Algebraic Number Theory

(PARI-GP version 2.13.2)

Binary Quadratic Forms

create $ax^2+bxy+cy^2$ (distance d)	`Qfb(a, b, c, {d})`
reduce x ($s=\sqrt{D}$, $l=\lfloor s \rfloor$)	`qfbred(x, {flag}, {D}, {l}, {s})`
return $[y,g]$, $g \in \mathrm{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced	`qfbredsl2(x)`
composition of forms	`x*y` or `qfbnucomp(x, y, l)`
n-th power of form	`x^n` or `qfbnupow(x, n)`
composition without reduction	`qfbcompraw(x, y)`
n-th power without reduction	`qfbpowraw(x, n)`
prime form of disc. x above prime p	`qfbprimeform(x, p)`
class number of disc. x	`qfbclassno(x)`
Hurwitz class number of disc. x	`qfbhclassno(x)`
solve $Q(x,y)=n$ in integers	`qfbsolve(Q, n)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1+\sqrt{x})/2$	`quadgen(x)`
minimal polynomial of ω	`quadpoly(x)`
discriminant of $\mathbf{Q}(\sqrt{x})$	`quaddisc(x)`
regulator of real quadratic field	`quadregulator(x)`
fundamental unit in real $\mathbf{Q}(\sqrt{D})$	`quadunit(D,{'w})`
class group of $\mathbf{Q}(\sqrt{D})$	`quadclassunit(D, {flag}, {t})`
Hilbert class field of $\mathbf{Q}(\sqrt{D})$	`quadhilbert(D, {flag})`
...using specific class invariant ($D<0$)	`polclass(D, {inv})`
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$	`quadray(D, f, {flag})`

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$. We denote $\theta = \bar{X}$ the canonical root of f in K. A *nf* structure contains a maximal order and allows operations on elements and ideals. A *bnf* adds class group and units. A *bnr* is attached to ray class groups and class field theory. A *rnf* is attached to relative extensions L/K.

init number field structure *nf*	`nfinit(f, {flag})`
known integer basis B	`nfinit([f, B])`
order maximal at $vp = [p_1, \dots, p_k]$	`nfinit([f, vp])`
order maximal at all $p \le P$	`nfinit([f, P])`
certify maximal order	`nfcertify(nf)`
nf members:	
a monic $F \in \mathbf{Z}[X]$ defining K	`nf.pol`
number of real/complex places	`nf.r1/r2/sign`
discriminant of *nf*	`nf.disc`
primes ramified in *nf*	`nf.p`
T_2 matrix	`nf.t2`
complex roots of F	`nf.roots`
integral basis of $\mathbf{Z}_K$ as powers of θ	`nf.zk`
different/codifferent	`nf.diff`, `nf.codiff`
index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$	`nf.index`
recompute *nf* using current precision	`nfnewprec(nf)`
init relative *rnf* $L = K[Y]/(g)$	`rnfinit(nf, g)`
init *bnf* structure	`bnfinit(f, 1)`
bnf members: same as *nf*, plus	
underlying *nf*	`bnf.nf`
class group, regulator	`bnf.clgp`, `bnf.reg`
fundamental/torsion units	`bnf.fu`, `bnf.tu`
add S-class group and units, yield *bnf*S	`bnfsunit(bnf, S)`
init class field structure *bnr*	`bnrinit(bnf, m, {flag})`
bnr members: same as *bnf*, plus	
underlying *bnf*	`bnr.bnf`
big ideal structure	`bnr.bid`
modulus m	`bnr.mod`
structure of $(\mathbf{Z}_K/m)^*$	`bnr.zkst`

