

Minimal Constraint Subsets Detected for differential trail in Table 7 DDH21+ and PT22

the 0 th Minimal Constraint set is:

$$y^{0_0} + [y^{0_{10}}] + [y^{0_{13}}] + [x^{1_0}] + k_0 = 0$$

$$y^{0_0} + [y^{0_{10}}] + x^{1_{12}} + k_0 = 0$$

$$[y^{0_7}] + [y^{0_{10}}] + x^{1_8} + k_7 = 0$$

$$y^{1_2} + x^{2_6} + k_8 = 0$$

$$y^{1_2} + y^{1_8} + x^{2_{14}} + k_8 = 0$$

$$y^{1_3} + y^{1_9} + y^{1_{12}} + x^{2_3} + k_{13} = 0$$

$$y^{1_3} + y^{1_9} + x^{2_{15}} + k_{13} = 0$$

$$y^{2_1} + y^{2_{11}} + y^{2_{14}} + x^{3_1} + k_7 = 0$$

$$y^{2_1} + y^{2_{11}} + x^{3_{13}} + k_7 = 0$$

$$y^{2_2} + y^{2_8} + y^{2_{15}} + [x^{3_2}] + k_0 = 0$$

$$y^{2_2} + y^{2_8} + [x^{3_{14}}] + k_0 = 0$$

$$y^{2_3} + y^{2_9} + x^{3_{15}} + k_5 = 0$$

$$y^{2_6} + y^{2_9} + x^{3_{11}} + k_4 = 0$$

$$y^{3_0} + y^{3_{10}} + y^{3_{13}} + x^{4_0} + k_{15} = 0$$

$$y^{3_0} + y^{3_{10}} + x^{4_{12}} + k_{15} = 0$$

$$y^{3_1} + y^{3_{11}} + [y^{3_{14}}] + [x^{4_1}] + k_{11} = 0$$

$$[y^{3_2}] + y^{3_8} + y^{3_{15}} + x^{4_2} + k_9 = 0$$

$$[y^{3_2}] + y^{3_8} + x^{4_{14}} + k_9 = 0$$

$$y^{3_3} + x^{4_7} + k_{14} = 0$$

$$y^{3_3} + y^{3_9} + x^{4_{15}} + k_{14} = 0$$

$$[y^{3_6}] + y^{3_9} + [x^{4_{11}}] + k_{10} = 0$$

$$y^{4_0} + y^{4_{10}} + x^{5_{12}} + k_7 = 0$$

$$[y^{4_1}] + [y^{4_{11}}] + y^{4_{14}} + x^{5_1} + k_3 = 0$$

$$[y^{4_1}] + [y^{4_{11}}] + x^{5_{13}} + k_3 = 0$$

$$y^{4_2} + y^{4_8} + y^{4_{15}} + x^{5_2} + k_1 = 0$$

$$y^{4_2} + x^{5_6} + k_1 = 0$$

$$y^{4_2} + y^{4_8} + x^{5_{14}} + k_1 = 0$$

$$y^{4_3} + y^{4_9} + y^{4_{12}} + x^{5_3} + k_6 = 0$$

$$y^{4_3} + y^{4_9} + x^{5_{15}} + k_6 = 0$$

$$y^{4_7} + y^{4_{10}} + x^{5_8} + k_5 = 0$$

$$y^{5_0} + y^{5_{10}} + y^{5_{13}} + x^{6_0} + k_{11} = 0$$

$$y^{5_0} + y^{5_{10}} + x^{6_{12}} + k_{11} = 0$$

$$y^{5_1} + y^{5_{11}} + y^{5_{14}} + x^{6_1} + k_{13} = 0$$

$$y^{5_1} + x^{6_5} + k_{13} = 0$$

$$y^{5_1} + y^{5_{11}} + x^{6_{13}} + k_{13} = 0$$

$$y^{5_2} + y^{5_8} + y^{5_{15}} + x^{6_2} + k_{15} = 0$$

$$y^{5_2} + x^{6_6} + k_{15} = 0$$

$$y^{5_2} + y^{5_8} + x^{6_{14}} + k_{15} = 0$$

