Recursive Ray Echoes in a Bounded Lattice

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1.1 From Origin Seed to Harmonic Echo Field (Byte 1 → Byte 8)

When a ray is launched inside an 8×8 bounded box, each wall reflection "encodes" the ray's direction into a growing interference pattern. We can think of **Byte 1** as the origin seed (the initial directional vector), and **Byte 8** as the resulting **echo field** after multiple recursive bounces. Each bounce mixes the ray's trajectory like a round of a hash function – the ray **reflects (folds)** at the boundaries, preserving information about its direction but scrambling its path across the lattice. Over successive reflections, the path's imprint evolves into a stable pattern that effectively **"remembers"** the initial vector. The final **Byte 8 output** is a complex lattice pattern encoding the input direction, much like a cryptographic hash encodes its input. This visual echo field acts as a form of **symbolic memory** of the ray's origin.

- **Reflection as a Hashing Operation:** Each wall bounce flips the ray's direction in that axis (like a bit flip) and constrains the coordinates modulo the box size. In code terms, one can simulate this by taking the position modulo `16` (since an 8-unit box reflected is like a 16-unit period) and folding values above 8 back down (`if x > 8: x = 16 - x; dx = -dx`). This is analogous to how a hashing round folds data within fixed bounds. The ray's direction vector is effectively **"hashed"** through recursive reflections – small changes in the input angle produce entirely different path patterns (an **avalanche effect**), yet the final pattern deterministically depends on the initial vector, preserving its "memory" in distributed form.

1.2 Lattice Resonances: When Paths Fill the Box vs. Collapse

As the ray ricochets, it may either **(a)** cover the lattice nearly uniformly, or **(b)** fall into a repeating orbit that traces a limited set of lines. The outcome depends on the **rationality of the direction vector's slope** (the ratio of its x and y components relative to the box). This is a classic result in billiard dynamics: *"a billiard trajectory with a rational slope is periodic, while one with an irrational slope is dense in the square."* In our context:

- **Harmonic Full-Field Coverage (Ergodic Trajectory):** If the ray's direction is **incommensurate** with the box dimensions (effectively an irrational slope), the path will **not repeat** and eventually **fill the 8×8 field densely**. Each bounce lands at a new location, and over time the trajectory "remembers" *every* corner of the lattice. The echo field becomes a nearly continuous web of lines – a **full memory field** where the ray visits almost every region. This is the ideal **"truth field"**: the input's influence is spread evenly, leaving a complete imprint. In practice, a direction vector with a high-order rational ratio (very large coprime integers) approximates this dense coverage. For example, an initial direction of `[3.0, 0.35]` (slope 0.1167) produces a path that **zig-zags through nearly the entire grid** without obvious repetition. The plot below shows how this ray **sweeps across almost all lattice lines**, creating a dense mesh of echoes:

Ray with input direction [3.0, 0.35] yielding near-complete coverage. The blue trajectory densely fills the 8×8 box with 576 segments, producing an echo field that "remembers" the input angle. Red dots mark bounce points on the boundaries.

Why 0.35 gives nearly perfect coverage: The angle corresponding to `dy/dx = 0.35/3.0 0.1167` (about 6.7°) is essentially a **high-order rational** – in fact 0.35 is 7/20, so the slope is 7/20 ÷ 3 = 7/60 (a rational with a large denominator 60). This means the ray must complete a long sequence of bounces (60 horizontal crossings for every 7 vertical crossings) before it even *begins* to repeat. In 576 steps it still hasn't closed a cycle, so it visits a great variety of points. The result is a nearly uniform lattice coverage with finely spaced lines. Each reflection adds a new "layer" to the pattern until the field is saturated with the ray's presence. In essence, **0.35 hits a sweet spot**: it's rational enough to eventually form a resonance, but with such large period that the interim trajectory appears ergodic (covering the field uniformly). The angle 0.35 acts like a **phase attractor** for full coverage – at or near this value, the system "converges" to a complete memory field before any repetition interferes.

