Qrlew: SQL Query Rewriting for Differential Privacy

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Abstract

In this paper we introduce *Qrlew*, an *open source* library that can parse SQL queries into *Relations* — an intermediate representation — that keeps track of rich data-types, value ranges and row ownership, so that they can easily be rewritten into *differentially-private* equivalent and turned back into SQL queries for execution in a standard data-store with virtually no technical integration.

With *Qrlew*, a *data practitionner* can express his data queries in standard SQL; the *data owner* can run the rewritten query without any technical integration and with strong privacy garantees on the output; and the query rewriting can be operated by a privacy-expert that has to be trusted by the owner, but may belong to a separate organizations.

1 Introduction

In recent years, the importance of safeguarding privacy when dealing with personal data has continuously increased. Traditional anonymization techniques have proven vulnerable to re-identification, as demonstrated by numerous works (Archie et al. 2018; Dwork et al. 2017; Narayanan and Shmatikov 2008; Sweeney, Abu, and Winn 2013). The total cost of data breaches has also significantly increased (IBM 2023) and governments have introduced stricter data protection laws. Yet, the collection, sharing, and utilization of data hold the potential to generate significant value across various industries, including healthcare, finance, transportation, and energy distribution.

To realize these benefits while managing privacy risks, researchers have turned to *differential privacy (DP)* (Wood et al. 2018; Dwork, Roth et al. 2014), which has become the *gold standard* for privacy protection since its introduction by Dwork et al. in 2006 (Dwork et al. 2006) due to its provable and automatic privacy guarantees.

Despite the availability of open-source tools, DP adoption remained limited. One of the reasons for this lack of adoption is the relative complexity of the existing tools considered the utility of the results. *Qrlew* (Grislain et al. 2023) has been designed to solve these problems by providing the following features:

***Qrlew* provides automatic output privacy guarantees** With *Qrlew* a *data owner* can let an analyst (*data practitionner*) with no expertise in privacy protection run arbitrary SQL queries with strong privacy garantees on the output.

***Qrlew* leverages existing infrastructures** *Qrlew* rewrites a SQL query into a *differentially private* SQL query that can be run on any data-store with a SQL interface: from lightweight DB to big-data stores. This removes the need for a custom execution engine and enables *differentially private analytics with virtually no technical integration*.

***Qrlew* leverages synthetic data** Synthetic data are an increasingly popular way of *privatizing* a dataset. Using jointly: *differentially private* mechanisms and *differentially private* synthetic data can be a simple, yet powerful, way of managing a privacy budget and reaching better utility-privacy tradeoffs.

2 Assumptions and Design Goals

In this work, we assume the *central model of differential privacy* (Near 2020), where a trusted central organization: hospital, insurance company, utility provider, called the *data owner*, collects and stores personal data in a secure database and whishes to let untrusted *data practitionners* run SQL queries on its data.

At a high level we pursued the following requirements:

- Ease of use for the *data practitionners*. The *data practitionners* are assumed to be a data experts but no privacy experts. They should be able to express their queries in a standard way. We chose SQL as the query language as it is very commonly used for analytics tasks.
- Ease of integration for the *data owner*. As SQL is a common language to express data analysis tasks, many data-stores support it from small embedded databases to big data stores.
- Simplicity for the *data owner* to setup privacy protection. Differential privacy is about capping the sensitivity of a result to the addition or removal of an individual that we call *protected entity*. *Qrlew* assumes that the *data owner* can tell if a table is public and, if it is not, that it can assign exactly one *protected entity* to each row of data. In the case there are multiple related tables, *Qrlew* enables to define easily the *protected entities* for each tables transitively.

- Simple integration with other privacy enhancing technologies such as *synthetic data*. To avoid repeated privacy losses or give result when a DP rewriting is not easily available (e.g. when the query is: `SELECT * FROM table`) *Qrlew* can use *synthetic data* to blend in the computation.

These requirements dictated the overall *query rewriting* architecture and many features, the most important of which, are detailed bellow.

