Orbital Angular Momentum States of two-photon entanglement

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Light is not just another form of energy; it is the basis of all life, the key to understanding the universe, and the foundation of modern technology.

Neutrino Astrophysics

John N. Bahcall
Astrophysicist

Outline

1. Overview
2. Motivation
   - Why OAM (Orbital Angular Momentum)
   - Latest Advancements
3. Plan of Work
4. Literature review
   - OAM (Orbital Angular Momentum)
   - LG Beam
   - Higher LG Modes
5. SPDC (Spontaneous parametric down conversion)
6. Detection probabilities
7. Correlation Function
8. Correlation through LG Modes
9. Future aims

Introduction

We study the OAM(orbital angular momentum) correlation and detection probabilities of a photon pair created in a spontaneous parametric down-conversion process. We quantify and analyze the correlation between different transverse modes of both photons Using Laguerre-Gaussian modes formalism. [1]

Sonja Franke-Arnold, Stephen M. Barnett, Miles J. Padgett, and L. Allen Phys. Rev. A 65, 033823

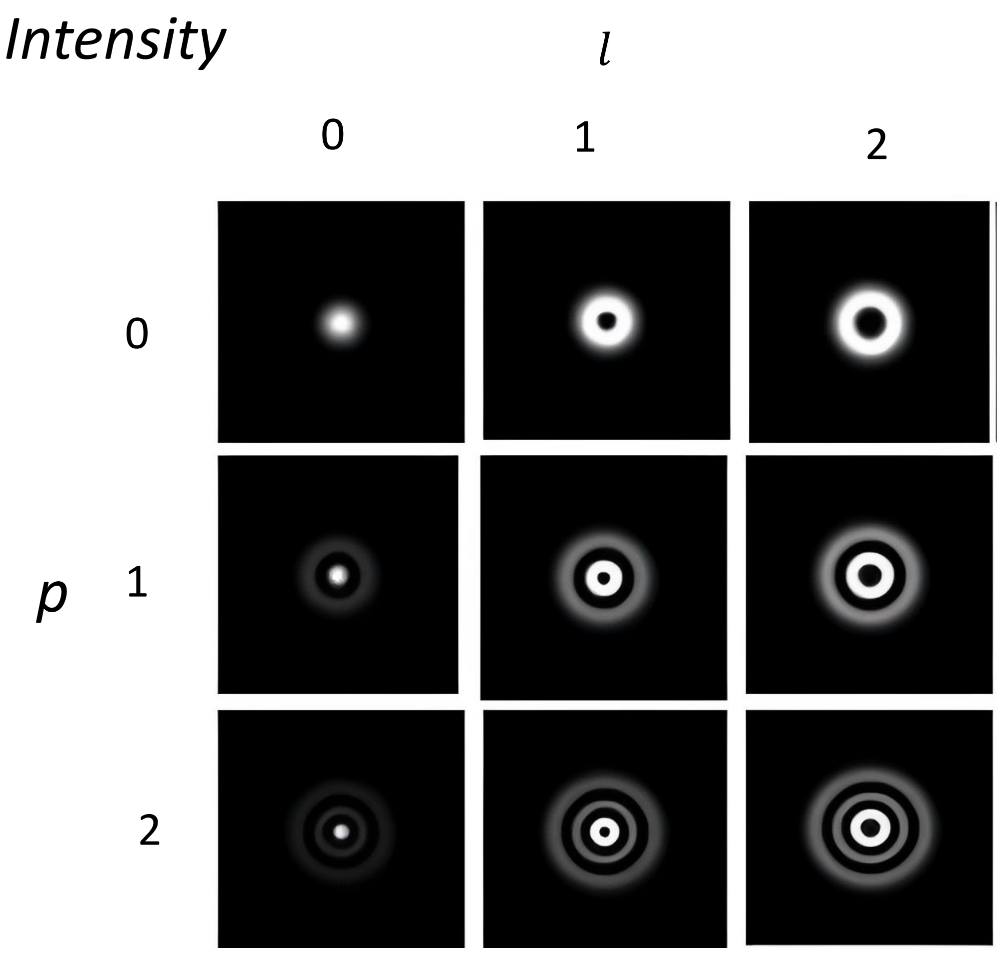
WHY OAM?

- Richer Structure
- High-Dimensional Entanglement[2]

$$dimensionality = N + 1$$

$$N = l + 2p$$

- Resistant to noise
- Resistant to decoherence



1- L. Allen, J. Courtial, and M.J. Padgett, Phys. Rev. E 60, 7497
2- Wen, H., Zeng, L., Ma, R., Kang, H., Liu, J., Qin, Z., & Su, X. (2022). Quantum coherence of an orbital angular momentum multiplexed continuous-variable entangled state. Optics Continuum

Latest Advancements

Orbital angular momentum photonic quantum interface

Zhi-Yuan Zhou[1,2]*, Yan Li[1,2]*, Dong-Sheng Ding[1,2], Wei Zhang[1,2], Shuai Shi[1,2], Bao-Sen Shi[1,2] and Guang-Can Guo[1,2]

Orbital angular momentum holography for high-security encryption

Xinyuan Fang[1,2,3,4], Haoran Ren[2,4] and Min Gu[1,2]*

Orbital Angular Momentum Based Sensing and Their Applications: A Review

Yi Weng ; Zhongqi Pan **All Authors**

Zhou, ZY., Li, Y., Ding, DS. et al. Orbital angular momentum photonic quantum interface. Light Sci Appl 5, e16019 (2016). https://doi.org/10.1038/lsa.2016.19
Y. Weng and Z. Pan, "Orbital Angular Momentum Based Sensing and Their Applications: A Review," in Journal of Lightwave Technology, vol. 41, no. 7, pp. 2007-2016, 1 April1, 2023, doi: 10.1109/JLT.2022.3202184.
Ma, Minghao ; Lian, Yudong ; Wang, Yulei ; Lu, Zhiwei https://ui.adsabs.harvard.edu/link_gateway/2021FrP.....9..703M/doi:10.3389/fphy.2021.773505

Plan of Work

Literature Review

(Completed)

- Understanding the system (SPDC)
- Understand Laguerre-Gaussian Beam.
- Understanding Orbital Angular Momentum OAM

Probability Calculations

(Completed)

- Individual and coincident detection probabilities
- Correlation wavefunction b/w photon pair

Laguerre-Gaussian formalism

(Partially Completed)

- Working out Laguerre-gaussian modes formalism and linking it to OAM entanglement
- Working out probability functions in LG Mode formalism
- Plotting intensity and probability functions graphs

Literature Review

Franke-Arnold, S. , Barnett, S. M. Padgett, M. and Allen, L.(2002) Two-photon entanglement of orbital angular momentum states. *Physical Review A:* 65(3), art 033823.

Miles Padgett, Johannes Courtial, Les Allen; Light's Orbital Angular Momentum. *Physics Today* 1 May 2004; 57 (5): 35–40. https://doi.org/10.1063/1.1768672

H. Kogelnik and T. Li, "Laser beams and resonators," in *Proceedings of the IEEE*, vol. 54, no. 10, pp. 1312-1329, Oct. 1966, doi: 10.1109/PROC.1966.5119.