$$y^{5_3} + y^{5_9} + y^{5_{12}} + x^{6_3} + k_{12} = 0$$

$$[y^{5_5}] + y^{5_8} + x^{6_{10}} + k_{10} = 0$$

$$\begin{aligned}
&y^5_6 + y^5_9 + x^6_{11} + k_8 = 0 \\
&y^6_0 + y^6_{10} + y^6_{13} + x^7_0 + k_3 = 0 \\
&y^6_1 + y^6_{11} + y^6_{14} + x^7_1 + k_5 = 0 \\
&y^6_2 + y^6_8 + x^7_{14} + k_7 = 0 \\
&y^6_3 + [y^6_9] + y^6_{12} + [x^7_3] + k_4 = 0 \\
&y^6_3 + x^7_7 + k_4 = 0 \\
&y^6_3 + [y^6_9] + [x^7_{15}] + k_4 = 0 \\
&y^6_5 + y^6_8 + x^7_{10} + k_2 = 0 \\
&y^6_6 + [y^6_9] + x^7_{11} + k_0 = 0 \\
&y^7_0 + x^8_4 + k_{13} = 0 \\
&y^7_1 + y^7_{11} + y^7_{14} + x^8_1 + k_{14} = 0 \\
&y^7_7 + y^7_{10} + x^8_8 + k_{12} = 0 \\
&y^8_1 + [y^8_{11}] + [x^9_{13}] + k_6 = 0 \\
&[y^8_2] + y^8_8 + [x^9_{14}] + k_3 = 0 \\
&y^8_4 + [y^8_{11}] + [x^9_9] + k_7 = 0 \\
&S(x^1_8) + y^1_8 = 0 \\
&S(x^1_{12}) + y^1_{12} = 0 \\
&S(x^2_3) + y^2_3 = 0 \\
&S(x^2_6) + y^2_6 = 0 \\
&S(x^2_{14}) + y^2_{14} = 0 \\
&S(x^2_{15}) + y^2_{15} = 0 \\
&S(x^3_1) + y^3_1 = 0 \\
&S(x^3_{11}) + y^3_{11} = 0 \\
&S(x^3_{13}) + y^3_{13} = 0 \\
&S(x^3_{15}) + y^3_{15} = 0 \\
&S(x^4_0) + y^4_0 = 0 \\
&S(x^4_2) + y^4_2 = 0 \\
&S(x^4_7) + y^4_7 = 0 \\
&S(x^4_{12}) + y^4_{12} = 0 \\
&S(x^4_{14}) + y^4_{14} = 0 \\
&S(x^4_{15}) + y^4_{15} = 0 \\
&S(x^5_1) + y^5_1 = 0 \\
&S(x^5_2) + y^5_2 = 0 \\
&S(x^5_3) + y^5_3 = 0 \\
&S(x^5_6) + y^5_6 = 0 \\
&S(x^5_8) + y^5_8 = 0 \\
&S(x^5_{12}) + y^5_{12} = 0 \\
&S(x^5_{13}) + y^5_{13} = 0 \\
&S(x^5_{14}) + y^5_{14} = 0 \\
&S(x^5_{15}) + y^5_{15} = 0 \\
&S(x^6_0) + y^6_0 = 0 \\
&S(x^6_1) + y^6_1 = 0 \\
&S(x^6_2) + y^6_2 = 0 \\
&S(x^6_3) + y^6_3 = 0
\end{aligned}$$

$$\begin{aligned}
S(x^6_5) + y^6_5 &= 0 \\
S(x^6_6) + y^6_6 &= 0 \\
S(x^6_{10}) + y^6_{10} &= 0 \\
S(x^6_{11}) + y^6_{11} &= 0 \\
S(x^6_{12}) + y^6_{12} &= 0 \\
S(x^6_{13}) + y^6_{13} &= 0 \\
S(x^6_{14}) + y^6_{14} &= 0 \\
S(x^7_0) + y^7_0 &= 0 \\
S(x^7_1) + y^7_1 &= 0 \\
S(x^7_7) + y^7_7 &= 0 \\
S(x^7_{10}) + y^7_{10} &= 0 \\
S(x^7_{11}) + y^7_{11} &= 0 \\
S(x^7_{14}) + y^7_{14} &= 0 \\
S(x^8_1) + y^8_1 &= 0 \\
S(x^8_4) + y^8_4 &= 0 \\
S(x^8_8) + y^8_8 &= 0
\end{aligned}$$