Fields, subfields, embeddings

Defining polynomials, embeddings

smallest poly defining $f=0$ (slow)	`polredabs(f, {flag})`
small poly defining $f=0$ (fast)	`polredbest(f, {flag})`
random Tschirnhausen transform of f	`poltschirnhaus(f)`
$\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic?	`nfisincl(f, g)`, `nfisisom`
reverse polmod $a = A(t) \bmod T(t)$	`modreverse(a)`
compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$	`polcompositum(f, g, {flag})`
compositum of $K[t]/(f)$, $K[t]/(g)$	`nfcompositum(nf, f, g, {flag})`
splitting field of K (degree divides d)	`nfsplitting(nf, {d})`
signs of real embeddings of x	`nfeltsign(nf, x, {pl})`
complex embeddings of x	`nfeltembed(nf, x, {pl})`
$T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$	`nfpolsturm(nf, T, {pl})`

Subfields, polynomial factorization

subfields (of degree d) of *nf*	`nfsubfields(nf, {d})`
maximal subfields of *nf*	`nfsubfieldsmax(nf)`
maximal CM subfield of *nf*	`nfsubfieldscm(nf)`
d-th degree subfield of $\mathbf{Q}(\zeta_n)$	`polsubcyclo(n, d, {v})`
roots of unity in *nf*	`nfrootsof1(nf )`
roots of g belonging to *nf*	`nfroots(nf, g)`
factor g in *nf*	`nffactor(nf, g)`

Linear and algebraic relations

poly of degree $\le k$ with root $x \in \mathbf{C}$	`algdep(x, k)`
alg. dep. with pol. coeffs for series s	`seralgdep(s, x, y)`
small linear rel. on coords of vector x	`lindep(x)`

Basic Number Field Arithmetic (nf)

Number field elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis `nf.zk`).

Basic operations

$x+y$	`nfeltadd(nf, x, y)`
$x \times y$	`nfeltmul(nf, x, y)`
x^n, $n \in \mathbf{Z}$	`nfeltpow(nf, x, n)`
x/y	`nfeltdiv(nf, x, y)`
$q = x\backslash/y :=$ `round`(x/y)	`nfeltdiveuc(nf, x, y)`
$r = x\%y := x - (x\backslash/y)y$	`nfeltmod(nf, x, y)`
... $[q,r]$ as above	`nfeltdivrem(nf, x, y)`
reduce x modulo ideal A	`nfeltreduce(nf, x, A)`
absolute trace $\mathrm{Tr}_{K/\mathbf{Q}}(x)$	`nfelttrace(nf, x)`
absolute norm $\mathrm{N}_{K/\mathbf{Q}}(x)$	`nfeltnorm(nf, x)`

Multiplicative structure of K^*; $K^*/(K^*)^n$

valuation $v_{\mathfrak{p}}(x)$	`nfeltval(nf, x, pr)`
... write $x = \pi^{v_{\mathfrak{p}}(x)} y$	`nfeltval(nf, x, pr, &y)`
quadratic Hilbert symbol (at $\mathfrak{p}$)	`nfhilbert(nf, a, b, {pr})`
b such that $xb^n = v$ is small	`idealredmodpower(nf, x, n)`

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$	`nfbasis(f)`
field discriminant of $\mathbf{Q}[x]/(f)$	`nfdisc(f)`
...and factorization	`nfdiscfactors(f)`
express x on integer basis	`nfalgtobasis(nf, x)`
express element x as a polmod	`nfbasistoalg(nf, x)`

Dedekind Zeta Function ζ_K, Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$ encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) < b$	`dirzetak(nf, b)`
init $\zeta_K^{(k)}(s)$ for $k \le n$	`L = lfuninit(bnf, R, {n = 0})`
compute $\zeta_K(s)$ (n-th derivative)	`lfun(L, s, {n = 0})`
compute $\Lambda_K(s)$ (n-th derivative)	`lfunlambda(L, s, {n = 0})`
init $L_K^{(k)}(s,\chi)$ for $k \le n$	`L = lfuninit([bnr, chi], R, {n = 0})`
compute $L_K(s,\chi)$ (n-th derivative)	`lfun(L, s, {n})`
Artin root number of K	`bnrrootnumber(bnr, chi, {flag})`
$L(1,\chi)$, for all χ trivial on H	`bnrL1(bnr, {H}, {flag})`

