3 PROBABILITY

3.07 Union

In this video I will introduce the concept of a union of sets and its relation to probability. While unification can in real life lead to an entity that gets new properties and is more than the sum of its parts, in set and probability theory that kind of magic doesn't exist. A union is simply the sum of the parts, where special care is taken that things are not counted double.

I will start with an example. You pick up three shells at random at a beach and there are only two types of shell around, Q and R. The two shell-types are equally abundant and innumerable. The sample space in this case comprises eight outcomes. And a tree diagram for the entire experiment is shown here. The probability to get any combination of three shells is $1/8^{th}$.

Let's consider the events of picking up one R-shell in total, event A, versus picking up two R-shells in total, event B. And let's assume that you would be interested in the combined event: event A occurs or event B occurs, or both A and B occur at the same time. The name for combining events in this way is a union. This is the shorthand for writing the union of events A and B. To calculate the probability of a union of events A and B, you have to sum the probability of both events, and then subtract the probability of the intersection of A and B. The reason for subtracting the intersection in this calculation is because it would be counted double otherwise. After all, the intersection of A and B is part of A as well as B, so by summing the probabilities for A and B this joint probability is effectively counted twice.

So let's calculate the probability of the union of events A and B. The probability of picking up one R-shell, event A, is $3/8^{th}$; and the probability of picking up two R-shells is the same. The sum of these is $6/8^{th}$, which is three quarters. As it turns out, events A and B do not share any outcomes – they are disjoint. So the probability of their intersection is zero. And therefore the probability of the union of A and B is just the sum of their separate probabilities, three quarters.

Now consider two different events. C: the first shell you pick-up would be an R-shell; and D: the last shell to pick-up is an R-shell. Clearly these two events are not disjoint, because they overlap.
The intersection of events C and D comprises the outcomes where the first shell you pick-up is an R-shell and also the third shell you pick-up is an R-shell, there are two of these outcomes.
In this case the probability of the union of C and D is found by adding the probability of C to that of D and subtracting the intersection of C and D. The result of that turns out to be three quarters.

At this point I have a question for you: could you tell the probability of the union of events A, B, C and D? […]

If you apply the equation for union mechanically, it gets a little bit tedious as it requires a number of additions and subtractions. But since there are not that many elementary events in the total sample space, there is a simpler approach. You can list the eight elementary events, and just tick them off one by one if they occur in any of the four combined events. In the end, only the outcome where you pick up no R shells appears to be absent from any of the four combined events, while the other seven outcomes are part of at least one combined event. So, the probability of the union of these four events is $7/8^{th}$.

I hope you have learned the following from this video.

- The union of several events is an event that contains all the outcomes from the original events without duplication
- The probability of the union of several events is the sum of the probabilities of the separate events minus the probability of the intersection among the events. For two events the equation is shown here.
- If events are disjoint, the probability of intersection is zero and the equation for union simplifies to only adding up the separate probabilities.