3 PROBABILITY

3.04 Quantifying probabilities with a tree diagram

When you have thought about a random phenomenon and put down your ideas on all the events in the sample space, with relationships between them as well as probabilities in a tree diagram. You can start to make probability calculations and assess the likelihood that a certain event occurs. Here I will explain how this quantification of probability with a tree diagram works in practice.

You are going to buy a soft-drink at a beach stand, which is almost sold out with ice-cream and only two bottles of soft drink left for sale. That's why sales are limited to one item per customer. Unfortunately, you find three persons lined-up in front of you. While waiting you are figuring out your chances to acquire your drink, and you make up this tree diagram. The diagram shows that you assume that there is an oh point five probability for choosing either ice-cream or soft drink by each customer.

The general probability rules apply to every node in the tree diagram. For instance, if we consider this tree diagram at the first customer, there are two branches, each with probability oh point five (summing to one), and at the second customer there are two pairs of branches (again each with an individual portability of oh point five). However, when considering the four branches for the 2nd customer jointly, we know that each of the four are equally likely. Therefore each should thus have a probability of one fourth ... so how does this relate to the probabilities of a halve? It makes sense if you consider the sequential nature of these events and the fact that each branch is independent of another. By going along one branch after the first customer, you have halved your probability of that particular outcome, which is again halved by the second customer. So in a tree diagram you multiply all the probabilities along a certain path to find the probability of the corresponding combined event.

Starting at the other end, you might ask yourself, what are the chances that after two customers there would still be at least one bottle of soft drink left. Here the event of interest consists of these three branches. One way to calculate the total probability in this case would be to calculate the probabilities of II, IS and SI and then sum these. This results in a total probability of oh point seventy five.

A quicker way to make the latter calculation is by using the fact that the total of all probabilities must sum to one. So one minus the probability of two bottles soft drink being sold gives the same result.

Now we give the third customer a closer look. There's something special here. She has two options in 3 out of 4 cases but in the situation where the two soft drinks have already been sold, she has only one option left – buy ice-cream. Also in this case, the rules set out previously have to be followed: the sum of the probabilities equals 1, so the probability assigned to this single branch is 1.

Let's now return to your chances for acquiring a drink. We have to sum the probabilities of the four branches, leading to a maximum of only one soft-drink being sold. The probability for each of the branches is 0.5^3, which is 0.125. Adding them up gives 0.5.

Having illustrated the convenience of using of a tree diagram to find probabilities, it is also important to point at a few caveats. The first is that while a tree diagram makes thinking about smaller problems easy, it is not a very suitable tool to understand random phenomena with many outcomes: it simply gets too large and won't help in keeping an overview anymore. Secondly, to actually apply the tree to quantify probabilities, requires specification of probabilities at each node. Sometimes this may be easy because you can assume that options are for instance equally likely and independent from previous choices. In other cases, however, it may be difficult. For instance, what if the second customer's choice for either ice cream or soft drink would be influenced by the first customer's choice?

Let me summarize what I explained in this video.

- In a tree diagram you make the sample space and assumptions about the various events explicit, including the probabilities per event and independence among the various events in sequence.
- You can calculate probabilities of combined events with tree diagrams:
- To find the probability from a starting point along a sequence of branches to a particular outcome, all probabilities along that path should be multiplied.
- And to find the probability of an event that includes multiple outcomes, the probabilities for all the outcomes in that event should be summed.