3 Overall Architecture of *Qrlew*

The *Qrlew* library, solves the problem of running a SQL query with DP guarantees in four steps. First the SQL query submitted by the *data practitionner* is parsed and converted into a *Relation*, this *Relation* is an intermediate representation that is designed to ease the tracking of data types ranges or possible values, to ease the tracking of the *protected entity* and to ease the rewriting into a DP *Relation*. Once the relation is rewriten into a DP one, it can be rendered as an SQL query string and submitted to the data store of the *data owner*. The output can then safely be shared with the *data practitionner*. This process is illustrated in Figure 2.

3.1 Qrlew Intermediate Representation

As the SQL language is very rich and complex, simply parsing a query into an abstract syntaxic tree does not produce a convenient representation for our needs. Therefore, it is converted into a simpler normalized representation with properties well aligned with the requirements of Differential Privacy: the `Relation`. A `Relation` is a collection of rows abiding a given *schema*. It may be:

A `Table` This is simply the collection of the rows of an SQL table.

A `Map` It takes an input `Relation`, filters the rows and transform them one by one. The filtering condition and row transforms are expressed by expressions similar to those of SQL. It acts as a `SELECT exprs FROM input WHERE expr LIMIT value` and therefore preserve the *protected entity* ownership structure.

A `Reduce` A `Reduce` takes an input `Relation` and aggregates some columns, possibly by group. It acts as a `SELECT aggregates FROM input GROUP BY expr`. This is where the rewriting into DP will happen as described in section **??**.

A `Join` This `Relation` combines two inputs `Relations` as a `SELECT * FROM left JOIN right ON expr` would do it. The provacy properties are more complex to propagate in this case.

It may also be a static list of values or a set operation between two `Relations`, but those are less important for our uses.

3.2 Range Propagation

Most DP mechanisms aggregating numbers require the knowledge of some sort of bounds on the values (see (Dwork, Roth et al. 2014)). Even if some bounds are known for some `Relations` like source `Tables`, it is not trivial to propagate these bounds through the steps of the computation.

To help with range propagation, *Qrlew* introduces two simple concepts.

- The concept of k-*Interval*, which are finite union of at most k closed intervals. A k-*Interval* can be noted:

$$I = \bigcup_{i=1}^{j \leq k} [a_i, b_i]$$

  Note that the union of k-*Interval*s may not be a k-*Interval* as it may be the union of more than k intervals. Unions of many intervals can be simplified into their convex enveloppe interval, which are often sufficient bounds approximations for our use case:

$$J = \bigcup_{i=1}^{j > k} [a_i, b_i] \subseteq \left[\min_i a_i, \max_i b_i\right]$$

- The concept of *piecewise-coordinatewise-monotonic-functions* or *piecewise-monotonic-functions*, which are functions $f : \mathbb{R}^n \to \mathbb{R}$ whose domain can be partitioned in product of intervals on which they are *coordinatewise-monotonic*. The image of a product of k-*Interval*s by a *piecewise-monotonic-function* can be easily computed as a k^n-*Interval*. If:

$$I = I_1 \times I_2 \times \ldots \times I_n = \bigcup_{\substack{1 \leq i_1 \leq k \\ \ldots \\ 1 \leq i_n \leq k}} [a_{i_1}, b_{i_1}] \times \ldots \times [a_{i_n}, b_{i_n}]$$

  then one can show that:

$$f(I) = \bigcup_{\substack{1 \leq i_1 \leq k \\ \ldots \\ 1 \leq i_n \leq k}} \text{Conv}\left(f\left(\{a_{i_1}, b_{i_1}\} \times \ldots \times \{a_{i_n}, b_{i_n}\}\right)\right)$$

  where $\text{Conv}\left(f\left(\{a_{i_1}, b_{i_1}\} \times \ldots \times \{a_{i_n}, b_{i_n}\}\right)\right)$ is efficiently computed in one pass thanks to the coordinatewise monotony of f.

