Trading strategies implemented on python - Equities (I)

Chenjie LI, Maxime LE FLOCH, ESILV Msc Financial Engineering

July 30, 2024

Abstract

In this paper, we introduce and implemented some trading strategies applied commonly on equities (there will be a part II on other strategies applied on equities), these strategies were back tested with real data from Yahoo Finance and each strategy is accompanied by a graph with high lightened long and short signals and the return calculated. You can find the Python implementation on: Chenjie's Github Trading strategies or Maxime's Github Trading strategies.

Introduction

Overview

Stock trading strategies are systematic approaches employed by traders and investors to maximize returns and manage risks in the financial markets. These strategies are based on various principles and can range from simple to complex, utilizing technical, fundamental, and quantitative analyses. Understanding these strategies is crucial for anyone looking to navigate the stock market effectively.
Having a well-defined trading strategy is essential for success in the stock market. It provides a clear framework for making decisions, helps manage emotions, and ensures consistency in approach. A robust strategy incorporates risk management techniques, such as stop-loss orders and position sizing, to protect against significant losses.
Developing and understanding various stock trading strategies is vital for navigating the complexities of the financial markets. Whether you're a novice trader or an experienced investor, a well-crafted strategy can enhance your ability to achieve your financial goals. By leveraging different approaches, traders can adapt to changing market conditions and improve their chances of success in the stock market.
For each strategy, we'll explain its mechanics, rationale, and how to backtest and optimize it. However, our main goal is to implement it in Python to enhance our knowledge and skills. While we may not apply all backtesting and optimization techniques to every strategy, we hope you find the process enjoyable nonetheless.

Contents

1 Conventional notation of our paper

1.1 variable used

- σ_i is the annualized volatility for a stock i.
- R_i is the average return of a stock.
- R_f is the risk-free rate.
- R_m is the average return of the market.
- $\text{Var}(R_i)$ is the variance of a stock i.
- $\text{Cov}(R_i, R_m)$ is the co-variance between a stock i and the market m.
- ϵ_i is the error term.

1.2 Metric used in our paper

- *Alpha* : Represents the excess return of an investment relative to the return of a benchmark index.
  - Formula: $\alpha = R_i - (R_f + \beta(R_m - R_f))$
  - $\alpha > 0$: Indicates that the investment has outperformed its benchmark, suggesting that the investment manager's strategy has added value.
  - $\alpha < 0$: Indicates that the investment has under-performed its benchmark, suggesting that the investment manager's strategy has detracted value.
  - $\alpha = 0$: Indicates that the investment has performed in line with its benchmark, suggesting that the investment manager's strategy has neither added nor detracted value.

- *Beta* : Measures the volatility or systematic risk of an investment relative to the overall market.
  - Formula: $\beta = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)}$
  - $\beta > 1$: Indicates that the investment is more volatile than the market, suggesting higher risk and potentially higher returns.
  - $\beta < 1$: Indicates that the investment is less volatile than the market, suggesting lower risk and potentially lower returns.
  - $\beta = 1$: Indicates that the investment's volatility is in line with the market, suggesting average market risk.

- *Correlation* : Measures the degree to which two securities move in relation to each other.
  - Formula: $\rho = \frac{\text{Cov}(R_i, R_m)}{\sigma_i \sigma_m}$
  - $\rho = 1$: Indicates a perfect positive correlation, suggesting that the two securities move in the same direction.
  - $\rho = -1$: Indicates a perfect negative correlation, suggesting that the two securities move in opposite directions.
  - $\rho = 0$: Indicates no correlation, suggesting that the movements of the two securities are unrelated.

- *SharpeRatio* : Measures the risk-adjusted return of an investment.
  - Formula: $\text{Sharpe Ratio} = \frac{R_i - R_f}{\sigma_i}$
  - Higher Sharpe Ratio: Indicates better risk-adjusted returns.
  - Lower Sharpe Ratio: Indicates worse risk-adjusted returns.

- $SortinoRatio$: Measures the risk-adjusted return of an investment, similar to the Sharpe Ratio, but only considers downside risk.
  - Formula: Sortino Ratio $= \frac{R_i - R_f}{\sigma_d}$
  - Higher Sortino Ratio: Indicates better risk-adjusted returns with respect to downside risk.
  - Lower Sortino Ratio: Indicates worse risk-adjusted returns with respect to downside risk.
- $UpsideCapture$: Measures a portfolio's performance in up markets relative to a benchmark.
  - Formula: Upside Capture $= \frac{\text{Portfolio Return in Up Markets}}{\text{Benchmark Return in Up Markets}} \times 100$
  - Upside Capture ¿ 100: Indicates the portfolio has outperformed the benchmark in up markets.
  - Upside Capture ¡ 100: Indicates the portfolio has under-performed the benchmark in up markets.
- $DownsideCapture$: Measures a portfolio's performance in down markets relative to a benchmark.
  - Formula: Downside Capture $= \frac{\text{Portfolio Return in Down Markets}}{\text{Benchmark Return in Down Markets}} \times 100$
  - Downside Capture ¡ 100: Indicates the portfolio has outperformed the benchmark in down markets.
  - Downside Capture ¿ 100: Indicates the portfolio has under-performed the benchmark in down markets.
- $AnnualizedReturn$: Measures the geometric average amount of money earned by an investment each year over a given time period.
  - Formula: Annualized Return $= \left(\prod_{t=1}^{T}(1 + R_t)\right)^{\frac{252}{T}} - 1$
  - Higher Annualized Return: Indicates better performance.
  - Lower Annualized Return: Indicates worse performance.
- $CumulativeReturn$: Measures the total change in the value of an investment over a set time period.
  - Formula: Cumulative Return $= \prod_{t=1}^{T}(1 + R_t) - 1$
  - Higher Cumulative Return: Indicates better performance.
  - Lower Cumulative Return: Indicates worse performance.
- $AnnualizedRisk$: Measures the annualized standard deviation of returns, representing the investment's volatility.
  - Formula: Annualized Risk $= \sigma_i \times \sqrt{252}$
  - Higher Annualized Risk: Indicates higher volatility and potential risk.
  - Lower Annualized Risk: Indicates lower volatility and potential risk.
- $MaximumDrawdown$: Measures the largest drop from a peak to a trough of an investment before a new peak is attained.
  - Formula: Maximum Drawdown $= \min\left(\frac{C_t - P_t}{P_t}\right)$ where C_t is the cumulative return at time t and P_t is the peak return before t.
  - Lower Maximum Drawdown: Indicates better performance in avoiding large losses.
  - Higher Maximum Drawdown: Indicates worse performance in avoiding large losses.