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually *bnr* (ray class field), *bnr*, H (congruence subgroup) or *bnr*, χ (character on `bnr.clgp`). Any of these define a unique abelian extension of K.

units / S-units	`bnfunits(bnf, {S})`
remove GRH assumption from *bnf*	`bnfcertify(bnf)`
expo. of ideal x on class gp	`bnfisprincipal(bnf, x, {flag})`
expo. of ideal x on ray class gp	`bnrisprincipal(bnr, x, {flag})`
expo. of x on fund. units	`bnfisunit(bnf, x)`
...on S-units, U is `bnfunits(bnf, S)`	`bnfisunit(bnfs, x, U)`
signs of real embeddings of `bnf.fu`	`bnfsignunit(bnf)`
narrow class group	`bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m	`bnrclassno(bnf, m)`
discriminant of class field	`bnrdisc(a1, {a2})`
ray class numbers, l list of moduli	`bnrclassnolist(bnf, l)`
discriminants of class fields	`bnrdisclist(bnf, l, {arch}, {flag})`
decode output from `bnrdisclist`	`bnfdecodemodule(nf, fa)`
is modulus the conductor?	`bnrisconductor(a1, {a2})`
is class field (bnr, H) Galois over K^G	`bnrisgalois(bnr, G, H)`
action of automorphism on `bnr.gen`	`bnrgaloismatrix(bnr, aut)`
apply `bnrgaloismatrix` M to H	`bnrgaloisapply(bnr, M, H)`
characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$	`bnrchar(bnr, g, {v})`
conductor of character χ	`bnrconductor(bnr, chi)`
conductor of extension	`bnrconductor(a1, {a2}, {flag})`
conductor of extension $K[Y]/(g)$	`rnfconductor(bnf, g)`
canonical projection $\mathrm{Cl}_F \to \mathrm{Cl}_f$, $f \mid F$	`bnrmap`
Artin group of extension $K[Y]/(g)$	`rnfnormgroup(bnr, g)`
subgroups of *bnr*, index $<= b$	`subgrouplist(bnr, b, {flag})`
class field defined by $H < \mathrm{Cl}_f$	`bnrclassfield(bnr, H)`
...low level equivalent, prime degree	`rnfkummer(bnr, H)`
same, using Stark units (real field)	`bnrstark(bnr, sub, {flag})`
is a an n-th power in K_v ?	`nfislocalpower(nf, v, a, n)`
cyclic L/K satisf. local conditions	`nfgrunwaldwang(nf, P, D, pl)`

Logarithmic class group

logarithmic ℓ-class group	bnflog(bnf, ℓ)
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	bnflogef(bnf, pr)
$\exp\deg_F(A)$	bnflogdegree(bnf, A, ℓ)
is ℓ-extension L/K locally cyclotomic	rnfislocalcyclo(rnf)

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	nfisideal(nf, id)
is x principal in bnf ?	bnfisprincipal(bnf, x)
give $[a,b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	idealtwoelt($nf, x, \{a\}$)
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	idealhnf($nf, a, \{b\}$)
norm of ideal x	idealnorm(nf, x)
minimum of ideal x (direction v)	idealmin(nf, x, v)
LLL-reduce the ideal x (direction v)	idealred($nf, x, \{v\}$)

Ideal Operations

add ideals x and y	idealadd(nf, x, y)
multiply ideals x and y	idealmul($nf, x, y, \{flag\}$)
intersection of ideal x with Q	idealdown(nf, x)
intersection of ideals x and y	idealintersect($nf, x, y, \{flag\}$)
n-th power of ideal x	idealpow($nf, x, n, \{flag\}$)
inverse of ideal x	idealinv(nf, x)
divide ideal x by y	idealdiv($nf, x, y, \{flag\}$)
Find $(a,b) \in x \times y$, $a+b=1$	idealaddtoone($nf, x, \{y\}$)
coprime integral A, B such that $x = A/B$	idealnumden(nf, x)

