NOTES AND CORRESPONDENCE

Simulation of Weatherlike Doppler Spectra and Signals

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30 April 1974 and 3 January 1975

ABSTRACT

A versatile algorithm to generate weatherlike spectra of any desired shape is described, and applications are briefly discussed.

Simulation techniques are used in engineering to determine systems' behavior under adverse conditions. Using fast digital computers, the power of these techniques has been increased. This note describes a specific simulation technique generating weatherlike Doppler spectra and signals that may be useful to radar meteorologists.

It is of interest to compare, under controlled conditions, the performance of various Doppler radar signal processing techniques (Sirmans and Doviak, 1973). Previous methods used an in-phase I and quadrature Q component generated from a Gaussian random number generator, and then filtered it to achieve a desired spectral shape (Benham *et al.*, 1972; Denenberg *et al.*, 1972).

In the filtering approach, a drawback is the difficulty of generating desired filter characteristics. Even for a simple Gaussian spectrum, only an approximation to the spectrum can be obtained through using a cascade of two recursive first-order filters. We present a method here that generates directly an arbitrarily shaped power spectrum with all the essential weatherlike signal properties.

The weather echo and receiver noise have very similar statistical properties, one difference between them being that the signal power is usually larger and its spectrum is not broadband. Hence, the ith sample in-phase, I, and quadrature phase, Q, components at one range gate can be written as

$$\left.\begin{aligned} I(i)&=s(i)\cos\phi(i)+n(i)\cos\psi(i)\\ Q(i)&=s(i)\sin\phi(i)+n(i)\sin\psi(i)\end{aligned}\right\}. \tag{1}$$

In the equations, $s(i)$ is the Rayleigh distributed signal envelope and $\phi(i)$ its uniformly distributed phase. Similarly, $n(i)$ and $\psi(i)$ are analogous quantities of radar noise.

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The equations can be expressed in terms of a discrete Fourier series summed over the spectral record of length T_{prf}^{-1}:

$$I(i)+jQ(i)=\frac{1}{n}\sum_{k=1}^{n}P_k^{\frac{1}{2}}\exp(j\theta_k)\exp\left(-j\frac{2\pi}{n}ki\right). \tag{2}$$

Here P_k is the exponentially distributed instantaneous power of the signal plus noise, in which the signal part is frequency dependent and the noise part is white; θ_k is a uniformly distributed phase; and P_k and θ_k are statistically independent. With N being the receiver's white noise power per discrete frequency (nN = total receiver noise) and S_k the frequency-dependent signal power density, the probability density of P_k can be written as

$$P(P_k)=\frac{1}{S_k+N}\exp[-P_k/(S_k+N)]. \tag{3}$$

This last is the basic equation for periodogram simulations. It suffices to generate a random variable P_k with the desired distribution $P(P_k)$, and an independent θ_k with uniform distribution. The average signal-to-noise ratio in such a spectrum is

$$\frac{\text{signal}}{\text{noise}}=(nN)^{-1}\sum_{k=1}^{n}S_k. \tag{4}$$

As most digital computers have available uniform cyclic random number generators, it is shown how the power distribution can be produced from these generators. Let X_k be a uniform random variable between 0 and 1. The basic probability law relates X_k and P_k through

$$P(P_k)dP_k=P_x(X_k)dX_k. \tag{5}$$

Integrating the left side between 0 and P_k and the right side between 0 and X_k, the transformation equation relating P_k and X_k is obtained:

$$P_k=-(S_k+N)\ln(1-X_k). \tag{6}$$

This can be simplified further to

$$P_k = -(S_k + N)\ln X_k. \tag{7}$$

The algorithm for spectral simulation is briefly summarized:

1) A desired signal power spectrum of arbitrary shape S_k is generated.

2) A noise power N is chosen such that it creates a desired signal-to-noise ratio according to (4).

3) The signal and noise powers at each frequency are added and multiplied with the logarithm of a uniformly distributed (0 to 1) random number X_k [according to Eq. (7)] to generate a desired power spectral component P_k.

4) If the I and Q components are desired, then the inverse discrete Fourier transform of the complex sequence $P_k^{\frac{1}{2}}\exp(j\theta_k)$ is required; θ_k can be generated from the same uniform random number generator but must be independent from P_k and uniformly distributed between 0 and 2π.

The algorithm described herein is direct and versatile since it creates spectra of any desired shape. Also, it is simple for digital computer implementation. In cases when time series are generated from spectra, the finite spectral window will create some correlation among the time samples in addition to the dependence related to the finite signal spectrum width. Although those effects can be minimized by proper window choice, it seems that this is not necessary for most applications.

The spectrum P_k can be used for signal and noise studies in Doppler radars. Specifically, the transition regions between the signal and the noise in the spectrum corresponding to maximum velocities can be simulated and analyzed. This could lead to some confidence criteria in the determination of maximum measured velocities. Moreover, the in-phase and quadrature time series components can be used for studies of receivers' nonlinearities on the spectral shape. The nonlinearities can be undesirable as in the case of clipping or desirable as in cases of square law or logarithmic receiver response.

At present the simulation is being used successfully for comparative studies among the pulse pair (Rummler, 1968), fast Fourier transform, and phase change (Doviak *et al.*, 1974) methods of spectral moment estimation.

Acknowledgments. The writer expresses his gratitude to Dr. R. J. Doviak of NSSL for numerous discussions and suggestions during the preparation of this note. The NSSL Doppler program, which supplied the impetus for this study, has been supported by the Atomic Energy Commission under Contract AT(49-5)-1289 and the Federal Aviation Administration under Contract DOT FA72-WAI-265.

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