Test Results in the Context of Quantum/Casimir Friction

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1 Response Functions

All response functions ($\underline{G}$, $\underline{\alpha}$, ϵ, etc.) have to fulfill a certain set of easily testable relations. First, they have to fulfill reality in time domain and thus the crossing relation in frequency domain

$$\begin{aligned}
\underline{G}(-\omega, \mathbf{r}, \mathbf{r'}) &= \underline{G}^\dagger(\omega, \mathbf{r}, \mathbf{r'})\,, && (1)\\
\underline{\alpha}(-\omega) &= \underline{\alpha}^*(\omega)\,, && (2)\\
\epsilon(-\omega) &= \epsilon^*(\omega)\,, && (3)\\
r(-\omega) &= r^*(\omega)\,. && (4)\\
& && (5)
\end{aligned}$$

Furthermore, in most cases one can rely on reciprocity of the Green's tensor

$$\underline{G}(\omega, -\mathbf{k}, z) = \underline{G}^\intercal(\omega, \mathbf{k}, z)\,. \tag{6}$$

In order to test the Green's tensor's reality, one can use that

$$\underline{G}_\Im(\omega, \mathbf{k}, z) = \frac{\underline{G}(\omega, \mathbf{k}, z) - \underline{G}^\dagger(\omega, \mathbf{k}, z)}{2\mathrm{i}} = \frac{\underline{G}^\dagger(-\omega, \mathbf{k}, z) - \underline{G}(-\omega, \mathbf{k}, z)}{2\mathrm{i}} = -\underline{G}_\Im(-\omega, \mathbf{k}, z)\,, \tag{7}$$

$$\underline{G}_\Re(\omega, \mathbf{k}, z) = \frac{\underline{G}(\omega, \mathbf{k}, z) + \underline{G}^\dagger(\omega, \mathbf{k}, z)}{2} = \frac{\underline{G}^\dagger(-\omega, \mathbf{k}, z) + \underline{G}(-\omega, \mathbf{k}, z)}{2} = \underline{G}_\Re(-\omega, \mathbf{k}, z)\,. \tag{8}$$

However, it is more feasible to compare the integrated quantities

$$\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\underline{G}_\Im(\omega, \mathbf{k}, z) = -\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\underline{G}_\Im(-\omega, \mathbf{k}, z)\,, \tag{9}$$

$$\tag{10}$$

2 Power Spectrum and Friction

Another relevant quantity in the context of the noncontact friction is the power spectrum $\underline{S}$ (see for example [**?**]). Here, we find the testable relation

$$\underline{S}^\dagger(\omega) = \underline{S}(\omega)\,. \tag{11}$$

Moreover, one should be able to retrieve the static case

$$\lim_{v\to 0} \underline{S}(\omega) \sim \frac{\hbar}{\pi}\theta(\omega)\underline{\alpha}_\Im(\omega)\,. \tag{12}$$

Additionally, one can test the integration of $\underline{\alpha}$ in the first order in α_0.

$$\int_0^{\omega_\mathrm{cut}} \mathrm{d}\omega\, \underline{\alpha}_\Im(\omega) \sim \begin{cases} \mathbb{1}\frac{\pi}{2}\alpha_0\omega_a & \text{for} \quad \omega_\mathrm{cut} \gg \omega_a \\ \alpha_0^2 \int_0^{\omega_\mathrm{cut}} \mathrm{d}\omega \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\underline{G}_\Im(\mathbf{k}, z_a, \omega + k_x v) & \text{for} \quad \omega_\mathrm{cut} \ll \omega_a \end{cases} \tag{13}$$

3 Test Case: Free Space

In free space for temperatures $k_{\mathrm{B}}T \gg \hbar\omega_a$ one finds (see `VacuumGreen.pdf`)

$$F = -\frac{v}{c}\frac{k_{\mathrm{B}}T\alpha_0}{6\pi\epsilon_0}\frac{\omega_a^4}{c^4} \tag{14}$$

Furthermore, one can test the individual Green's tensor integrations via the relations given in the `VacuumGreen.pdf`. With respect to the polarizability one finds

$$\int_0^{\omega_{\mathrm{cut}}} \mathrm{d}\omega\, \underline{\alpha}_{\Im}(\omega) \sim \begin{cases} \mathbb{1}\frac{\pi}{2}\alpha_0\omega_a & \text{for} \quad \omega_{\mathrm{cut}} \gg \omega_a \\ \alpha_0^2\frac{\omega_{\mathrm{cut}}^4}{8\pi\epsilon_0}\mathrm{diag}\left[\frac{c}{3(c^2-v^2)^2}, \frac{c\left(c^2+v^2\right)}{3(c^2-v^2)^3}, \frac{c\left(c^2+v^2\right)}{3(c^2-v^2)^3}\right] \overset{v\ll c}{\approx} \mathbb{1}\alpha_0^2\frac{\omega_{\mathrm{cut}}^4}{24\pi\epsilon_0 c^3} & \text{for} \quad \omega_{\mathrm{cut}} \ll \omega_a \end{cases} \tag{15}$$