Primes and Multiplicative Structure

check whether x is a maximal ideal	idealismaximal(nf, x)
factor ideal x in $\mathbf{Z}_K$	idealfactor(nf, x)
expand ideal factorization in K	idealfactorback($nf, f, \{e\}$)
is ideal A an n-th power ?	idealispower(nf, A, n)
expand elt factorization in K	nffactorback($nf, f, \{e\}$)
decomposition of prime p in $\mathbf{Z}_K$	idealprimedec(nf, p)
valuation of x at prime ideal pr	idealval(nf, x, pr)
weak approximation theorem in nf	idealchinese(nf, x, y)
$a \in K$, s.t. $v_{\mathfrak{p}}(a) = v_{\mathfrak{p}}(x)$ if $v_{\mathfrak{p}}(x) \neq 0$	idealappr(nf, x)
$a \in K$ such that $(a \cdot x, y) = 1$	idealcoprime(nf, x, y)
give bid =structure of $(\mathbf{Z}_K/id)^*$	idealstar($nf, id, \{flag\}$)
structure of $(1+\mathfrak{p})/(1+\mathfrak{p}^k)$	idealprincipalunits(nf, pr, k)
discrete log of x in $(\mathbf{Z}_K/bid)^*$	ideallog(nf, x, bid)
idealstar of all ideals of norm $\leq b$	ideallist($nf, b, \{flag\}$)
add Archimedean places	ideallistarch($nf, b, \{ar\}, \{flag\}$)
init modpr structure	nfmodprinit($nf, pr, \{v\}$)
project t to $\mathbf{Z}_K/pr$	nfmodpr($nf, t, modpr$)
lift from $\mathbf{Z}_K/pr$	nfmodprlift($nf, t, modpr$)

Galois theory over Q

conjugates of a root θ of nf	nfgaloisconj($nf, \{flag\}$)
apply Galois automorphism s to x	nfgaloisapply(nf, s, x)
Galois group of field $\mathbf{Q}[x]/(f)$	polgalois(f)
initializes a Galois group structure G	galoisinit($pol, \{den\}$)
character table of G	galoischartable(G)
conjugacy classes of G	galoisconjclasses(G)
$\det(1-\rho(g)T)$, χ character of ρ	galoischarpoly($G, \chi, \{o\}$)
$\det(\rho(g))$, χ character of ρ	galoischardet($G, \chi, \{o\}$)
action of p in nfgaloisconj form	galoispermtopol($G, \{p\}$)
identify as abstract group	galoisidentify(G)
export a group for GAP/MAGMA	galoisexport($G, \{flag\}$)
subgroups of the Galois group G	galoissubgroups(G)
is subgroup H normal?	galoisisnormal(G, H)

Algebraic Number Theory

(PARI-GP version 2.13.2)

subfields from subgroups	galoissubfields($G, \{flag\}, \{v\}$)
fixed field	galoisfixedfield($G, perm, \{flag\}, \{v\}$)
Frobenius at maximal ideal P	idealfrobenius(nf, G, P)
ramification groups at P	idealramgroups(nf, G, P)
is G abelian?	galoisisabelian($G, \{flag\}$)
abelian number fields/$\mathbf{Q}$	galoissubcyclo(N,H,{flag},{v})

The galpol package

query the package: polynomial	galoisgetpol(a,b,{s})
...: permutation group	galoisgetgroup(a,b)
...: group description	galoisgetname(a,b)

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	rnfequation($nf, T, \{flag\}$)
is L/K abelian?	rnfisabelian(nf, T)
relative nfalgtobasis	rnfalgtobasis(rnf, x)
relative nfbasistoalg	rnfbasistoalg(rnf, x)
relative idealhnf	rnfidealhnf(rnf, x)
relative idealmul	rnfidealmul(rnf, x, y)
relative idealtwoelt	rnfidealtwoelt(rnf, x)

