3 PROBABILITY

3.09 Conditional probability

In this video I will explain the concept of conditional probability and show how it can be calculated. The term conditional means 'depending on something else' and has more or less the same meaning in the context of probability as it has in every-day language. Its formal definition is as follows: the probability of an event, given that another event occurs. And this is its mathematical notation: the probability of event A, given that B occurs or 'conditional on B' – the vertical line is shorthand for the words 'given' or 'conditional'.

A conditional probability of A given B is calculated by the probability that events A and B occur together, divided by the probability that event B occurs. So: the joint probability of A and B divided by the probability of B. It can also be illustrated with this Venn diagram, which emphasizes that the joint probability of A and B can only be smaller than or equal to the probability of B.
Let's apply the equation to a familiar example: you have made a count of different activities by the people on a beach, and you also distinguished people by gender. You have turned the results into table with probabilities, here it is.

So with these variables, activity and gender, what would be an example of a conditional probability? It are those probabilities for the case where you know the outcome of one variable and then then want to calculate the probability of the other variable occurring. Let's pick a concrete example: you would estimate the probability that a person is resting if you know that person is male.

To calculate it, you would apply the equation and divide the joint probability of resting and male by the probability of resting, hence 0.3 divided by 0.45. To calculate the conditional probabilities for the other activities given that a person is male, the equivalent calculation is made, as shown here. And for females, the conditional probabilities for the activities would be calculated by dividing with 0.55. The resulting conditional probabilities for the activities, given gender, are shown here.

Now I have a question for you. Can you calculate the conditional probabilities of gender, given activity? This is the original data, which provides you with the required information. […]

What you should do here, is to consider a joint probability, for example the case of being male & resting, and then divide this joint probability by the marginal probability for the relevant activity (in this case resting). Next, this should be done for the remaining five joint probabilities. This gives you the following six conditional probabilities. So based on joint and marginal probabilities you can calculate conditional probabilities.

But the equation for conditional probability, simple as it is, has a little bit more to offer, it is at the same time the general equation for calculating joint probability, for both independent and non-independent events. Take a look at the equation again. If you multiply both sides with the marginal probability for event B, the probability of B disappears from the right-hand side of the equation. By subsequently interchanging the parts at the left and right, you have an expression to calculate joint probability. This implies that if you were given the task to find a joint probability for events A and B where you cannot assume independence between A and B, you would need the probability for A as well as the conditional probability for A given B.

Let me summarize what I have explained in this video.

- Conditional probability is the probability of an event, given that another event occurs. While it may not seem that special on the face of it, it is at the heart of many probability calculations.
- Mathematically, the conditional probability of A given B equals the joint probability of A and B, divided by the probability of event B.
- The definition for conditional probability is at the same time the general definition for joint probability of both independent and dependent events.
- The Probability of A conditional on B can be considered as the probability of A in the reduced sample space where B occurred, to which all rules of a sample space apply.