Uniqueness in Logic Puzzles

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Pure deduction puzzles typically have a single unique solution. However, some puzzle setters argue that challenges with multiple solutions are also valid, if they can be solved by eliminating choices that lead to ambiguous states. This paper considers the arguments for and against this position, and presents a counterexample that demonstrates the danger of using uniqueness to decide between multiple solutions.

1 Introduction

A characteristic of pure deduction puzzles, such as Japanese logic puzzles, is that each challenge has a single unique solution. This allows such challenges to be solved by deduction rather than guesswork [1].

I was therefore surprised to find a Kakuro challenge with multiple solutions in a publication as respectable as *The Guardian* [2]. This was the first time that I had ever encountered such a case in print. The aim in Kakuro is to fill each cell with a digit in the range 1–9, such that each horizontal and vertical run adds to the hint total shown, and no digit is repeated within each run [3].

Figure 1 shows the relevant section of the Kakuro challenge in question (all other values have been resolved). Possible values for the final few unresolved cells are shown in small print, and a key cell with possible values 4 or 5 is circled. This challenge has three possible solutions, depending on whether this key cell takes the value 4 or 5, as shown.

After alerting the UK setter of this challenge to what appeared to be a flawed design with no deducible solution, I was also surprised by his response. He maintained that this challenge was indeed valid, and could be solved by deduction based on *relative* uniqueness.

2 The Case For Ambiguity

The setter of the ambiguous Kakuro challenge argued as follows:

> *Any move M that leads to multiple solutions can be eliminated.*

For instance, the value of the circled cell in Figure 1 cannot be 4, as such a move would allow multiple solutions (top row). This cell must therefore take the value 5, producing the single ‘correct’ solution (bottom row).

This argument of *deduction by relative uniqueness*, for selecting among multiple solutions, seems fair enough at first glance. It adds some much-needed depth to Kakuro, by allowing an additional solution strategy. It also increases the number of possible challenges that can be devised, by allowing cases with multiple solutions that traditional setters would not allow.

However, Japanese publisher Nikoli, the inventor and major supplier of Kakuro, categorically state that uniqueness should not be exploited in this way to solve Kakuro, or any of their other pure deduction puzzles.[1] We now consider the argument for absolute rather than relative uniqueness.

3 The Case For Uniqueness

A serious problem with deduction by relative uniqueness is that it does not work unless the solver also knows that this rule is in force, but uniqueness is generally assumed for such puzzles rather than explicitly stated. For example, the Kakuro rules provided by *The Guardian* make no mention of uniqueness, making those rules insufficient to solve the ambiguous challenge shown in Figure 1 [2].

Further, there is an obvious corollary to the argument (1) made above:

[1] Strongly worded personal correspondence.

Kirk, J. T., Scott, M., and McCoy, L., ‘Uniqueness in Logic Puzzles,’ *Game & Puzzle Design*, vol. 1, no. 1, 2015, pp. 1–3.

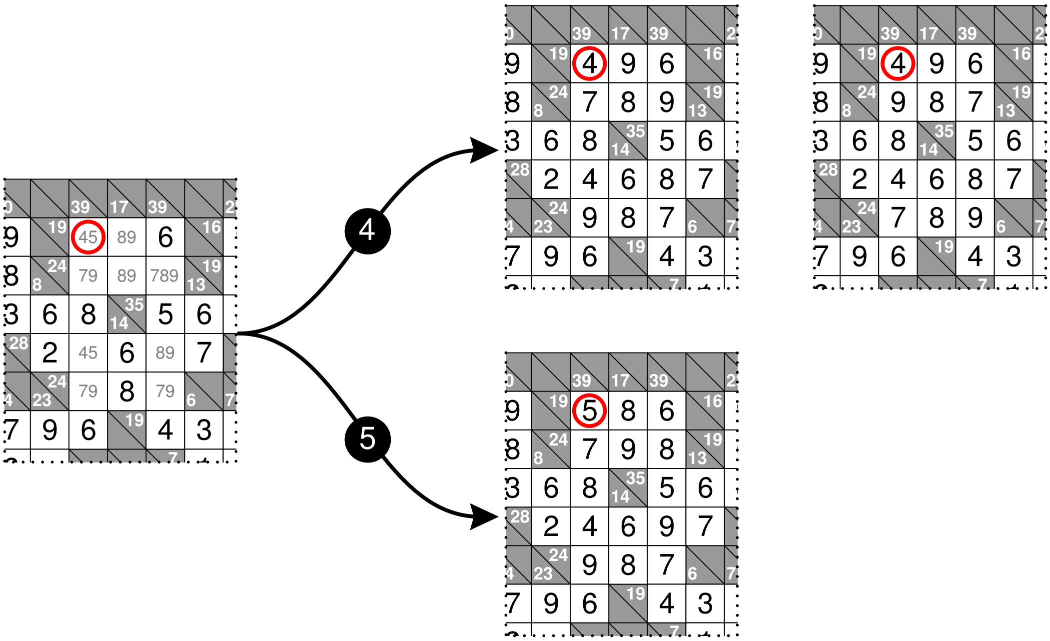

Figure 1. A Kakuro challenge with three solutions. The circled cell can take the value 4 or 5.