The notion of k-*Interval* is convenient for tracking value bounds as it can express natural patterns in SQL such as:

- `WHERE x>0 AND x<=1`, which translates into the implied $x \in [0, 1]$;
- `WHERE x IN (1,2,3)`, which is also easily expressed as a *k-Interval*: $x \in [1, 1] \cup [2, 2] \cup [3, 3]$

The idea of *piecewise-monotonic-function* is also very useful as in SQL many standard arithmetic operators (+, −, ⋆, /, <, >, =, !=, ...) and functions (`EXP`, `LOG`, `ABS`, `SIN`, `COS`, `LEAST`, `GREATEST`, ...) are trivially *piecewise-monotonic-function* (in one, two or many variables).

Most of the range propagation in *Qrlew* is based on these concepts. It enables a rather simple and efficient range propagation mechanism, leading to better utility / privacy tradeoffs.

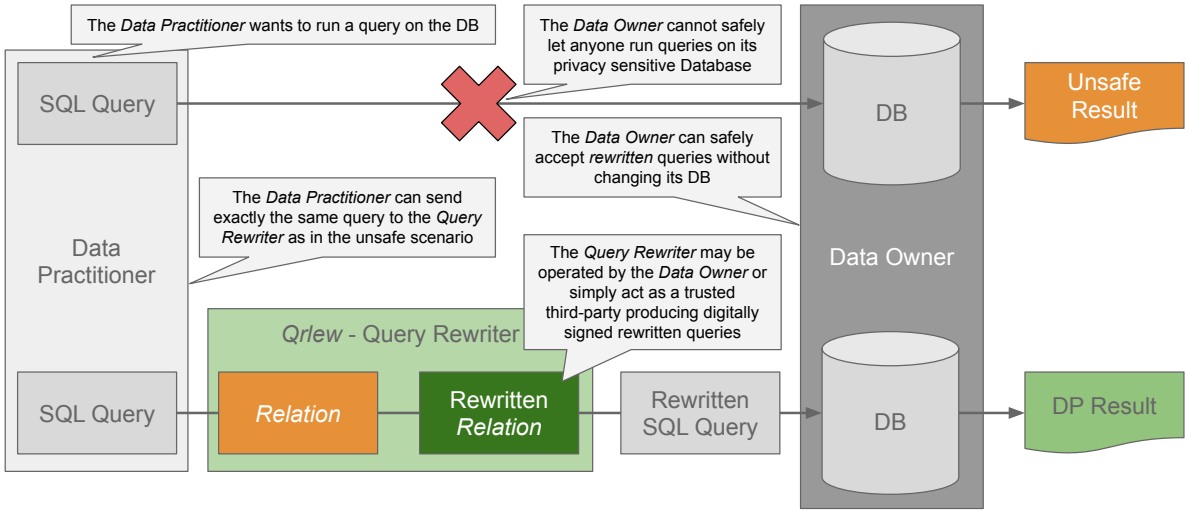

Figure 1: The rewritting process occurs in three stages: The *data practitionner*'s query is parsed into a *Relation*, which is rewritten into a DP equivalent and finally executed by the the *data owner* which returns the privacy-safe result.

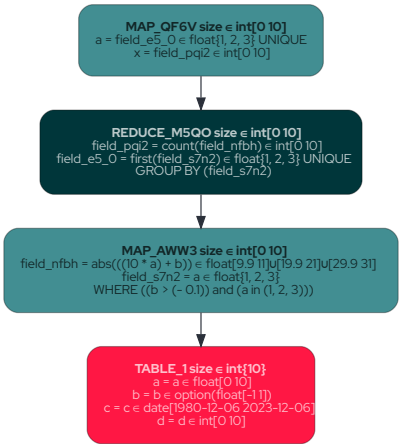

Figure 2: `Relation` (`Map`) associeted to the query: `SELECT a, count(abs(10*a+b)) AS x FROM table_1 WHERE b>-0.1 AND a IN (1,2,3) GROUP BY a`. The arrows point to the inputs of each `Relation`. Note the propagation of the data type ranges.

