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Zeno-like phenomena in STIRAP processes

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Abstract

The presence of a continuous measurement quantum Zeno effect in a stimulated Raman adiabatic passage is studied, exploring in detail a sort of self-competition of the damping, which drives the system toward a loss of population and, at the same time, realizes the conditions for optimizing the adiabatic passage.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

The quantum Zeno effect [1] is the inhibition of the dynamics of a physical system due to repeated measurements. Its occurrence has been demonstrated in different physical systems and contexts [2, 3]. In the standard treatment, measurements are assumed to be 'pulsed', i.e. happening at specific instants of time, and are mathematically described by actions of projection operators. Nevertheless, in some cases the measurement can be 'continuous', and in such situations a different description is required [4–8]. A typical example is provided by a decaying level. Indeed, spontaneous decay can be thought of as an observation process, since a decay corresponds to the emission of radiation that can be revealed by a detector. Apart from interpretations, it is a matter of fact that a very strong damping is able to hinder dynamical effects [9]. More precisely, if a decaying state is coupled to another state and if the decay rate tremendously exceeds the relevant coupling constant (i.e. the associated Rabi frequency), then any transition from one of these two states to the other is forbidden. This circumstance could be very helpful in some applications, where the Zeno effect associated with quantum noise can compensate for the detrimental effects of the noise itself. As an example, we will consider a stimulated Raman adiabatic passage (STIRAP) (for reviews see [10]), where a transfer of population from the initial state to the final one is realized through an adiabatic evolution following [11], i.e. an adiabatic evolution [12] where an eigenstate of the Hamiltonian coincides with the initial state of the system in the initial time and then gradually transforms into the target state. This procedure has been widely demonstrated and exploited [13–16]. Since the process involves an auxiliary state, it can be negatively affected when such an auxiliary state is decaying.

The effects of quantum noise have been investigated through suitable phenomenological models [17–19]. Very recently, a microscopic model for describing STIRAP in the presence of environmental effects has been analyzed in detail [20]. The analysis of the model proposed by Scala *et al* has revealed a strange behavior of the efficiency of population transfer versus the decay rate of the auxiliary state. In particular, for very small damping the effects of the environment are negligible, which is as expected. For stronger damping there is a visible diminishing of the efficiency, which is still reasonable. But, surprisingly, for larger values of the decay rate, the efficiency increases and reaches the maximum value, which is maintained even in the limit of infinite damping. The reason for this behavior has been traced back to a dissipation-induced dynamical decoupling through qualitative arguments.

In this paper, we show in detail how the presence of strong environmental effects can restore the maximum efficiency of a STIRAP process because of the occurrence of Zeno-like phenomena. In the next section, we show that a strong damping is responsible for a dynamical decoupling of the relevant state, which can preserve the population of a non-decaying state against transitions toward other states. In section 3, we consider the special case of STIRAP in a Λ-system in the presence of dissipation, discussing an improvement of the efficiency of STIRAP in the strong damping limit. Finally, in section 4, we give some concluding remarks.

2. Dynamical decoupling induced by dissipation

It is known that a strong damping can hinder the dynamics induced by coherent couplings [4, 5, 9]. In order to better clarify this point beyond specific examples, let us consider an N-level system described by a time-dependent non-Hermitian Hamiltonian that takes into account possible decays ($\hbar = 1$):

$$H(t) = H_{\rm sys}(t) - \mathrm{i}\sum_k \lambda\gamma_k(t)|k\rangle\langle k|, \qquad (1)$$

where $H_{\rm sys}(t)$ is the Hermitian part of the total Hamiltonian and describes the system in the absence of dissipation, $\{|k\rangle\}$ is a basis of the Hilbert space, $\gamma_k(t)$ are the relevant decay rates and λ is a dimensionless parameter. Assume that each γ_k is either identically vanishing or always strictly positive. We rewrite the Hamiltonian in the following form:

$$H(t) = H_0(t) + H_1(t), \qquad (2)$$

with

$$H_0(t) = \sum_k [\omega_k(t) - \mathrm{i}\lambda\gamma_k(t)]|k\rangle\langle k|, \qquad (3)$$

where $\omega_k(t)$ are the diagonal matrix elements (not necessarily the eigenvalues) of $H_{\rm sys}(t)$.