Paraxial Helmholtz Equation

Wave eq. of electric field

$$E = E_0 e^{i(kz-\omega t)} \qquad \Delta E = \frac{1}{c^2}\frac{\partial E^2}{\partial t^2}$$

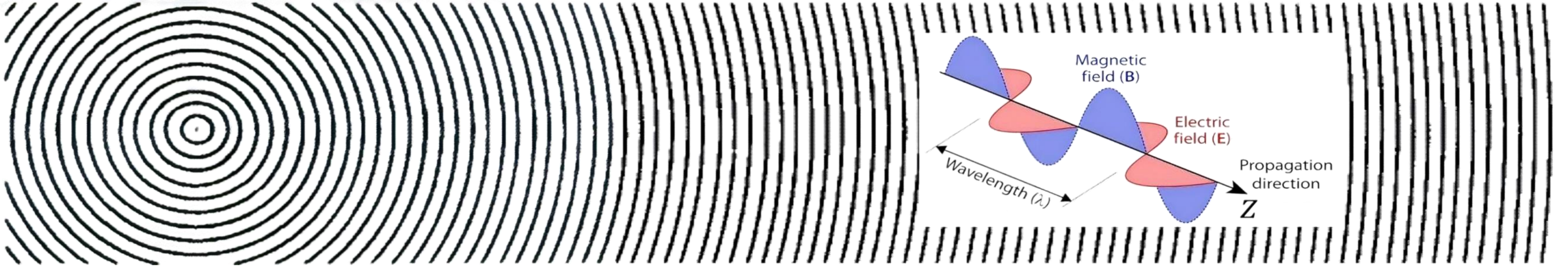
Helmholtz eq.

$$\Delta u + k^2 u = 0$$

$$u = f(x,y,z)e^{ikz}$$

Paraxial Helmholtz Equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + 2ik\frac{\partial f}{\partial z} = 0$$



Circular

$$u(r) = \frac{A}{r}e^{ikr}$$

Paraboloidal

$$u(r) = \frac{A}{z}\exp[ik\left(\frac{x^2+y^2}{2z}\right)]e^{ikz}$$

Planer

$$u(r) = Ae^{ikz}$$

Gaussian beam

Paraboloidal wave

$$u(r) = \frac{A}{z-iz_0} \exp\left[ik\left(\frac{x^2+y^2}{2(z-iz_0)}\right)\right] e^{ikz}$$ is a solution.

The Gaussian Beam becomes $\rho = \sqrt{x^2+y^2}$ $\varsigma(z) = -tan^{-1}(\frac{z_0}{z})$

$$u = f e^{ikz} = \boxed{A_0 \frac{w_0}{w(z)} \exp\left[-\frac{\rho^2}{w^2(z)}\right]} \boxed{\exp\left[ikz - i\varsigma(z) + ik\frac{\rho^2}{2R(z)}\right]}$$

Amplitude

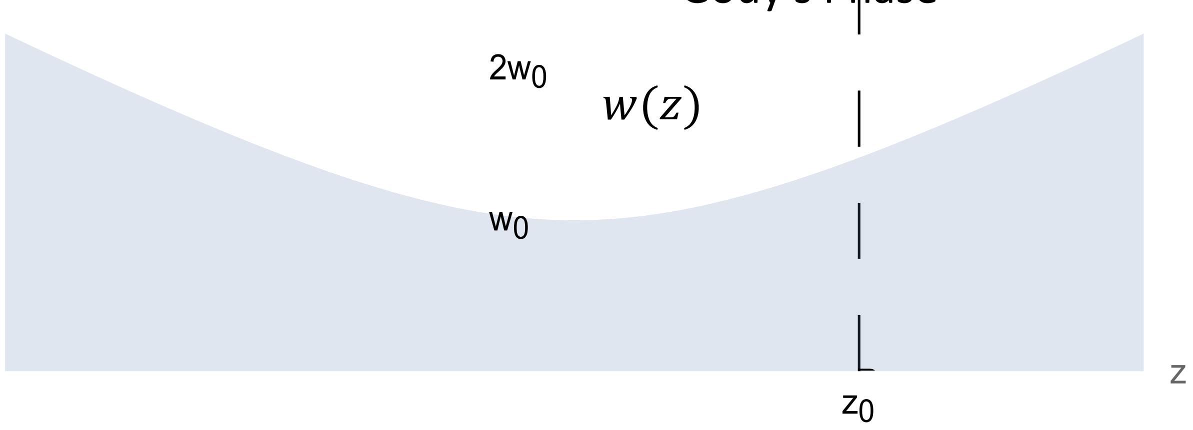
Gouy's Phase

Beam Radius

$$w(z) = w_0\sqrt{[1+\left(\frac{z}{z_0}\right)^2]} \quad , \; w_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

