Dual Polarization Radar Parameters

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1 Introduction

For a fast alternating horizontal (H) and vertical (V) transmit polarization radar, the received copolar (HH, VV) and crosspolar (VH, HV) time series (i.e., I and Q samples) are designated as:

$$HH_i, VH_i, VV_i, HV_i \tag{1}$$

Figure 1 shows the four complex time series for length N. Note that the pair HH_i and VH_i are not simultaneous samples with the pair VV_i and HV_i and are separated in time by the PRT. The powers are calculated by

$$R_{HHHH}(0) = P_H^r = \frac{1}{N}|HH_i|^2; \qquad \text{transmit H receive H copolar power} \tag{2}$$

$$R_{VHVH}(0) = P_{XH}^r = \frac{1}{N}\sum_{i=1}^{N}|VH_i|^2; \qquad \text{trc1Nansmit H receive V crosspolar power} \tag{3}$$

$$R_{VVVV}(0) = P_V^r = \frac{1}{N}\sum_{i=1}^{N}|VV_i|^2; \qquad \text{transmit V receive V copolar power} \tag{4}$$

$$R_{HVHV}(0) = P_{XV}^r = \frac{1}{N}\sum_{i=1}^{N}|HV_i|^2; \qquad \text{transmit V receive crosspolar H power} \tag{5}$$

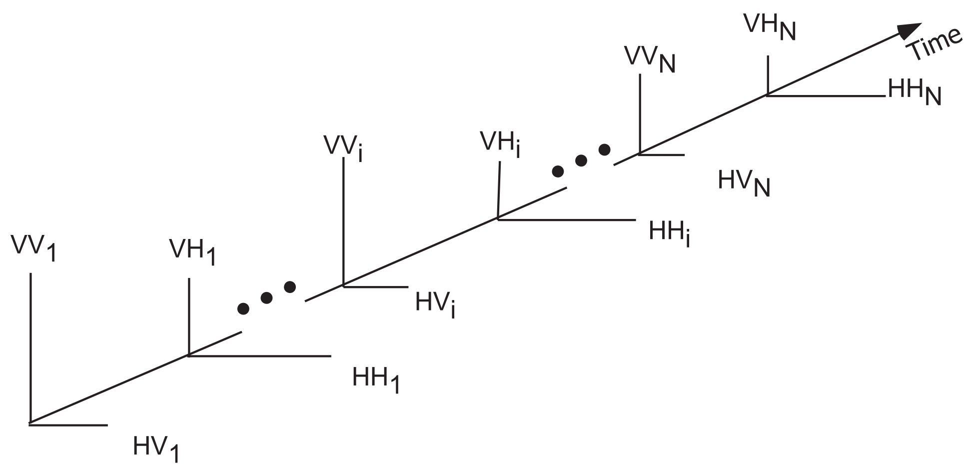

Figure 1: *Diagram of the four dual polarization time series.*

where the superscript “r” indicates raw powers. These powers are unit-less and need to be calibrated and scaled to dBZ values.

2 Complex covariance

The Doppler phase shift can be calculated is several ways. One way is to use either of the copolar time series, e.g.,

$$\hat{\phi}_{vel} = \arg\{\sum_{i=1}^{N-1} HH_{i+1}HH_i^*\} \tag{6}$$

where "*" denotes complex conjugation and "ˆ" denotes estimate. However, the resulting Nyquist velocity will be based on twice the PRT instead of the PRT (i.e., the maximum Nyquist velocity is not realized). A better estimator of the velocity phase uses both copolar time series and yields the Nyquist velocity based on the PRT; however, the differential propagation phase (and phase upon backscatter) are both present and need to be separated from the velocity phase. Thus, the two first lag estimates of cross-correlation function between HH and VV are used:

$$R_{HHVV}(-1) = R_a = \frac{1}{N}\sum_{i=1}^{N} HH_iVV_i^* \tag{7}$$

$$R_{VVHH}(1) = R_b = \frac{1}{N-1}\sum_{i=1}^{N-1} VV_{i+1}HH_i^* \tag{8}$$

The arguments of R_a and R_b are

$$\arg\{R_a\} = +\hat{\phi}_{vel} - \hat{\Psi}_{co} \tag{9}$$

$$\arg\{R_b\} = +\hat{\phi}_{vel} + \hat{\Psi}_{co} \tag{10}$$

The calculation of the differential phase, Ψ_{co}, and velocity phase, ϕ_{vel}, appears to be straight forward:

$$\hat{\phi}_{vel} = [\arg\{R_a\} + \arg\{R_b\}]/2 \tag{11}$$

$$\hat{\Psi}_{co} = [\arg\{R_b\} - \arg\{R_a\}]/2 \tag{12}$$

however, if one wishes to minimize possible phase wrapping, the phase calculations need special consideration.