- **Periodic Lattice Resonance (Partial Coverage):** If the ray's direction slope is a **simple rational** (small integer ratio), the path will eventually **close on itself** and start repeating a fixed cycle. In these cases, the trajectory resonates with the box dimensions, forming a **stable periodic orbit** – a harmonic pattern – but it only covers a subset of the lattice. The memory field is **partial**: the ray keeps revisiting the same lines, leaving portions of the box untouched. For example, an initial direction `[1.0, 0.5]` (slope = 0.5, i.e. 1/2) yields a periodic orbit. This slope equals 1/2, meaning the ray's path aligns such that after a few bounces it returns to its starting point. The result is a **striped echo pattern**: the ray oscillates between a small number of track lines. The figure below illustrates this **partial collapse** – the path forms repeating diagonal bands and neglects other regions:

  *Ray with input direction [1.0, 0.50] (slope = 1/2). The trajectory is periodic (1088 segments shown over many cycles) and confined to a set of parallel bands. Red markers indicate the repeating bounce positions. This is a stable **lattice resonance** but covers only part of the box.*

  In this case, the slope 0.5 = 1/2 leads to a short 4-bounce cycle that closes on itself (hitting a corner and retracing). The pattern is a **harmonic standing wave** in the box – analogous to a resonant mode where only certain "nodes" are hit. The ray's memory field here is *stable but incomplete*: it **remembers** the input direction as a simple repeating pattern. Mathematically, such a trajectory corresponds to a rational slope (p/q with small integers). It will hit the same boundary points in a fixed sequence, forever cycling. The path has **collapsed** to a limited set of lattice lines (a **partial memory field**).

- **Degenerate Interference (Self-Collapse):** In extreme cases, the path can retrace itself exactly, yielding almost no coverage. For instance, a 45° angle (slope = 1) will bounce corner-to-corner and follow the same diagonal line back and forth. This **destructive interference** is a trivial resonance: the ray keeps hitting its own path. The echo field reduces to a single line (or a small set of lines) repeatedly traced – the system has **minimal memory** of the initial input (basically just one line's worth). Similarly, purely horizontal or vertical inputs

(0° or 90°) simply oscillate back and forth along one axis. These are edge cases where the trajectory's "echo" contains little information – the input wasn't **trust-aligned** to excite the field. In terms of memory, these inputs produce a nearly **empty** or redundant field (all bounces overwrite the same track).

Why do some angles fill the box while others don't? It comes down to **frequency matching** with the 8×8 geometry. A ray direction can be seen as having "notes" or frequencies in the x and y directions. If those frequencies are rationally related (forming a simple ratio), the motion in X and Y will **sync up periodically**, creating a repeating orbit (like a repeating melody). If they are irrationally related, the motion never quite syncs, and the ray will sample new positions indefinitely – akin to a **quasi-random** sequence. In wave terms, the box supports certain **standing wave harmonics**; a ray whose angle corresponds to those harmonics will resonate in a fixed pattern. Non-harmonic angles cause a **superposition of many modes**, effectively spreading the ray everywhere. This is why a slight change in angle can cause a **bifurcation** from ordered to disordered behavior.

1.3 Phase Thresholds and Symbolic Convergence

Not all "full coverage" trajectories are equal – some achieve a *more uniform* distribution of bounces than others. We observe special **phase thresholds** (critical angles) at which the echo field qualitatively changes. The angle 0.35 (as a fraction of the horizontal) appears to be one such threshold that produces a **symbolic convergence**: the ray's path converges to a stable, **all-encompassing pattern** (the "truth field"). We call 0.35 a **collapse attractor** because at this angle the system collapses any randomness into a coherent coverage of the lattice. Nearby angles may exhibit two distinct regimes (a bifurcation): for example, an angle just off 0.35 might start to wander in a more complex way or split the coverage into two interleaved patterns (a **superposition-like outcome**). At 0.35 exactly, the pattern "locks in" to a clean resonance that still covers the field completely. In effect, the ray finds a **harmonic equilibrium** between repeating and exploring. This can be interpreted as the input being **trust-aligned** – tuned precisely to the box's dimensions so that the output field is an orderly reflection of the input.