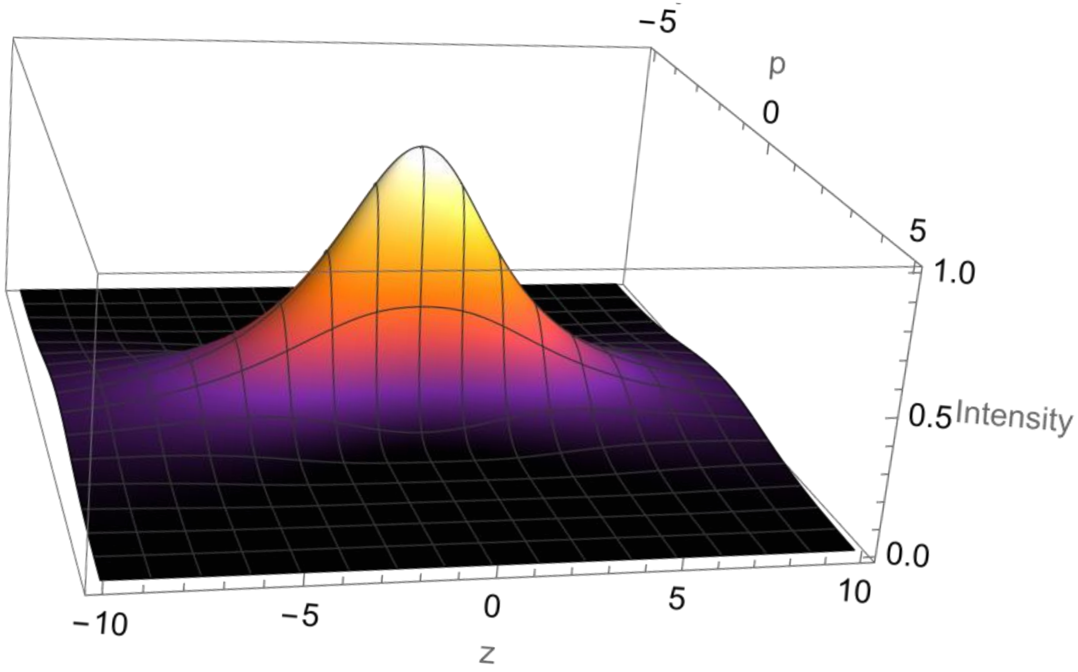
Beam waist

$$w(0) = w_0, \qquad w(z_0) = w_0\sqrt{2}$$



Gaussian beam Parameters

Intensity of beam $$I = I_0[\frac{w_0}{w(z)}]^2 \exp[\frac{-2\rho^2}{w^2(z)}]$$

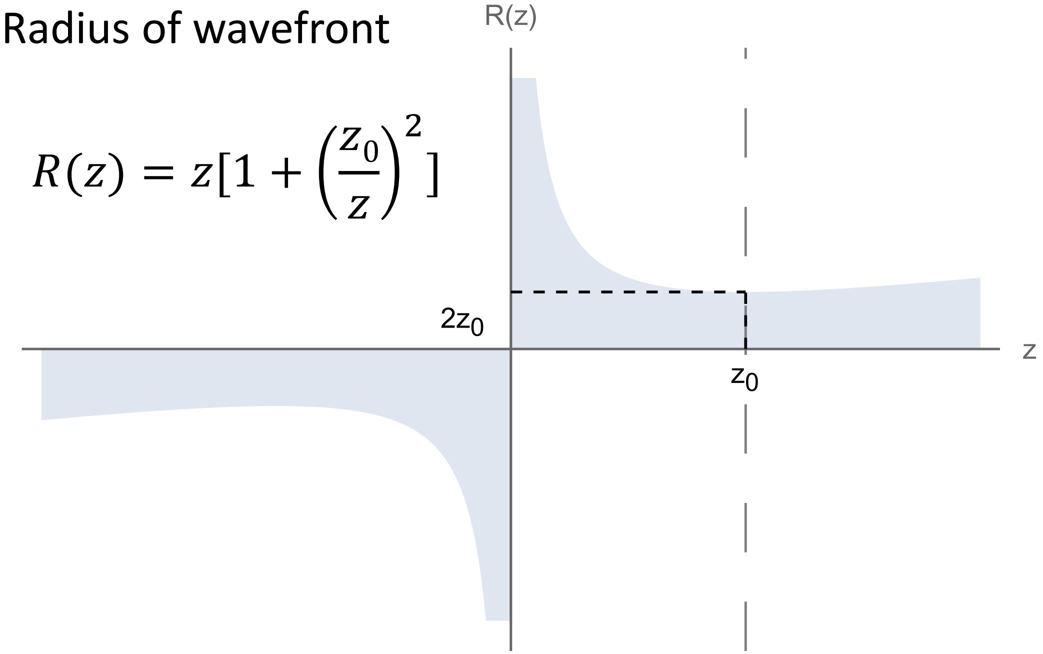


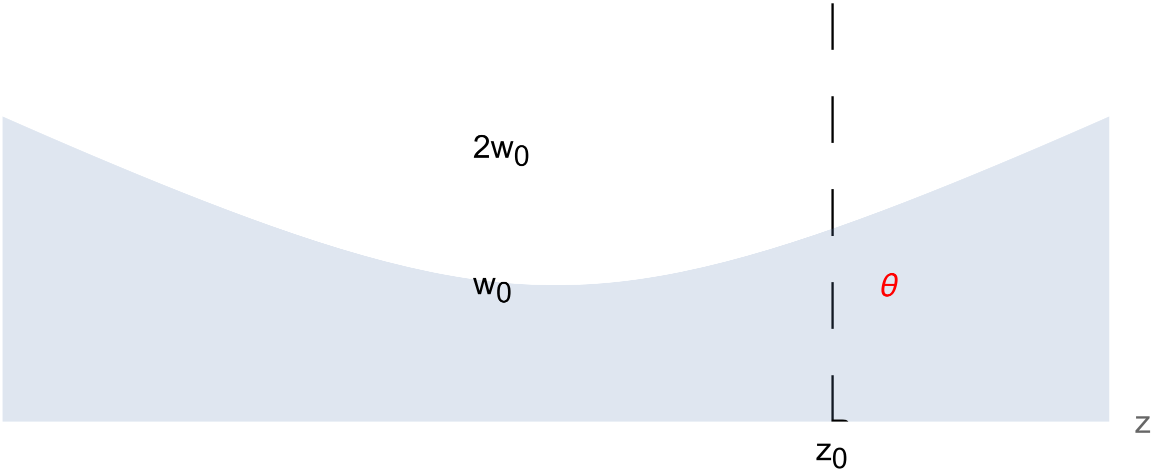
Rayleigh range z_0 , Beam

Divergence $\theta_0 = w_0/z_0$

Radius of wavefront

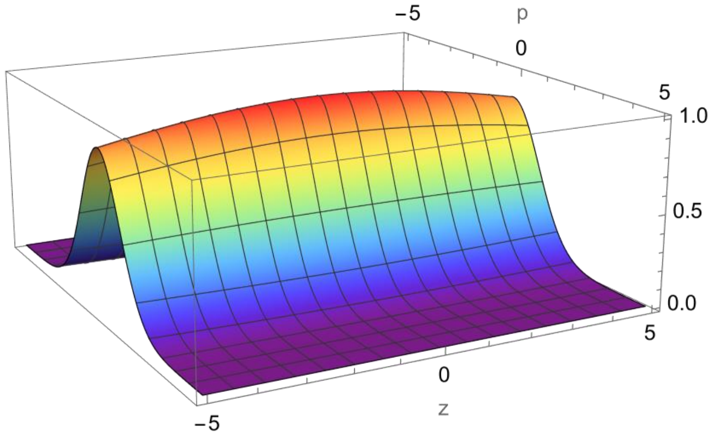
$$R(z) = z[1 + \left(\frac{z_0}{z}\right)^2]$$



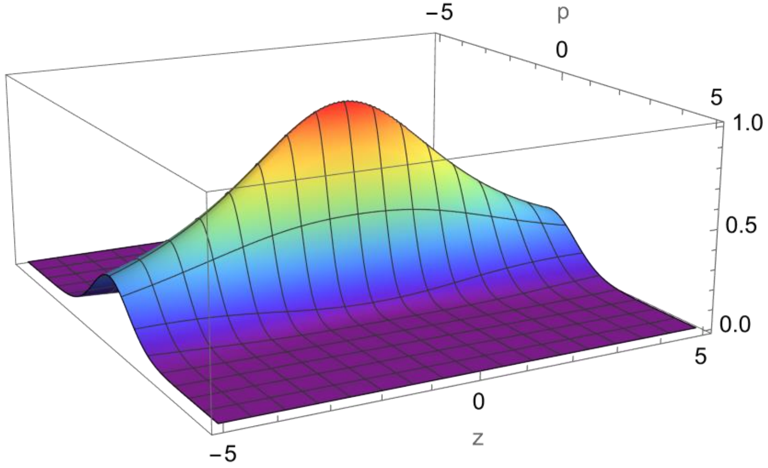


Gaussian beam

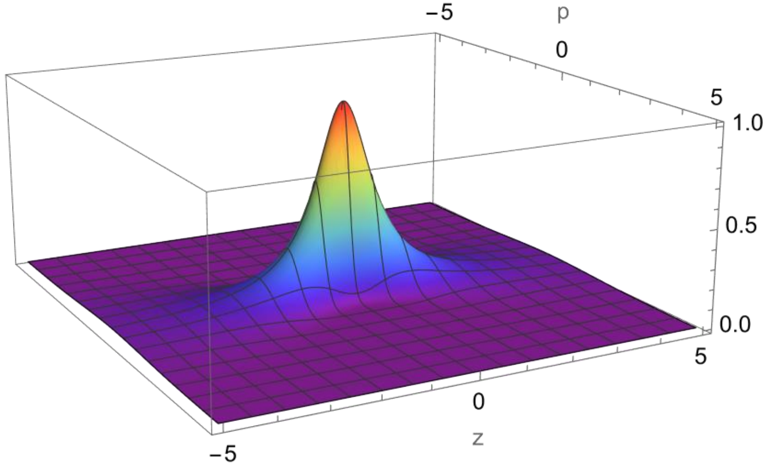
Beam can't be focused until $w_0 < \lambda$, $\Delta z \Delta pz > \hbar/2$



$w_0 = 2\ \lambda$



$w_0 = \lambda$



$w_0 = 0.5\lambda$

Laguerre-Gaussian Modes

As, the paraxial Helmholtz eq.