3.3 Protected Entity Definition

3.4 Query Rewriting

Rewriting in *Qrlew*, as detailed in Section **??**, refers to the process of altering parts of a *Relation* by substituting them with different components to alter the privacy properties of the result. This substitution aims to achieve specific objectives, such as ensuring privacy through the incorporation of differential privacy mechanisms. To facilitate this work, we decompose the SQL query in the form of a computation graph where each node (a `Relation`) is the result of transformations representing part of the SQL queries.

The main goals of the differential privacy rewriting are to modify SQL queries to ensure compliance with differential privacy frameworks, protecting sensitive data, and to guarantee that these modifications are consistent and deterministic, adhering to established privacy standards.

The challenge lies in accurately identifying sections for modification across a complex array of potential transformations, vigilantly tracking the integrity of data rows tied to protected entities during extensive aggregations, and adeptly applying the correct rewriting rules tailored to the relation dynamics of the query, such as Joins, Maps, or Reduces, while considering the original configuration of the sensitive data.

Rewriting is a recursive, two-step process: for any given relation, we begin by rewriting the parents to alter their privacy properties, and then we modify the transforms from which the relation is derived, ensuring the desired privacy property based on the privacy properties of the parents. The recursion's base step involves deciding which tables to utilize—either the private tables or the synthetic ones.

To make the rewriting process more intelligible, we separate global privacy properties accounting for the computation graph and local rewriting rules concerning only the relation and the associated transforms to understand how the rewriting process could unfold recursively.

Global Privacy Properties Let's sum up the global privacy properties:

- **Protected Entity (PE)**: The definition of Differential Privacy (DP) is based on a notion of distance between datasets (see OpenDP work). Literature often formalizes datasets as multisets and uses the size of their symmetric difference as the distance between them (or equivalently, the ℓ_1 norm of the difference of their histograms). In our clients' datasets, an individual's data is often described by several rows of data. To adapt the distance to this requirement, Sarus labels each data row with an identifier:

the Protected Entity ID (PEID) and defines the distance between datasets as the size of the symmetric difference of the set of PEIDs from each dataset.

- **Protected Entity Preserving (PEP)**: A dataset is PEP if each data row is associated with a single PE. A dataset transformation is said to be PEP if each row labeled with a PEID results from calculations that do not depend on rows labeled with another PEID.
- **Differential Privacy (DP)**: A result will be qualified as DP if it comes from a DP mechanism applied to a PEP relation. A DP relation can be published if the associated privacy loss has been accurately accounted for.
- **Synthetic (SD)**: A relation derived only from transformation from the synthetic table.
- **Public (Pub)**: A relation derived from public tables is labeled as such and does not require DP protections.
- **Published (Pubd)**: A relation is considered 'published' if the overall mechanism from the private table to the relation complies with the differential privacy framework, ensuring that the privacy of the private data is maintained provided the dataset is not inherently public. This is achieved when all parent relations of a given relation are either published, public, synthetic themselves, or use a differential privacy mechanism.

3.5 Rewriting Rules and Their Application

Rewriting rules are the main component to understand how to rewrite the computation graph. A rewriting rule is a local property explaining how to rewrite a relation to achieve global properties, being either DP, PEP, or synthetic, depending on some list of requirements on the privacy characteristics of the parents of the relation. In essence, a rewriting rule is a recursive step in the rewriting process. It explains how we could rewrite a relation given how we can rewrite the parents. The base case in the recursion is always: a table could either be private, public, or synthetic.

Relations can appear in various forms, including Join, Map, Reduce, Relation, Set, Table, and Values. Each type has its unique set of possible rewriting rules, outlining potential transformations and the associated global properties. An exhaustive list of the rewriting rules for each type of relation is provided in the annex.

Key Rewriting Rules Let's detail two crucial rewriting rules:

Relation with PEP $\rightarrow$ PEP and PEP $\rightarrow$ DP Rewriting Rules The first rule, PEP $\rightarrow$ PEP, concerns transferring the protected entity preserving property from one transformation stage to the next without mixing protected entities. The second rule, PEP $\rightarrow$ DP, means that if the parent of the relation can be rewritten as PEP, then we can rewrite the relation to be DP by applying a DP mechanism to publish the result. Both types of rewriting rules necessitate having parents that are capable of preserving protected entities. Some transformations, such as REDUCE, can aggregate data from multiple rows and thus can mix protected entities in the process. Such relations could be rewritten mostly only into published or synthetic, but it is possible that some REDUCE operations only aggregate rows related to the same protected entity and therefore could be rewritten into a PEP Relation.

