3 PROBABILITY

3.05 Mutually exclusive, collectively exhaustive, intersection

In this video I will introduce a few important concepts relating to sets – collections if items. These are very useful to understand probability and also to derive calculation rules for probabilities. They are also a bit special because they are used in logic as well as probability calculus.

Let's get started: a sample space is the collection of all possible outcomes for a random phenomenon, for example these four possible outcomes of tossing a coin two times. And an event is a subset of the sample space, for example the cases where your first coin-toss would result in Heads.

Now there can be two or more events in a sample space that do not share any outcomes, for example the cases where your double toss would result in zero, one or two Heads.
These events are said to be disjoint. Another term for this is mutually exclusive.

A special pair of disjoint events is the combination of an event and its opposite, the case that this event is not happening. This opposite case is called the complement. An example of an event and its complement could be 'no Heads' versus 'one or more Heads'.

You can also have multiple events which together fill-up the complete sample space, these events are called collectively or jointly exhaustive. Collectively exhaustive events can but don't have to be disjoint.

The sum of the probabilities associated with disjoint events will be smaller than or equal to 1, but the sum of the probabilities associated with disjoint collectively exhaustive events will be equal to 1.

These concepts can be intuitively understood with so-called Venn-diagrams – combinations of simple geometric shapes that represent sets or parts of sets. This rectangle depicts the sample space. Inside this space there is the event A, and the complement of A is all the rest. Another event B inside this sample space and not-overlapping with A is a disjoint event.

If we apply this Venn-diagram to the double coin-toss, could you think of an arrangement of the four different outcomes in the diagram and describe the events? […]
Having a single Heads as outcome could be event A, and having two Heads could be event B. The complement of A would then contain both Tails-Tails and Heads-Heads.

This could be another example of a Venn-diagram for the double coin toss.
Try to arrange the four different outcomes in this diagram as well. […]
Here the two events A and B do in fact overlap. A could be the event of getting Heads as second result, and B could be the event of having just a single Heads. The outcome Tails-Heads falls in both events. The outcome Tails-Tails, is also part of the sample space but not part of either of the events. The overlapping part of two events is called intersection. This is the shorthand for referring to the intersection of events A and B.

Now let's find the probabilities for an intersection of two events. If two events are disjoint , things are easy: then the probability for the intersection of two events is zero , it is namely impossible for an outcome to be part of both events at the same time. If two events are not disjoint - they overlap - things are slightly more complex. Here we will assume that we are dealing with independent events - this means that the probability for one event (in the example getting Heads as second) is not influenced by another event (getting Heads only once). For independent events A and B, the probability for their intersection is the product of the separate probabilities. There are two cases in event A, so the probability that A occurs is two-fourth. And the same holds for event B. The probability of the intersection results from multiplying the two probabilities, leading to one fourth.

Let me summarize what I have explained in this video.

- events in a sample space that do not share any outcomes are called disjoint or mutually exclusive
- multiple events that together fill-up the sample space are called collectively or jointly exhaustive
- if there are just two disjoint and collectively exhaustive events in a sample space, they are each other's complement
- the sum of the probabilities associated with disjoint events will be smaller than or equal to 1
- the sum of the probabilities associated with collectively exhaustive events is 1
- the intersection of events A and B is a subset of both events, it contains outcomes that are part of A as well as B
- The probability of the intersection of independent events A and B is calculated by multiplying the probability of A with that of B; for disjoint events this probability is zero by definition.