2 Global challenges for our trading strategies

All our strategies implemented and back tested with real data from Yahoo Finance face similar challenges listed and each strategy represents specific risks and will be detailed on each section.

1. **Data Quality Issues:**
    - *Incomplete or Incorrect Data*: Stock volatility data from sources like Yahoo Finance or Bloomberg may be incomplete or incorrect, which can affect the accuracy of the low and high volatility portfolios.
2. **Market Efficiency:**
    - *Efficient Market Hypothesis*: According to the efficient market hypothesis, stock prices already reflect all available information. This could limit the effectiveness of the low volatility anomaly strategy, or quaterly published earning as market efficiency might diminish the potential advantage of investing in low volatility stocks.
3. **Market Conditions:**
    - *Market Trends*: Market trends and macroeconomic conditions can affect stock volatility and thereby impact the performance of the low and high volatility portfolios. For example, during periods of high market stress, the performance of low volatility stocks may not align with historical trends.
    - *Sector Rotation*: Shifts in sector performance may influence the volatility of stocks and affect the strategy's effectiveness.
4. **Stock-Specific Factors:**
    - *Company-Specific Events*: Earnings reports, management changes, or other significant company-specific events can cause abrupt changes in stock volatility, affecting portfolio stability.
    - *Sector-Specific Volatility*: Stocks within certain sectors may exhibit higher or lower volatility due to sector-specific factors, which could impact the results of the low volatility strategy.
5. **Behavioral Biases:**
    - *Investor Behavior*: Investor biases, such as overreacting to short-term market movements or news, can affect stock volatility and potentially distort the outcomes of the strategy.
    - *Herding Effect*: The tendency for investors to follow market trends or other investors can impact the volatility and performance of the stocks in the portfolio.

3 Stocks Trading Strategies

3.1 Price-Momentum

Concept

Momentum trading is a strategy that capitalizes on the continuance of existing trends in the market. The premise of this strategy is that assets which have performed well in the past will continue to perform well, while those that have performed poorly will continue to underperform. This strategy relies on the persistence of trends and is a form of trend-following strategy.

Construction

- **Selection Criteria:**
  - Identify a universe of assets, typically from a broad market index such as the S&P 500 or a basket of global equities, commodities, or currencies.
  - Determine the look-back period (h) for calculating momentum, commonly ranging from 3 to 12 months.
- **Momentum Calculation:**
  - Calculate the momentum ($M_{i,t}$) of each asset i at time t using the formula:

    $$M_{i,t} = \frac{S_{i,t} - S_{i,t-h}}{S_{i,t-h}} \tag{1}$$

    Where:
    * $M_{i,t}$ is the momentum of asset i at time t.
    * $S_{i,t}$ is the historical price of asset i at time t.
    * $S_{i,t-h}$ is the historical price of asset i at time $t-h$.
    * h is the horizon used for calculating the momentum.
- **Ranking:**
  - Rank the assets based on their calculated momentum, from highest to lowest.
  - Select the top percentage (e.g., top 10-20%) of assets with the highest positive momentum for long positions.
  - Select the bottom percentage (e.g., bottom 10-20%) of assets with the most negative momentum for short positions.
- **Portfolio Formation:**
  - Construct a diversified portfolio by equally or proportionally weighting the selected long and short assets.
  - Ensure diversification to avoid concentration risk, particularly in similar sectors or asset classes.
- **Rebalancing:**
  - Periodically rebalance the portfolio to maintain the momentum tilt, typically on a monthly or quarterly basis.
  - Adjust the holdings based on updated momentum calculations and asset performance.

Expected Performance

The Momentum Trading Strategy aims to provide several key benefits:

- **Capitalizing on Trends:** By focusing on assets with strong recent performance, the strategy seeks to benefit from the persistence of trends.
- **Higher Potential Returns:** Momentum strategies can potentially deliver higher returns during trending markets compared to other strategies.
- **Diversification Benefits:** Including both long and short positions can offer diversification benefits and potential risk mitigation.

Practical Considerations

While the Momentum Trading Strategy offers compelling advantages, it also presents certain challenges and considerations:

- **Market Reversals:** Momentum strategies can suffer during market reversals, as trends may abruptly change.
- **High Turnover:** Frequent rebalancing can lead to higher transaction costs, which can erode overall returns.
- **Volatility Sensitivity:** The strategy may be sensitive to market volatility, potentially leading to larger drawdowns during turbulent periods.

Mathematical Formula

- **Momentum:**
$$M_{i,t} = \frac{S_{i,t} - S_{i,t-h}}{S_{i,t-h}}$$
- **Trading Signal:**
$$\begin{cases} M_{i,t} > 0 \Rightarrow \text{go long on asset } i \\ M_{i,t} < 0 \Rightarrow \text{go short on asset } i \end{cases}$$
- **Annualized Volatility:** $\sigma_i = \sqrt{252 \times \mathrm{Var}(R_i)}$
- **Sharpe Ratio:** $\text{Sharpe Ratio}_i = \frac{R_i - R_f}{\sigma_i}$
- **Beta Calculation:** $\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}$
- **Factors Models:** $R_i - R_f = \alpha_i + \beta_{iM}(R_m - R_f) + \beta_{iSMB}\mathrm{SMB} + \beta_{iHML}\mathrm{HML} + \beta_{iMOM}\mathrm{MOM} + \epsilon_i$
  - **SMB** stands for Small Minus Big (size factor),
  - **HML** stands for High Minus Low (value factor),
  - **MOM** is the Momentum factor.

Backtesting and Optimization

Lastly, we need to backtest the strategy and optimize the portfolio. Common metrics for backtesting include cumulative returns, maximum drawdown, and the Sharpe ratio. Optimization can involve techniques like quadratic programming or heuristic algorithms to find the optimal set of weights.