Feasible Rewriting Rules A rewriting rule of a relation is said to be feasible if there exists a rewriting of the relation satisfying the rule. This is a global property of the relation, meaning that it depends on all the parents of the relation and not only on the transforms of the relation. Thus, the rewriting rule PEP $\rightarrow$ DP could be feasible for a relation if there exists a rewriting where the parent of the relation is in practice PEP. Then, we can change the transform and apply a DP mechanism to publish the relation. All rewriting rules about synthetic data are feasible because it is always possible in Sarus to build a synthetic variant of the relation using the synthetic table. The main goal of the DP rewriting is then to list for the relation that we want to obtain all the feasible rewriting rules and to select one where the outcome is DP. If not, it would mean that there does not exist a way to obtain this value using DP mechanisms, and we would need to use synthetic data.

Complexity of the Rewriting In the computation graph, while each node's multiple rewriting rules might suggest a combinatorial explosion in the number of possible paths, this is mitigated in practice. The pruning of infeasible rules, dictated by the requirement for most relations to have a PEP input for a DP or PEP outcome, significantly reduces the complexity. Hence, despite the theoretical breadth of possibilities, the actual number of feasible paths remains manageable, avoiding substantial computational problems.

3.6 Implementation of the Rewriting Algorithm

The goal is to have a deterministic algorithm that rewrites SQL queries into a privacy-preserving form. During the *Qrlew* rewriting process, the algorithm systematically goes through the computation graph, applying the following steps:

1. Transform the SQL query into a computation graph composed of relations.
2. Tagging rewriting rules: Initially tag all relations with their rewriting rules depending on the type of relation and the parameters of the relation from the list in the annex (see Figure 3 for an illustration of Rule Setting).
3. Filtering Out Infeasible Rules: Identify and remove recursively any rewriting rules that are not feasible, ensuring that only viable transformations are considered (refer to Figure 4 for a depiction of Rule Elimination).
4. Applying Rewriting Rules: Starting from the last final node, for each node with multiple feasible rewriting rules, we select the best one based on a scoring system. The simple score system encodes the preferences among different privacy properties: we prefer to have a DP result than a synthetic one, that is why we give a better score for a feasible rule resulting in a DP result than a synthetic one (Figures 5 and 6 illustrate examples of SD and DP Allocation, respectively).
5. Building the Final Query: Once all relations have been transformed, convert the computation graph back into an

SQL query that adheres to differential privacy standards. The budget is split among the different DP mechanisms involved in the computation graph.

4 Privacy algorithms

At this stage, we examine a `Relation` that takes one or two protected `Relation` inputs, where the entity to be protected is specified. To secure the outcome of the SQL query, it is imperative to protect not only the results obtained from aggregation functions but also the grouping keys. We will briefly describe these two steps in the following section.

Protecting aggregation results The protection of aggregation functions is carried out in three sequential steps. Given that all currently supported aggregations (`COUNT`, `SUM`, `AVG`, `VARIANCE` `STDDEV`) can be expressed as compositions of sums, our focus will be on the `SUM` aggregation. Let's consider the scenario where we aim to compute the sum of a column.

1. **Limit the contribution per user within groups**: We represent the contribution of each user by a vector whose each component is the sum of the contributions within one group then we calculate its ℓ_2 norm. Subsequently, we constrain the contributions of a specific user by scaling $\mathbf{x}$ with a factor chosen to maintain the original observations if the ℓ_2 norm of the user's observations does not exceed the clipping factor c. Conversely, if the ℓ_2 norm surpasses c, the ℓ_2 norm of the rescaled observations is restricted to c. More technical details can be found in the appendix 6.3.
2. **Add random noise**: The sum of the original data is substituted with the sum of the scaled data with the addition of Gaussian noise. The level of noise is parameterized by the clipping and privacy parameters.
3. **Restrict differentially private aggregation**: The final operation involves confining the differentially private aggregation within the bounds automatically computed for the `SUM`($\mathbf{x}$).

