7 SIGNIFICANCE TESTS

7.07 Example

Imagine you're a diver interested in whale sharks. More specifically, you want to know what the average length of these gigantic animals is. Also suppose you have spent years and years in different parts of the world to study these creatures. Over the years you have encountered and measured 258 whale sharks. Because you have measured whale sharks all over the world, we assume for now that these 258 whale sharks can be understood as a simple random sample. It turns out that the mean length equals 8.3 meters. The sample standard deviation is 3.4 meters. It also turns out that the distribution of whale shark length is approximately normal. In this video we'll test three alternative hypotheses against the null hypothesis that the mean whale shark-length in the population equals 8 meters. The first one is that the population mean differs from 8 meters. The second one is that it is larger than 8 meters. And the third one is that the population mean is smaller than 8 meters. In all three cases we set the significance level alpha at 0.10.

First we'll have to check our assumptions. As I've said before, the selection of whale sharks can be understood as a simple random sample. So that's fine. We've also seen that our *sample* distribution of whale shark length is approximately normal. We have, therefore, no reason to expect that the *population* distribution seriously deviates from normality. Moreover, this is not much of an issue anyway, because we have a rather large n.

Let's now compute the test statistic. The value of the test statistic is the same for all three tests. After all, the sample mean and the null hypothesis mean do not differ between tests. This is the formula we use. That leads to the following computation: 8.3 minus 8 divided by 3.4 divided by the square root of 258. That equals about 1.42.

Now, let's start with the first alternative hypothesis. This one. It claims that the population mean differs from 8. First we draw the relevant sampling distribution and show the null hypothesis value. We have to do a two-tailed test, based on an alpha of 0.10. The rejection region is about here. We can look up the critical values in the t-table. The critical values are -1.66 and 1.66. So our distribution looks like this. The test statistic of t equals 1.42 is not located in the rejection region so we do not reject the null hypothesis. This means that, based on an alpha of 0.10, we cannot conclude that the population mean differs from 8.

The second alternative hypothesis is that the population mean is larger than 8. The sampling distribution looks the same, but now we do a one- instead of a two-tailed test. That looks like this: We still have an alpha of 0.10, so these are the cumulative probabilities. We look up the critical value in the t-table. That's 1.29. Now our test statistic is located within the rejection region, so in this case we do reject our null hypothesis. And we conclude that the population mean is indeed larger than 8.

The final alternative hypothesis is that the population mean is smaller than 8. In this case we do a left-tailed test. That looks like this. We have a cumulative probability of 0.10 at the left side of the distribution. This is exactly the mirror image of our previous right-tailed test. So the critical value which corresponds to this rejection region is minus 1.29. Now our test statistic (which is 1.42) is a value more extreme than our critical value. However, it is located at the other side of the distribution. This means that it is not located in the rejection region and that we therefore don't reject our null hypothesis. We cannot conclude that the population mean is smaller than 8.
This final example shows that it is important to always draw the sampling distribution. Otherwise you might well fail to notice that the test statistic is located on the other side of the distribution than your critical value.

Whatever the outcome of this test, There are two things I know for sure: (1) whale sharks are huge! And (2) I’m going to pack my bags and hope that I will see one of them during my scuba-diving holiday. Adios!