# KTH - Royal Institute of Technology Pattern Recognition Assignment 1

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## 1 Markov Chain Random Model

HMM Model with $\lambda = q, A, B$ with

$$q = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix} A = \begin{pmatrix} 0.99 & 0.01 \\ 0.03 & 0.97 \end{pmatrix};$$

We want to prove

$$P_t = \begin{pmatrix} P(S_t = 1) \\ P(S_t = 2) \end{pmatrix} = q = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}$$

To start

$$P_2 = (q^T A)^T = (\begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix} \begin{pmatrix} 0.99 & 0.01 \\ 0.03 & 0.97 \end{pmatrix})^T = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix} = q$$

Because the transition matrix is constant over time this means that

$$P_t = \begin{pmatrix} P(S_t = 1) \\ P(S_t = 2) \end{pmatrix} = (P_{t-1}^T A)^T = q = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}$$

rand of Markov Chain class to generate 10000 State Using the following Code

```
A = [0.99, 0.01; 0.03, 0.97];
B(1) = GaussD('Mean',0,'StDev',1);
B(2) = GaussD('Mean',3,'StDev',2);
T = 10000;
mc = MarkovChain(q,A);
randMc = rand(mc,T);
tabulate(randMc(:))

Output:

  Value    Count    Percent
      1     7660     76.60%
      2     2340     23.40%
```

Which confirm with our computation

# 2 HMM Random Model

$$
\begin{aligned}
E[X_t] &= P(S=1)[X_t \mid S=1] + P(S=2)E[X_t \mid S=2] \\
&= 0.75 * 0 + 0.25 * 3 \\
&= 0.75
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
Var[X_t] &= P(S=1)Var[S=1]^2 + P(S=2)Var[S=2]^2 \\
&+ [P(S=1)\mu_{S=1}^2 + P(S=2)\mu_{S=2}^2 - (P(S=1)\mu_{S=1} + P(S=2)\mu_{S=2})^2] \\
&= 0.75 * 1^2 + 0.25 * 2^2 + [0.75 * 0^2 + 0.25 * 3^2 - (0 + 0.25 * 3)^2] \\
&= 3.4375
\end{aligned} \tag{2}
$$

Following code is used to verify the result

```
hmm = HMM(mc,B);
randhmm = rand(hmm,T);
meanhmm = mean(randhmm);
varhmm = var(randhmm);
disp(['mean:',num2str(meanhmm),'  variance:',num2str(varhmm)]);

Output:
    mean: 0.78244 variance: 3.4712
```

Which confirm with out computation

# 3 HMM Behaves

Use HMM/rand Code to generate a series of 500 contiguous Xt give us

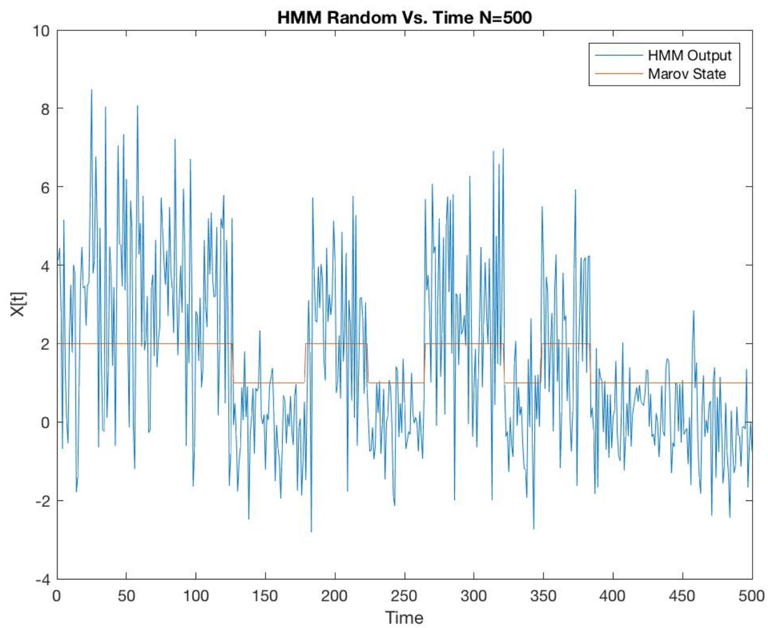


Figure 1: 500 random output from the HMM rand function. The blue line is the HMM output from the while the red line is the HMM State at that time

From the result we can easily observe that when the state is at 1, the outputs are around 3, when the state is at 2, the outputs are around 0,state 1 has bigger output variation than state 2