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2}+2ik\frac{\partial\psi}{\partial z}=0$$

Taking a trial solution,

$$\psi=g\left(\frac{r}{w(z)}\right)\exp\left\{-iP(z)+\frac{ik}{2q(z)}r^2-il\varphi\right\}=gF(r,\varphi,z)$$

Doing extensive calculation and taking analogy with Gaussian Beam, we get

$$\psi(r,\varphi,z)=\mathrm{A}\frac{w_0}{w(z)}\boxed{\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|}}\boxed{L_p^l\left(\frac{2r^2}{w^2(z)}\right)}\boxed{\exp\left(-\frac{r^2}{w^2(z)}\right)}\boxed{\exp\left[ik\frac{r^2}{2R(z)}-il\varphi-i(2n+l+1)\psi(z)\right]}$$

Donut Shape

Laguerre Polynomial

Gaussian Behavior

Phase

Laguerre-Gaussian Modes

Where, A is normalization constant

$$A = \sqrt{\frac{2p!}{\pi(|l| + p)}}\,\frac{1}{w(z)} \qquad and \qquad \psi(z) = \arctan(\frac{z}{z_0})$$

Where, solving associated Laguerre differential Equation

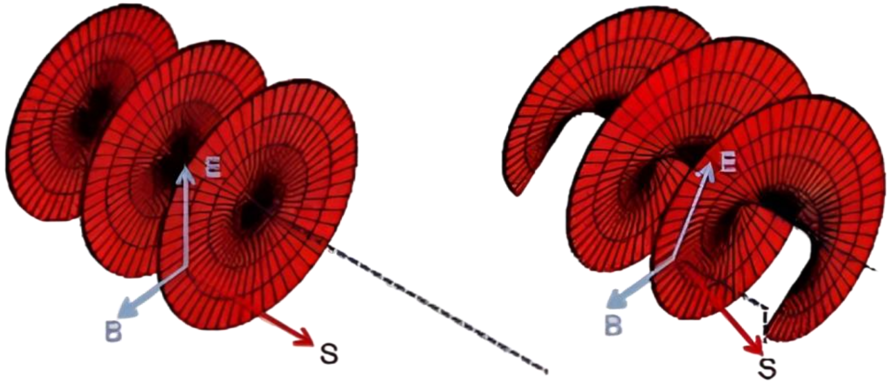
$$xL_p^{l''} + (l + 1 - x)L_p^{l'} + nL_p^{l} = 0$$

Gives, L_n^l

$$L_p^l = \sum_{m=0}^{p} (-1)^m\, C_{p-m}^{p+l}\, \frac{r^m}{m!}$$

Orbital Angular Momentum

- Association with **twisted light** $\boldsymbol{L} = \boldsymbol{l}\hbar$
  - Helical phase structure
  - Azimuthal phase dependence $e^{il\varphi}$
  - Phase singularity



1-Ph.D. thesis, Leopold Franzens Universitat Innsbruck (2000)A. E. Mair, A. Vaziri, G. Weihs, and Anton Zeilinger, Nature !London" 412, 313 !2001“

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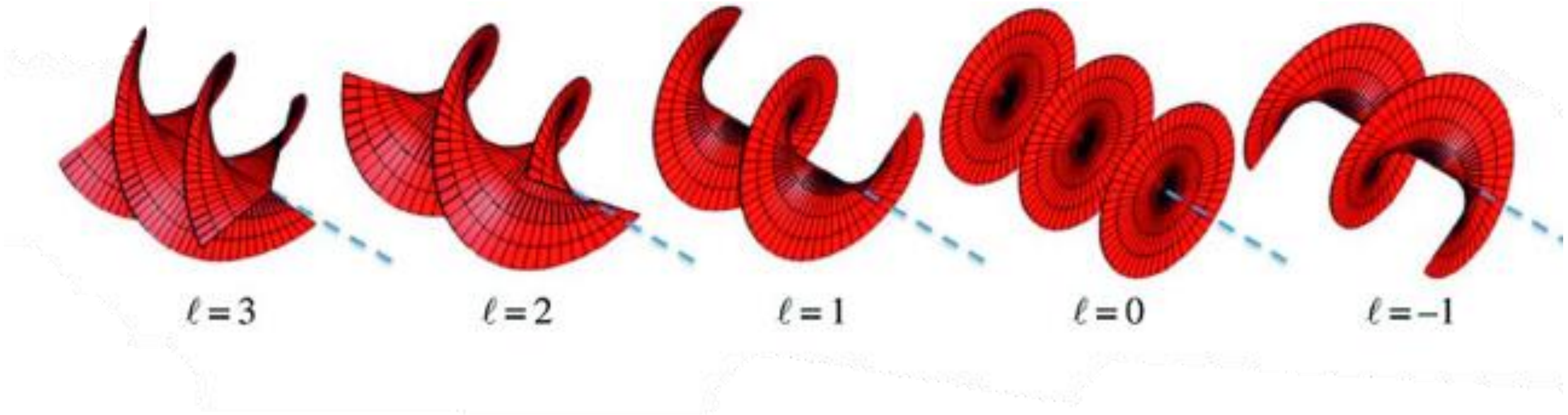
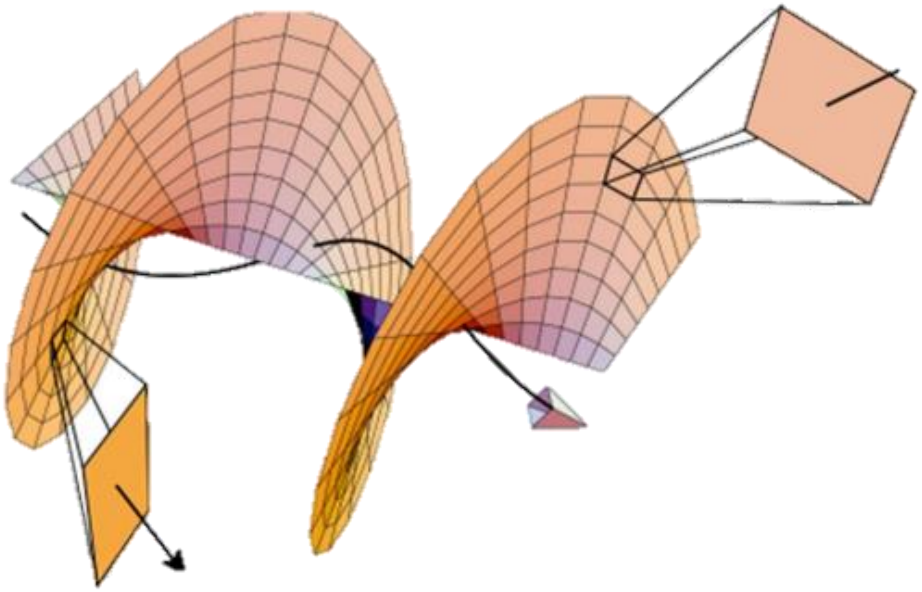