Protecting grouping keys As explained by Wilson et al., making grouping keys public could result in privacy breaches. The treatment of grouping keys varies depending on whether the analyst knows them.

- **Public Grouping Keys.**We categorize grouping keys as public when the analyst specifies them in the `WHERE` clause. In this scenario, all user-specified keys must be disclosed, even if they do not exist in the database. If a key is absent in the database, differentially private results will include these missing keys, and the corresponding aggregations will be random noise.
- **Revealing Private Keys through τ-Thresholding** As introduced by Wilson et al., tau-thresholding involves releasing keys whose differentially private noise surpasses a threshold determined by privacy parameters.

When the SQL query involves both public and private grouping keys, we perform cross joins on the grouping keys obtained through the two algorithms.

5 Comparison to other systems

The Google differential-privacy extension for ZetaSQL enables differentially private analysis through SQL tools. Users can incorporate privacy guarantees by including a privacy clause in their queries, which replaces aggregations with their differentially private equivalent, limits the number of grouping keys per user, and applies tau-thresholding. While this framework does not support complex queries with joins or inner subqueries, it offers a straightforward integration with SQL databases.

Smartnoise SQL is a Python framework, built on the top of Open-DP, for doing SQL analysis with differential privacy guarantees. The framework is easy of use for non-experts, but it does not support all SQL grammar, and complex queries with joins or subqueries are not supported. Additionally, the execution of SQL analysis requires a Python post-processing step, potentially leading to increased execution times.

OpenDP is a Rust library with Python bidings designed for building SQL-like transformations with differential privacy guarantees. Each part of the code is well documented with mathematical proofs but its utilization can be challenging for non experts.

Similarly, Tumult Analytics does not directly handle SQL queries; instead, it utilizes transformations that closely resemble SQL syntax. While offering support for budgeting various privacy tasks, it is important to note that the analysis must be executed by the data owner.

Chorus is the closest framework to *Qrlew*, and in the work by Johnson et al., there is a strong emphasis on conducting all analyses directly within the database. However, it's worth noting that the code for Chorus is no longer being actively maintained.

6 Known limitations

Qrlew relies on the random number generator of the SQL engine used. It is usually not a cryptographic noise.

Qrlew uses the floating-point numbers of the host SQL engine, therefore our system is liable to the vulnerabilities described in Casacuberta et al..

Useful links

6.1 PPAI

Last year papers: https://aaai-ppai23.github.io/#sp2 This year program: https://ppai-workshop.github.io/

6.2 Comparable open-source projects

- Paszke et al. 2017 - Automatic differentiation in PyTorch https://openreview.net/pdf?id=BJJsrmfCZ
- Frostig et al. 2018 - Compiling machine learning programs via high-level tracing https://mlsys.org/Conferences/2019/doc/2018/146.pdf

6.3 Comparable DP SQL papers

- Lessons Learned: Surveying the Practicality of Differential Privacy in the Industry (Garrido et al. 2022)

- Tumult Analytics: a robust, easy-to-use, scalable, and expressive framework for differential privacy (Berghel et al. 2022)
- Differentially Private SQL with Bounded User Contribution (Wilson et al. 2019)
- CHORUS: a Programming Framework for Building Scalable Differential Privacy Mechanisms (Johnson et al. 2020)
- Towards Practical Differential Privacy for SQL Queries (Johnson, Near, and Song 2018)

Thank you for reading these instructions carefully. We look forward to receiving your electronic files!

