Minimal Constraint Subsets Detected for differential trail in Table 9 DDH21+

```
the 0 th Minimal Constraint set is:
 y^0_0 + x^1_4 + k_1_0 + k_2_0 + k_3_0_0 = 0
 y^0_0 + y^0_1 + x^1_1 + k_1 + k_2 + k_2 + k_3 = 0
 y^0_2 + [y^0_8] + [y^0_15] + x^1_2 + k_1_2 + k_2_2_0 + k_3_2_0 = 0
 y^0 2 + x^1 6 + k1 2 + k2 2 0 + k3 2 0 = 0
 [y^0_7] + y^0_10 + x^1_8 + k_1^7 + k_2^7_0 + k_3^7_0 = 0
 y^1 + y^1 
 y^1_1 + x^2_5 + k_{115} + k_{215} + k_{315} = 0
 y^1 2 + y^1 8 + x^2 14 + k1 8 + k2 8 0 + k3 8 0 = 0
 y^1_3 + y^1_9 + y^1_12 + [x^2_3] + k_{113} + k_{213}0 + k_{313}0 = 0
 y^1_3 + [x^2_7] + k_1_13 + k_2_13_0 + k_3_13_0 = 0
 y^1_4 + y^1_11 + x^2_9 + k_1_10 + k_2_10_0 + k_3_10_0 = 0
 y^1_6 + y^1_9 + x^2_{11} + k_{112} + k_{212}0 + k_{312}0 = 0
 v^2 0 + x^3 4 + k1 1 + k2 1 1 + k3 1 1 = 0
 y^2_0 + y^2_10 + x^3_12 + k_11 + k_21_1 + k_31_1 = 0
 [y^2 1] + y^2 11 + y^2 14 + x^3 1 + k1 7 + k2 7 1 + k3 7 1 = 0
 y^2 = + x^3 = + k_1 = 0 + k_2 = 0 + k_3 = 0
 y^2_2 + y^2_8 + x^3_14 + k_10 + k_20_1 + k_30_1 = 0
 [y^2_3] + y^2_9 + x^3_15 + k_1^5 + k_2^5_1 + k_3^5_1 = 0
 y^2_5 + y^2_8 + x^3_10 + k_{16} + k_{26}1 + k_{36}1 = 0
 [y^2 7] + y^2 10 + [x^3 8] + k1 3 + k2 3 1 + k3 3 1 = 0
 y^3_0 + x^4_4 + k_{115} + k_{215} + k_{315} = 0
 y^3_0 + y^3_1 + x^4_1 + k_{115} + k_{215} + k_{315} = 0
 y^3_1 + y^3_1 + y^3_1 + y^3_1 + x^4_1 + k_{111} + k_{211} + k_{311} = 0
 [y^3_2] + [y^3_8] + y^3_15 + x^4_2 + k_1_9 + k_2_9_1 + k_3_9_1 = 0
 y^3_3 + y^3_9 + y^3_{12} + x^4_3 + k_{114} + k_{214} + k_{314} = 0
 y^3_4 + y^3_11 + x^4_9 + k_18 + k_28_1 + k_38_1 = 0
 y^3_6 + y^3_9 + x^4_{11} + k_{11} + k_{2101} + k_{3101} = 0
 y^4_1 + x^5_5 + k_1^3 + k_2^3_2 + k_3^3_2 = 0
 y^4_2 + x^5_6 + k_1_1 + k_2_1_2 + k_3_1_2 = 0
 y^4_3 + y^4_9 + y^4_12 + x^5_3 + k_{16} + k_{262} + k_{362} = 0
 y^4_4 + y^4_11 + x^5_9 + k_1_0 + k_2_0_2 + k_3_0_2 = 0
 y^4 - 7 + [y^4 - 10] + x^5 - 8 + k_1 - 5 + k_2 - 5 - 2 + k_3 - 5 - 2 = 0
 y^5_3 + x^6_7 + k_{112} + k_{2122} + k_{3122} = 0
 [y^5_4] + [y^5_11] + x^6_9 + k_1_9 + k_2_9_2 + k_3_9_2 = 0
 y^5_5 + y^5_8 + x^6_10 + k_110 + k_210_2 + k_310_2 = 0
 y^5_6 + y^5_9 + x^6_{11} + k_{18} + k_{282} + k_{382} = 0
 y^6_0 + y^6_10 + [y^6_13] + [x^7_0] + k_1_3 + k_2_3_3 + k_3_3_3 = 0
 y^6_0 + y^6_10 + x^7_12 + k_13 + k_233 + k_333 = 0
 [y^6_1] + y^6_11 + x^7_13 + k_15 + k_25_3 + k_35_3 = 0
 y^6 3 + x^7 7 + k1 4 + k2 4 3 + k3 4 3 = 0
```

```
y^6_3 + y^6_9 + x^7_{15} + k_{14} + k_{243} + k_{343} = 0
y^6 7 + y^6 10 + x^7 8 + k1 6 + k2 6 3 + k3 6 3 = 0
[y^7_0] + y^7_10 + y^7_13 + x^8_0 + k_{113} + k_{2133} + k_{3133} = 0
y^7_2 + y^7_8 + y^7_15 + x^8_2 + k_111 + k_211_3 + k_311_3 = 0
y^7_2 + x^8_6 + k_1_{11} + k_2_{11}_3 + k_3_{11}_3 = 0
y^7_3 + y^7_9 + y^7_{12} + x^8_3 + k_{110} + k_{2103} + k_{3103} = 0
y^7 3 + y^7 9 + x^8 15 + k1 10 + k2 10 3 + k3 10 3 = 0
y^7_7 + y^7_10 + x^8_8 + k_{112} + k_{2123} + k_{3123} = 0
v^8 0 + x^9 4 + k1 5 + k2 5 4 + k3 5 4 = 0
y^8_2 + y^8_8 + y^8_15 + x^9_2 + k_1^3 + k_2^3_4 + k_3^3_4 = 0
y^8 3 + y^8 9 + x^9 15 + k1 2 + k2 2 4 + k3 2 4 = 0
y^8_6 + y^8_9 + x^9_{11} + k_{11} + k_{214} + k_{314} = 0
y^9_2 + y^9_8 + y^9_15 + [x^10_2] + k_{113} + k_{2134} + k_{3134} = 0
y^9_2 + [x^10_6] + k_{113} + k_{2134} + k_{3134} = 0
y^9_2 + y^9_8 + [x^10_14] + k_113 + k_213_4 + k_313_4 = 0
y^9_4 + y^9_11 + x^10_9 + k_111 + k_211_4 + k_311_4 = 0
[y^10_6] + y^10_9 + [x^11_1] + k_17 + k_27_5 + k_37_5 = 0
S(x^1 2) + y^1 2 = 0
S(x^1_4) + y^1_4 = 0
S(x^1_6) + y^1_6 = 0
S(x^1_8) + y^1_8 = 0
S(x^1_12) + y^1_12 = 0
S(x^2 5) + y^2 5 = 0
S(x^2_9) + y^2_9 = 0
S(x^2_11) + y^2_11 = 0
S(x^2_14) + y^2_14= 0
S(x^3_1) + y^3_1 = 0
S(x^3_4) + y^3_4 = 0
S(x^3_6) + y^3_6 = 0
S(x^3_10) + y^3_10 = 0
S(x^3_12) + y^3_12 = 0
S(x^3_14) + y^3_14 = 0
S(x^3_15) + y^3_15 = 0
S(x^4 1) + y^4 1 = 0
S(x^4_2) + y^4_2 = 0
S(x^4_3) + y^4_3 = 0
S(x^4_4) + y^4_4 = 0
S(x^4_7) + y^4_7 = 0
S(x^4_9) + y^4_9 = 0
S(x^4 11) + y^4 11 = 0
S(x^4_12) + y^4_12 = 0
S(x^5_3) + y^5_3 = 0
S(x^5_5) + y^5_5 = 0
S(x^5 6) + y^5 6 = 0
```

```
S(x^5_8) + y^5_8 = 0
   S(x^5 9) + y^5_9 = 0
   S(x^6_7) + y^6_7 = 0
   S(x^6_9) + y^6_9 = 0
   S(x^6_10) + y^6_10 = 0
   S(x^6_11) + y^6_11 = 0
   S(x^7 7) + y^7 7 = 0
   S(x^7_8) + y^7_8 = 0
   S(x^7 12) + v^7 12 = 0
   S(x^7_13) + y^7_13 = 0
   S(x^7 15) + y^7 15 = 0