- Quantization of OAM

$$l \quad \& \quad p$$

- Global Property
  - Communication Security

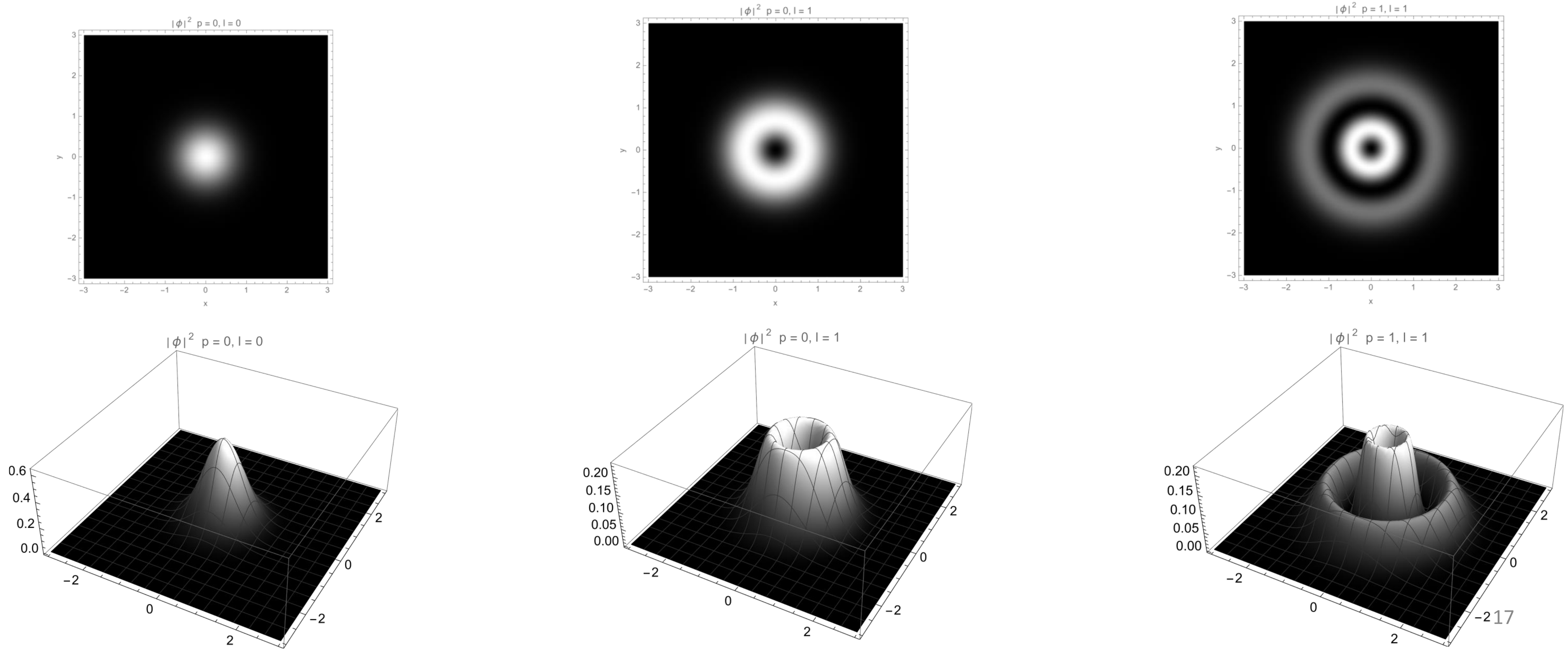


1-Ph.D. thesis, Leopold Franzens Universitat Innsbruck (2000)A. E. Mair, A. Vaziri, G. Weihs, and Anton Zeilinger, Nature !London" 412, 313 !2001“

Laguerre-Gaussian Modes

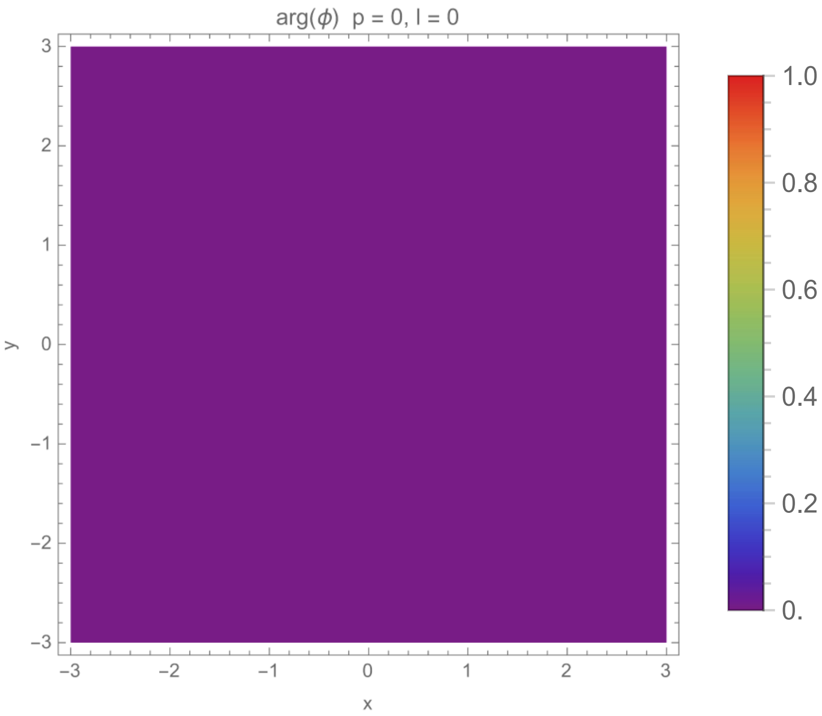
Whereas Laguerre-Gaussian modes equation in polar coordinates would simplify to

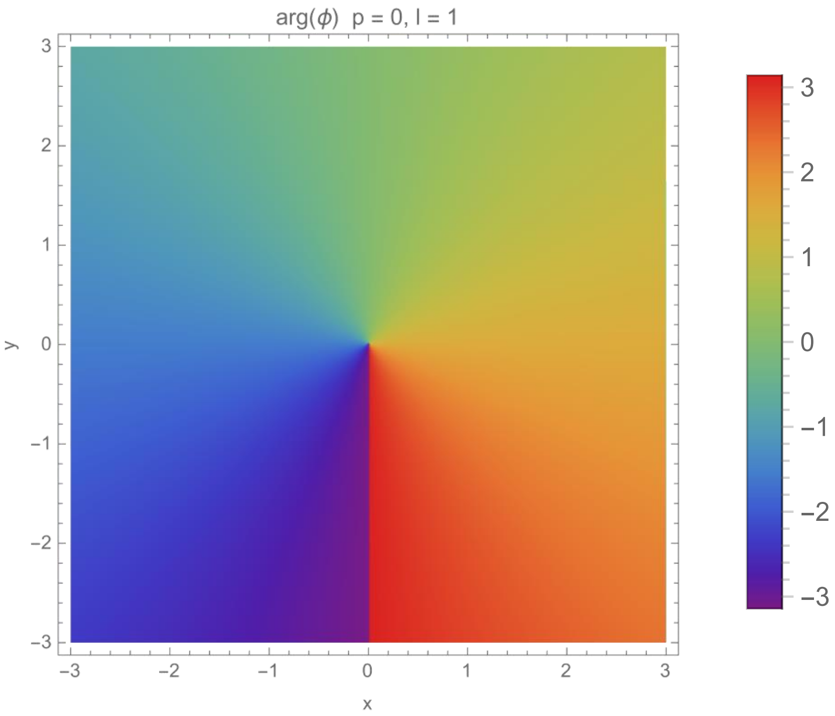
$$\Phi_{p,l}(r,\varphi) = \sqrt{\frac{2p!}{\pi(|l|+p)}}\,\frac{1}{w}\left(\frac{r\sqrt{2}}{w}\right)^{|l|} L_p^l\left(\frac{2r^2}{w^2}\right)e^{-\frac{r^2}{w^2}}\,e^{-il\varphi}$$