The strong damping limit is defined by $\lambda \to \infty$. In this limit, we consider $H_0(t)$ as the unperturbed part and $H_1(t) = \| v_{mn}(t) \|$ as a perturbation. Here $v_{mn}(t) = \langle m|H_1(t)|n\rangle$. Once the transformation $|\tilde{\psi}\rangle = U(t)|\psi\rangle$ is performed, with

$$U(t) = \exp\left[\mathrm{i}\int_0^t H_0(s)\,\mathrm{d}s\right] = \sum_k \alpha_k(t)|k\rangle\langle k|,$$
$$\alpha_k(t) = \exp\left[\mathrm{i}\int_0^t \omega_k(s)\mathrm{d}s\right]\exp\left[\lambda\int_0^t \gamma_k(s)\,\mathrm{d}s\right], \qquad (4)$$

where we have used $[H_0(t), H_0(t')] = 0$, one obtains the following Schrödinger equation:

$$\mathrm{i}\frac{\mathrm{d}|\tilde{\psi}\rangle}{\mathrm{d}t} = U(t)\,H_1(t)\,U^{-1}(t)\,|\tilde{\psi}\rangle. \qquad (5)$$

Expanding the state $|\tilde{\psi}\rangle$ in the basis $\{|k\rangle\}$, $|\tilde{\psi}\rangle = \sum_k \tilde{c}_k|k\rangle$, one obtains the equation for the generic coefficient $\tilde{c}_k$, $\mathrm{i}\,\mathrm{d}\tilde{c}_k/\mathrm{d}t = \alpha_k \sum_l \alpha_l^{-1}\, v_{kl}\,\tilde{c}_l$, whose formal solution is

$$\tilde{c}_k(t) = \tilde{c}_k(0) - \mathrm{i}\int_0^t \mathrm{d}s\,\sum_l \alpha_k(s)\,\alpha_l^{-1}(s)\,v_{kl}(s)\,\tilde{c}_l(s). \qquad (6)$$

By using this it is possible to define the following iteration-based sequence:

$$\tilde{c}_k^{(n+1)}(t) = \tilde{c}_k(0) - \mathrm{i}\int_0^t \mathrm{d}s\,\sum_l \alpha_k(s)\,\alpha_l^{-1}(s)\,v_{kl}(s)\,\tilde{c}_l^{(n)}(s), \qquad (7)$$

which gives rise to the first-order (i.e. one-iteration) approximated solution

$$\tilde{c}_k^{(1)}(t) = c_k(0) - \mathrm{i}\int_0^t \mathrm{d}s\,\sum_l \alpha_k(s)\,\alpha_l^{-1}(s)\,v_{kl}(s)\,c_l(0), \qquad (8)$$

where we have used $\tilde{c}_k^{(0)}(t) = \tilde{c}_k(0) = c_k(0)$.

Therefore, the coefficients of the expansion $|\psi\rangle = \sum_k c_k|k\rangle$ are given, to first order, by the following expression:

$$c_k^{(1)}(t) = \alpha_k^{-1}(t)\left[c_k(0) - \mathrm{i}\sum_l\int_0^t \mathrm{d}s\,\alpha_k(s)\,\alpha_l^{-1}(s)v_{kl}(s)\,c_l(0)\right]. \qquad (9)$$

Now, if $\gamma_k(t) \neq 0$ and λ increases, then $c_k(0)\alpha_k^{-1}(t) \to 0$, and the second contribution in the right-hand side vanishes as well (see the appendix). Instead, if $\gamma_k(t) = 0$, then $c_k(0)$ acquires a possible time-dependent phase factor related to $\omega_k(t)$, whereas in the $\lambda \to \infty$ limit, the only surviving terms in the summation over l are those for which $\gamma_l(t) = 0$. Summarizing, for large λ,

$$\begin{cases} c_k^{(1)}(t) \approx 0\,, & \text{if } \gamma_k(t) \neq 0, \\ c_k^{(1)}(t) \approx \mathrm{e}^{-\mathrm{i}\int_0^t \omega_k(s)\,\mathrm{d}s}\Big[c_k(0) - \mathrm{i}\sum\limits_{l:\gamma_l(t)=0} c_l(0) & \\ \quad\times\int_0^t \mathrm{e}^{\mathrm{i}\int_0^s \omega_{kl}(r)\,\mathrm{d}r}\, v_{kl}(s)\,\mathrm{d}s\Big], & \text{if } \gamma_k(t) = 0, \end{cases} \qquad (10)$$

with $\omega_{kl}(t) = \omega_k(t) - \omega_l(t)$.