   S(x^8_0) + y^8_0 = 0
   S(x^8_2) + y^8_2 = 0
   S(x^8_3) + y^8_3 = 0
   S(x^8_6) + y^8_6 = 0
   S(x^8_8) + y^8_8 = 0
   S(x^8_15) + y^8_15 = 0
   S(x^9 2) + y^9 2 = 0
   S(x^9_4) + y^9_4 = 0
   S(x^9_11) + y^9_11 = 0
   S(x^9_15) + y^9_15 = 0
   S(x^10_9) + y^10_9 = 0
the 1 th Minimal Constraint set is:
  y^0_0 + x^1_4 + k_1_0 + k_2_0 + k_3_0 = 0
  y^0_0 + y^0_1 + x^1_1 + k_1 + k_2 + k_2 = 0 + k_3 = 0
   [y^0_7] + y^0_10 + x^1_8 + k_1^7 + k_2^7_0 + k_3^7_0 = 0
  y^1_1 + y^1_1 + [y^1_14] + [x^2_1] + k_{115} + k_{215}0 + k_{315}0 = 0
  y^1_1 + x^2_5 + k_{115} + k_{215} + k_{315} = 0
  y^1_2 + x^2_6 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
  y^1_2 + y^1_8 + x^2_14 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
  y^1_3 + y^1_9 + y^1_12 + [x^2_3] + k_{113} + k_{213}0 + k_{313}0 = 0
  y^1_3 + y^1_9 + [x^2_{15}] + k_{113} + k_{213}0 + k_{313}0 = 0
  y^1_4 + y^1_11 + x^2_9 + k_{11}0 + k_{210}0 + k_{310}0 = 0
   y^2_0 + x^3_4 + k_{11} + k_{211} + k_{311} = 0
  y^2_0 + y^2_10 + x^3_12 + k_11 + k_21_1 + k_31_1 = 0
   [y^2] + y^2] + y^2 + y^2 + x^3 + k^3 + k^2 + k^3 + k
   [y^2_1] + y^2_1 + x^3_1 + k_1 + k_2 + k_2 + k_3 + k_1 + k_2 + k_3 + k_3 + k_1 + k_2 + k_3 + k_3 + k_1 + k_3 + k_
  y^2_2 + x^3_6 + k_1_0 + k_2_1 + k_3_1 = 0
  y^2_2 + y^2_8 + x^3_14 + k_1_0 + k_2_0_1 + k_3_0_1 = 0
   [y^2_3] + x^3_7 + k_1^5 + k_2^5_1 + k_3^5_1 = 0
  y^2_5 + y^2_8 + x^3_10 + k_1_6 + k_2_6_1 + k_3_6_1 = 0
  y^2_6 + y^2_9 + x^3_{11} + k_1_4 + k_2_4_1 + k_3_4_1 = 0
   [y^2_7] + y^2_10 + [x^3_8] + k_1_3 + k_2_3_1 + k_3_3_1 = 0
   y^3 0 + y^3 10 + y^3 13 + x^4 0 + k1 15 + k2 15 1 + k3 15 1 = 0
```

```
y^3_0 + y^3_1 + x^4_1 + k_{115} + k_{215} + k_{315} = 0
y^3 1 + y^3 11 + y^3 14 + x^4 1 + k1 11 + k2 11 1 + k3 11 1 = 0
y^3_1 + x^4_5 + k_1_1 + k_2_1_1 + k_3_1_1 = 0
y^3_1 + y^3_1 + x^4_1 + k_11 + k_21_1 + k_31_1 = 0
y^3_3 + y^3_9 + y^3_{12} + x^4_3 + k_{114} + k_{214} + k_{314} = 0
y^3_3 + x^4_7 + k_114 + k_2141 + k_3141 = 0
y^3 4 + y^3 11 + x^4 9 + k1 8 + k2 8 1 + k3 8 1 = 0
y^3_6 + y^3_9 + x^4_11 + k_1_10 + k_2_10_1 + k_3_10_1 = 0
y^3_7 + y^3_10 + x^4_8 + k_{113} + k_{2131} + k_{3131} = 0
y^4_0 + [y^4_10] + y^4_13 + x^5_0 + k_1^7 + k_2^7_2 + k_3^7_2 = 0
y^4 0 + [x^5 4] + k17 + k272 + k372 = 0
y^4 0 + [y^4 10] + x^5 12 + k1 7 + k2 7 2 + k3 7 2 = 0
y^4_1 + y^4_1 + x^5_1 + k^3_2 + k^3_3 = 0
y^4_3 + y^4_9 + y^4_12 + x^5_3 + k_{16} + k_{262} + k_{362} = 0
y^4_5 + y^4_8 + x^5_10 + k_14 + k_24_2 + k_34_2 = 0
v^4 7 + [v^4 10] + x^5 8 + k1 5 + k2 5 2 + k3 5 2 = 0
y^5_0 + y^5_10 + y^5_13 + x^6_0 + k1_{11} + k2_{11}_2 + k3_{11}_2 = 0
y^5 2 + x^6 6 + k1 15 + k2 15 2 + k3 15 2 = 0
y^5_2 + y^5_8 + x^6_14 + k_1_15 + k_2_15_2 + k_3_15_2 = 0
y^5_3 + y^5_9 + y^5_12 + x^6_3 + k_1_12 + k_2_12_2 + k_3_12_2 = 0
y^5_3 + x^6_7 + k_{112} + k_{2122} + k_{3122} = 0
y^5_3 + y^5_9 + x^6_{15} + k_{112} + k_{2122} + k_{3122} = 0
[y^5 4] + [y^5 11] + x^6 9 + k1 9 + k2 9 2 + k3 9 2 = 0
y^6_0 + y^6_10 + [y^6_13] + [x^7_0] + k_1_3 + k_2_3_3 + k_3_3_3 = 0
[y^6_1] + y^6_11 + y^6_14 + x^7_1 + k_1_5 + k_2_5_3 + k_3_5_3 = 0
[y^6_1] + y^6_11 + x^7_13 + k_15 + k_253 + k_353 = 0
y^6_2 + y^6_8 + y^6_15 + x^7_2 + k_1^7 + k_2^7_3 + k_3^7_3 = 0
y^6_2 + y^6_8 + x^7_14 + k_1^7 + k_2^7_3 + k_3^7_3 = 0
y^6_3 + x^7_7 + k_1_4 + k_2_4_3 + k_3_4_3 = 0
y^6_6 + y^6_9 + x^7_{11} + k_{10} + k_{203} + k_{303} = 0
y^6_7 + y^6_10 + x^7_8 + k_{16} + k_{263} + k_{363} = 0
[y^7_0] + y^7_10 + y^7_13 + x^8_0 + k_{113} + k_{213} + k_{313} = 0
[y^7_0] + x^8_4 + k_{113} + k_{2133} + k_{3133} = 0
y^7_1 + y^7_1 + y^7_1 + y^7_1 + x^8_1 + k_114 + k_214_3 + k_314_3 = 0
y^7_2 + y^7_8 + x^8_14 + k_1_11 + k_2_11_3 + k_3_11_3 = 0
y^7_7 + y^7_10 + x^8_8 + k_1_12 + k_2_12_3 + k_3_12_3 = 0
y^8_0 + x^9_4 + k_{15} + k_{254} + k_{354} = 0
y^8_1 + y^8_1 + y^8_1 + y^8_1 + x^9_1 + k_{16} + k_{264} + k_{364} = 0
y^8_2 + x^9_6 + k_{13} + k_{234} + k_{334} = 0
y^8 2 + y^8 8 + x^9 14 + k1 3 + k2 3 4 + k3 3 4 = 0
y^8_4 + y^8_11 + x^9_9 + k_1^7 + k_2^7_4 + k_3^7_4 = 0
y^9_1 + y^9_11 + y^9_14 + x^10_1 + k_112 + k_212_4 + k_312_4 = 0
y^9_4 + y^9_11 + x^10_9 + k_111 + k_211_4 + k_311_4 = 0
y^9 6 + y^9 9 + x^10 11 + k1 15 + k2 15 4 + k3 15 4 = 0
```

```
y^10_1 + y^10_1 + [y^10_14] + [x^11_1] + k_14 + k_245 + k_345 = 0
```