the 1 th Minimal Constraint set is:

$$\begin{aligned}
y^0_0 + [y^0_{10}] + [y^0_{13}] + [x^1_0] + k_0 &= 0 \\
y^0_0 + [y^0_{10}] + x^1_{12} + k_0 &= 0 \\
y^1_3 + y^1_9 + y^1_{12} + x^2_3 + k_{13} &= 0 \\
y^1_3 + y^1_9 + x^2_{15} + k_{13} &= 0 \\
y^2_2 + y^2_8 + y^2_{15} + [x^3_2] + k_0 &= 0 \\
y^2_2 + y^2_8 + [x^3_{14}] + k_0 &= 0 \\
y^2_3 + x^3_7 + k_5 &= 0 \\
y^3_0 + x^4_4 + k_{15} &= 0 \\
y^3_0 + y^3_{10} + x^4_{12} + k_{15} &= 0 \\
y^3_1 + y^3_{11} + [y^3_{14}] + [x^4_1] + k_{11} &= 0 \\
y^3_1 + y^3_{11} + x^4_{13} + k_{11} &= 0 \\
[y^3_2] + x^4_6 + k_9 &= 0 \\
y^3_7 + y^3_{10} + x^4_8 + k_{13} &= 0 \\
y^4_0 + y^4_{10} + y^4_{13} + x^5_0 + k_7 &= 0 \\
y^4_0 + y^4_{10} + x^5_{12} + k_7 &= 0 \\
y^4_2 + x^5_6 + k_1 &= 0 \\
y^4_2 + y^4_8 + x^5_{14} + k_1 &= 0 \\
y^4_3 + y^4_9 + y^4_{12} + x^5_3 + k_6 &= 0 \\
y^4_3 + x^5_7 + k_6 &= 0 \\
y^4_4 + [y^4_{11}] + x^5_9 + k_0 &= 0 \\
y^4_6 + y^4_9 + x^5_{11} + k_2 &= 0 \\
y^5_0 + y^5_{10} + x^6_{12} + k_{11} &= 0 \\
y^5_1 + y^5_{11} + y^5_{14} + x^6_1 + k_{13} &= 0 \\
y^5_1 + x^6_5 + k_{13} &= 0 \\
y^5_2 + x^6_6 + k_{15} &= 0 \\
y^5_2 + y^5_8 + x^6_{14} + k_{15} &= 0 \\
y^5_3 + y^5_9 + y^5_{12} + x^6_3 + k_{12} &= 0
\end{aligned}$$