On the basis of this, we can say that there are no transitions from the decaying states toward the non-decaying ones, and vice versa. Moreover, the dynamics within the non-decaying subspace is essentially unitary, according to the first-order relation between the coefficients and the matrix elements of the interaction Hamiltonian, $v_{kl}(s)$. Therefore, the total population of the non-decaying subspace is conserved since there are no such processes as transitions to decaying states and consequent losses of population. Instead, the decaying states lose all their initial population, as one could expect. As a special case, in the strong damping limit, if there is a single non-decaying state, then the time evolution preserves the population of such a state through all of the process, while all the other states become empty if they are initially populated.

3. STIRAP and dissipation

In a recent paper, Scala *et al* [20] have analyzed a STIRAP in a Λ-system and in the presence of dissipation. The Hamiltonian model describing the system driven by pulses in the absence of dissipation, expressed in the basis $\{|1\rangle, |2\rangle, |3\rangle\}$, is (with $\hbar = 1$)

$$H_0(t) = \begin{bmatrix} \omega_1 & \Omega_p(t)\mathrm{e}^{\mathrm{i}(\omega_{21}-\Delta)t} & 0 \\ \Omega_p(t)\mathrm{e}^{-\mathrm{i}(\omega_{21}-\Delta)t} & \omega_2 & \Omega_s(t)\mathrm{e}^{-\mathrm{i}(\omega_{23}-\Delta)t} \\ 0 & \Omega_s(t)\mathrm{e}^{\mathrm{i}(\omega_{23}-\Delta)t} & \omega_3 \end{bmatrix}, \qquad (11)$$

where ω_k are the free energies of the three states, and $\Omega_p(t) = \Omega_0 f_p(t)$ and $\Omega_s(t) = \Omega_0 f_s(t)$ are the edges of the two pulses coupling the state $|2\rangle$ with states $|1\rangle$ and $|3\rangle$, respectively. Moreover, $\omega_{mn} = \omega_m - \omega_n$ and Δ is the detuning between each of the two laser frequencies and the Bohr frequency of the relevant transition.

In the interaction picture associated with the transformation $A(t) = e^{i\omega_1 t}|1\rangle\langle 1| + e^{i(\omega_2-\Delta)t}|2\rangle\langle 2| + e^{i\omega_3 t}|3\rangle\langle 3|$, the Hamiltonian $H_0(t)$ becomes

$$H_s(t) = \begin{bmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & \Delta & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{bmatrix}, \qquad (12)$$

whose instantaneous eigenstates are [17]

$$\begin{aligned} |+\rangle &= \sin\varphi \sin\theta|1\rangle + \cos\varphi|2\rangle + \sin\varphi\cos\theta|3\rangle, \\ |0\rangle &= \cos\theta|1\rangle - \sin\theta|3\rangle, \\ |-\rangle &= \cos\varphi\sin\theta|1\rangle - \sin\varphi|2\rangle + \cos\varphi\cos\theta|3\rangle, \end{aligned}$$

where

$$\begin{aligned} &\tan\theta(t) = \frac{\Omega_p(t)}{\Omega_s(t)}, \quad \tan 2\varphi(t) = \frac{2\Omega(t)}{\Delta}, \\ &\Omega(t) = \Omega_0\sqrt{f_p(t)^2 + f_s(t)^2}. \end{aligned} \qquad (13)$$

The corresponding eigenvalues are $\omega_+(t) = \Omega(t)\cot\varphi(t)$, 0 and $\omega_-(t) = -\Omega(t)\tan\varphi(t)$.

It is clear that when the pulse $\Omega_p(t)$ precedes the pulse $\Omega_s(t)$, it turns out that the angle $\theta(t)$ is $\pi/2$ at the beginning of the experiment, say $t = 0$, and equal to 0 at the end of the experiment, say $t = T$. Consequently, at $t = 0$ it is $|0\rangle = |1\rangle$ and at $t = T$ it turns out to be $|0\rangle = |3\rangle$. Therefore, if the Hamiltonian changes adiabatically, the *dark state* $|0\rangle$ can transport the population of state $|1\rangle$ to state $|3\rangle$.

In a realistic situation the auxiliary state is a decaying one, and the environmental effects can reduce the efficiency of this adiabatic passage. In [20], the quantum noise has been taken into account through a dipolar coupling involving the auxiliary state $|2\rangle$ and an 'external' (to the Λ-system) state $|4\rangle$ (whose eigenenergy ω_4 is assumed to be quite far from the other three eigenenergies), in the presence of radiation. In this model, the free Hamiltonian describing the environment is $H_E = \sum_k \omega_k a_k^\dagger a_k$, while the system–environment interaction in the Schrödinger picture is modelled by

$$H_{\text{noise}} = \sum_k g_k(a_k + a_k^\dagger) \otimes [|2\rangle\langle 4| + |4\rangle\langle 2|]. \qquad (14)$$