Our Implementation

Figure 1: Price Momentum applied on Air Liquide

Figure 2: Price Momentum applied on Airbus

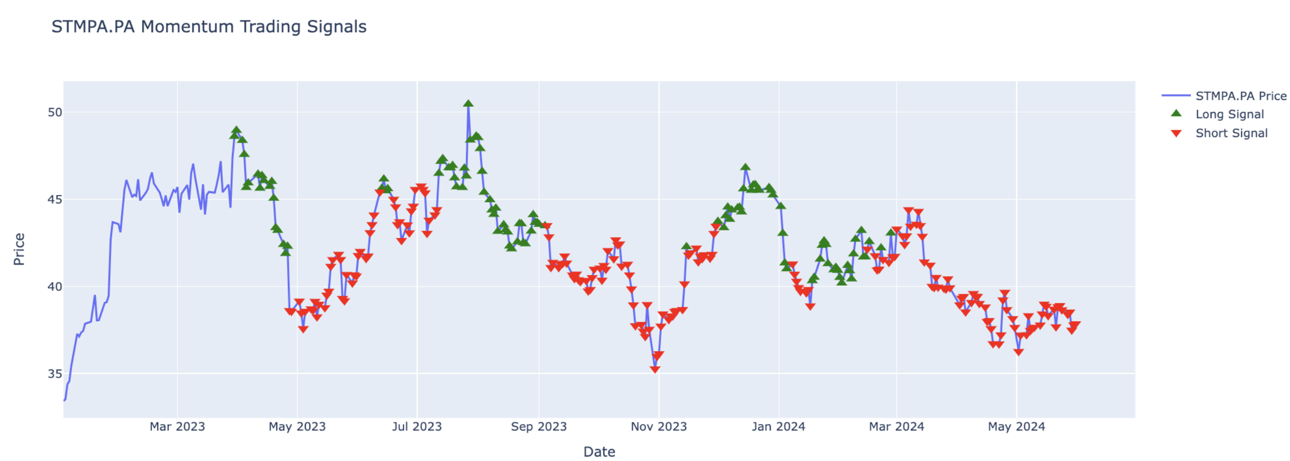


Figure 3: Price Momentum applied on STMicroelectronics

Ticker	Total Return (%)	Annualized Volatility	Sharpe Ratio
AI.PA	11.27	0.62	0.52
AIR.PA	19.54	0.90	6.30
STMPA.PA	-37.23	1.03	-12.53

Table 1: Performance Metrics

Commentary

- **AI.PA:**
  - **Total Return (%):** 11.27% - This indicates a positive return over the period.
  - **Annualized Volatility:** 0.62 - Indicates moderate volatility.
  - **Sharpe Ratio:** 0.52 - Indicates a moderate risk-adjusted return.
- **AIR.PA:**
  - **Total Return (%):** 19.54% - This shows an even higher positive return.
  - **Annualized Volatility:** 0.90 - Indicates higher volatility.
  - **Sharpe Ratio:** 6.30 - Indicates an excellent risk-adjusted return.
- **STMPA.PA:**
  - **Total Return (%):** -37.23% - This reflects a significant loss over the period.
  - **Annualized Volatility:** 1.03 - Indicates the highest volatility among the three.
  - **Sharpe Ratio:** -12.53 - Indicates a very poor risk-adjusted return, as the negative value suggests the returns are lower than the risk-free rate, factoring in the volatility.

Limit of this model

- **Earnings Surprises:** Unexpected earnings announcements can cause significant price movements, impacting the strategy.
- **High Turnover:** Frequent rebalancing can lead to higher transaction costs, which can erode overall returns.
- **Volatility Sensitivity:** The strategy may be sensitive to market volatility, potentially leading to larger drawdowns during turbulent periods.
- **looking back period estimation**: The momentum trading strategy needs a rigorous estimation of looking back period, It seriously affects the precision of prediction model. The next step will be evaluating the precision of algorithm in function of different looking back period and find a local maximum to fit the right model.
- **Marlket Neutral Trend**: Another challenge for this model would be the evaluation during market neutral period, as we can see through the graph STMPA, there was a range and price floating between 50 and 35 without a clear trend which clearly mistakes the prediction.
- **High Turnover and Transaction Costs:**
  - *Frequent Trading*: Earnings momentum strategies may require frequent trading, leading to high transaction costs that can erode profits.
  - *Slippage*: The difference between expected trade prices and actual trade prices can reduce the strategy's effectiveness, especially in less liquid stocks.

3.2 Earnings-Momentum

Concept

Earnings-Momentum trading is a strategy that capitalizes on the persistence of earnings trends. The premise of this strategy is that companies with strong earnings growth will continue to perform well, while those with declining earnings will continue to underperform. This strategy relies on the continuation of earnings trends and is a form of fundamental momentum trading.

Construction

Selection Criteria:

- Identify a universe of assets, typically from a broad market index such as the S&P 500 or a basket of global equities.
- Determine the look-back period (e.g., last 4 quarters) for evaluating earnings momentum.

Earnings Momentum Calculation:

- Calculate the earnings momentum ($EM_{i,t}$) of each asset i at time t using the formula:

$$EM_{i,t} = \frac{E_{i,t} - E_{i,t-h}}{E_{i,t-h}}$$

  Where:

  - $EM_{i,t}$ is the earnings momentum of asset i at time t.
  - $E_{i,t}$ is the most recent earnings report of asset i at time t.
  - $E_{i,t-h}$ is the earnings report of asset i at time $t - h$.
  - h is the horizon used for calculating the earnings momentum.

Ranking:

- Rank the assets based on their calculated earnings momentum, from highest to lowest.
- Select the top percentage (e.g., top 10-20%) of assets with the highest positive earnings momentum for long positions.
- Select the bottom percentage (e.g., bottom 10-20%) of assets with the most negative earnings momentum for short positions.

Portfolio Formation:

- Construct a diversified portfolio by equally or proportionally weighting the selected long and short assets.
- Ensure diversification to avoid concentration risk, particularly in similar sectors or asset classes.

Rebalancing:

- Periodically rebalance the portfolio to maintain the earnings momentum tilt, typically on a quarterly or semi-annual basis.
- Adjust the holdings based on updated earnings momentum calculations and asset performance.