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Appendix

Appendix: Limit the contribution per user within the aggregations

The observations can be represented as a matrix whose first dimension is the user and second dimension is the group:

$$\begin{array}{c} \overbrace{\phantom{x_{11}}}^{\text{Group 1}}\ \overbrace{\phantom{x_{12}}}^{\text{Group 2}}\quad\quad\quad\overbrace{\phantom{x_{1n}}}^{\text{Group n}} \\ \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \begin{array}{l} \} \text{User 1} \\ \} \text{User 2} \\ \\ \} \text{User } m \end{array} \end{array}$$

The coordinate x_{ij} is the sum of all the observations of user i within the group j.

For each user i, we compute the scale factor that constrains his contribution to a maximum value of c:

$$s_i = \frac{1}{\max(1, \frac{1}{c}\ell_2(x_{i,.}))} \text{ with } \ell_2(x_{i,.}) = \sqrt{\sum_j x_{ij}^2}. \quad (1)$$

The original data is then rescaled by multiplying each observation by the corresponding user's scale factor. This process ensures that the ℓ_2 contribution of each user is restricted to c within the rescaled data.

In our algorithm, the clipping value c is given by:

$$c = \max(|\min \mathbf{x}|, |\max \mathbf{x}|), \quad (2)$$

where $\min \mathbf{x}$ and $\max \mathbf{x}$ are the automatically computed bounds of $\mathbf{x}$.

Description of the Rewriting Rules for Each Type of Relation

Map Function: Transforming a Single Relation

- Pub $\rightarrow$ Pub: A relation could stay public if it starts off public.
- Pubd $\rightarrow$ Pubd: A relation could stay published if it starts off published.
- PEP $\rightarrow$ DP (with specific budget): A relation could become differentially private, adopting a specific budget, if it starts off preserving protected entities.

- SD → SD: A relation could remain as synthetic data if it starts off as synthetic data.
- SD → Pubd (with specific synthetic data parameters): A relation could transform into published if it starts off as synthetic data, depending on specific synthetic data parameters.

Reduce Function: Transforming a Single Relation

- Pub → Pub: A relation could stay public if its parent is public.
- Pubd → Pubd: A relation could stay published if its parent is published.
- PEP → DP (with specific budget): A relation could become differentially private, adopting a specific budget, if it is preserving protected entities.
- SD → SD: A relation could remain as synthetic data if its parent is synthetic data.
- SD → Pubd (with specific synthetic data parameters): A relation could transform into published if it is synthetic data, depending on specific synthetic data parameters.

Join Function: Combining Two Relations

- Pub + Pub → Pub: Combining two public relations could result in a public relation.
- Pubd + Pubd → Pubd: Combining two published relations could result in a published relation.
- Pubd + PEP → PEP (with specific parameters): Combining a published relation and a relation that preserves protected entities could result in a relation that preserves protected entities, adopting specific parameters.
- PEP + PEP → PEP (with specific parameters): Combining two relations that preserve protected entities could result in a relation that preserves protected entities, adopting specific parameters.
- DP + PEP → PEP (with specific parameters): Combining a differentially private relation and a relation that preserves protected entities could result in a relation that preserves protected entities, adopting specific parameters.
- SD + SD → SD: Combining two synthetic data relations could result in a synthetic data relation.

Set Function: Merging Two Relations

- Pub + Pub → Pub: Combining two public relations could result in a public relation.
- Pubd + Pubd → Pubd: Combining two published relations could result in a published relation.
- PEP + PEP → PEP (with specific parameters): Combining two relations that preserve protected entities could result in a relation that preserves protected entities, adopting specific parameters.
- SD + SD → SD (with specific parameters): Combining two synthetic data relations could result in a synthetic data relation, adopting specific parameters.

Values Function: Creating a New Relation

- → SD (with specific parameters): A new relation could be created as synthetic data, depending on specific parameters.
- → Pub: A new relation could be created as public.

Visual Depiction of Algorithmic Steps

The computation graph figures illustrate the step-by-step transformation of relations within *Qrlew*. The rectangles (relations) represent tables or intermediate results, and the circles represent the rewriting rules (feasible or not) of these relations. Here is how to understand the components and read the figures:

- **Rectangles** symbolize the relations in the computation graph, which could be a base table or derived through operations such as Join, Map, Reduce, etc.
- **Circles** denote the rewriting rules that can be applied to these relations. The color and label of each circle indicate the type of rewriting rule:
- The **arrows** connecting the circles to the rectangles show which rewriting rules can be applied to which relations.
- A sequence of transformations is shown as a path through the graph, beginning with the source relations at the bottom and culminating in the final transformed query at the top.