$$[y^10_6] + y^10_9 + [x^11_1] + k_17 + k_27_5 + k_37_5 = 0$$

- $S(x^1_4) + y^1_4 = 0$
- $S(x^1_8) + y^1_8 = 0$
- $S(x^1_12) + y^1_12 = 0$
- $S(x^2_5) + y^2_5 = 0$
- $S(x^2_6) + y^2_6 = 0$
- $S(x^2_9) + y^2_9 = 0$
- $S(x^2_14) + y^2_14= 0$
- $S(x^3_1) + y^3_1 = 0$
- $S(x^3_4) + y^3_4 = 0$
- $S(x^3_6) + y^3_6 = 0$
- $S(x^3_7) + y^3_7 = 0$
- $S(x^3_10) + y^3_10 = 0$
- $S(x^3_11) + y^3_11 = 0$
- $S(x^3_12) + y^3_12 = 0$
- $S(x^3_13) + y^3_13 = 0$
- $S(x^3 14) + y^3 14 = 0$
- $S(x^4_0) + y^4_0 = 0$
- $S(x^4_1) + y^4_1 = 0$
- $S(x^4_3) + y^4_3 = 0$
- $S(x^4_5) + y^4_5 = 0$
- $S(x^4 7) + y^4 7 = 0$
- $S(x^4_8) + y^4_8 = 0$
- $S(x^4_9) + y^4_9 = 0$
- $S(x^4_11) + y^4_11 = 0$
- $S(x^4_12) + y^4_12 = 0$
- $S(x^4_13) + y^4_13 = 0$
- $S(x^5_0) + y^5_0 = 0$
- $S(x^5_3) + y^5_3 = 0$
- $S(x^5_8) + y^5_8 = 0$
- $S(x^5_10) + y^5_10 = 0$
- $S(x^5_12) + y^5_12 = 0$
- $S(x^5_13) + y^5_13 = 0$
- $S(x^6_0) + y^6_0 = 0$
- $S(x^6_3) + y^6_3 = 0$
- $S(x^6_6) + y^6_6 = 0$
- $S(x^6_7) + y^6_7 = 0$
- $S(x^6_9) + y^6_9 = 0$
- $S(x^6_14) + y^6_14= 0$
- $S(x^6_15) + y^6_15 = 0$
- $S(x^7_1) + y^7_1 = 0$
- $S(x^7_2) + y^7_2 = 0$
- $S(x^7_7) + y^7_7 = 0$

```
S(x^7_8) + y^7_8 = 0
  S(x^7_11) + y^7_11 = 0
  S(x^7_13) + y^7_13 = 0
  S(x^7_14) + y^7_14 = 0
  S(x^8_0) + y^8_0 = 0
  S(x^8_1) + y^8_1 = 0
  S(x^8 4) + y^8 4 = 0
  S(x^8_8) + y^8_8 = 0
  S(x^8 14) + y^8 14 = 0
  S(x^9_1) + y^9_1 = 0
  S(x^9 4) + y^9 4 = 0
  S(x^9 6) + y^9 6 = 0
  S(x^9_9) + y^9_9 = 0
  S(x^9_14) + y^9_14 = 0
  S(x^10_1) + y^10_1 = 0
  S(x^10 9) + v^10 9 = 0
  S(x^10_11) + y^10_11 = 0
the 2 th Minimal Constraint set is:
 y^0_0 + x^1_4 + k_1_0 + k_2_0 + k_3_0 = 0
 y^0_0 + y^0_1 + x^1_1 + k_0 + k_0 + k_0 = 0
  [y^0_7] + y^0_10 + x^1_8 + k_1^7 + k_2^7_0 + k_3^7_0 = 0
 y^1_1 + x^2_5 + k_{115} + k_{215} + k_{315} = 0
 y^1 1 + y^1 11 + x^2 13 + k1 15 + k2 15 0 + k3 15 0 = 0
 y^1_2 + x^2_6 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
 y^1_2 + y^1_8 + x^2_14 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
 y^1_3 + y^1_9 + y^1_12 + [x^2_3] + k_1_13 + k_2_13_0 + k_3_13_0 = 0
 y^1_3 + y^1_9 + [x^2_{15}] + k_{113} + k_{213}0 + k_{313}0 = 0
 y^1 4 + y^1 11 + x^2 9 + k1 10 + k2 10 0 + k3 10 0 = 0
 y^2_0 + y^2_10 + y^2_13 + x^3_0 + k_11 + k_21_1 + k_31_1 = 0
 y^2_0 + y^2_10 + x^3_12 + k_11 + k_21_1 + k_31_1 = 0
  [y^2_1] + y^2_1 + y^2_1 + y^2_1 + x^3_1 + k_1^7 + k_2^7_1 + k_3^7_1 = 0
  [y^2_1] + y^2_1 + x^3_1 + k_1 + k_2 + k_2 + k_3 + k_1 + k_2 + k_3 + k_3 + k_1 + k_2 + k_3 + k_3 + k_1 + k_3 + k_
 y^2_2 + x^3_6 + k_1_0 + k_2_1 + k_3_1 = 0
 y^2_2 + y^2_8 + x^3_14 + k_1_0 + k_2_0_1 + k_3_0_1 = 0
 y^2_5 + y^2_8 + x^3_10 + k_1_6 + k_2_6_1 + k_3_6_1 = 0