- **Delta-Phase Map:** One way to visualize this is to consider a **map of coverage vs. angle**. As the ray's angle (phase) varies from 0° to 45°, the coverage goes from zero (just sliding along a wall) up to maximal (some irrational angle). Rational slopes act like "spikes" or resonances on this map – at those angles the pattern repeats (dropping coverage). For extremely high-order rationals (angles very close to irrational), the coverage is nearly full because the repetition period is huge. Thus the coverage graph oscillates with finer structure as angle changes. Angles like 0.35 (7/20 of horizontal component in our normalized units) stand out because they combine a **moderately high denominator** with alignment to the grid: the trajectory hits every possible horizontal strip (20 distinct heights in 8 units, each 0.4 apart) before repeating. This **granularity** yields a uniformly filled field. A tiny change off 0.35 breaks the perfect spacing – the ray might skip some strips until a longer cycle fills them, or it might never exactly align, leading to a different interference pattern. We can say the system undergoes a **phase transition** at such values, from one harmonic regime to another. These thresholds signal where the **symbolic memory** (the echo field) transitions between patterns.

1.4 Reverse-Engineering the Harmonic Structure

The beauty of this echo system is that the final pattern (Byte 8) encodes the initial direction (Byte 1) in a decipherable way. By examining the harmonic structure of the echo field, we can **infer properties of the origin seed**:

- If the echo field is **fully saturated (uniform web)**, we deduce the input angle was *not a simple rational*. A densely filled lattice of echoes implies the ray did not find a short cycle – likely an irrational or high-order rational input. The *grain* of the pattern (how fine the mesh is) hints at the denominator of the rational approximation. For example, a very fine criss-cross pattern suggests a high-frequency input (numerator/denominator large), whereas a coarser grid suggests a moderately lower order. In the [3.0, 0.35] example above, the fine spacing of lines (dozens of thin zig-zags) indicates a high-order resonance, consistent with the 60/7 ratio embedded in 0.35.
- If the echo field shows a **limited set of distinct stripes or loops**, we know the input vector had a **low-order rational slope**. A clearly repeating motif (like the bands for 1:2 slope, or a diagonal loop for 1:1) reveals the exact ratio. We can essentially read off the ratio p/q by counting the hits: e.g., the [1.0, 0.5] pattern has 2 vertical bounces for 1 horizontal bounce in each cycle, matching 1/2. In general, a periodic pattern that hits, say, q distinct heights and p distinct vertical positions corresponds to a slope ~q/p. (In formal terms, an unfolded trajectory that goes through a lattice point (2p, 2q) corresponds to a rational slope q/p, and if p,q are coprime, that trajectory is the fundamental cycle.) If the pattern appears to be two superimposed cycles alternating, it means the input ratio was not in lowest terms – the trajectory has a common factor and essentially traces a smaller orbit multiple times. In other words, the echo field reveals whether the direction vector's components had a common factor (causing a **multiple orbit**). This is the **superposition-like bifurcation** we discussed: the pattern looks like two (or more) identical sub-patterns overlaid, indicating the ray's fundamental period was shorter and repeated.
- If the echo field is just a single line or a very simple X-shape, the initial direction was aligned to a symmetry (like 45° or axis-aligned). Such a degenerate echo tells us the input was a "resonant chord" of the square – the pattern collapsed completely due to destructive self-interference. These are easy to recognize (e.g., a perfect diagonal means equal x,y components).

In summary, by **"reading" the Byte 8 output pattern**, one can work backwards to the Byte 1 seed: the distribution of echoes (uniform vs patterned), the count of distinct bounce points, and the symmetry of the lines all encode the rational structure of the original direction. The echo field serves as a **symbolic fingerprint** of the input vector.

