

A Compilation of Circuit Identities and Decomposition

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1 Gate Identities

1.1 Conjugation by Hadamard

$$\text{---} \boxed{X} \text{---} = \text{---} \boxed{H} \text{---} \boxed{Z} \text{---} \boxed{H} \text{---}$$

$$\text{---} \boxed{Y} \text{---} = \text{---} \boxed{H} \text{---} \boxed{-Y} \text{---} \boxed{H} \text{---}$$

$$\text{---} \boxed{Z} \text{---} = \text{---} \boxed{H} \text{---} \boxed{X} \text{---} \boxed{H} \text{---}$$

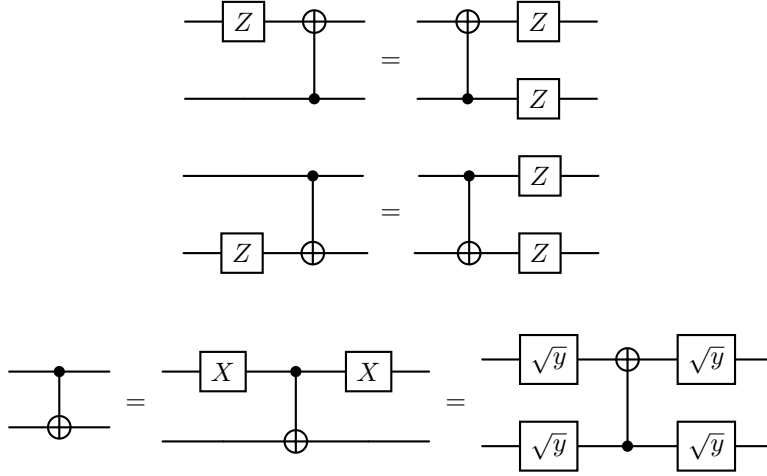
1.2 Two-qubit gates

$$\begin{array}{c} \bullet \\ | \\ \text{---} \boxed{Z} \text{---} \end{array} = \begin{array}{c} \text{---} \boxed{Z} \text{---} \\ | \\ \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \text{---} \boxed{H} \text{---} \oplus \text{---} \boxed{H} \text{---} \end{array}$$

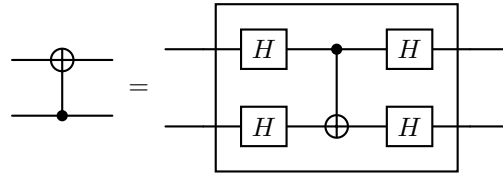
$$\begin{array}{c} \text{---} \boxed{X} \text{---} \bullet \\ | \\ \text{---} \oplus \text{---} \end{array} = \begin{array}{c} \bullet \text{---} \boxed{X} \text{---} \\ | \\ \oplus \text{---} \boxed{X} \text{---} \end{array}$$

$$\begin{array}{c} \text{---} \boxed{X} \text{---} \oplus \\ | \\ \text{---} \bullet \text{---} \end{array} = \begin{array}{c} \oplus \text{---} \boxed{X} \text{---} \\ | \\ \bullet \text{---} \end{array}$$

$$\begin{array}{c} \text{---} \boxed{Z} \text{---} \bullet \\ | \\ \text{---} \oplus \text{---} \end{array} = \begin{array}{c} \bullet \text{---} \boxed{Z} \text{---} \\ | \\ \oplus \text{---} \end{array}$$

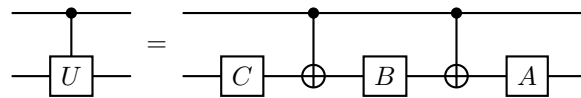


1.3 Phase kickback



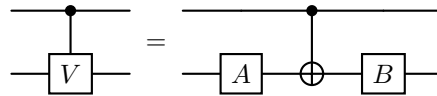
2 Decompositions

- See Barenco et al. Lemma 5.1 for a proof.



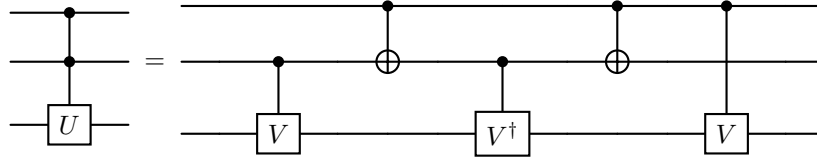
Where $U = e^{ia}AXBXC$, with $ABC = I$. See Corollary 4.2 of Nielsen and Chuang on page 176, or Barenco et al. Lemma 4.3 for a proof.

- See Barenco et al. Lemma 5.5 for a proof.



Where $V = R_z(\alpha) \cdot R_y(\theta) \cdot R_z(\alpha) \cdot \sigma_x$.

- See Barenco et al. Lemma 6.1 for a proof.



Where $V^2 = U$. The Toffoli gate is a special case in which $V = (1 - i)(I + iX)/2$.

3 References

- 1 Nielsen MA, Chuang IL. 2010. Quantum Computation and Quantum Information. New York: Cambridge Univ. Press. 10th Anniv. Ed. - Chapter 4.
- 2 Barenco, A., Bennett, C.H., Cleve, R., DiVincenzo, D.P., Margolus, N., Shor, P., Sleator, T., Smolin, J.A. and Weinfurter, H. (1995) Elementary gates for quantum computation. Phys. Rev. A 52, 3457–3467.