$$\begin{aligned}
&y^5_3 + x^6_7 + k_{12} = 0 \\
&y^5_3 + y^5_9 + x^6_{15} + k_{12} = 0 \\
&[y^5_5] + y^5_8 + x^6_{10} + k_{10} = 0 \\
&y^5_6 + y^5_9 + x^6_{11} + k_8 = 0 \\
&y^5_7 + y^5_{10} + x^6_8 + k_{14} = 0 \\
&y^6_1 + y^6_{11} + y^6_{14} + x^7_1 + k_5 = 0 \\
&y^6_2 + y^6_8 + y^6_{15} + x^7_2 + k_7 = 0 \\
&y^6_2 + y^6_8 + x^7_{14} + k_7 = 0 \\
&y^6_3 + [y^6_9] + y^6_{12} + [x^7_3] + k_4 = 0 \\
&y^6_3 + x^7_7 + k_4 = 0 \\
&y^6_3 + [y^6_9] + [x^7_{15}] + k_4 = 0 \\
&y^6_5 + y^6_8 + x^7_{10} + k_2 = 0 \\
&y^6_6 + [y^6_9] + x^7_{11} + k_0 = 0 \\
&y^6_7 + y^6_{10} + x^7_8 + k_6 = 0 \\
&y^7_1 + y^7_{11} + y^7_{14} + x^8_1 + k_{14} = 0 \\
&y^7_2 + y^7_8 + x^8_{14} + k_{11} = 0 \\
&y^7_7 + y^7_{10} + x^8_8 + k_{12} = 0 \\
&y^8_1 + [y^8_{11}] + y^8_{14} + [x^9_1] + k_6 = 0 \\
&[y^8_2] + y^8_8 + [x^9_{14}] + k_3 = 0 \\
&S(x^{11}_2) + y^{11}_2 = 0 \\
&S(x^{12}_3) + y^{12}_3 = 0 \\
&S(x^{15}_2) + y^{15}_2 = 0 \\
&S(x^{17}_3) + y^{17}_3 = 0 \\
&S(x^{19}_4) + y^{19}_4 = 0 \\
&S(x^{21}_6) + y^{21}_6 = 0 \\
&S(x^{23}_8) + y^{23}_8 = 0 \\
&S(x^{25}_{12}) + y^{25}_{12} = 0 \\
&S(x^{27}_{13}) + y^{27}_{13} = 0 \\
&S(x^{30}_5) + y^{30}_5 = 0 \\
&S(x^{33}_3) + y^{33}_3 = 0 \\
&S(x^{35}_6) + y^{35}_6 = 0 \\
&S(x^{37}_7) + y^{37}_7 = 0 \\
&S(x^{39}_9) + y^{39}_9 = 0 \\
&S(x^{41}_{11}) + y^{41}_{11} = 0 \\
&S(x^{43}_{12}) + y^{43}_{12} = 0 \\
&S(x^{45}_{14}) + y^{45}_{14} = 0 \\
&S(x^{47}_1) + y^{47}_1 = 0 \\
&S(x^{49}_3) + y^{49}_3 = 0 \\
&S(x^{51}_5) + y^{51}_5 = 0 \\
&S(x^{53}_6) + y^{53}_6 = 0 \\
&S(x^{55}_7) + y^{55}_7 = 0 \\
&S(x^{57}_8) + y^{57}_8 = 0 \\
&S(x^{59}_{10}) + y^{59}_{10} = 0 \\
&S(x^{61}_{11}) + y^{61}_{11} = 0
\end{aligned}$$

$$S(x^{6}_{12}) + y^{6}_{12} = 0$$

$$S(x^{6}_{14}) + y^{6}_{14} = 0$$

$$S(x^{6}_{15}) + y^{6}_{15} = 0$$

$$S(x^{7}_{1}) + y^{7}_{1} = 0$$

$$S(x^{7}_{2}) + y^{7}_{2} = 0$$

$$S(x^{7}_{7}) + y^{7}_{7} = 0$$

$$S(x^{7}_{8}) + y^{7}_{8} = 0$$

$$S(x^{7}_{10}) + y^{7}_{10} = 0$$

$$S(x^{7}_{11}) + y^{7}_{11} = 0$$

$$S(x^{7}_{14}) + y^{7}_{14} = 0$$

$$S(x^{8}_{1}) + y^{8}_{1} = 0$$

$$S(x^{8}_{8}) + y^{8}_{8} = 0$$

$$S(x^{8}_{14}) + y^{8}_{14} = 0$$

the 2 th Minimal Constraint set is:

$$y^0_0 + [y^0_{10}] + [y^0_{13}] + [x^1_0] + k_0 = 0$$

$$y^0_0 + [y^0_{10}] + x^1_{12} + k_0 = 0$$

$$[y^1_0] + x^2_4 + k_9 = 0$$

$$y^1_3 + y^1_9 + y^1_{12} + x^2_3 + k_{13} = 0$$

$$y^1_3 + y^1_9 + x^2_{15} + k_{13} = 0$$

$$y^2_1 + x^3_5 + k_7 = 0$$

$$y^2_1 + y^2_{11} + x^3_{13} + k_7 = 0$$

$$y^2_2 + y^2_8 + y^2_{15} + [x^3_2] + k_0 = 0$$

$$y^2_2 + y^2_8 + [x^3_{14}] + k_0 = 0$$

$$y^2_3 + x^3_7 + k_5 = 0$$

$$y^2_4 + y^2_{11} + x^3_9 + k_2 = 0$$

$$y^3_0 + y^3_{10} + y^3_{13} + x^4_0 + k_{15} = 0$$

$$y^3_0 + x^4_4 + k_{15} = 0$$

$$[y^3_2] + x^4_6 + k_9 = 0$$

$$[y^3_2] + y^3_8 + x^4_{14} + k_9 = 0$$

$$y^3_3 + x^4_7 + k_{14} = 0$$

$$y^3_3 + y^3_9 + x^4_{15} + k_{14} = 0$$

$$y^3_5 + y^3_8 + x^4_{10} + k_{12} = 0$$

$$y^3_7 + y^3_{10} + x^4_8 + k_{13} = 0$$

$$y^4_0 + x^5_4 + k_7 = 0$$

$$[y^4_1] + [y^4_{11}] + y^4_{14} + x^5_1 + k_3 = 0$$

$$y^4_2 + y^4_8 + y^4_{15} + x^5_2 + k_1 = 0$$

$$y^4_2 + x^5_6 + k_1 = 0$$

$$y^4_3 + x^5_7 + k_6 = 0$$

$$y^4_3 + y^4_9 + x^5_{15} + k_6 = 0$$

$$y^4_4 + [y^4_{11}] + x^5_9 + k_0 = 0$$

$$y^4_6 + y^4_9 + x^5_{11} + k_2 = 0$$

$$y^4_7 + y^4_{10} + x^5_8 + k_5 = 0$$

$$y^5_0 + x^6_4 + k_{11} = 0$$

$$y^5_0 + y^5_{10} + x^6_{12} + k_{11} = 0$$

$$\begin{aligned}
&y^5_1 + x^6_5 + k_{13} = 0 \\
&y^5_2 + y^5_8 + y^5_{15} + x^6_2 + k_{15} = 0 \\
&y^5_4 + y^5_{11} + [x^6_9] + k_9 = 0 \\
&y^5_6 + y^5_9 + x^6_{11} + k_8 = 0 \\
&y^5_7 + y^5_{10} + x^6_8 + k_{14} = 0 \\
&y^6_2 + x^7_6 + k_7 = 0 \\
&y^6_3 + [y^6_9] + y^6_{12} + [x^7_3] + k_4 = 0 \\
&y^6_3 + x^7_7 + k_4 = 0 \\
&y^6_4 + y^6_{11} + x^7_9 + k_1 = 0 \\
&y^6_5 + y^6_8 + x^7_{10} + k_2 = 0 \\
&y^7_6 + y^7_9 + [x^8_{11}] + k_9 = 0 \\
&y^7_7 + y^7_{10} + x^8_8 + k_{12} = 0 \\
&[y^8_2] + y^8_8 + [x^9_{14}] + k_3 = 0 \\
&S(x^1_{12}) + y^1_{12} = 0 \\
&S(x^2_3) + y^2_3 = 0 \\
&S(x^2_4) + y^2_4 = 0 \\
&S(x^2_{15}) + y^2_{15} = 0 \\
&S(x^3_5) + y^3_5 = 0 \\
&S(x^3_7) + y^3_7 = 0 \\
&S(x^3_9) + y^3_9 = 0 \\
&S(x^3_{13}) + y^3_{13} = 0 \\
&S(x^4_0) + y^4_0 = 0 \\
&S(x^4_4) + y^4_4 = 0 \\
&S(x^4_6) + y^4_6 = 0 \\
&S(x^4_7) + y^4_7 = 0 \\
&S(x^4_8) + y^4_8 = 0 \\
&S(x^4_{10}) + y^4_{10} = 0 \\
&S(x^4_{14}) + y^4_{14} = 0 \\
&S(x^4_{15}) + y^4_{15} = 0 \\
&S(x^5_1) + y^5_1 = 0 \\
&S(x^5_2) + y^5_2 = 0 \\
&S(x^5_4) + y^5_4 = 0 \\
&S(x^5_6) + y^5_6 = 0 \\
&S(x^5_7) + y^5_7 = 0 \\
&S(x^5_8) + y^5_8 = 0 \\
&S(x^5_9) + y^5_9 = 0 \\
&S(x^5_{11}) + y^5_{11} = 0 \\
&S(x^5_{15}) + y^5_{15} = 0 \\
&S(x^6_2) + y^6_2 = 0 \\
&S(x^6_4) + y^6_4 = 0 \\
&S(x^6_5) + y^6_5 = 0 \\
&S(x^6_8) + y^6_8 = 0 \\
&S(x^6_{11}) + y^6_{11} = 0 \\
&S(x^6_{12}) + y^6_{12} = 0
\end{aligned}$$