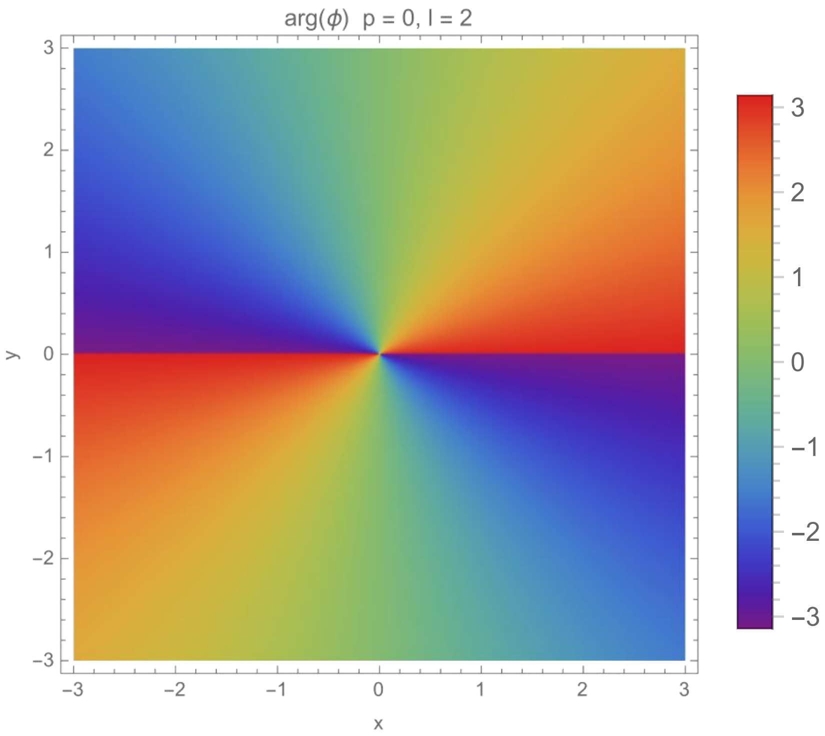


Laguerre-Gaussian Modes

Where argument changes as follow,





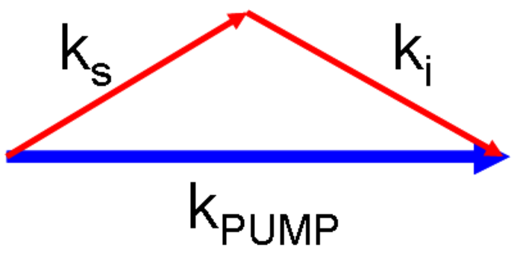


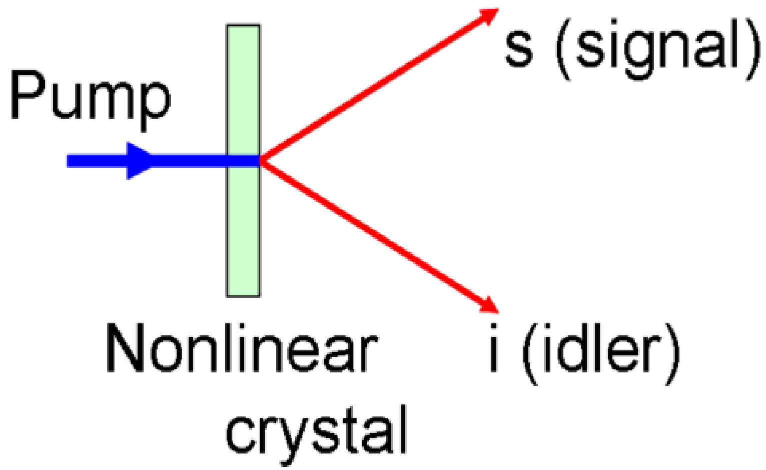
SPDC(Spontaneous parametric down-conversion)

- Kleinman 1968
- Non-Linear Crystal
- Phase Matching

$$\hbar kp = \hbar ks + \hbar ki$$

$$k_p = ks + ki$$

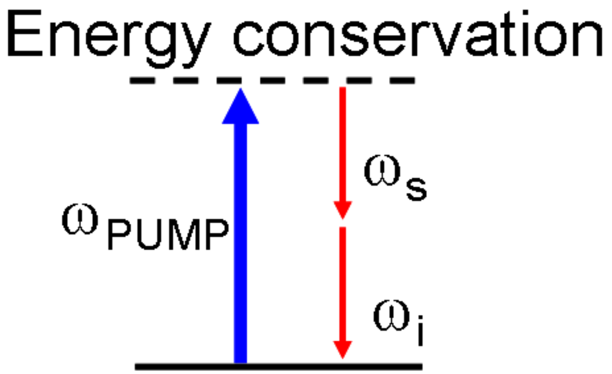




- Energy Conservation

$$\hbar w_p = \hbar ws + \hbar w_i$$

$$w_p = w_s + w_i$$



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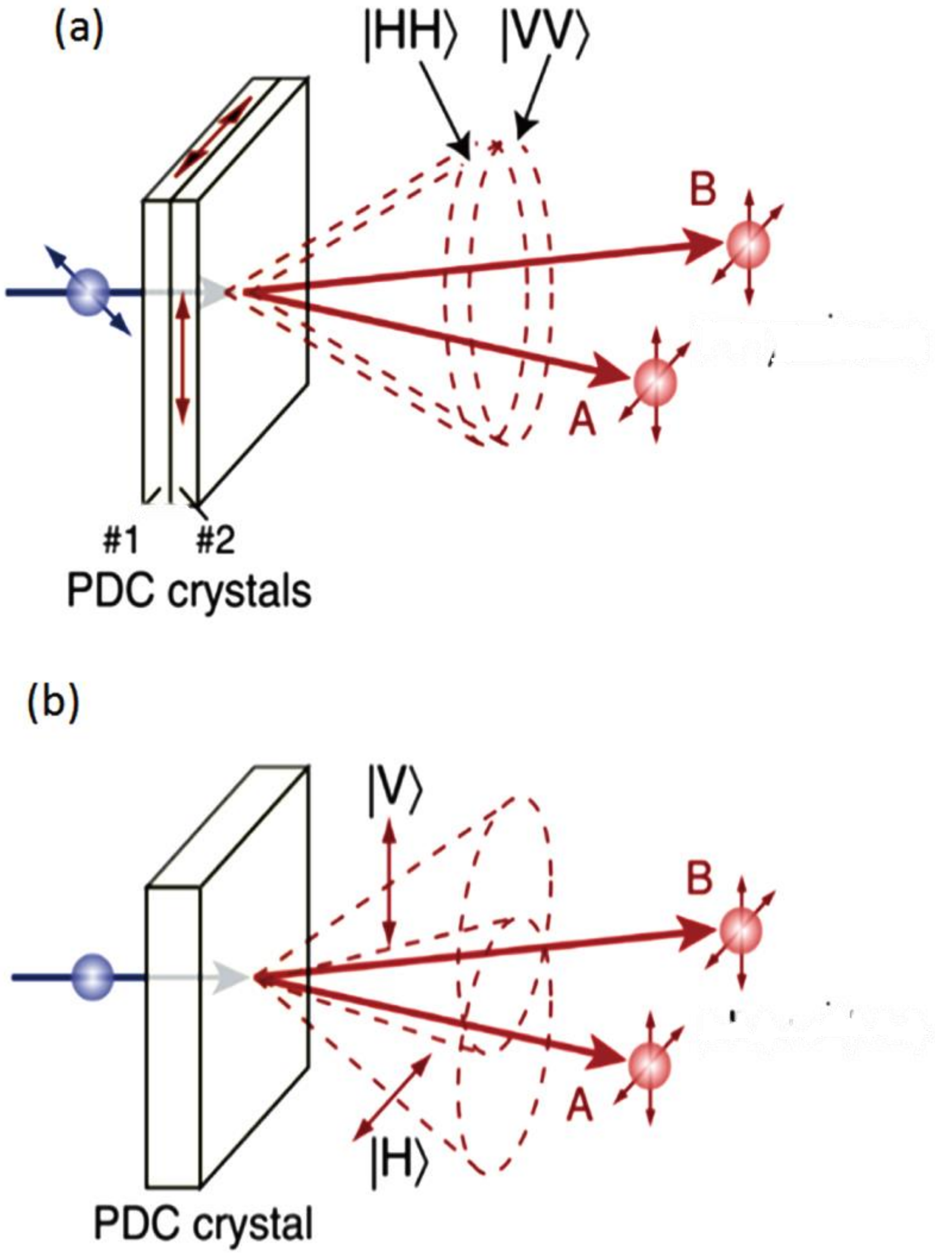
Types

