3 PROBABILITY

3.06 Practice with sets

Probability concepts sometimes appear to be a bit abstract. However, they are quite practical as well: knowing them helps a lot to see the most important structure in seemingly complex problems and express your thoughts clearly. Therefore you should learn to recognize and describe practical situations and problems as experiments with trials, outcomes and events that make up a sample space. Here I will present a number of exercises that you can use to further your understanding of basic probability concepts.

Imagine this situation where you pick up three shells at random at a beach with only two types of shells around, which I will call Q and R. Can you describe this activity with the terms experiment, trial, outcome, random variable and event? Try to explain to which part of the shell-collecting activity each term applies. […]

The whole enterprise, picking three shells at random, is the experiment. Each time you pick up a shell is a random trial, so the experiment consists of three trials. Each trial leads to an outcome: you will have picked up either a shell of type Q or R. At the end of the experiment you have outcomes that consist of combinations of three shells, for example QQR or RQR. You can define any outcome or combination of outcomes as an event, for example all cases where you have picked up at least one Q-shell. The random variable in this experiment (and also in each trial of this experiment) is the type of shell you will pick-up.

Can you list the sample space for this experiment? […]

The sample space of an experiment comprises all the possible outcomes. In this case the following eight outcomes are possible.

Try to draw a tree diagram that shows the structure of this experiment, with all its outcomes. […]

Your tree diagram for the entire experiment should resemble this one. You have three levels of nodes: the first, second and third time you pick up a shell. And at each node you have one branch which represents picking up a Q-shell and one for an R-shell. At the end of the branches the final outcomes are listed.

Now that you have visualized the experiment in a tree diagram, can you define two disjoint events for the outcome of this experiment that are not each other's complement? […]

An example of this would be event A that you would only have picked up one R-shell in total and event B that you would have picked up two R-shells in total. These are disjoint but not each other's complement, because there is some part of the sample space that belongs to neither of the two events.

Can you also give an example of an event for the outcome of this experiment and its complement as well? […]

An example of complementary events could be event A where you would have picked-up two or more R-shells versus event B of picking up one or zero R-shells in total.

Now consider the event that the first shell you pick-up would be an R-shell and another event that the last shell to pick-up is an R-shell. Can you describe the relation between these two events? […]

Clearly these two events are not disjoint, because they overlap. So these two events are said to intersect each other. The intersection of the two events is a subset of both events: cases where the first shell you pick-up is an R-shell and also the third shell you pick-up is an R-shell.

Let's now try to assign probabilities to the various events. Let's assume that R-shells are two times as abundant as Q-shells, while they both occur in large quantities. What would then be the probability of picking up a Q-shell? […]

The probability is found by considering the relative frequency of both shells, which is already given in this problem. One out of three shells is a Q shell and two out of three shells is an R shell, so the probability to pick up a Q shell is one third.

Now you know the probabilities for a single event of picking up a shell it is possible to put the probabilities for every event in the tree diagram and start to calculate the probabilities for combined events. For example, what would be the probability to pick-up at least two Q-shells? […]

The event of picking up at least two Q-shells contains the sequences QQR, QRQ, RQQ and QQQ as outcome. The probability for the first three of these sequences is one third times one third times two thirds which is two twenty-seventh. The probability for the sequence QQQ is one twenty-seventh. Adding-up these four probabilities gives seven twenty-seventh.

The case were the shells would not be abundant would change the nature of the system considerably. Let's assume that there are only 4 Q shells and 6 R shells at the entire beach, but that you would still collect 3 out of these randomly. Now, put the probabilities for this experiment in the tree diagram again. […]

This is how your tree diagram should look. When you pick up the first shell, there is a probability of four tenth to choose a Q shell and a probability of six tenth to choose an R shell. If your first shell was a Q-shell, the subsequent probabilities are $3/9^{th}$ and 6/9th to select respectively a Q or R shell, and thereafter the probabilities change again. While the denominators in the probabilities decrease from ten, to nine and eight shells at each time a shell is picked up, the numerators change depending on how many shells of that type are still left on the beach.

Can you, for this new situation with only a handful of shells, also calculate the probability to pick-up at least two Q-shells? […]

While the probabilities in the tree diagram have changed, its structure is still the same. The event of picking up at least two Q-shells still contains the sequences QQR, QRQ, RQQ and QQQ as outcomes. The probability for each of these is given here. And the total probability to get any of these outcomes is $1/3^{rd}$.

This was the last exercise I had in stock for you. I hope they did show how to effectively apply tree diagrams for calculating probabilities and helped you to get on grips with the various probability concepts.