Expected Performance

The Earnings-Momentum Trading Strategy aims to provide several key benefits:

- **Capitalizing on Earnings Trends:** By focusing on assets with strong recent earnings growth, the strategy seeks to benefit from the persistence of earnings trends.
- **Higher Potential Returns:** Earnings momentum strategies can potentially deliver higher returns during periods of earnings growth compared to other strategies.
- **Diversification Benefits:** Including both long and short positions can offer diversification benefits and potential risk mitigation.

Practical Considerations

While the Earnings-Momentum Trading Strategy offers compelling advantages, it also presents certain challenges and considerations:

Mathematical Formula

Earnings Momentum:

$$EM_{i,t} = \frac{E_{i,t} - E_{i,t-h}}{E_{i,t-h}}$$

Trading Signal:

$$\begin{cases} EM_{i,t} > 0 \Rightarrow \text{go long on asset } i \\ EM_{i,t} < 0 \Rightarrow \text{go short on asset } i \end{cases}$$

Annualized Volatility:

$$\sigma_i = \sqrt{252 \times \text{Var}(R_i)}$$

Sharpe Ratio:

$$\text{Sharpe Ratio}_i = \frac{R_i - R_f}{\sigma_i}$$

Beta Calculation:

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)}$$

Factors Models:

$$R_i - R_f = \alpha_i + \beta_{iM}(R_m - R_f) + \beta_{iSMB}SMB + \beta_{iHML}HML + \beta_{iMOM}MOM + \epsilon_i$$

- SMB stands for Small Minus Big (size factor),
- HML stands for High Minus Low (value factor),
- MOM is the Momentum factor.

Backtesting and Optimization

Model Overfitting:

- *Overfitting to Historical Data*: The model may perform well on historical data but fail to generalize to unseen data due to overfitting.
- *Parameter Sensitivity*: The model's performance can be highly sensitive to parameter choices, such as the lookback period or rebalancing frequency.

Lastly, we need to backtest the strategy and optimize the portfolio. Common metrics for backtesting include cumulative returns, maximum drawdown, and the Sharpe ratio. Optimization can involve techniques like quadratic programming or heuristic algorithms to find the optimal set of weights.

Our Implementation

The implementation on this strategy meets several challenges and limits :

1. **Data Quality Issues:**
   - *Incomplete or Incorrect Data*: Earnings data from sources like Yahoo Finance may be incomplete or incorrect, leading to inaccurate momentum calculations.
   - *Data Lag*: Earnings announcements may not be immediately reflected in the data, causing delays in signal generation.
2. **Market Efficiency:**
   - *Efficient Market Hypothesis*: If markets are efficient, all available information, including earnings, is already reflected in stock prices, reducing the effectiveness of earnings-based momentum strategies.
3. **Lookback Period Selection:**
   - *Suboptimal Lookback Period*: Choosing an inappropriate lookback period (e.g., too long or too short) can lead to poor momentum signal accuracy and trading performance. Especially when earning are released quarterly and
4. **Timing of Earnings Announcements:**
   - *Earnings Release Schedule*: Companies release earnings at different times, leading to asynchronous signals that can complicate portfolio management.
   - *Earnings Surprises*: Unexpected earnings announcements (both positive and negative) can cause significant volatility, impacting the strategy's performance.
5. **Short-Term Focus:**
   - *Quarterly Earnings Focus*: Earnings-momentum strategies typically focus on quarterly results, which may not capture long-term fundamental trends and company performance.
   - *Volatility*: Short-term focus can lead to higher portfolio volatility and larger drawdowns during periods of market stress.
6. **External Factors:**
   - *Macroeconomic Events*: Market reactions to earnings may be overshadowed by larger macroeconomic events, political changes, or global crises, rendering the earnings momentum signal ineffective.
   - *Sector-Specific Factors*: Sector-specific news or trends can impact stock prices independently of earnings momentum.
7. **Market Sentiment and Behavioral Biases:**
   - *Investor Sentiment*: Market sentiment can impact stock prices independently of earnings momentum, leading to false signals.
   - *Behavioral Biases*: Investor behavior, such as overreaction or underreaction to earnings announcements, can create challenges for the strategy.

3.3 Low-Volatility Anomaly

Concept

The Low-Volatility Anomaly refers to the observation that portfolios of low-volatility stocks have historically delivered higher risk-adjusted returns than portfolios of high-volatility stocks. This phenomenon contradicts conventional financial theory, which suggests that higher risk (volatility) should be compensated with higher returns. The Low-Volatility Anomaly strategy aims to capitalize on this inconsistency.
The fundamental concept behind the Low-Volatility Anomaly strategy is that stocks with lower volatility tend to perform better on a risk-adjusted basis compared to their higher volatility counterparts. This strategy focuses on identifying and investing in stocks that exhibit less price fluctuation, thereby aiming to achieve steadier returns while mitigating risk.

Construction

- **Selection Criteria:**
  - Identify a universe of stocks, typically from a broad market index such as the S&P 500.
  - Calculate the historical volatility of each stock over a specified period, such as the past 6-12 months. Volatility can be measured using standard deviation of daily or monthly returns.
- **Ranking:**
  - Rank the stocks based on their calculated volatility, from lowest to highest.
  - Select the top percentage (e.g., top 10-20%) of stocks with the lowest volatility for inclusion in the portfolio.
- **Portfolio Formation:**
  - Construct a diversified portfolio by equally or proportionally weighting the selected low-volatility stocks.
  - Ensure sector diversification to avoid concentration risk, as low-volatility stocks may be clustered in specific sectors.
- **Rebalancing:**
  - Periodically rebalance the portfolio to maintain the low-volatility tilt, typically on a quarterly or annual basis.
  - Adjust the holdings based on updated volatility calculations and stock performance.

Expected Performance

The Low-Volatility Anomaly strategy aims to provide several key benefits:

- **Higher Risk-Adjusted Returns:** By focusing on stocks with lower volatility, the strategy seeks to achieve better returns per unit of risk compared to high-volatility stocks.
- **Reduced Drawdowns:** Low-volatility stocks often exhibit less severe declines during market downturns, leading to more stable portfolio performance.
- **Steadier Growth:** The strategy emphasizes consistency, aiming for gradual and sustained portfolio growth rather than rapid, unpredictable gains.