- ***Type I:***
  - Same Polarization

$$|\psi\rangle = a|H\rangle_A\,|H\rangle_B + b|V\rangle_A\,|V\rangle_B$$

- ***Type II:***
  - Orthogonal Polarization

$$|\psi\rangle = a|H\rangle_A\,|V\rangle_B + b|V\rangle_A\,|H\rangle_B$$



Catalano, Jesse, "Spontaneous Parametric Down-Conversion and Quantum Entanglement" (2014). University Honors Theses. Paper 474

Wavefunctions

Coincident wavefunction

$$|\psi\rangle = \int dk_0 \int dk_1 \int dk_2 \Phi_0(\, k_0) \hat{a}_1^\dagger(k_1)\, \hat{a}_2^\dagger(k_2)\, \Delta(k_1 - k_2) \delta(k_0 - k_1 - k_2)\, |0\rangle$$

Fourier transform

$$\Phi_0\,(k_0) = \frac{1}{2\pi} \int dx_0\ \Phi_0\,(x_0) exp(ik_0 x_0)$$

$$\hat{a}_{1,2}^\dagger\left(k_{1,2}\right) = \frac{1}{2\pi} \int dx\ \hat{a}_{1,2}^\dagger\left(x_{1,2}\right) exp(ik_{1,2} x_{1,2})$$

$$|\psi\rangle = \int dx_0 \int dx_1 \int dx_2 \Phi_0(\, x_0)\, \hat{a}_1^\dagger(x_1)\, \hat{a}_2^\dagger(x_2)\, \Delta(x_1 - x_2)\, |0\rangle$$

Individual wavefunction

$$|\psi_{1,2}\rangle = \int dx_{1,2} \Phi_{1,2}(\, x_{1,2})\, \hat{a}_{1,2}^\dagger(x_{1,2})\, |0\rangle \ldots \ldots \ldots (1)$$

Detection Probabilities

Coincident detection probability

$$P(\Phi_1, \Phi_2) = |\langle\psi_1, \psi_2|\psi\rangle|^2 = |\int dy \Delta(y)\,|^2\,|\int dx\, \Phi_1^*(x)\Phi_2^*(x)\Phi_0(x)\,|^2$$

Individual detection probability

$$P(\Phi_2) = |\langle\psi_2|\psi\rangle|^2 = |\int dy \Delta(y)\,|^2\,|\int dx\, \Phi_2^*(x)\Phi_0(x)\,|^2$$

$$P(\Phi_1) = |\int dy \Delta(y)\,|^2\,|\int dx\, \Phi_1^*(x)\Phi_0(x)\,|^2$$

$$P_N(\Phi_1, \Phi_2) = \frac{|\int dx\, \Phi_1^*(x)\Phi_2^*(x)\Phi_0(x)\,|^2}{\sqrt{|\int dx\, \Phi_1^*(x)\Phi_0(x)\,|^2 |\int dx\, \Phi_2^*(x)\Phi_0(x)\,|^2}}$$

Correlation wavefunction

For a given pump mode and detected signal mode, the state of idler photon collapses into

$$|\psi_2\rangle = \int dy \Delta(y) \int dx_2 \; \Phi_1^*(x_2)\Phi_0(x_2)\hat{a}^\dagger(x_2)|0\rangle$$

Comparing it with (1)

$$\int dx_2 \Phi_2(\, x_2)\, \hat{a}_2^\dagger(x_2)\, |0\rangle = \int dy \Delta(y) \int dx_2 \; \Phi_1^*(x_2)\Phi_0(x_2)\hat{a}^\dagger(x_2)|0\rangle$$