From Eq. (12) it is seen that when the velocity forces either R_a or R_b across the 180° phase boundary (assuming the function used to return the phase of a complex number give values between ±180°) a phase wrap will occur in Ψ_{co}. To avoid this, Ψ_{co} can be calculated as

$$\hat{\psi}_{co} = \arg\{R_b\,R_a^*\}/2 \tag{13}$$

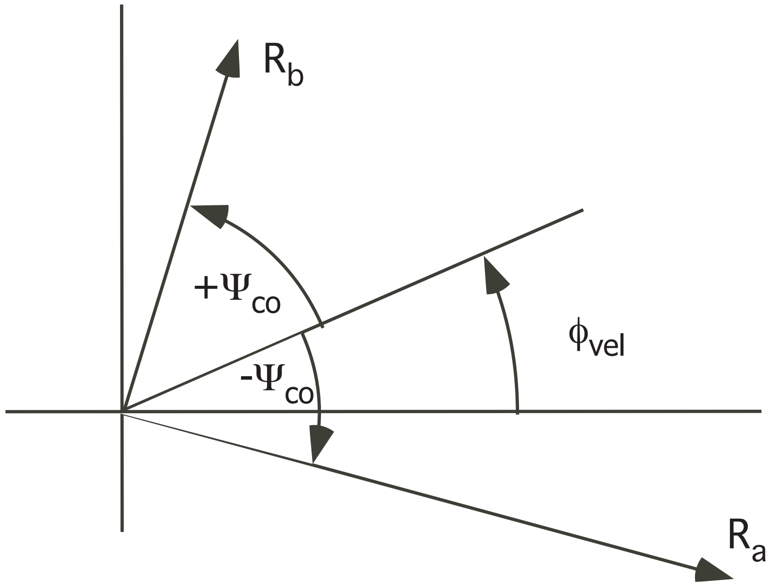

Figure 2: R_a and R_b in the complex plane.

which will yield an angle between $\pm 90°$. As shown in Fig. 2, $\hat{\Psi}_{co}$ is half the interior angle (the more acute angle) between R_a and R_b. Note that this is not the case if one would use $\hat{\Psi}_{co} = \frac{1}{2}(\arg\{\hat{R}_b\} - \arg\{\hat{R}_a\})$ which would yield phase discontinuities as either R_a or R_b passes through the 180° phase boundary in the complex plane. As Ψ_{co} increases beyond 90°, the estimate $\hat{\Psi}_{co}$ will wrap to -90° and, importantly, a phase wrap will also occur in the velocity estimate. For meteorological targets, Ψ_{co} is typically an increasing function with respect to range. For long paths of precipitation or at shorter radar wavelengths, it is possible that Ψ_{co} will increase beyond 90° and wrap back to -90°. This phase interval of 90° can, however, be nearly doubled.

All dual polarized radars have a inherent system phase offset, ϕ_{of}, between the two copolar channels. If, for example, $\phi_{of} = 50°$, $\phi_{vel} \approx 30°$ and $\psi_{co} = 0°$, the covariances R_a and R_b would be positioned approximately as shown in Fig. 2. As can be seen, if Ψ_{co} increases 40°, $\hat{\Psi}_{co}$ will wrap back to -90°. In order to have the maximum range through which Ψ_{co} can increase without encountering a wrap in $\hat{\Psi}_{co}$, one can force the ϕ_{of} to be -90° thus allowing Ψ_{co} to increase 180° before $\hat{\Psi}_{co}$ would wrap from 90° back to -90°. In practice the offset value should allow for statistical fluctuations in Ψ_{co} so that there are not unnecessary $\pm 90°$ phase wrappings due to the random phase variations. If 15° is allowed for random phase fluctuations, then the maximum range is reduced to 150° which will be adequate for many meteorological targets at S- and C-bands. If a greater range of phase is necessary then additional software can be used to extend the range. For example, one could check for three consecutive values of $\hat{\Psi}_{co}$ which exceed, say, 60°. Subsequent negative values of $\hat{\Psi}_{co}$ could then be shifted by 180° thus increasing the maximum range by about 90° to 240°.