$$S(x^{7_6}) + y^{7_6} = 0$$

$$S(x^{7_7}) + y^{7_7} = 0$$

$$S(x^{7_9}) + y^{7_9} = 0$$

$$S(x^{7_{10}}) + y^{7_{10}} = 0$$

$$S(x^{8_8}) + y^{8_8} = 0$$

the 3 th Minimal Constraint set is:

$$[y^{0_7}] + [y^{0_{10}}] + x^{1_8} + k_7 = 0$$

$$y^{1_2} + x^{2_6} + k_8 = 0$$

$$y^{1_2} + y^{1_8} + x^{2_{14}} + k_8 = 0$$

$$y^{2_1} + y^{2_{11}} + y^{2_{14}} + x^{3_1} + k_7 = 0$$

$$y^{2_1} + y^{2_{11}} + x^{3_{13}} + k_7 = 0$$

$$y^{2_3} + x^{3_7} + k_5 = 0$$

$$y^{2_3} + y^{2_9} + x^{3_{15}} + k_5 = 0$$

$$y^{2_6} + y^{2_9} + x^{3_{11}} + k_4 = 0$$

$$y^{3_0} + y^{3_{10}} + y^{3_{13}} + x^{4_0} + k_{15} = 0$$

$$y^{3_0} + x^{4_4} + k_{15} = 0$$

$$y^{3_1} + y^{3_{11}} + x^{4_{13}} + k_{11} = 0$$

$$[y^{3_2}] + y^{3_8} + y^{3_{15}} + x^{4_2} + k_9 = 0$$

$$[y^{3_2}] + x^{4_6} + k_9 = 0$$

$$[y^{3_2}] + y^{3_8} + x^{4_{14}} + k_9 = 0$$

$$y^{3_3} + x^{4_7} + k_{14} = 0$$

$$y^{3_3} + y^{3_9} + x^{4_{15}} + k_{14} = 0$$

$$[y^{3_6}] + y^{3_9} + [x^{4_{11}}] + k_{10} = 0$$

$$y^{3_7} + y^{3_{10}} + x^{4_8} + k_{13} = 0$$

$$y^{4_0} + y^{4_{10}} + y^{4_{13}} + x^{5_0} + k_7 = 0$$

$$[y^{4_1}] + [y^{4_{11}}] + y^{4_{14}} + x^{5_1} + k_3 = 0$$

$$[y^{4_1}] + [y^{4_{11}}] + x^{5_{13}} + k_3 = 0$$

$$y^{4_2} + y^{4_8} + y^{4_{15}} + x^{5_2} + k_1 = 0$$

$$y^{4_3} + x^{5_7} + k_6 = 0$$

$$y^{4_3} + y^{4_9} + x^{5_{15}} + k_6 = 0$$

$$y^{4_4} + [y^{4_{11}}] + x^{5_9} + k_0 = 0$$

$$y^{4_6} + y^{4_9} + x^{5_{11}} + k_2 = 0$$

$$y^{4_7} + y^{4_{10}} + x^{5_8} + k_5 = 0$$

$$y^{5_0} + y^{5_{10}} + y^{5_{13}} + x^{6_0} + k_{11} = 0$$

$$y^{5_1} + y^{5_{11}} + x^{6_{13}} + k_{13} = 0$$

$$y^{5_2} + y^{5_8} + y^{5_{15}} + x^{6_2} + k_{15} = 0$$

$$y^{5_3} + x^{6_7} + k_{12} = 0$$

$$y^{5_3} + y^{5_9} + x^{6_{15}} + k_{12} = 0$$

$$y^{5_7} + y^{5_{10}} + x^{6_8} + k_{14} = 0$$