In [20], the relevant master equation and an effective non-Hermitian Hamiltonian have been derived. Figure 1 shows the efficiency of population transfer (i.e. the final population of state $|3\rangle$ after a time T corresponding to the total duration of the experiment) as a function of the decay rate of state $|2\rangle$ toward $|4\rangle$. In accordance with [17, 20], the two pulse edges are assumed to be the following: $\Omega_p(t) = \Omega_0/(\tau\sqrt{2})\,\text{sech}((t–T/2)/\tau)\sin[(\tanh((t-T/2)/\tau)+1)\pi/4]$ and $\Omega_s(t) = \Omega_0/(\tau\sqrt{2})\,\text{sech}((t-T/2)/\tau)\cos[(\tanh((t-T/2)/\tau)+1)\pi/4]$. The values of the relevant parameters are $\Omega_0\tau = 10$ and $\tau\Delta = 0.1$, while the total duration of the experiment is assumed to be $T = 10\tau$. It is clear that for small damping, i.e. $\Gamma \to 0$, the efficiency is close to unity. Then, for increasing Γ, the efficiency reasonably decreases, but for larger values of Γ, the predicted final population of state $|3\rangle$ is again close to 1.

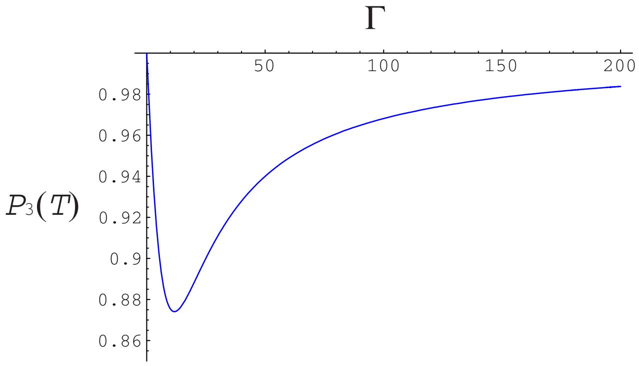

Figure 1. The final population $P_3(T)$ of state $|3\rangle$ after a time T corresponding to the total duration of the experiment, as a function of the parameter Γ (in units of τ^{-1}). The values of the relevant parameters are such that $\Omega_0\tau = 10$, $\tau\Delta = 0.1$ and $T = 10\tau$, where τ represents the width of each of the two pulses. In the intermediate region there is a decrease of efficiency in the population transfer, while in the limit $\Gamma \to \infty$ the efficiency tends to unity.

The behavior of the population $P_3(T)$ can be explained with the results of section 2. In fact, in [20] it has been proven that the effective Hamiltonian describing the dissipating system in the basis $\{|+\rangle, |0\rangle, |-\rangle\}$ is

$$\begin{aligned} H_{\text{eff}} = \lambda &\begin{bmatrix} -i\gamma_+(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -i\gamma_-(t) \end{bmatrix} \\ &+ \begin{bmatrix} \Omega\cot\varphi & i\dot\theta\sin\varphi & i\dot\varphi \\ -i\dot\theta\sin\varphi & 0 & -i\dot\theta\cos\varphi \\ -i\dot\varphi & i\dot\theta\cos\varphi & -\Omega\tan\varphi \end{bmatrix}, \end{aligned} \qquad (15)$$

with $\lambda = \Gamma/\Omega_0$, $\gamma_+(t) = \Omega_0\cos^2\varphi > 0$, $\gamma_0(t) = 0$, $\gamma_-(t) = \Omega_0\sin^2\varphi > 0$, and where explicit dependence of Ω, θ and φ on t has been omitted. Now, since state $|0\rangle$ carries the population and for $\lambda \to \infty$ the damping does not affect the dynamics of such a state, then in the limit of strong damping a perfect population transfer from $|1\rangle$ toward $|3\rangle$ is restored.

It is worth commenting on the presence of the minimum in the figure. In fact, its occurrence relies on the facts that $P_3(T) \to 1$ both for $\Gamma \to 0$ and for $\Gamma \to \infty$ and that for small non-vanishing values of Γ (when the Zeno dynamics cannot be established) the damping has only detrimental effects.