> *Any move leading to ambiguous move M can therefore also be eliminated.*

Hence, chaining backwards from ambiguous move *M*, every prior move can also be said to lead to ambiguity and hence be eliminated, until the challenge has no valid moves. Or can it? There is no clear answer to this question, which depends on the setter's and solver's interpretations.

3.1 Counterexample

The following counterexample demonstrates the dangers of deduction by relative uniqueness. Slitherlink is a deduction puzzle in which a simple closed path must be traced through orthogonal vertices of a square grid, to visit the number of sides indicated on each numbered cell [4]. For example, Figure 2 shows a simple 2×3 Slitherlink challenge with three valid solutions: *a*, *b* and *c*.

Given that four edges can be deduced as shown in Figure 3 (top), consider the move indicated by the dotted line. If there is *not* an edge between these vertices then two possible solutions exist (left), hence this move must be an edge and *c* must be the 'correct' solution (right).

However, if the same process is applied to the move indicated in Figure 4 (top, dotted), then *b* is deduced to be the 'correct' solution (right).

Deduction by relative uniqueness therefore gives two conflicting 'correct' solutions, *b* and *c*, depending on processing order. To derive the same solution as the setter, the solver would have to follow the same sequence of decisions in the exact same order, but there is no way to enforce this in practice. Deduction by relative uniqueness is not guaranteed to yield the same solution from among multiple solutions in all cases.

This Slitherlink counterexample could be said to have one valid solution (depending on the order in which the solver made their deductions), two equally valid solutions (through deduction by relative uniqueness) or three equally valid solutions (which it does, after all—see Figure 2). This is clearly an unsatisfactory state of affairs. But if absolute uniqueness is enforced, and such cases of multiple solutions avoided, then all of these problems simply go away, at no real cost. As expert puzzle designer Hiroshi Higashida points out:

> *Puzzle creators, not only solvers, mustn't defy rules, either* [5, p216].

4 Conclusion

The characteristic of pure deduction puzzles to have a single unique solution is not only elegant, but performs a vital practical function. It guarantees that challenges can be solved by deduction alone, without guesswork or ambiguity, and

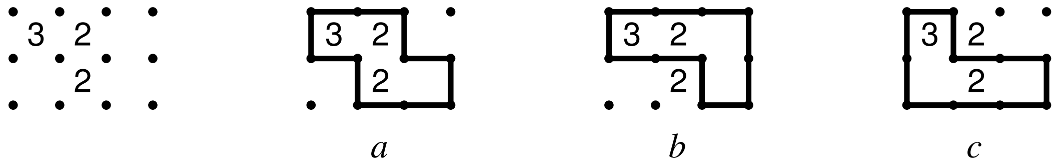

Figure 2. A 2×3 Slitherlink challenge (left) with three solutions (*a*, *b* and *c*).

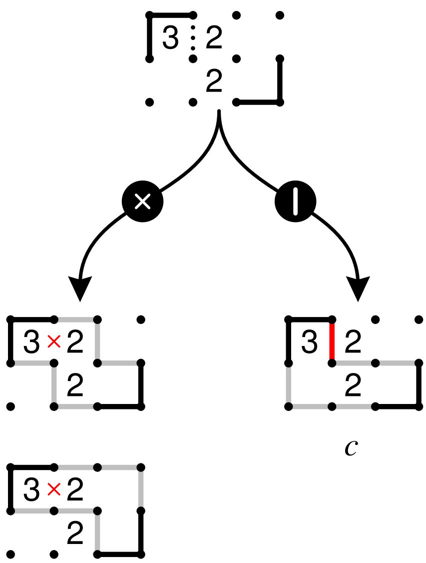

Figure 3. Deduction by uniqueness yields *c*.

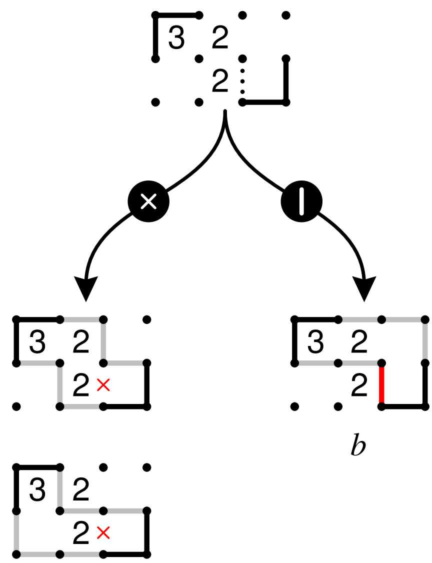

Figure 4. Deduction by uniqueness yields *b*.

means that the setter and solver are both playing from the same rule set without the need to make assumptions about implied or hidden rules. Further, uniqueness makes challenges self-checking; if the player has deduced a solution, then it must be the correct one. As tempting as it may be to relax this constraint of absolute uniqueness and instead exploit relative uniqueness as a solution strategy, this is best avoided in pure deduction puzzles.

Acknowledgements

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References

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