$$y^{6_0} + y^{6_{10}} + y^{6_{13}} + x^{7_0} + k_3 = 0$$

$$y^{6_2} + y^{6_8} + y^{6_{15}} + x^{7_2} + k_7 = 0$$

$$y^{6_7} + y^{6_{10}} + x^{7_8} + k_6 = 0$$

$$y^{7_0} + x^{8_4} + k_{13} = 0$$

$$y^{7_2} + y^{7_8} + [y^{7_{15}}] + [x^{8_2}] + k_{11} = 0$$

$$y^{8_4} + [y^{8_{11}}] + [x^{9_9}] + k_7 = 0$$

$$S(x^{1_8}) + y^{1_8} = 0$$

$$S(x^{2_6}) + y^{2_6} = 0$$

$$S(x^{2_{14}}) + y^{2_{14}} = 0$$

$$S(x^{3_1}) + y^{3_1} = 0$$

$$S(x^{3_7}) + y^{3_7} = 0$$

$$S(x^{3_{11}}) + y^{3_{11}} = 0$$

$$S(x^{3_{13}}) + y^{3_{13}} = 0$$

$$S(x^{3_{15}}) + y^{3_{15}} = 0$$

$$S(x^{4_0}) + y^{4_0} = 0$$

$$S(x^{4_2}) + y^{4_2} = 0$$

$$S(x^{4_4}) + y^{4_4} = 0$$

$$S(x^{4_6}) + y^{4_6} = 0$$

$$S(x^{4_7}) + y^{4_7} = 0$$

$$S(x^{4_8}) + y^{4_8} = 0$$

$$S(x^{4_{13}}) + y^{4_{13}} = 0$$

$$S(x^{4_{14}}) + y^{4_{14}} = 0$$

$$S(x^{4_{15}}) + y^{4_{15}} = 0$$

$$S(x^{5_0}) + y^{5_0} = 0$$

$$S(x^{5_1}) + y^{5_1} = 0$$

$$S(x^{5_2}) + y^{5_2} = 0$$

$$S(x^{5_7}) + y^{5_7} = 0$$

$$S(x^{5_8}) + y^{5_8} = 0$$

$$S(x^{5_9}) + y^{5_9} = 0$$

$$S(x^{5_{11}}) + y^{5_{11}} = 0$$

$$S(x^{5_{13}}) + y^{5_{13}} = 0$$

$$S(x^{5_{15}}) + y^{5_{15}} = 0$$

$$S(x^{6_0}) + y^{6_0} = 0$$

$$S(x^{6_2}) + y^{6_2} = 0$$

$$S(x^{6_7}) + y^{6_7} = 0$$

$$S(x^{6_8}) + y^{6_8} = 0$$

$$S(x^{6_{13}}) + y^{6_{13}} = 0$$

$$S(x^{6_{15}}) + y^{6_{15}} = 0$$

$$S(x^{7_0}) + y^{7_0} = 0$$

$$S(x^{7_2}) + y^{7_2} = 0$$

$$S(x^{7_8}) + y^{7_8} = 0$$

$$S(x^{8_4}) + y^{8_4} = 0$$

the 4 th Minimal Constraint set is:

$$y^{0_0} + [y^{0_{10}}] + [y^{0_{13}}] + [x^{1_0}] + k_0 = 0$$

$$y^{0_0} + [x^{1_4}] + k_0 = 0$$

the 5 th Minimal Constraint set is:

$$[y^{4_1}] + [x^{5_5}] + k_3 = 0$$

the 6 th Minimal Constraint set is:

$$[y^{8_2}] + [x^{9_6}] + k_3 = 0$$