Practical Considerations

While the Low-Volatility Anomaly strategy offers compelling advantages, it also presents certain challenges and considerations:

- **Sector Bias:** Low-volatility stocks may be concentrated in specific sectors such as utilities or consumer staples, leading to potential sector bias.
- **Underperformance in Bull Markets:** During strong bull markets, low-volatility stocks may underperform compared to high-volatility, high-growth stocks.
- **Transaction Costs:** Regular rebalancing can incur higher transaction costs, which can erode the overall returns of the strategy.

Mathematical Formula

- **Annualized Volatility:** $\sigma_i = \sqrt{252 \times \mathrm{Var}(R_i)}$
- **Low-Volatility Portfolio Construction:** Minimize $\quad \sigma_p = \sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}$
  - $\mathbf{w}$ is the vector of portfolio weights.
  - $\boldsymbol{\Sigma}$ is the covariance matrix of stock returns.
  - $\mathbf{w}^T \mathbf{1} = 1$
  - $\mathbf{w}^T \mathbf{R} \geq \mathbf{R}_{\text{target}}$
- **Factors Models:** $R_i - R_f = \alpha_i + \beta_{iM}(R_m - R_f) + \beta_{iSMB}\text{SMB} + \beta_{iHML}\text{HML} + \beta_{iLV}\text{LV} + \epsilon_i$
  - **SMB** stands for Small Minus Big (size factor),
  - **HML** stands for High Minus Low (value factor),
  - **LV** is the Low-Volatility factor.

Backtesting and Optimization

Model Overfitting:

- *Overfitting to Historical Data*: The low volatility anomaly strategy might show strong performance on historical data used for backtesting. However, this does not guarantee that the strategy will perform equally well on future, unseen data. Overfitting occurs when the strategy is too closely tailored to historical patterns and may not generalize effectively to new market conditions.
- *Parameter Sensitivity*: The strategy's effectiveness can be highly sensitive to the parameters chosen, such as the length of the lookback period for calculating volatility and the frequency of portfolio rebalancing. Small changes in these parameters can significantly impact the performance of the low volatility portfolios, potentially leading to suboptimal results if not carefully optimized.

Lastly, we need to backtest the strategy and optimize the portfolio. Common metrics for backtesting include cumulative returns, maximum drawdown, and the Sharpe ratio. Optimization can involve techniques like quadratic programming or heuristic algorithms to find the optimal set of weights.

Our Implementation

The implementation of the Low Volatility Anomaly Strategy presents several challenges and limitations, as outlined below:

1. **Data Quality Issues:**
   - *Incomplete or Incorrect Data*: Stock volatility data from sources like Yahoo Finance or Bloomberg may be incomplete or incorrect, which can affect the accuracy of the low and high volatility portfolios.
   - *Data Lag*: Volatility calculations may not immediately reflect recent market changes, causing delays in portfolio adjustments and signal generation.
2. **Market Efficiency:**
   - *Efficient Market Hypothesis*: According to the efficient market hypothesis, stock prices already reflect all available information. This could limit the effectiveness of the low volatility anomaly strategy, as market efficiency might diminish the potential advantage of investing in low volatility stocks.
3. **Portfolio Rebalancing:**
   - *Quarterly Rebalancing*: Our strategy involves rebalancing portfolios on a quarterly basis to account for changes in the FTSE 100 index and stock volatility. This frequent rebalancing may incur transaction costs and impact portfolio performance.
   - *Weighting Scheme*: Each stock's weight is inversely proportional to its volatility, which requires precise calculation and adjustment of weights. Misestimation or data errors in volatility can lead to suboptimal portfolio performance.
4. **Market Conditions:**
   - *Market Trends*: Market trends and macroeconomic conditions can affect stock volatility and thereby impact the performance of the low and high volatility portfolios. For example, during periods of high market stress, the performance of low volatility stocks may not align with historical trends.
   - *Sector Rotation*: Shifts in sector performance may influence the volatility of stocks and affect the strategy's effectiveness.
5. **Stock-Specific Factors:**
   - *Company-Specific Events*: Earnings reports, management changes, or other significant company-specific events can cause abrupt changes in stock volatility, affecting portfolio stability.
   - *Sector-Specific Volatility*: Stocks within certain sectors may exhibit higher or lower volatility due to sector-specific factors, which could impact the results of the low volatility strategy.
6. **Behavioral Biases:**
   - *Investor Behavior*: Investor biases, such as overreacting to short-term market movements or news, can affect stock volatility and potentially distort the outcomes of the strategy.
   - *Herding Effect*: The tendency for investors to follow market trends or other investors can impact the volatility and performance of the stocks in the portfolio.

We have chosen to apply this strategy to the FTSE 100 Index, which tracks the performance of the 100 largest companies listed on the London Stock Exchange. The objective is to construct two portfolios: one comprising the 25 stocks with the lowest annualized volatility's and another with the 25 stocks exhibiting the highest annualized volatility's.
Each stock's weight within the portfolios is inversely proportional to its volatility. The strategy is rolled quarterly to align with the quarterly review of FTSE 100 constituent companies. We then compare the returns of the Low-Volatility, High-Volatility, and FTSE 100 portfolios and compute various performance metrics.