 y^2_6 + y^2_9 + x^3_{11} + k_{14} + k_{24} + k_{34} = 0
 y^3_0 + y^3_10 + y^3_13 + x^4_0 + k_{115} + k_{215} + k_{315} = 0
 y^3_1 + y^3_1 + y^3_1 + x^4_1 + k_{111} + k_{211} + k_{311} = 0
 y^3_3 + y^3_9 + y^3_12 + x^4_3 + k_114 + k_214_1 + k_314_1 = 0
 y^3_3 + x^4_7 + k_{114} + k_{214} + k_{314} = 0
 y^3_6 + y^3_9 + x^4_{11} + k_{11} + k_{2101} + k_{3101} = 0
 y^4_0 + [y^4_10] + x^5_12 + k_17 + k_27_2 + k_37_2 = 0
 y^4_1 + y^4_1 + x^5_1 + k^3_2 + k^3_2 = 0
 y^4 3 + y^4 9 + x^5 15 + k1 6 + k2 6 2 + k3 6 2 = 0
```

```
[y^4_6] + y^4_9 + [x^5_11] + k_1^2 + k_2^2_2 + k_3^2_2 = 0
y^4 7 + [y^4 10] + x^5 8 + k1 5 + k2 5 2 + k3 5 2 = 0
y^5_0 + y^5_10 + y^5_13 + x^6_0 + k_111 + k_211_2 + k_311_2 = 0
y^5_0 + y^5_10 + x^6_12 + k_111 + k_211_2 + k_311_2 = 0
y^5_1 + x^6_5 + k_{113} + k_{2132} + k_{3132} = 0
y^5_1 + [y^5_1] + [x^6_1] + k_113 + k_213_2 + k_313_2 = 0
y^5 2 + y^5 8 + y^5 15 + x^6 2 + k1 15 + k2 15 2 + k3 15 2 = 0
y^5_2 + x^6_6 + k_{115} + k_{215}^2 + k_{315}^2 = 0
y^5 3 + y^5 9 + y^5 12 + x^6 3 + k_1 12 + k_2 12 2 + k_3 12 2 = 0
y^5_3 + y^5_9 + x^6_{15} + k_{112} + k_{2122} + k_{3122} = 0
[y^5 4] + [y^5 11] + x^6 9 + k1 9 + k2 9 2 + k3 9 2 = 0
y^6_0 + y^6_10 + [y^6_13] + [x^7_0] + k_1_3 + k_2_3_3 + k_3_3_3 = 0
y^6_0 + x^7_4 + k_{1_3} + k_{2_{3_3}} + k_{3_{3_3}} = 0
y^6_0 + y^6_1 + x^7_1 + k_1^3 + k_2^3 + k_3^3 = 0
[y^6_1] + x^7_5 + k_1^5 + k_2^3 + k_3^5_3 = 0
v^6 2 + v^6 8 + v^6 15 + x^7 2 + k1 7 + k2 7 3 + k3 7 3 = 0
y^6_3 + y^6_9 + y^6_12 + x^7_3 + k_1_4 + k_2_4_3 + k_3_4_3 = 0
y^6 3 + x^7 7 + k1 4 + k2 4 3 + k3 4 3 = 0
y^6_3 + y^6_9 + x^7_{15} + k_{14} + k_{243} + k_{343} = 0
y^6_5 + y^6_8 + x^7_10 + k_1^2 + k_2^3 + k_3^2_3 = 0
y^6_6 + y^6_9 + x^7_{11} + k_{10} + k_{203} + k_{303} = 0
y^7_2 + y^7_8 + y^7_15 + x^8_2 + k_111 + k_211_3 + k_311_3 = 0
y^7 3 + y^7 9 + y^7 12 + x^8 3 + k1 10 + k2 10 3 + k3 10 3 = 0
y^7_3 + x^8_7 + k_{110} + k_{2103} + k_{3103} = 0
y^7_3 + y^7_9 + x^8_15 + k_1_10 + k_2_10_3 + k_3_10_3 = 0
y^7_4 + y^7_11 + x^8_9 + k_1_15 + k_2_15_3 + k_3_15_3 = 0
y^7_5 + y^7_8 + x^8_10 + k_18 + k_28_3 + k_38_3 = 0
y^7 7 + y^7 10 + x^8 8 + k1 12 + k2 12 3 + k3 12 3 = 0
y^8_2 + y^8_3 + y^8_15 + x^9_2 + k_1^3 + k_2^3_4 + k_3^3_4 = 0
y^8_3 + y^8_9 + x^9_{15} + k_{12} + k_{224} + k_{324} = 0
y^8_7 + y^8_10 + x^9_8 + k_14 + k_244 + k_344 = 0
y^9_2 + y^9_8 + y^9_{15} + [x^10_2] + k_{113} + k_{2134} + k_{3134} = 0
S(x^1_4) + y^1_4 = 0
S(x^1_8) + y^1_8 = 0
S(x^1_12) + y^1_12 = 0
S(x^2_5) + y^2_5 = 0
S(x^2 6) + v^2 6 = 0
S(x^2_9) + y^2_9 = 0
S(x^2_13) + y^2_13 = 0
S(x^2 14) + y^2 14 = 0
S(x^3_0) + y^3_0 = 0
S(x^3_1) + y^3_1 = 0
S(x^3_6) + y^3_6 = 0
S(x^3 10) + y^3 10 = 0
```

```
S(x^3_11) + y^3_11 = 0
```

