3 PROBABILITY

3.02 Probability

Persistence pays off. Even though there are already so many proverbs and quotes on this idea, I would like to add one: persistence beats the odds! In this video I will explain why and also how it leads us describing randomness quantitatively, by using probability.

The human brain may not be particularly fit to deal with the nuances of randomness, but fortunately there is a fundamental mechanism at work which helps to simplify our lives enormously: randomness changes from something which is variable and unpredictable when considering a few cases , to something that is very constant and predictable when considering many.
There is even a mathematical proof for this: the 'law of large numbers'. It relies on independence, which means that the outcome of a random phenomenon should not be influenced by previous outcomes.

Let me give an example to illustrate how the law of large numbers looks like in real life. You are on a beach and have decided to search for shells. You quickly find out that there are only four types of shells at the beach, distributed randomly and in equal quantities.
Now, if you were to count the fraction of Q-shells, it could be done by selecting for example 20 shells at random and counting the number of Q-shells in that sample. So that's what you do and here is the result. As you see only two out of twenty are Q-shells. The relative frequency is one tenth. This is nowhere near the fraction of one-fourth that you would expect based on your reasoning ... but then you also know that irregularity with small samples is the very nature of randomness. So you would just stay calm, not jump to conclusions, and carry on to select more shells and see how the running record of the proportion (the so-called cumulative proportion) evolves to one fourth. Each time you select a new shell at random is considered as a small chance-experiment with as possible outcomes shell Q, R, S or T. In probability jargon the outcome, the type of shell you just picked up, is called an event, and the act of selecting a shell at random called an independent trial. The whole undertaking of doing many trials is called an experiment.
Because you are dividing the counts per shell type by the total number of shells when you calculate relative frequencies, two properties always hold: each probability will be larger or equal to zero and smaller or equal to one. and the sum of the probabilities for all the possible outcomes of the random phenomenon will be one.

But now back to reality: life isn't a beach, and certainly not one with randomly distributed shells.
In every-day life purely independent trials are not very common. There is usually some interdependence between random events. Nonetheless, probabilities can often be quantified quite well by using this simplifying assumption. On top of that you would need to take a sufficient number of samples to have the law of large numbers do its work and ensure that the probability you estimate will be close to it's real value.

Let me summarize what I hope you understood from this video:

- Probability is a way to quantify randomness. It can be expressed by means of relative frequencies.
- From this definition it follows that probabilities are always larger or equal to zero and smaller or equal to one; and also that the sum of the probabilities for all possible events equals one.
- A formal definition makes use of the concepts of 'experiment', 'event' and 'independent trial': the probability of a certain event is calculated by the relative frequency of that event occurring in an experiment. The experiment consists of independent trials which are

repeated many times. For example throwing one with a die, is an event; and each throw would be an independent trial.

- A good way to translate the law of large numbers to a practical guideline for estimating probabilities is: stay calm and carry on doing trials until a cumulative probability is not changing much anymore.