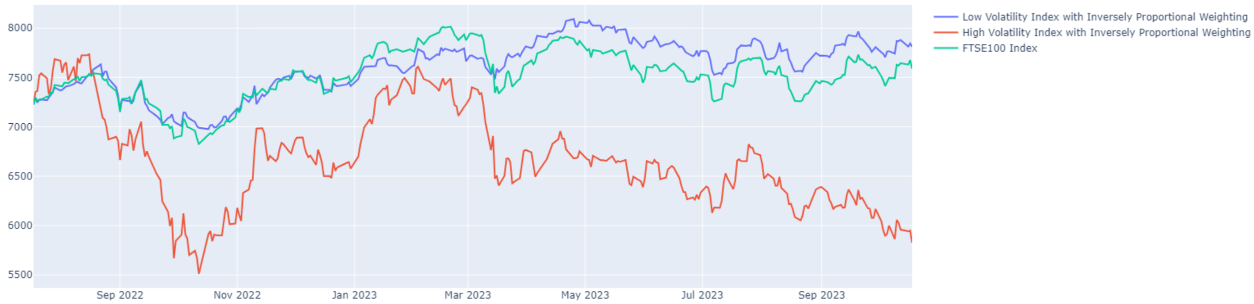


Figure 4: Low & High Volatility's Index based on FTSE100

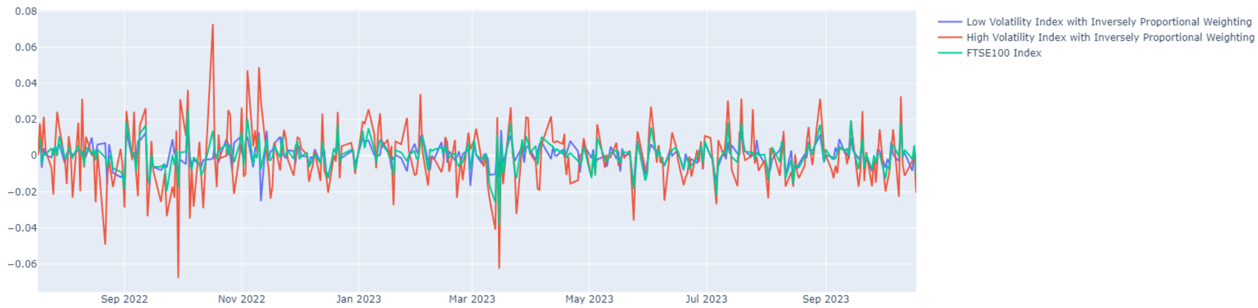


Figure 5: Returns of Volatility's Index based on FTSE100

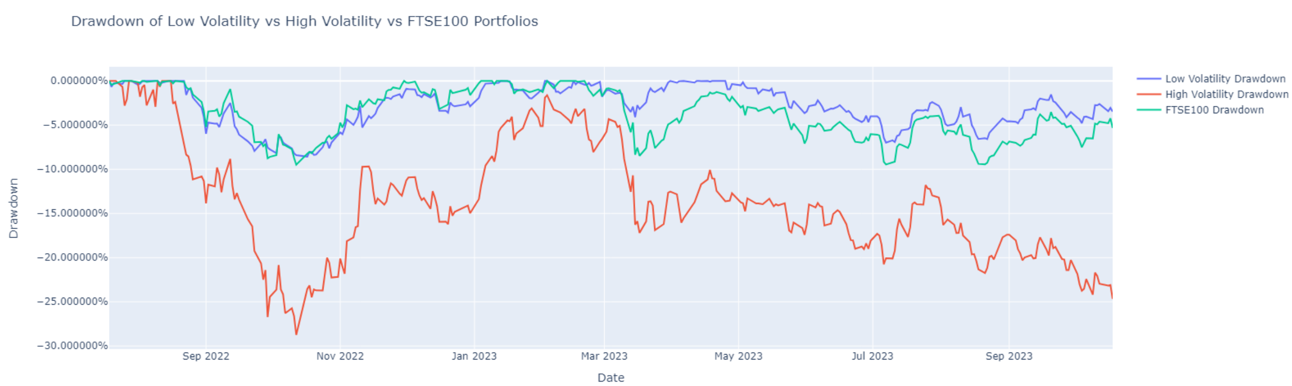


Figure 6: Draw-downs of Volatility's Index based on FTSE100

Metric	Low Volatility	High Volatility	FTSE100
Alpha (%)	1.577702	-8.921167	0.000000
Beta	0.632749	1.641548	1.000000
Correlation	0.764572	0.756823	1.000000
Sharpe Ratio	0.599245	-0.584568	0.316854
Sortino Ratio	0.854012	-0.881498	0.427245
Upside Capture (%)	66.3081	158.0838	100.0000
Downside Capture (%)	61.0745	188.3747	100.0000
Annualized Return (%)	6.765819	-16.467701	4.211184
Cumulative Return (%)	8.133616	-19.339769	5.050390
Annualized Risk (%)	10.108346	26.492693	12.214245
Maximum Drawdown (%)	-8.574278	-28.747048	-9.489647

Table 2: Metrics for Low Volatility, High Volatility Portfolios, and FTSE100

Limits of the Implied Volatility Strategy

1. **High Transaction Costs:**
   - *Frequent Rebalancing*: The low volatility strategy requires quarterly rebalancing to adjust the portfolio based on the latest volatility data. This frequent adjustment can lead to higher transaction costs that might reduce the overall returns of the strategy.
   - *Slippage*: The difference between expected and actual trade prices, known as slippage, can further impact the effectiveness of the strategy, especially in stocks with lower liquidity.
2. **Market Sensitivity:**
   - *Volatility of the Market*: The performance of the low volatility strategy can be influenced by broader market volatility. During periods of high market stress, the strategy may experience larger drawdowns compared to calmer periods.
3. **Rebalancing Period:**
   - *Optimal Frequency*: The choice of rebalancing frequency is crucial. Quarterly adjustments align with the review cycle of the FTSE 100 index but may not always be optimal for capturing the most current volatility trends.
4. **Market Trends:**
   - *Performance During Flat Markets*: The strategy may struggle in flat or sideways markets where price movements are minimal, leading to potentially misleading signals and reduced strategy performance.
5. **Volatility Measurement:**
   - *Precision of Volatility Estimates*: Accurate measurement of stock volatility is essential for the strategy. Inaccurate estimates can lead to poor portfolio construction and diminished performance.
6. **Strategy Specificity:**
   - *Focus on Volatility*: The strategy's focus on volatility may not account for other factors influencing stock performance, potentially missing out on gains from other fundamental or technical signals.

3.4 Implied Volatility

Concept

The Implied Volatility Strategy leverages the empirical finding that stocks with significant increases in call implied volatilities over the past month often see higher future returns, while those with increases in put implied volatilities typically exhibit lower future returns.
To capitalize on this, a trader can construct a dollar-neutral portfolio by buying stocks in the top decile for call IV increases and shorting stocks in the top decile for put IV increases. Alternatively, the strategy could involve buying stocks based on the difference between call and put IV changes.
This approach aims to exploit the relationship between implied volatility changes and subsequent stock performance for potentially higher returns.