Lifts and Push-downs

absolute $\to$ relative representation for x	rnfeltabstorel(rnf, x)
relative $\to$ absolute representation for x	rnfeltreltoabs(rnf, x)
lift x to the relative field	rnfeltup(rnf, x)
push x down to the base field	rnfeltdown(rnf, x)

idem for x ideal: (rnfideal)reltoabs, abstorel, up, down

Norms and Trace

relative norm of element $x \in L$	rnfeltnorm(rnf, x)
relative trace of element $x \in L$	rnfelttrace(rnf, x)
absolute norm of ideal x	rnfidealnormabs(rnf, x)
relative norm of ideal x	rnfidealnormrel(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	bnfisintnorm(bnf, x)
is $x \in \mathbf{Q}$ a norm from K?	bnfisnorm($bnf, x, \{flag\}$)
initialize T for norm eq. solver	rnfisnorminit($K, pol, \{flag\}$)
is $a \in K$ a norm from L?	rnfisnorm($T, a, \{flag\}$)
initialize t for Thue equation solver	thueinit(f)
solve Thue equation $f(x,y) = a$	thue($t, a, \{sol\}$)
characteristic poly. of a mod T	rnfcharpoly($nf, T, a, \{v\}$)

Factorization

factor ideal x in L	rnfidealfactor(rnf, x)
$[S,T]$: $T_{i,j} \mid S_i$; S primes of K above p	rnfidealprimedec(rnf, p)

Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$-module

relative polredbest	rnfpolredbest(nf, T)
relative polredabs	rnfpolredabs(nf, T)
relative Dedekind criterion, prime pr	rnfdedekind(nf, T, pr)
discriminant of relative extension	rnfdisc(nf, T)
pseudo-basis of $\mathbf{Z}_L$	rnfpseudobasis(nf, T)

General $\mathbf{Z}_K$-modules: $M = [\text{matrix}, \text{vec. of ideals}] \subset L$

relative HNF / SNF	nfhnf(nf, M), nfsnf
multiple of det M	nfdetint(nf, M)
HNF of M where $d = nfdetint(M)$	nfhnfmod(x, d)
reduced basis for M	rnflllgram(nf, T, M)
determinant of pseudo-matrix M	rnfdet(nf, M)
Steinitz class of M	rnfsteinitz(nf, M)
$\mathbf{Z}_K$-basis of M if $\mathbf{Z}_K$-free, or 0	rnfhnfbasis(bnf, M)
n-basis of M, or $(n+1)$-generating set	rnfbasis(bnf, M)
is M a free $\mathbf{Z}_K$-module?	rnfisfree(bnf, M)

Associative Algebras

A is a general associative algebra given by a multiplication table mt (over $\mathbf{Q}$ or $\mathbf{F}_p$); represented by al from algtableinit.

create al from mt (over $\mathbf{F}_p$)	algtableinit($mt, \{p=0\}$)
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	alggroup($G, \{p=0\}$)
center of group algebra	alggroupcenter($G, \{p=0\}$)

Properties

is (mt, p) OK for algtableinit?	algisassociative($mt, \{p=0\}$)
multiplication table mt	algmultable(al)
dimension of A over prime subfield	algdim(al)
characteristic of A	algchar(al)
is A commutative?	algiscommutative(al)
is A simple?	algissimple(al)
is A semi-simple?	algissemisimple(al)
center of A	algcenter(al)
Jacobson radical of A	algradical(al)
radical J and simple factors of A/J	algsimpledec(al)

Operations on algebras

create A/I, I two-sided ideal	algquotient(al, I)
create $A_1 \otimes A_2$	algtensor($al1, al2$)
create subalgebra from basis B	algsubalg(al, B)
quotients by ortho. central idempotents e	algcentralproj(al, e)
isomorphic alg. with integral mult. table	algmakeintegral(mt)
prime subalgebra of semi-simple A over $\mathbf{F}_p$	algprimesubalg(al)
find isomorphism $A \cong M_d(\mathbf{F}_q)$	algsplit(al)