$$\boxed{\Phi_2 \;= \left(\int dy\, \Delta(y)\right) \Phi_1^* \Phi_0}$$

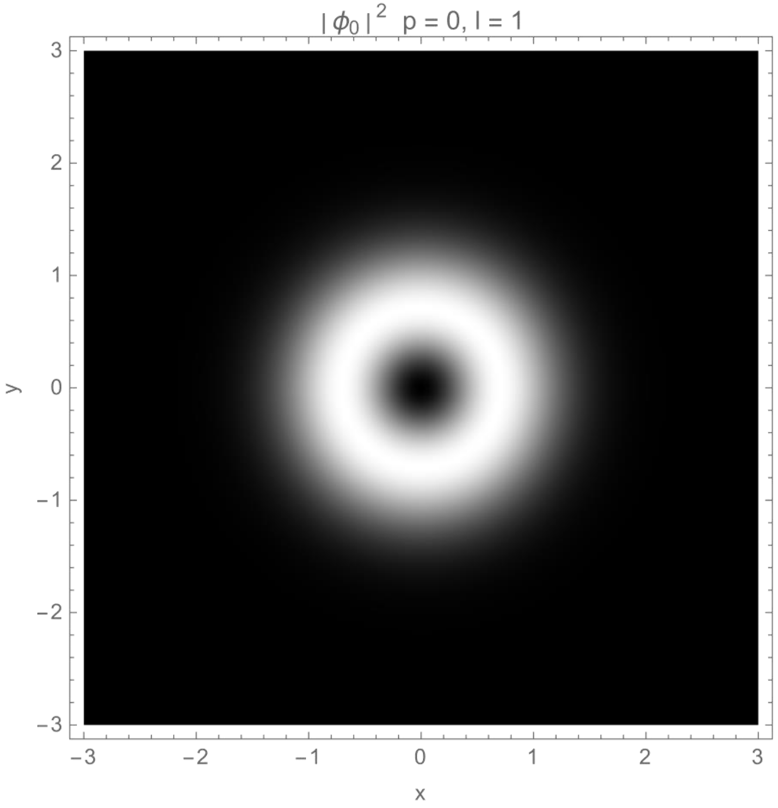
Correlation with LG Modes

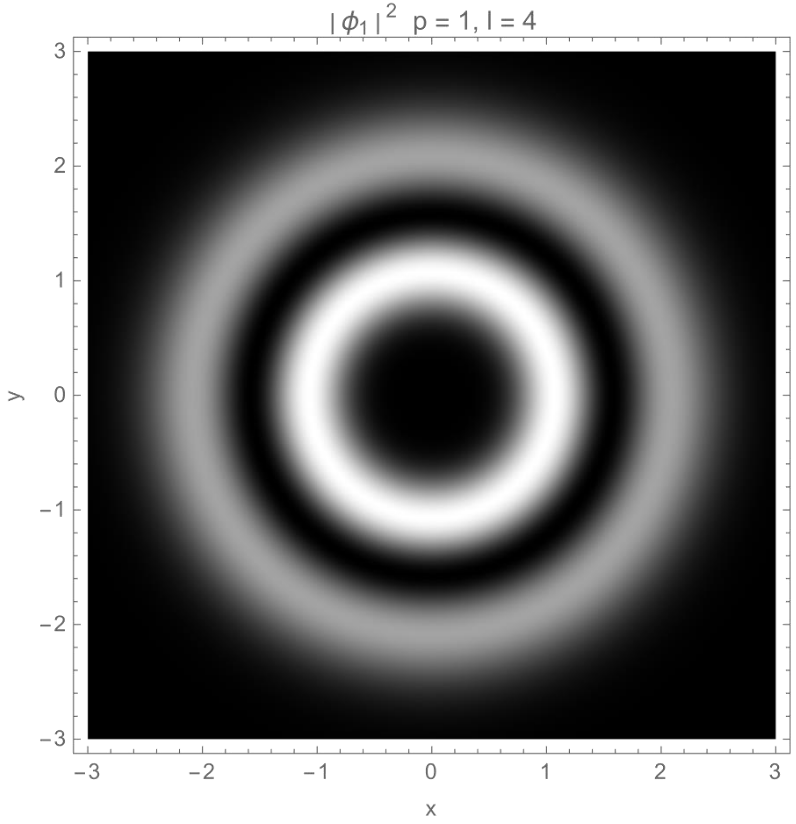
So, we have the correlation function as

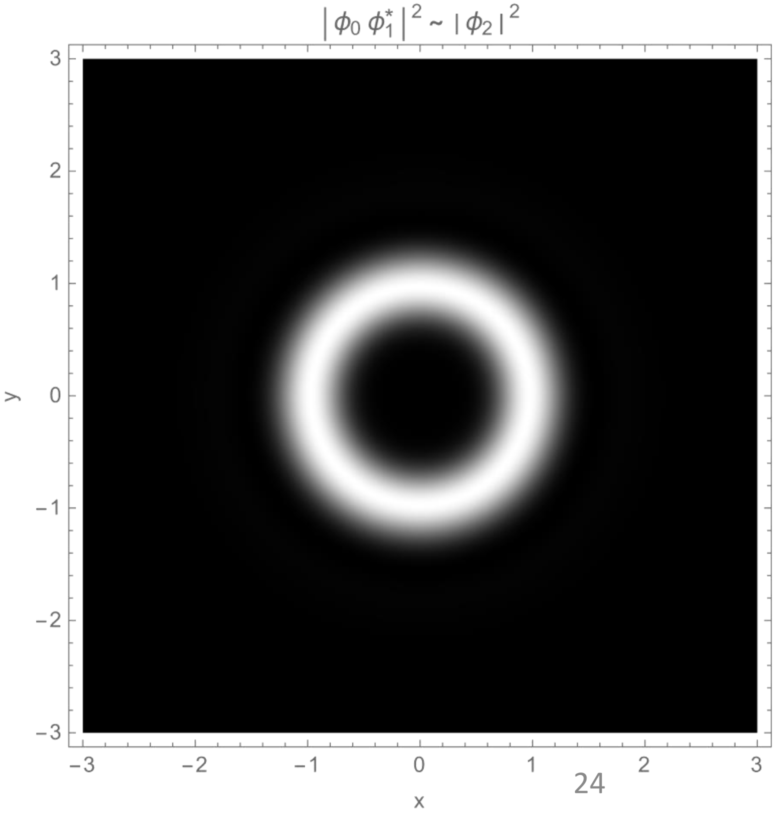
$$\Phi_2 \ = \left(\int dy\, \Delta(y)\right) \Phi_1^* \Phi_0$$

Taking mode square would be

$$|\Phi_2|^2 = \left|\left(\int dy\, \Delta(y)\right)\right|^2 |\Phi_1^* \Phi_0|^2 \sim |\Phi_1^* \Phi_0|^2$$

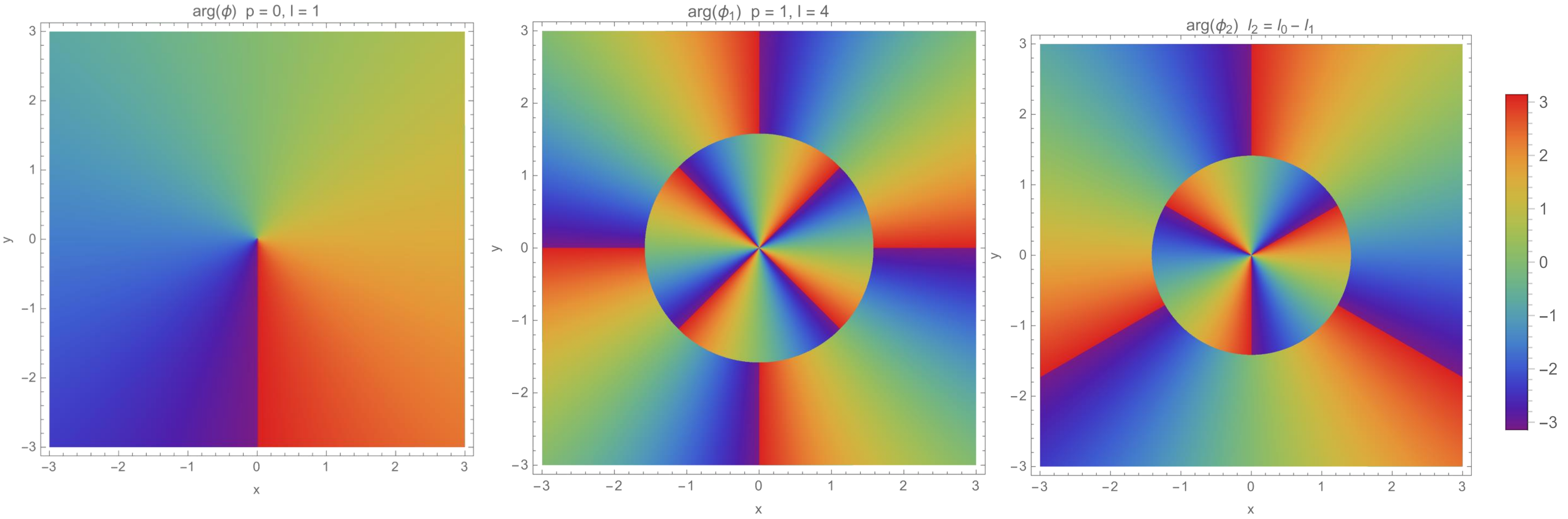






Correlation with LG Modes

Same would be the case with argument, stating $L_{pump} = L_{signal} + L_{idler} = 4 - 3 = 1$



Conclusion

Laguerre-Gaussian modes can be used to generate a pair of entangled photons specifically entangled in their orbital angular momentum through SPDC. Whereas this entanglement is the result of phase matching and energy conservation in non-linear crystal.

To be continued

- Workout the detection probabilities using Laguerre-Gaussian Formalism
- Analyzing and Plotting detection probabilities
- Workout OAM based Poincare’s Sphere formalism

Gant Chart

Month	Feb - March 2023	April - May 2023	June - July 2023	Aug - Sep 2023	Oct - Nov 2023	Dec - Jan 2023
Literature Review + Understanding SPDC and OAM						
Detection Probabilities and correlation function Calculation						
Working out Laguerre-Gaussian modes formalism						
Graphical analysis of LG Modes, and Correlation through LG modes						
Probability calculations using LG modes formalism and plots analysis						
Conclusions + Thesis						

THANK YOU