We next address the problem of velocity estimation when using (11). First note that (11) will give $\hat{\phi}_{vel}$ a range of $\pm 180°$ where as the alternate estimator, $\hat{v} = \frac{1}{2}\arg\left\{\hat{R}_a\hat{R}_b\right\}$, only gives $\hat{\phi}_{vel}$ a range of $\pm 90°$ and thus reduces the Nyquist velocity by one half. As can be seen from Fig. 2 if $\hat{\Psi}_{co}$ is zero then the range through which ϕ_{vel} can vary is $\pm 180°$ without $\hat{\phi}_{vel}$ wrapping. But when $\hat{\Psi}_{co}$ is non-zero then this range is reduced to $\pm(180° - \hat{\Psi}_{co})$. Thus the actual velocity at which $\hat{\phi}_{vel}$ will wrap is a function of $\hat{\Psi}_{co}$ if (11) is used. This dependence can be eliminated if ϕ_{vel} is estimated with

$$\hat{\phi}_{vel} = \arg\left\{\hat{R}_a \, e^{j\hat{\Psi}_{co}}\right\} \tag{14}$$

The velocity can be identically estimated with

$$\hat{\phi}_{vel} = \arg\left\{\hat{R}_b \, e^{-j\hat{\Psi}_{co}}\right\}. \tag{15}$$

Both estimators of velocity can be averaged to reduced statistical fluctuation but since the two estimates are highly correlated, this benefit is likely minimal.

3 Estimation of Copolar Cross Correlation

The correlation of the copolar time sequences, HH_i and VV_i, *at zero lag* indicates the degree of similarity between the vertical and horizontal scattering amplitudes of the particles in the resolution volume. Since the HH and VV are not sampled simultaneously, only the first lag correlation is directly available as given by Eqs. (7) and (8). If the spectra of HH and VV are Gaussian, the first lag correlations can be normalized by an estimate of the second lag correlation of either HH of VV to estimate the desired zero lag correlation coefficient. The zero lag estimator is given by

$$\hat{\rho}_{hv}(0) = \frac{(|R_a| + |R_b|)/2}{[P_H^r \, P_V^r]^{0.5} \rho_{hv}^{0.25}(2)} \tag{16}$$

where

$$\rho_{hv}(2) = R_{HHHH}(2) = \frac{1}{N-1} \sum_{i=1}^{N-1} HH_i HH_{i+1}^* \tag{17}$$

The other covariances of interest are

$$R_{HHVH}(0) = \frac{1}{N} \sum_{i=1}^{N} HH_i VH_i^* \tag{18}$$

$$R_{VVHV}(0) = \frac{1}{N} \sum_{i=1}^{N} VV_i HV_i^* \tag{19}$$

$$R_{VHHV}(1) = \frac{1}{N} \sum_{i=1}^{N} VH_i HV_i^* \tag{20}$$

3.1 Archived covariances

The the list of covariance and information to be archived is

$R_{HHHH}(0)$ H copolar power (21)

$$
\begin{aligned}
R_{VHVH}(0) & \quad \text{transmit H crosspolar power} && (22)\\
R_{VVVV}(0) & \quad \text{V copolar power} && (23)\\
R_{HVHV}(0) & \quad \text{transmit V crosspolar power} && (24)\\
R_a & \quad \text{first lag co-to-copolar covariance} && (25)\\
R_b & \quad \text{other first lag co-to-copolar covariance} && (26)\\
R_{HHVH}(0) & \quad \text{transmit H co-to- crosspolar covariance} && (27)\\
R_{VVHV}(0) & \quad \text{transmit V co-to- crosspolar covariance} && (28)\\
R_{VHHV}(1) & \quad \text{first lag cross-to- crosspolar covariance} && (29)\\
R_{HHHH}(2) & \quad \text{``second lag'' H copolar covariance} && (30)\\
R_{VVVV}(2) & \quad \text{``second lag'' V copolar covariance} && (31)\\
R_{HH} & = \sum_{i=1}^{N} HH_i; \text{Refractivity H} && (32)\\
R_{VV} & = \sum_{i=1}^{N} VV_i; \text{Refractivity V} && (33)\\
\text{mag}\{HH\} & = \sum_{i=1}^{N} |HH_i|; \text{Clutter quality indicator} && (34)\\
noisefloorH & && (35)\\
noisefloorV & && (36)
\end{aligned}
$$

CMD (Clutter Mitigation Decision) algorithm will be applied to time series data so that a clutter filter can be applied to clutter contaminated data in real time. The above parameters Eq. (21) to (31) are calculated after the application of the clutter filter. What non clutter filtered covariances should be archived? I suggest just the four powers. The characteristics of the used clutter filter should be archived also.