Operations on lattices in algebras

lattice generated by cols. of M	alglathnf(al, M)
...by the products xy, $x \in lat1$, $y \in lat2$	alglatmul($al, lat1, lat2$)
sum $lat1 + lat2$ of the lattices	alglatadd($al, lat1, lat2$)
intersection $lat1 \cap lat2$	alglatinter($al, lat1, lat2$)
test $lat1 \subset lat2$	alglatsubset($al, lat1, lat2$)
generalized index $(lat2 : lat1)$	alglatindex($al, lat1, lat2$)
$\{x \in al \mid x \cdot lat1 \subset lat2\}$	alglatlefttransporter($al, lat1, lat2$)
$\{x \in al \mid lat1 \cdot x \subset lat2\}$	alglatrighttransporter($al, lat1, lat2$)
test $x \in lat$ (set $c =$ coord. of x)	alglatcontains($al, lat, x, \{\&c\}$)
element of lat with coordinates c	alglatelement(al, lat, c)

Operations on elements

$a+b$, $a-b$, $-a$	algadd(al, a, b), algsub, algneg
$a \times b$, a^2	algmul(al, a, b), algsqr
a^n, a^{-1}	algpow(al, a, n), alginv
is x invertible ? (then set $z = x^{-1}$)	algisinv($al, x, \{\&z\}$)
find z such that $x \times z = y$	algdivl(al, x, y)
find z such that $z \times x = y$	algdivr(al, x, y)
does z s.t. $x \times z = y$ exist? (set it)	algisdivl($al, x, y, \{\&z\}$)
matrix of $v \mapsto x \cdot v$	algtomatrix(al, x)
absolute norm	algnorm(al, x)
absolute trace	algtrace(al, x)
absolute char. polynomial	algcharpoly(al, x)
given $a \in A$ and polynomial T, return $T(a)$	algpoleval(al, T, a)
random element in a box	algrandom(al, b)

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩

Central Simple Algebras

A is a central simple algebra over a number field K; represented by al from `alginit`; K is given by a nf structure.

create CSA from data	`alginit`$(B, C, \{v\}, \{maxord = 1\})$
multiplication table over K	$B = K$, $C = mt$
cyclic algebra $(L/K, \sigma, b)$	$B = rnf$, $C = [sigma, b]$
quaternion algebra $(a, b)_K$	$B = K$, $C = [a, b]$
matrix algebra $M_d(K)$	$B = K$, $C = d$
local Hasse invariants over K	$B = K$, $C = [d, [PR, HF], HI]$

Properties

type of al (mt, CSA)	`algtype`(al)
dimension of A over $\mathbf{Q}$	`algdim`$(al, 1)$
dimension of al over its center K	`algdim`(al)
degree of A $(= \sqrt{\dim_K A})$	`algdegree`(al)
al a cyclic algebra $(L/K, \sigma, b)$; return σ	`algaut`(al)
... return b	`algb`(al)
... return L/K, as an rnf	`algsplittingfield`(al)
split A over an extension of K	`algsplittingdata`(al)
splitting field of A as an rnf over center	`algsplittingfield`(al)
multiplication table over center	`algrelmultable`(al)
places of K at which A ramifies	`algramifiedplaces`(al)
Hasse invariants at finite places of K	`alghassef`(al)
Hasse invariants at infinite places of K	`alghassei`(al)
Hasse invariant at place v	`alghasse`(al, v)
index of A over K (at place v)	`algindex`$(al, \{v\})$
is al a division algebra? (at place v)	`algisdivision`$(al, \{v\})$
is A ramified? (at place v)	`algisramified`$(al, \{v\})$
is A split? (at place v)	`algissplit`$(al, \{v\})$

Operations on elements

reduced norm	`algnorm`(al, x)
reduced trace	`algtrace`(al, x)
reduced char. polynomial	`algcharpoly`(al, x)
express x on integral basis	`algalgtobasis`(al, x)
convert x to algebraic form	`algbasistoalg`(al, x)
map $x \in A$ to $M_d(L)$, L split. field	`algtomatrix`(al, x)

Orders

$\mathbf{Z}$-basis of order $\mathcal{O}_0$	`algbasis`(al)
discriminant of order $\mathcal{O}_0$	`algdisc`(al)
$\mathbf{Z}$-basis of natural order in terms $\mathcal{O}_0$'s basis	`alginvbasis`(al)