A Compilation of Circuit Identities and Decomposition

Lucas Camponogara Viera

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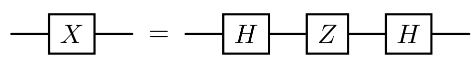
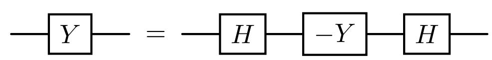
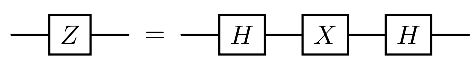
1 Gate Identities

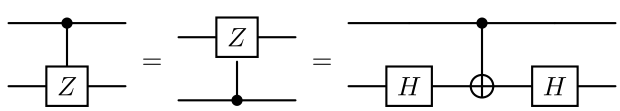
1.1 Conjugation by Hadamard

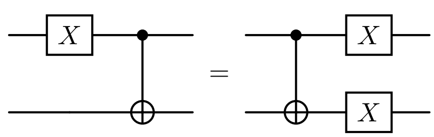


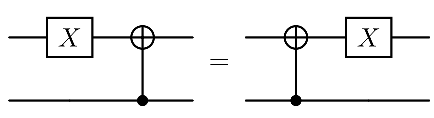


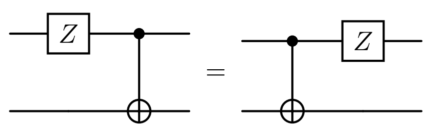


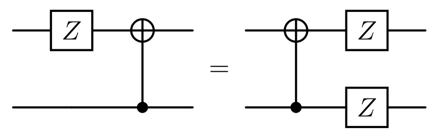
1.2 Two-qubit gates

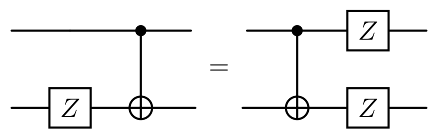


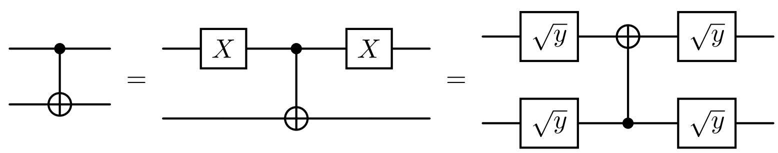




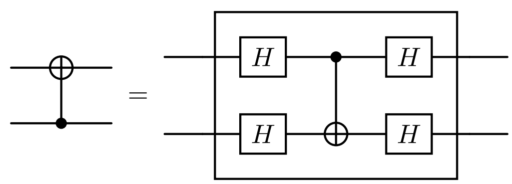






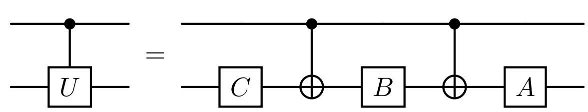


1.3 Phase kickback



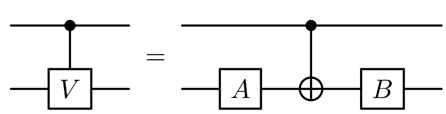
2 Decompositions

- See Barenco et al. Lemma 5.1 for a proof.



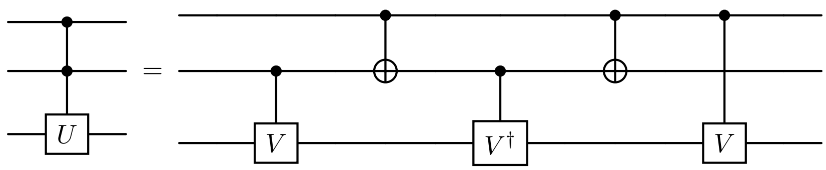
Where $U = e^{ia}AXBXC$, with $ABC = I$. See Corollary 4.2 of Nielsen and Chuang on page 176, or Barenco et al. Lemma 4.3 for a proof.

- See Barenco et al. Lemma 5.5 for a proof.



Where $V = R_z(\alpha) \cdot R_y(\theta) \cdot R_z(\alpha) \cdot \sigma_x$.

- Sleator-Weinfurter construction. See Barenco et al. Lemma 6.1 for a proof.



Where $V^2 = U$. The Toffoli gate is a special case in which $V = (1-i)(I + iX)/2$.

3 References

1 Nielsen MA, Chuang IL. 2010. Quantum Computation and Quantum Information. New York: Cambridge Univ. Press. 10th Anniv. Ed. - Chapter 4.

2 Barenco, A., Bennett, C.H., Cleve, R., DiVincenzo, D.P., Margolus, N., Shor, P., Sleator, T., Smolin, J.A. and Weinfurter, H. (1995) Elementary gates for quantum computation. Phys. Rev. A 52, 3457–3467.