$$S(x^3 12) + y^3 12 = 0$$

$$S(x^3_13) + y^3_13 = 0$$

$$S(x^3_14) + y^3_14= 0$$

$$S(x^4_0) + y^4_0 = 0$$

$$S(x^4_1) + y^4_1 = 0$$

$$S(x^4_3) + y^4_3 = 0$$

$$S(x^4_7) + y^4_7 = 0$$

$$S(x^4_11) + y^4_11 = 0$$

$$S(x^5_8) + y^5_8 = 0$$

$$S(x^5_12) + y^5_12 = 0$$

$$S(x^5_13) + y^5_13 = 0$$

$$S(x^5_15) + y^5_15 = 0$$

$$S(x^6_0) + y^6_0 = 0$$

$$S(x^6_2) + y^6_2 = 0$$

$$S(x^6_3) + y^6_3 = 0$$

$$S(x^6_5) + y^6_5 = 0$$

$$S(x^6_6) + y^6_6 = 0$$

$$S(x^6_9) + y^6_9 = 0$$

$$S(x^6_12) + y^6_12 = 0$$

$$S(x^6_15) + y^6_15 = 0$$

$$S(x^7_2) + y^7_2 = 0$$

$$S(x^7_3) + y^7_3 = 0$$

$$S(x^7_4) + y^7_4 = 0$$

$$S(x^{5} - 5) + y^{5} = 0$$

$$S(x^7_7) + y^7_7 = 0$$

$$S(x^7_10) + y^7_10 = 0$$

$$S(x^7_11) + y^7_11 = 0$$

$$S(x^7_12) + y^7_12 = 0$$

$$S(x^7_15) + y^7_15 = 0$$

$$S(x^8_2) + y^8_2 = 0$$

$$S(x^8_3) + y^8_3 = 0$$

$$S(x^8_7) + y^8_7 = 0$$

$$S(x^8_8) + y^8_8 = 0$$

$$S(x^8_9) + y^8_9 = 0$$

$$S(x^8_10) + y^8_10 = 0$$

$$S(x^8_15) + y^8_15 = 0$$

$$S(x^9_2) + y^9_2 = 0$$

$$S(x^9_8) + y^9_8 = 0$$

$$S(x^9_15) + y^9_15 = 0$$

the 3 th Minimal Constraint set is:

$$y^0_0 + x^1_4 + k_1_0 + k_2_0 + k_3_0 = 0$$

$$y^0_0 + y^0_1 + x^1_1 + k_0 + k_0 + k_0 = 0$$

$$y^0 2 + x^1 6 + k1 2 + k2 2 0 + k3 2 0 = 0$$

```
y^0_2 + [y^0_8] + [x^1_14] + k_1^2 + k_2^2_0 + k_3^2_0 = 0
[y^0 7] + y^0 10 + x^1 8 + k1 7 + k2 7 0 + k3 7 0 = 0
y^1_1 + x^2_5 + k_{115} + k_{150} + k_{150} = 0
y^1_1 + y^1_1 + x^2_1 + k_1_1 + k_2_1 + k_2_1 = 0
y^1_2 + x^2_6 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
y^1_2 + y^1_8 + x^2_14 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
y^1 3 + y^1 9 + y^1 12 + [x^2 3] + k1 13 + k2 13 0 + k3 13 0 = 0
y^1_3 + [x^2_7] + k_{113} + k_{213} + k_{313} = 0
y^1_4 + y^1_11 + x^2_9 + k_1_10 + k_2_10_0 + k_3_10_0 = 0
y^1_6 + y^1_9 + x^2_{11} + k_{11}^2 + k_{21}^0 + k_{31}^0 = 0
y^2 0 + y^2 10 + y^2 13 + x^3 0 + k1 1 + k2 1 1 + k3 1 1 = 0
y^2_0 + y^2_10 + x^3_12 + k_11 + k_21_1 + k_31_1 = 0
[y^2_1] + y^2_1 + y^2_1 + y^2_1 + x^3_1 + k_1^7 + k_2^7_1 + k_3^7_1 = 0
y^2_2 + x^3_6 + k_1_0 + k_2_1 + k_3_1 = 0
y^2_2 + y^2_8 + x^3_14 + k_10 + k_20_1 + k_30_1 = 0
v^2 5 + v^2 8 + x^3 10 + k1 6 + k2 6 1 + k3 6 1 = 0
y^2_6 + y^2_9 + x^3_{11} + k_1_4 + k_2_4_1 + k_3_4_1 = 0
y^3 0 + y^3 10 + x^4 12 + k1 15 + k2 15 1 + k3 15 1 = 0
y^3_1 + y^3_1 + y^3_1 + y^3_1 + x^4_1 + k_11 + k_21_1 + k_31_1 = 0
[y^3_2] + [y^3_8] + x^4_14 + k_1_9 + k_2_9_1 + k_3_9_1 = 0
y^3_3 + y^3_9 + y^3_12 + x^4_3 + k_{114} + k_{214} + k_{314} = 0
y^3_3 + x^4_7 + k_114 + k_2141 + k_3141 = 0
y^3 6 + y^3 9 + x^4 11 + k1 10 + k2 10 1 + k3 10 1 = 0
y^4_1 + y^4_1 + y^4_1 + x^5_1 + k_3 + k_2_2 + k_3_2 = 0
y^4_3 + y^4_9 + y^4_12 + x^5_3 + k_{16} + k_{262} + k_{362} = 0
y^4_3 + x^5_7 + k_1_6 + k_2_6_2 + k_3_6_2 = 0
y^4_3 + y^4_9 + x^5_15 + k_1_6 + k_2_6_2 + k_3_6_2 = 0
y^4_7 + [y^4_10] + x^5_8 + k_1^5 + k_2^5_2 + k_3^5_2 = 0
y^5_0 + x^6_4 + k_{111} + k_{2112} + k_{3112} = 0
y^5_0 + y^5_10 + x^6_12 + k_111 + k_211_2 + k_311_2 = 0
y^5_1 + x^6_5 + k_{113} + k_{2132} + k_{3132} = 0
y^5_2 + y^5_8 + y^5_15 + x^6_2 + k_115 + k_215_2 + k_315_2 = 0
y^5_2 + x^6_6 + k_{1_15} + k_{2_15_2} + k_{3_15_2} = 0
y^5_3 + x^6_7 + k_{112} + k_{2122} + k_{3122} = 0
y^5_7 + y^5_10 + x^6_8 + k_1_14 + k_2_14_2 + k_3_14_2 = 0
y^6_0 + x^7_4 + k_1_3 + k_2_3_3 + k_3_3_3 = 0
y^6_0 + y^6_10 + x^7_12 + k_13 + k_233 + k_333 = 0
[y^6_1] + y^6_11 + x^7_13 + k_15 + k_25_3 + k_35_3 = 0
y^6_2 + x^7_6 + k_1^7 + k_2^7_3 + k_3^7_3 = 0
y^6_3 + y^6_9 + y^6_12 + x^7_3 + k_1_4 + k_2_4_3 + k_3_4_3 = 0
y^6_3 + x^7_7 + k_1_4 + k_2_4_3 + k_3_4_3 = 0
y^6_4 + y^6_11 + x^7_9 + k_11 + k_21_3 + k_31_3 = 0
y^6_5 + y^6_8 + x^7_10 + k_1^2 + k_2^3 + k_3^2_3 = 0
y^6 6 + y^6 9 + x^7 11 + k1 0 + k2 0 3 + k3 0 3 = 0
```

```
y^6_7 + y^6_10 + x^7_8 + k_{16} + k_{263} + k_{363} = 0
[y^7 \ 0] + y^7 \ 10 + y^7 \ 13 + x^8 \ 0 + k1 \ 13 + k2 \ 13 \ 3 + k3 \ 13 \ 3 = 0
[y^7_0] + x^8_4 + k_{113} + k_{2133} + k_{3133} = 0
[y^7_0] + y^7_10 + x^8_12 + k_113 + k_213_3 + k_313_3 = 0
y^7_2 + x^8_6 + k_1_{11} + k_2_{11}_3 + k_3_{11}_3 = 0
y^7_2 + y^7_8 + x^8_14 + k_1_11 + k_2_11_3 + k_3_11_3 = 0
y^7 3 + y^7 9 + y^7 12 + x^8 3 + k1 10 + k2 10 3 + k3 10 3 = 0
y^7_3 + x^8_7 + k_{110} + k_{2103} + k_{3103} = 0
v^7 3 + v^7 9 + x^8 15 + k1 10 + k2 10 3 + k3 10 3 = 0
y^7_4 + y^7_11 + x^8_9 + k_1_15 + k_2_15_3 + k_3_15_3 = 0
y^7 6 + y^7 9 + x^8 11 + k1 9 + k2 9 3 + k3 9 3 = 0
y^7 7 + y^7 10 + x^8 8 + k1 12 + k2 12 3 + k3 12 3 = 0
y^8_0 + y^8_1 + x^9_1 + k_5 + k_5 + k_5 + k_5 = 0
y^8_1 + y^8_1 + y^8_1 + y^8_1 + x^9_1 + k_1_6 + k_2_6_4 + k_3_6_4 = 0
y^8_1 + y^8_1 + x^9_1 + x^9_1 + k_1^6 + k_2^6_4 + k_3^6_4 = 0
y^8_2 + y^8_8 + y^8_15 + x^9_2 + k_1_3 + k_2_3_4 + k_3_3_4 = 0
y^8_2 + x^9_6 + k_1^3 + k_2^3_4 + k_3^3_4 = 0
y^8 3 + y^8 9 + y^8 12 + x^9 3 + k1 2 + k2 2 4 + k3 2 4 = 0
y^8_3 + x^9_7 + k_1_2 + k_2_4 + k_3_4 = 0
y^8_3 + y^8_9 + x^9_15 + k_1^2 + k_2^4 + k_3^2_4 = 0
y^8_4 + y^8_11 + x^9_9 + k_1^7 + k_2^7_4 + k_3^7_4 = 0
y^8_6 + y^8_9 + x^9_{11} + k_{11} + k_{214} + k_{314} = 0
y^8 7 + y^8 10 + x^9 8 + k1 4 + k2 4 4 + k3 4 4 = 0
y^9_0 + y^9_10 + y^9_13 + x^10_0 + k_114 + k_2144 + k_3144 = 0
y^9_0 + x^1_0_4 + k_1_1_4 + k_2_1_4_4 + k_3_1_4_4 = 0
y^9_1 + y^9_1 + x^1_0_1 + x^1_1 + x^1_1 + x^1_2 + x^1_2 + x^1_2 + x^1_3 + x^1_4 + x^1_4 + x^1_5 + x^
y^9_2 + y^9_8 + y^9_15 + [x^10_2] + k_{113} + k_{2134} + k_{3134} = 0
y^9_3 + y^9_9 + y^9_{12} + x^10_3 + k_18 + k_28_4 + k_38_4 = 0
y^9_3 + x^10_7 + k_{18} + k_{284} + k_{384} = 0
y^9_3 + y^9_9 + x^10_15 + k_18 + k_28_4 + k_38_4 = 0
y^9_6 + y^9_9 + x^10_{11} + k_{115} + k_{2154} + k_{3154} = 0
y^9_7 + y^9_10 + x^10_8 + k_110 + k_210_4 + k_310_4 = 0
y^10_0 + y^10_1 + y^10_1 + x^11_0 + k_16 + k_26_5 + k_36_5 = 0
[y^10_2] + y^10_8 + y^10_15 + x^11_2 + k_15 + k_25_5 + k_35_5 = 0
[y^10_2] + x^11_6 + k_15 + k_25_5 + k_35_5 = 0
y^10_3 + y^10_9 + x^11_15 + k_10 + k_20_5 + k_30_5 = 0
y^10_4 + y^10_11 + x^11_9 + k_1_3 + k_2_3_5 + k_3_3_5 = 0
[y^10_6] + y^10_9 + [x^11_1] + k_17 + k_27_5 + k_37_5 = 0
y^10_7 + y^10_10 + x^11_8 + k_1^2 + k_2^5 + k_3^2_5 = 0
y^1_1_0 + x^1_4 + k_1_1_2 + k_2_1_5 + k_3_1_5 = 0
y^11_2 + y^11_8 + y^11_15 + x^12_2 + k_114 + k_214_5 + k_314_5 = 0
y^1_1_6 + y^1_9 + x^1_1_1 + k_1_1 + k_2_1_5 + k_3_1_5 = 0
y^12_2 + x^13_6 + k_16 + k_26_6 + k_36_6 = 0
y^12 4 + y^12 11 + x^13 9 + k1 5 + k2 5 6 + k3 5 6 = 0
```

```
y^13_6 + y^13_9 + [x^14_11] + k_113 + k_213_6 + k_313_6 = 0
```