Construction

- **Data Collection:**
  - Obtain historical implied volatilities for both call and put options, and stock price data for a universe of stocks.
- **Calculation of Implied Volatility Changes:**
  - Compute the percentage change in call implied volatility (ΔIV_{call}) over a specific period, such as the past month:
$$\Delta IV_{call} = \frac{IV_{call,t} - IV_{call,t-1}}{IV_{call,t-1}}$$
  - Compute the percentage change in put implied volatility (ΔIV_{put}) over the same period:
$$\Delta IV_{put} = \frac{IV_{put,t} - IV_{put,t-1}}{IV_{put,t-1}}$$
- **Ranking:**
  - Rank stocks based on their increase in call implied volatility (ΔIV_{call}) and select the top 10% for potential long positions.
  - Rank stocks based on their increase in put implied volatility (ΔIV_{put}) and select the top 10% for potential short positions.
  - Alternatively, compute the difference between changes in call and put implied volatilities:
$$\text{IV Difference} = \Delta IV_{call} - \Delta IV_{put}$$
  - Rank stocks based on this difference and select the top stocks with the largest positive differences for long positions and the largest negative differences for short positions.
- **Portfolio Formation:**
  - For the dollar-neutral approach, buy stocks in the top decile for call implied volatility increases and short stocks in the top decile for put implied volatility increases.
  - Alternatively, construct a portfolio by buying stocks with the highest positive IV difference and shorting stocks with the lowest IV difference.
  - Ensure equal dollar amounts are allocated to long and short positions to maintain neutrality.
- **Rebalancing:**
  - Periodically rebalance the portfolio, such as on a monthly basis, to adjust for changes in implied volatilities and maintain dollar-neutrality.
  - Update the holdings based on the latest volatility changes and stock performance.
- **Risk Management:**
  - Diversify the portfolio across different sectors to mitigate concentration risk.
  - Adjust position sizes according to the volatility of individual stocks and overall portfolio risk.

Expected Performance

The Implied Volatility Strategy aims to provide several key benefits:

- **Potential for Higher Returns:** By focusing on stocks with significant increases in call implied volatility, the strategy targets stocks that are expected to outperform, potentially leading to higher returns.
- **Enhanced Dollar-Neutral Exposure:** Constructing a dollar-neutral portfolio by balancing long positions in high call IV stocks and short positions in high put IV stocks aims to reduce market risk and isolate volatility-based returns.
- **Systematic Alpha Generation:** By exploiting the empirical relationship between changes in implied volatilities and future stock performance, the strategy seeks to generate alpha through a systematic approach to volatility shifts.

Practical Considerations

While the Implied Volatility Strategy offers potential benefits, it also presents certain challenges and considerations:

- **Market Impact:** Stocks with extreme changes in implied volatility may experience liquidity issues or higher bid-ask spreads, affecting execution and overall strategy performance.
- **Volatility Clustering:** Implied volatility may exhibit clustering, where periods of high or low volatility persist, potentially leading to prolonged periods of underperformance or increased risk.
- **Model Risk:** The effectiveness of the strategy relies on the accurate calculation and ranking of implied volatility changes. Errors in data or assumptions could impact the expected outcomes.

Backtesting and Optimization

- **Model Overfitting:**
  - *Overfitting to Historical Data*: The implied volatility strategy may show strong performance on historical data used for backtesting. However, this past performance does not guarantee future results. Overfitting occurs when the strategy is overly adapted to historical patterns and may not perform well under new market conditions.
  - *Parameter Sensitivity*: The strategy's effectiveness can be sensitive to parameters such as the lookback period for calculating changes in implied volatilities and the frequency of portfolio rebalancing. Small adjustments to these parameters can significantly affect the performance, potentially leading to suboptimal results if not carefully calibrated.
- **Backtesting:**
  - Evaluate the strategy using historical data to assess performance metrics such as cumulative returns, maximum drawdown, and the Sharpe ratio.
  - Analyze how the strategy performs across different market conditions to ensure robustness.
- **Optimization:**
  - Utilize techniques like quadratic programming or heuristic algorithms to optimize the portfolio weights and allocation.
  - Fine-tune the parameters for calculating implied volatility changes and portfolio rebalancing to enhance the strategy's performance.

Our Implementation

The implementation of the Implied Volatility Strategy presents several challenges and limitations, as outlined below:

1. **Data Quality Issues:**
   - *Incomplete or Incorrect Data*: Implied volatility data sourced from platforms like Yahoo Finance or Bloomberg may be incomplete or inaccurate, affecting the precision of volatility changes used to construct the portfolio.
   - *Data Lag*: Changes in implied volatility may not be immediately reflected in available data, leading to delays in portfolio adjustments and potentially impacting signal generation.
2. **Market Efficiency:**
   - *Efficient Market Hypothesis*: According to the efficient market hypothesis, all available information is already reflected in stock prices. This could limit the effectiveness of the implied volatility strategy if market efficiency reduces the potential advantage of targeting stocks based on volatility shifts.
3. **Portfolio Construction:**
   - *Selection of Stocks*: With a limited universe of only 5 stocks from the SP 500, the strategy may lack diversification, potentially impacting risk and return profiles.
   - *Dollar-Neutral Implementation*: Ensuring that the portfolio remains dollar-neutral by balancing long positions in high call IV increase stocks with short positions in high put IV increase stocks requires precise calculations and frequent adjustments.
4. **Market Conditions:**
   - *Market Trends*: Broader market trends and macroeconomic factors can influence implied volatilities and may affect the performance of the stocks in the portfolio. For instance, during market stress, the performance of stocks based on implied volatility changes may not align with historical expectations.
   - *Sector Performance*: Changes in sector performance can impact the implied volatility of stocks, potentially affecting the strategy's effectiveness.
5. **Stock-Specific Factors:**
   - *Company-Specific Events*: Significant events such as earnings releases or management changes can cause abrupt shifts in implied volatility, impacting portfolio stability.
   - *Sector-Specific Volatility*: Stocks within certain sectors may experience unique volatility patterns, influencing the strategy's performance based on sector-specific dynamics.
6. **Behavioral Biases:**
   - *Investor Behavior*: Investor biases and reactions to short-term news or market movements can affect implied volatility and potentially skew the outcomes of the strategy.
   - *Herding Effect*: The tendency of investors to follow trends or other investors can impact the implied volatility and performance of the selected stocks.