1.5 Connection to BBP -ray Addressing

Interestingly, the fully filled echo fields draw a parallel to sequences like the digits of . An irrational angle (like those related to) produces an **ergodic trajectory** – one that in theory hits every region of the box with uniform frequency. This is akin to how the digits of are (conjecturally) uniformly distributed. We can imagine a **-ray**, an ideal ray whose slope is an irrational number like /1 (a very steep angle ~72°). Such a ray would never repeat and would densely cover the square in a pseudo-random pattern. Each bounce coordinate (when folded into the 0–8 range) would appear random, but collectively the path encodes the irrational number. In fact, one could see the

bounce sequence as "addressing" the digits of : each reflection computes another piece of the fractional position, much like the Bailey–Borwein–Plouffe (BBP) formula computes binary digits of directly. The BBP algorithm allows extracting the nth digit of in base-16 without computing previous digits; analogously, a ray with an irrational slope "jumps" around the square accessing new locations without repeating old ones, effectively sampling an irrational coordinate sequence. This connection is conceptual – it suggests that a **bounded ray can serve as a read-write address system** for irrational numbers, with the walls enforcing a modulo operation similar to arithmetic in a finite base.

In practical terms, the concept of **BBP -ray addressing** underscores how thoroughly an irrational-angle ray explores the space. The ray's path is like a pointer moving through memory addresses (the lattice cells); given enough bounces, it will visit addresses in a pattern dictated by the irrational number. A rational-angle ray, in contrast, gets "stuck" cycling through a fixed subset of addresses (much like a repeating decimal). Thus, to create a **truth field** – one that encodes maximal information – we want an angle that behaves like : non-repeating, uniformly covering. The angle 0.35 we highlighted behaves in that spirit, as a rational approximation that yields uniform coverage. It's an angle that effectively **aligns trust** (predictability of coverage) with **truth** (completeness of coverage). The ray's recursive echoes become a **harmonic field of truth** when the input phase is chosen to avoid destructive self-interference and instead produce a wide, stable superposition of path harmonics.

1.6 Echo Harmonics: Visual and Symbolic Model

In the end, we can model the 8×8 recursive ray system as a kind of **oscillator** that can support certain harmonic patterns. The **visual patterns** correspond to the superposition of **echo harmonics** (each bounce can be seen as adding a reflection of the initial wave). If the input direction matches a harmonic of the 8×8 "cavity," the result is a stable standing wave (periodic orbit). If the input is off-harmonic, the echoes interfere to fill the space (an ergodic superposition of many modes). We can symbolize this as follows:

- Let the input direction be represented by a vector angle . Define its "frequency" components as *(p, q)* proportional to the travel in X and Y per reflection (in units of half-box-lengths). The **resonance condition** is that *(p, q)* are integers – in which case the trajectory corresponds to a standing wave mode with that many half-wavelengths fitting in the box. When *(p, q)* are co-prime integers (e.g. p=60,q=7 for giving 0.35 slope), the ray executes a **prime periodic orbit** covering p distinct horizontal crossings and q distinct vertical crossings. If those numbers are large, the "mode" involves many nodes and the pattern looks like a dense lattice. When *(p, q)* have a common factor, the mode is degenerate (multiple smaller orbit superimposed), leading to fewer distinct nodes.

- We can draw a **symbolic phase diagram** where stable harmonic orbits are like points on a lattice (given by rational = q/p) and chaotic or dense orbits fill the gaps. The **memory resonance** is maximal near those high-order rational points – there the system still eventually repeats (so it's a resonance), but only after covering the space (so it stores the input thoroughly before converging). At lower-order rationals, memory "collapses" quickly into a simple cycle, losing coverage.