- $S(x^1_4) + y^1_4 = 0$
- $S(x^1_6) + y^1_6 = 0$
- $S(x^1_8) + y^1_8 = 0$
- $S(x^1_12) + y^1_12 = 0$
- $S(x^2_5) + y^2_5 = 0$
- $S(x^2_6) + y^2_6 = 0$
- $S(x^2_9) + y^2_9 = 0$
- $S(x^2_11) + y^2_11 = 0$
- $S(x^2_13) + y^2_13 = 0$
- $S(x^2_14) + y^2_14= 0$
- $S(x^3_0) + y^3_0 = 0$
- $S(x^3_1) + y^3_1 = 0$
- $S(x^3_6) + y^3_6 = 0$
- $S(x^3_10) + y^3_10 = 0$
- $S(x^3_11) + y^3_11 = 0$
- $S(x^3_12) + y^3_12 = 0$
- $S(x^3_14) + y^3_14 = 0$
- $S(x^4_1) + y^4_1 = 0$
- $S(x^4_3) + y^4_3 = 0$
- $S(x^4_7) + y^4_7 = 0$
- $S(x^4_11) + y^4_11 = 0$
- $S(x^4 12) + y^4 12 = 0$
- $S(x^4_14) + y^4_14 = 0$
- $S(x^5_1) + y^5_1 = 0$
- $S(x^5_3) + y^5_3 = 0$
- $S(x^5_7) + y^5_7 = 0$
- $S(x^5_8) + y^5_8 = 0$
- $S(x^5_15) + y^5_15 = 0$
- $S(x^6_2) + y^6_2 = 0$
- $S(x^6_4) + y^6_4 = 0$
- $S(x^6_5) + y^6_5 = 0$
- $S(x^6_6) + y^6_6 = 0$
- $S(x^6_7) + y^6_7 = 0$
- $S(x^6_8) + y^6_8 = 0$
- $S(x^6_12) + y^6_12 = 0$
- $S(x^{7}_3) + y^{7}_3 = 0$
- $S(x^7_4) + y^7_4 = 0$
- $S(x^{7}_6) + y^{7}_6 = 0$
- $S(x^7_7) + y^7_7 = 0$
- $S(x^7_8) + y^7_8 = 0$
- $S(x^7_9) + y^7_9 = 0$
- $S(x^7_10) + y^7_10 = 0$
- $S(x^7_11) + y^7_11 = 0$