## HMM Behaves with $\mu 1 = \mu 2 = 0$

Use HMM/rand code again to generate a series of 500 contiguous Xt give us that

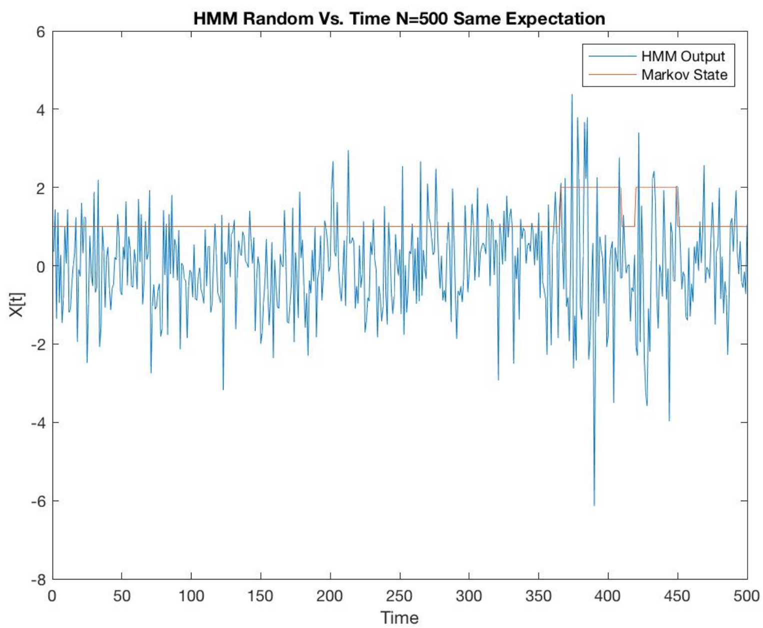


Figure 2: The caption

It's harder to distinguish between State1 and 2, However; State 2 has larger variation than in state 1. We can observe the variation between outputs to distinguish between states

# 4 Finite HMM Behave

To test HMM/rand behaves under finite duration, I used new transition

$$A = \begin{pmatrix} 0.79 & 0.01 & 0.2 \\ 0.03 & 0.77 & 0.2 \end{pmatrix}$$

with probability of exit state being equal to 0.2, I am expecting my HMM/rand function will return a average HMM length of 5

```
mc = MarkovChain([0.75; 0.25], [0.79 0.01 0.2; 0.03 0.77 0.2]);

B(1) = GaussD('Mean', 0, 'StDev', 1);
B(2) = GaussD('Mean', 3, 'StDev', 2);

h = HMM(mc, B);

N = 10000; average = 0;

for i=1:N
      [X, S] = rand(h, 500);
      average = average + length(S);
end
display(average/N);
```

give me the output average = 4.9827 which close to 5

# 5 Vector HMM Verification

In this test I used

$$B = \begin{pmatrix} N([0,10], C1) \\ N([30,55], C2) \end{pmatrix} C1 = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} C2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

the code to verify HMM/rand function is

```
q = [0.75;0.25];
A = [0.99, 0.01; 0.03, 0.97];
B(1) = GaussD('Mean',[0,10],'Covariance',[2 1;1 4]);
B(2) = GaussD('Mean',[30,55],'Covariance',[3 1;1 3]);
mc = MarkovChain(q,A);
hmm = HMM(mc,B);
N = 500;
[X,S] = rand(hmm,N);
plot(X);
hold on;
plot(S);
xlabel('Time');
ylabel('X[t]');
legend('HMM Output', 'Marov State')
title('HMM Random Vs. Time N=500');
```

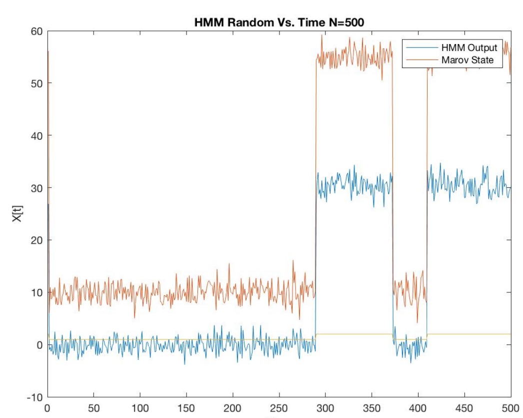


Figure 3: HMM generated with random vector

From figure 3, we can easily to distinguish between the two Gaussian distributions when there is a state transition.