4. Discussion and concluding remarks

To summarize, in this paper we have given a detailed explanation of the restoration of a high efficiency of population transfer realized through a STIRAP process, in the presence of quantum noise. Environmental effects initially reduce the efficiency because of a certain loss of probability of the dark state existence that carries the population from the initial to the target one. Since the dark state is not directly decaying (the relevant diagonal matrix element of the Hamiltonian does not possess an imaginary part) this loss of probability is due to small transitions to the other states provoked by the off-diagonal terms of the Hamiltonian, followed by spontaneous decay. When the damping increases, a dynamical decoupling is induced. In fact, the differences between the three diagonal terms of the Hamiltonian (the complex counterparts of the Bohr frequencies) become much larger than the off-diagonal elements, avoiding transitions. Therefore, the dynamics of the dark state is affected neither by the interaction with the other states nor by their decay. In a sense, we can say that quantum noise competes with itself. Indeed, on the one hand, it tends to provoke loss of population, while on the other hand, a strong damping perfectly isolates the only non-decaying state, then realizing the conservation of the probability of the population carrier.

We conclude with some comments on the approximate solution in equation (9) and the consequent behavior expressed in equation (10). In fact, it is possible to arrive at the same conclusion at any order. Indeed, looking at equation (7), one sees that, whatever be the approximation order, the integrand is always the product of an exponential and a bounded function, and the statement proven in appendix A is still applicable. Therefore, at any order, in the $\lambda \to \infty$ limit, most of the coefficients (those corresponding to states with non-vanishing decay rates) approach zero, while the others (those corresponding to states with vanishing decay rates) are modified as if a unitary dynamics governs the relevant subspace.

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Appendix

In this appendix, we prove some statements that have been exploited throughout this paper.

Statement. *Let* $\Gamma(t)$ *be a continuous monotonic increasing and non-negative function defined in the interval* $[0, T]$*:* $\Gamma(t) \geqslant 0$*;* $t > t' \Rightarrow \Gamma(t) > \Gamma(t')$*. Then the integral*

$$I(\lambda, t) = \mathrm{e}^{-\lambda\Gamma(t)} \int_0^t \mathrm{e}^{\lambda\Gamma(s)}\, \mathrm{d}s \tag{A.1}$$

has the following property:

$$\lim_{\lambda\to\infty} I(\lambda, T) = 0. \tag{A.2}$$

Proof. We want to prove that $\forall \xi > 0 \quad \exists \lambda_\xi \colon \lambda > \lambda_\xi \Rightarrow 0 < I(\lambda, T) < \xi$.

It is obvious that the quantity $I(\lambda, t)$ is always strictly positive. Moreover, for any $\eta > 0$ the following occurs:

$$\begin{aligned} I(\lambda, T) &= \mathrm{e}^{-\lambda\Gamma(T)} \int_0^{T-\eta} \mathrm{e}^{\lambda\Gamma(s)}\, \mathrm{d}s + \mathrm{e}^{-\lambda\Gamma(T)} \int_{T-\eta}^{T} \mathrm{e}^{\lambda\Gamma(s)}\, \mathrm{d}s \\ &\leqslant (T-\eta)\mathrm{e}^{-\lambda(\Gamma(T)-\Gamma(T-\eta))} + \eta \\ &\leqslant \eta + T\, \mathrm{e}^{-\lambda(\Gamma(T)-\Gamma(T-\eta))}. \end{aligned} \tag{A.3}$$

Now, choosing $\eta = \xi/2$ and $\lambda_0 = -[\Gamma(T) - \Gamma(T - \xi/2)]^{-1} \ln(\xi/2T)$, for any $\lambda > \lambda_0$, one has

$$0 < I(\lambda, T) < \xi, \tag{A.4}$$

which proves the assertion. □

On the basis of this, one can immediately prove the following:

Statement. *Given a function* $\Gamma(t)$ *satisfying the previous statement, and a complex and integrable function* f *whose modulus is bounded in the interval* $[0, T]$*, it turns out that*

$$\lim_{\lambda\to\infty} \mathrm{e}^{-\lambda\Gamma(T)} \int_0^T \mathrm{e}^{\lambda\Gamma(s)} f(s)\, \mathrm{d}s = 0. \tag{A.5}$$

Proof. The modulus of the limit argument is smaller than $I(\lambda, T) \times \max_{[0,T]}|f|$.

Because of the structure of the α_k coefficients given by equation (4), equation (A.5) proves that the second term in equation (9) goes to zero when $\lambda \to \infty$ and $\gamma_k(t) > 0$. Indeed, for $\gamma_k(t) > 0$ (state $|k\rangle$ is decaying) and $\gamma_l(t) \geqslant 0$ (state $|l\rangle$ is decaying or not) it turns out that the function $\Gamma(t) = \int_0^t \gamma_k(s)\, \mathrm{d}s$ satisfies the previous statement and that $\alpha_l^{-1}(s) v_{kl}(s)$ is a bounded integrable function. □

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