{RV_0S_0}{(a+1)V}\left[\left(\frac{S}{S_0}\right)^{a+1} - 1\right] - RS_0 \ln\frac{V}{V_0} + U_0$$

又由:

$$\left(\frac{\partial p}{\partial S}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_S = \frac{RV_0}{V^2}\left(\frac{S}{S_0}\right)^a$$

$$\Rightarrow \quad p(S,V) = \frac{RV_0S_0}{(a+1)V^2}\left[\left(\frac{S}{S_0}\right)^{a+1} - 1\right] + \frac{RS_0}{V}$$

$$W(S, V_0 \to V) = \int_{V_0}^{V} p(S,V')\mathrm{d}V' = \frac{RV_0S_0}{a+1}\left[\left(\frac{S}{S_0}\right)^{a+1} - 1\right]\left(\frac{1}{V_0} - \frac{1}{V}\right) + RS_0 \ln\frac{V}{V_0}\,.$$