3 PROBABILITY

3.03 Sample space, event, the probability of an event and tree diagrams

Beaches can be eventful places. Especially when the weather is good, there are some people around and there are things to do and see. This type of beach is the backdrop for this video, where I am going to explain some basic concepts that help to find probabilities, and also a visual aid to do so: a tree diagram.

It has been a warm afternoon and you could use a refreshment. Luckily, on your beach there is a stand which sells these. It is almost sold out, with only one type of ice-cream and two bottles of soft drink left for sale. A bit unfortunate you also find three persons lined-up in front of you. But on the positive side the stand-owner has limited sales to one item per customer. Since you are desperately longing for a cold drink, you start to wonder: what are my chances. You have no idea about choices that the other customers will make, so these are random events for you. The first customer chooses either a soft drink (S) or an ice-cream (I), and after this choice, the second has the same options. Next it's the third customer's turn. If the previous customers have both bought soft-drinks, she has only one option left. In the other situations she might still choose between ice-cream and soft drink. And finally it would be your turn. Through this tree diagram you have ordered all the possible outcomes of this random trial, there appear to be seven: all possible combinations of soft-drinks and ice-creams in a group of 3, except for three soft-drinks.

The name of this list with all possible outcomes of a random phenomenon is the sample space. And if a random trial has discrete outcomes, as in this case, a convenient way to picture the sample space is by listing all the possibilities via a tree diagram as we have done here.

In a tree diagram there are intermediate outcomes, at each split, as well as outcomes at the final branch. An outcome or a combination of outcomes is called an event. So an event is a subset of the sample space. In this particular case you are of course interested in the events: unlucky, no soft-drink left for you versus lucky: you're able to buy a soft drink.

Any random event has a probability associated with it, also these two events that are in fact a combination of three smaller events. A way to quantify probabilities for these events is through an experiment. This could consist of observing the frequency of ice-cream or soft drink sales over a sufficiently long period. But there's no time for that, while the first customer is making up his mind, you 'd just like to have your answer now. So you resort to the alternative strategy: to specify plausible assumptions about outcomes in the sample space. You are going to assume that both outcomes for each elementary event are equally likely, and assign a probability of oh point five for choosing either ice-cream or soft drink by each customer.

In any case, when assigning these probabilities per event, you can rely on the general probability rules: probability lays between zero and one and the total probability of all possible outcomes, for example at all the options at each node in a tree diagram, equals one.

This will quickly give the answer you are looking for. The answer may help you to decide whether you'd be optimistic and cheerfully wait for your turn, or start considering other ways to buy a drink. However, what's important to remember: even after the events have unfolded themselves. You will never know whether your assessment of the probability was correct. You would have collected information over three trials from the customers in front of you – not many trials and therefore not leading to a very accurate probability estimate.

Let me summarize what I explained in this video.

- A sample space is the collection of all possible outcomes for a random phenomenon, and an event is a subset of the sample space - it corresponds to a particular outcome of a random variable or a group of possible outcomes.
- Each event has a probability and to find such probabilities you can use a tree diagram.
- In a tree diagram you make the sample space and assumptions about the various events explicit.
- To quantify the probabilities for each event in a tree diagram, you can conduct experiments. In the absence of these you can sometimes specify plausible assumptions about outcomes in the sample space and estimate the probabilities based on reasoning.
- In any case, the general probability rules also apply to tree diagrams: the probability of any event lays between zero and one and the total probability of all possible outcomes at a node in a tree diagram equals one.