- Each echo (bounce) is like iterating a function: *F(x) = |x| mod 8 with direction sign flips.* Symbolically, this is similar to iterating a hashing round or a linear congruential generator. The long-term behavior (pattern) can thus be analyzed with number theory: rational slopes yield eventually cyclic sequences (the state machine enters a cycle), irrational slopes yield sequences that pseudo-randomly traverse the state space. The **delta between phases** ( ) can dramatically change the sequence – a hallmark of chaotic systems and also of cryptographic hash functions (small in input yields large changes in output). In our ray system, a small tweak in can shift the path from one resonance to another (a bifurcation), or from periodic to ergodic. This sensitivity reinforces that the **echo field is a unique signature** of the exact input angle.

1.7 Conclusion: Harmonic Truth Fields Through Aligned Inputs

By analyzing the recursive ray-trace in the 8×8 box, we see that **bounded echoes become harmonic truth fields when the input is properly aligned**. An input like 0.35 (with the right rational characteristics) acts as a **convergence point** where the ray's path encodes the input almost completely into the lattice – the pattern is rich and covers the "memory" of the box. In this state, the **echo field functions as symbolic memory**: it retains the influence of the initial vector in every corner of the grid. We likened this to a cryptographic hash or a resonant mode – in both cases, the output is a complex, stable representation of the input.

To formalize the findings:

- **Direction vectors collapse into stable lattice resonances** when their slope is rational. The box acts like a filter that causes such rays to retrace periodically, forming geometric "harmonics" (standing wave patterns). Simpler ratios collapse faster (fewer distinct reflections), whereas complex ratios take longer, producing richer patterns before closing.
- **Certain angles (like 0.35) yield near-perfect field coverage** because they combine a favorable rational ratio with the box size, resulting in a very large fundamental period. Before the ray ever repeats, it has effectively painted the entire lattice with its trajectory. These angles sit at **phase thresholds** where the system transitions from partial to full coverage – they serve as **attractors for stable, complete patterns**.
- **SHA-style reflection models the bounce echo** by treating each wall impact as a deterministic "mixing" operation. Just as a SHA hash expands and folds input bits through rounds, the ray's reflections fold the trajectory within the box. Despite the chaotic appearance, the process preserves the ray's initial "bits" (direction components) in the final pattern. Thus the echo field is a **persistent memory** of the input, analogous to how a hash digest retains input uniqueness.
- **Symbolic convergence occurs at phase thresholds** where the pattern stops getting more complex and settles into a resonance that nevertheless spans the whole domain. At these points, the echo field represents a **unified truth** of the input conditions – all parts of the field "agree" with the input, as the coverage is uniform and stable. Slightly off these phases, the field might split into multiple sub-patterns (like alternating orbits) or fail to cover completely, reflecting a less unified memory of the input (some information is lost or compartmentalized in separate loops).

In essence, a **bounded recursive echo system** (like our ray in the 8×8 box) behaves like a **trustable memory** when the input is aligned to the correct harmonics. The case of 0.35 shows how a carefully chosen direction can turn the entire box into a **harmonic truth field** – a field of echoes that stably and completely encodes the input's "truth" (its directional signature) in the lattice. Through this lens, the bouncing ray isn't just a physical path, but a **symbolic computation**: it's hashing an input vector into a pattern, it's oscillating in resonant modes, and it's addressing memory locations like the digits of . By reverse-engineering these patterns, we decode the rules of this system: when input and boundary are in harmony, the output is maximal and orderly; when they are not, the output fragments or collapses. This deep interplay of geometry, number theory, and symbolic representation is what underlies the recursive ray-trace simulation's behavior – turning simple bounces into a **field of harmonic echoes** that we can interpret and trust as the memory of the initial seed.

Sources: The analysis above is informed by principles of dynamical billiards (periodicity of rational vs. irrational slopes) and the theory of periodic orbits in rectangular tables, as well as analogies to cryptographic hashing and signal harmonics. The examples and images were produced from custom simulations of a ray in an 8×8 grid with various input vectors (e.g. [3, 0.35], [1, 0.5]) to illustrate partial vs. full lattice coverage. The connection to the BBP formula for is conceptual, highlighting how an irrational-angle trajectory can uniformly explore a space akin to how 's digits uniformly distribute.

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