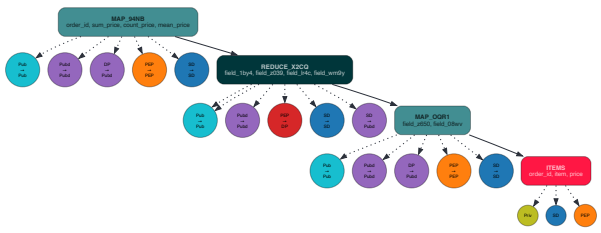

Figure 3: Rule Setting: Each node in the graph (rectangle) is assigned the applicable rules (circle) for that type of node. In blue, the *→SD rules, in orange the *→PEP type rules, in purple the *→Pubd rules, and in red the *→DP rules.

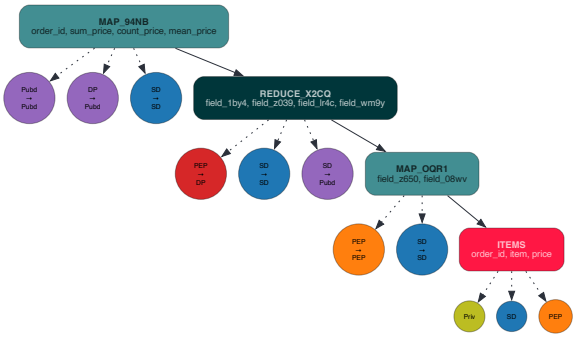

Figure 4: Rule Elimination: Only feasible rules are retained.

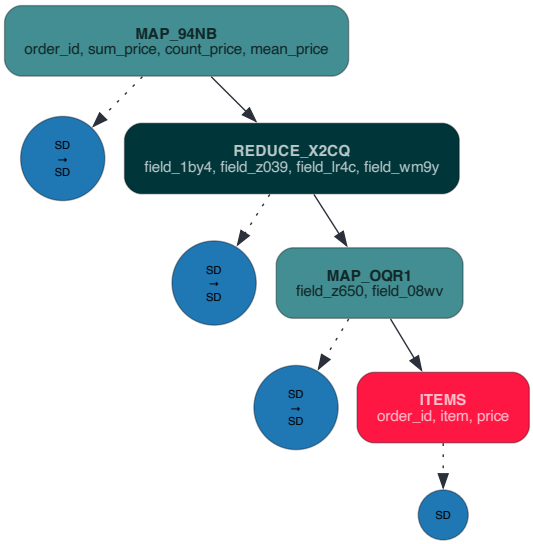

Figure 5: Example of SD Allocation: The sources of the graph are substituted with their synthetic equivalent.

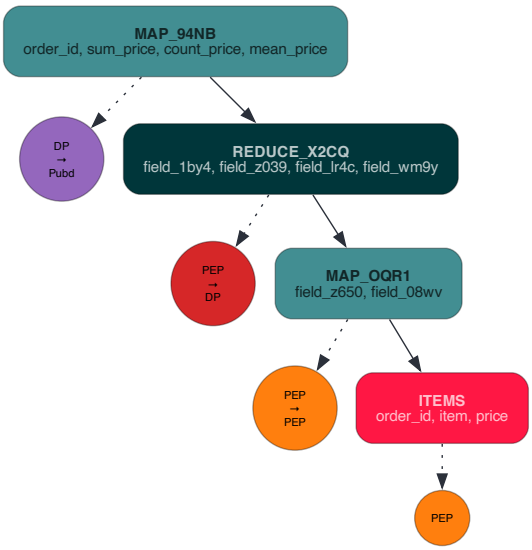

Figure 6: Example of DP Allocation: Rewrite in DP as soon as necessary (and possible).