- $S(x^7_12) + y^7_12 = 0$
- $S(x^7_13) + y^7_13 = 0$
- $S(x^8_0) + y^8_0 = 0$
- $S(x^8_3) + y^8_3 = 0$
- $S(x^8_4) + y^8_4 = 0$
- $S(x^8_6) + y^8_6 = 0$
- $S(x^8_7) + y^8_7 = 0$
- $S(x^8_8) + y^8_8 = 0$
- $S(x^8_9) + y^8_9 = 0$
- $S(x^8_11) + y^8_11 = 0$
- $S(x^8_12) + y^8_12 = 0$
- $S(x^8_14) + y^8_14= 0$
- $S(x^8_15) + y^8_15 = 0$
- $S(x^9_1) + y^9_1 = 0$
- $S(x^9_2) + y^9_2 = 0$
- $S(x^9_3) + y^9_3 = 0$
- $S(x^9_6) + y^9_6 = 0$
- $S(x^9 7) + y^9 7 = 0$
- $S(x^9_8) + y^9_8 = 0$
- $S(x^9_9) + y^9_9 = 0$
- $S(x^9_11) + y^9_11 = 0$
- $S(x^9_12) + y^9_12 = 0$
- $S(x^9 13) + y^9 13 = 0$
- $S(x^9_15) + y^9_15 = 0$
- $S(x^10_0) + y^10_0 = 0$
- $S(x^10_3) + y^10_3 = 0$
- $S(x^10_4) + y^10_4 = 0$
- $S(x^10_7) + y^10_7 = 0$
- $S(x^10_8) + y^10_8 = 0$
- $S(x^10_11) + y^10_11 = 0$
- $S(x^10_13) + y^10_13 = 0$
- o(..._o__o_
- $S(x^10_15) + y^10_15 = 0$
- $S(x^11_0) + y^11_0 = 0$
- $S(x^11_2) + y^11_2 = 0$
- $S(x^1_{-6}) + y^1_{-6} = 0$
- $S(x^11_8) + y^11_8 = 0$
- $S(x^11_9) + y^11_9 = 0$
- $S(x^11_15) + y^11_15 = 0$
- $S(x^12_2) + y^12_2 = 0$
- $S(x^12_4) + y^12_4 = 0$
- $S(x^12_11) + y^12_11 = 0$
- $S(x^13_6) + y^13_6 = 0$
- $S(x^13_9) + y^13_9 = 0$

the 4 th Minimal Constraint set is:

```
y^0_2 + [y^0_8] + [y^0_15] + x^1_2 + k_1_2 + k_2_2_0 + k_3_2_0 = 0
y^0_2 + x^1_6 + k_1_2 + k_2_0 + k_3_0 = 0
y^1_1 + y^1_1 + [y^1_1] + [x^2_1] + k_1_1 + k_2_1 + k_2_1 = 0
y^1_1 + y^1_1 + x^2_1 + k_1_1 + k_2_1 + k_2_1 = 0
y^1_2 + x^2_6 + k_1_8 + k_2_8_0 + k_3_8_0 = 0
y^1_3 + [x^2_7] + k_{113} + k_{2130} + k_{3130} = 0
y^1 3 + y^1 9 + [x^2 15] + k1 13 + k2 13 0 + k3 13 0 = 0
y^1_6 + y^1_9 + x^2_11 + k_1_12 + k_2_12_0 + k_3_12_0 = 0
v^2 0 + v^2 10 + v^2 13 + v^3 0 + k1 1 + k2 1 1 + k3 1 1 = 0
y^2_0 + x^3_4 + k_{11} + k_{211} + k_{311} = 0
 [y^2 1] + y^2 11 + x^3 13 + k1 7 + k2 7 1 + k3 7 1 = 0
 [y^2_3] + x^3_7 + k_1_5 + k_2_5_1 + k_3_5_1 = 0
 [y^2_3] + y^2_9 + x^3_15 + k_1^5 + k_2^5_1 + k_3^5_1 = 0
y^2_6 + y^2_9 + x^3_{11} + k_{14} + k_{24} + k_{34} = 0
[y^2-7] + y^2-10 + [x^3-8] + k_1-3 + k_2-3-1 + k_3-3-1=0
y^3 0 + y^3 10 + y^3 13 + x^4 0 + k1 15 + k2 15 1 + k3 15 1 = 0
[y^3_2] + [y^3_8] + y^3_15 + x^4_2 + k_1_9 + k_2_9_1 + k_3_9_1 = 0
y^3 4 + y^3 11 + x^4 9 + k1 8 + k2 8 1 + k3 8 1 = 0
y^3_7 + y^3_10 + x^4_8 + k_1_{13} + k_2_{13}_1 + k_3_{13}_1 = 0
y^4_0 + [x^5_4] + k_1^7 + k_2^7_2 + k_3^7_2 = 0
y^4_2 + y^4_8 + x^5_14 + k_11 + k_21_2 + k_31_2 = 0
[y^4_6] + y^4_9 + [x^5_11] + k_1^2 + k_2^2_2 + k_3^2_2 = 0
y^5 1 + [y^5 11] + y^5 14 + [x^6 1] + k1 13 + k2 13 2 + k3 13 2 = 0
y^5_1 + [y^5_11] + [x^6_13] + k_1_13 + k_2_13_2 + k_3_13_2 = 0
 S(x^1_2) + y^1_2 = 0
 S(x^1_6) + y^1_6 = 0
 S(x^2_6) + y^2_6 = 0
 S(x^2 11) + y^2 11 = 0
 S(x^2 13) + y^2 13 = 0
 S(x^3_0) + y^3_0 = 0
 S(x^3_4) + y^3_4 = 0
 S(x^3_1) + y^3_1 = 0
 S(x^3_11) + y^3_11 = 0
 S(x^3_13) + y^3_13 = 0
 S(x^3_15) + y^3_15 = 0
 S(x^4_0) + y^4_0 = 0
 S(x^4_2) + y^4_2 = 0
 S(x^4_8) + y^4_8 = 0
 S(x^4_9) + y^4_9 = 0
 S(x^5 14) + y^5 14 = 0
the 5 th Minimal Constraint set is:
[y^2_1] + x^3_5 + k_1^7 + k_2^7_1 + k_3^7_1 = 0
y^3_5 + [y^3_8] + [x^4_10] + k_112 + k_2121 + k_3121 = 0
 S(x^3 5) + y^3 5 = 0
```

the 6 th Minimal Constraint set is:

$$[y^3_2] + [x^4_6] + k_1_9 + k_2_9_1 + k_3_9_1 = 0$$

the 7 th Minimal Constraint set is:

$$[y^11_1] + [x^12_5] + k1_10 + k2_10_5 + k3_10_5 = 0$$