4 Simultaneous H and V Transmit Mode

In simultaneous transmit mode there are no crosspolar time series available, only the two copolar time series, HH_i and VV_i, are measured *simultaneously*. Figure 3 shows the two time series for a particular range gate. The powers are calculated as given previously in Eq. (2) and (4). The velocity phase is calculated via Eq. (6). The differential phase is calculated as

$$\hat{\psi}_{co} = \arg[\sum_{i=1}^{N} VV_i HH_i^*] \tag{37}$$

and the copolar correlation coefficient is calculated as

$$\hat{\rho}_{co} = \frac{|\sum_{i=1}^{N} VV_i HH_i^*]\,|}{(P_H^r P_V^r)^{0.5}} \tag{38}$$

The radar receiver doesn't "care" what the polarization state of the transmit wave form is. Comparing Figs. 1 and 3 it is seen that the VH_i time series of the fast alternating mode becomes

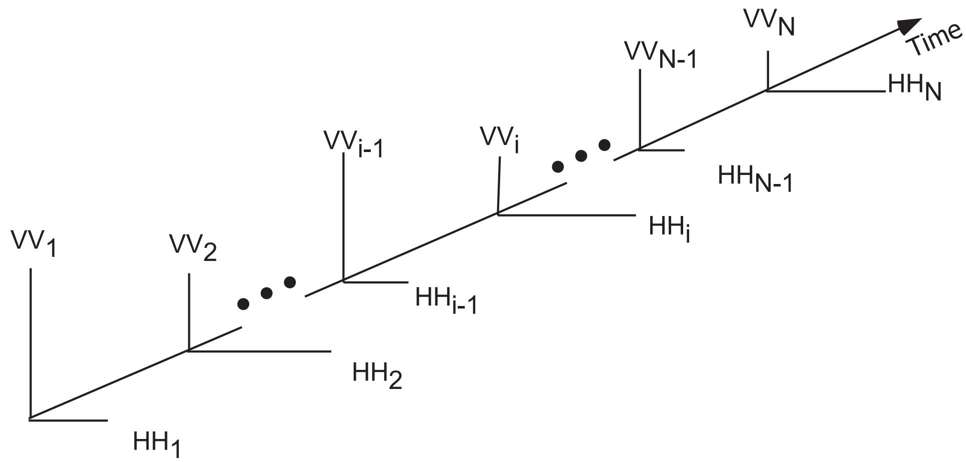

Figure 3: *Diagram of the two dual polarization time series for simultaneous H and V transmission for a particular range gate.*

part of the VV_i time series of the simultaneous H/V transmission mode and the HV_i time series of the fast alternating mode becomes part of the HH_i time series of the simultaneous H/V transmission mode. It may be easier to simply calculate the simultaneous mode radar parameters in terms of the existing fast alternating mode parameters. The following equations give the simultaneous mode radar parameters *in terms of the time series of the simultaneous transmit mode give above.* The archived parameters are

$$R_{HHHH}(0) = \frac{1}{N}\sum_{i=1}^{N}|HH_i|^2; \quad \text{Simultaneous H/V transmit receive H power} \tag{39}$$

$$R_{VVVV}(0) = \frac{1}{N}\sum_{i=1}^{N}|VV_i|^2; \quad \text{Simultaneous H/V transmit receive V power} \tag{40}$$

$$R_{VVHH}(0) = \frac{1}{N}\sum_{i=1}^{N}VV_iHH_i^*; \quad \text{complex copolar correlation} \tag{41}$$

$$R_{HHHH}(1) = \frac{1}{N-1}\sum_{i=1}^{N}HH_iHH_{i+1}^*; \quad \text{First lag H correlation} \tag{42}$$

$$R_{VVVV}(1) = \frac{1}{N-1}\sum_{i=1}^{N}VV_iVV_{i+1}^*; \quad \text{First lag V correlation} \tag{43}$$

$$R_{HH} = \sum_{i=1}^{N}HH_i; \quad \text{Refractivity H} \tag{44}$$

$$R_{VV} = \sum_{i=1}^{N}VV_i; \quad \text{Refractivity V} \tag{45}$$

$$\text{mag}\{HH\} = \sum_{i=1}^{N}|HH_i|; \quad \text{Clutter quality indicator} \tag{46}$$

$$Hnoisefloor \tag{47}$$

$$Vnoisefloor \tag{48}$$