Limits of the Implied Volatility Strategy

1. **High Transaction Costs:**
   - *Frequent Adjustments*: Implementing the implied volatility strategy may require frequent portfolio rebalancing based on the latest changes in implied volatilities. This frequent adjustment can result in higher transaction costs, potentially diminishing overall returns.
   - *Market Impact and Slippage*: Trading in stocks with lower liquidity or large volumes can cause slippage, where the execution price deviates from the expected price, further impacting the strategy's effectiveness.
2. **Sensitivity to Market Conditions:**
   - *Volatility Clustering*: The performance of the strategy may be affected by periods of high or low market volatility. During such periods, the relationship between changes in implied volatility and subsequent stock performance may deviate from historical patterns, leading to unpredictable results.
3. **Rebalancing Frequency:**
   - *Optimal Timing Issues*: Determining the optimal frequency for rebalancing is crucial. While more frequent adjustments aim to capture recent volatility shifts, they may not always align with the best timing for reflecting meaningful performance changes.
4. **Performance in Different Market Regimes:**
   - *Flat or Trending Markets*: The strategy may underperform in flat or trending markets where implied volatility changes are less indicative of future returns. This can lead to misleading signals and reduced effectiveness in capturing potential returns.
5. **Accuracy of Volatility Estimates:**
   - *Dependence on Data Quality*: The success of the strategy relies heavily on the accuracy of implied volatility data. Errors or outdated information can lead to incorrect stock ranking and selection, adversely affecting overall performance.
6. **Focus on Volatility Alone:**
   - *Limited Scope*: Concentrating solely on implied volatility may ignore other important factors such as fundamental indicators or broader market trends, which could influence stock performance and potentially enhance returns.

3.5 Pairs Trading

Concept

Pair trading is a market-neutral strategy that involves matching a long position with a short position in two stocks with high correlation. The idea is to capitalize on the relative price movement between the two assets, assuming that the spread between their prices will revert to the mean over time. Here we are going to focus on two variants which are DTW base on mean-reversion principal approach and also a regression approach

Construction

- **Selection Criteria:**
  - Identify a universe of assets, typically from the same industry or sector to ensure a strong correlation.
  - Use statistical methods such as cointegration or correlation to identify pairs of assets with historically stable relationships.
- **Spread Calculation:**
  - Calculate the spread ($S_{i,j,t}$) between the prices of the two assets i and j at time t using the formula:

$$S_{i,j,t} = P_{i,t} - \beta P_{j,t} \tag{2}$$

    Where:
    * $S_{i,j,t}$ is the spread between assets i and j at time t.
    * $P_{i,t}$ is the price of asset i at time t.
    * $P_{j,t}$ is the price of asset j at time t.
    * β is the hedge ratio, typically determined by a regression of the prices of the two assets.
- **Trading Signal:**
  - Establish entry and exit thresholds for the spread.
  - Generate a buy (long) signal for asset i and a sell (short) signal for asset j when the spread widens beyond the entry threshold.
  - Close the positions when the spread reverts to the mean or crosses the exit threshold.
- **Portfolio Formation:**
  - Construct a market-neutral portfolio by taking equal but opposite positions in the paired assets.
  - Ensure diversification by forming multiple pairs to avoid concentration risk.
- **Rebalancing:**
  - Monitor the spreads and rebalance the portfolio as necessary to maintain the market-neutral stance.
  - Adjust the positions based on updated spread calculations and asset performance.

Expected Performance

The Pair Trading Strategy aims to provide several key benefits:

- **Market Neutrality:** By matching long and short positions, the strategy aims to be less affected by market-wide movements.
- **Relative Value:** It focuses on the relative performance of the paired assets, rather than their absolute performance.
- **Diversification:** Multiple pairs can be formed to enhance diversification and reduce risk.

3.5.1 Pair Trading Strategy with DTW

Mathematical Formulation

Dynamic Time Warping (DTW) is a method that calculates an optimal match between two given sequences (time series) with certain restrictions. The objective is to align the sequences in a way that minimizes the total distance between them. This distance is calculated using a cost function.

- **DTW Distance Calculation:**

  Let $ts1 = \{ts1_1, ts1_2, \ldots, ts1_n\}$ and $ts2 = \{ts2_1, ts2_2, \ldots, ts2_m\}$ be two time series of lengths n and m respectively. The DTW distance $D(ts1, ts2)$ is defined by the following steps:

  1. Initialize the DTW matrix $DTW_{n+1,m+1}$ with dimensions $(n + 1) \times (m + 1)$:

  $$DTW_{i,j} = \begin{cases} 0 & \text{if } i = 0 \text{ and } j = 0 \\ \infty & \text{if } i = 0 \text{ or } j = 0 \\ 0 & \text{otherwise} \end{cases}$$

  2. For each element (i, j) in the DTW matrix, compute the cost $cost(i, j)$ as the absolute difference between the corresponding elements of the time series:

  $$cost(i, j) = |ts1_i - ts2_j|$$

  3. Update the DTW matrix:

  $$DTW_{i,j} = cost(i, j) + \min(DTW_{i-1,j}, DTW_{i,j-1}, DTW_{i-1,j-1})$$

  4. The DTW distance between the two time series is the value in the bottom-right cell of the matrix:

  $$D(ts1, ts2) = DTW_{n,m}$$

  The Python implementation of the DTW distance calculation is as follows:

- **Spread Calculation:**

  $$S_{i,j,t} = P_{i,t} - \beta P_{j,t}$$

  Where:

  - $S_{i,j,t}$ is the spread between assets i and j at time t.
  - $P_{i,t}$ is the price of asset i at time t.
  - $P_{j,t}$ is the price of asset j at time t.
  - β is the hedge ratio, typically determined by a regression of the prices of the two assets.

- **Trading Signal:**

  $$\begin{cases} D(ts1, ts2) > \text{Threshold} \Rightarrow \text{Buy shares of } i \\ D(ts1, ts2) < \text{Threshold} \Rightarrow \text{Sell shares of } i \end{cases}$$

- **Portfolio Value:**

  $$V_t = C_t + S_t \cdot P_{i,t}$$

  Where:

  - V_t is the portfolio value at time t.
  - C_t is the cash balance at time t.
  - S_t is the number of shares held at time t.
  - $P_{i,t}$ is the price of the held asset i at time t.

Correlation and Cointegration Analysis Implementation in Python

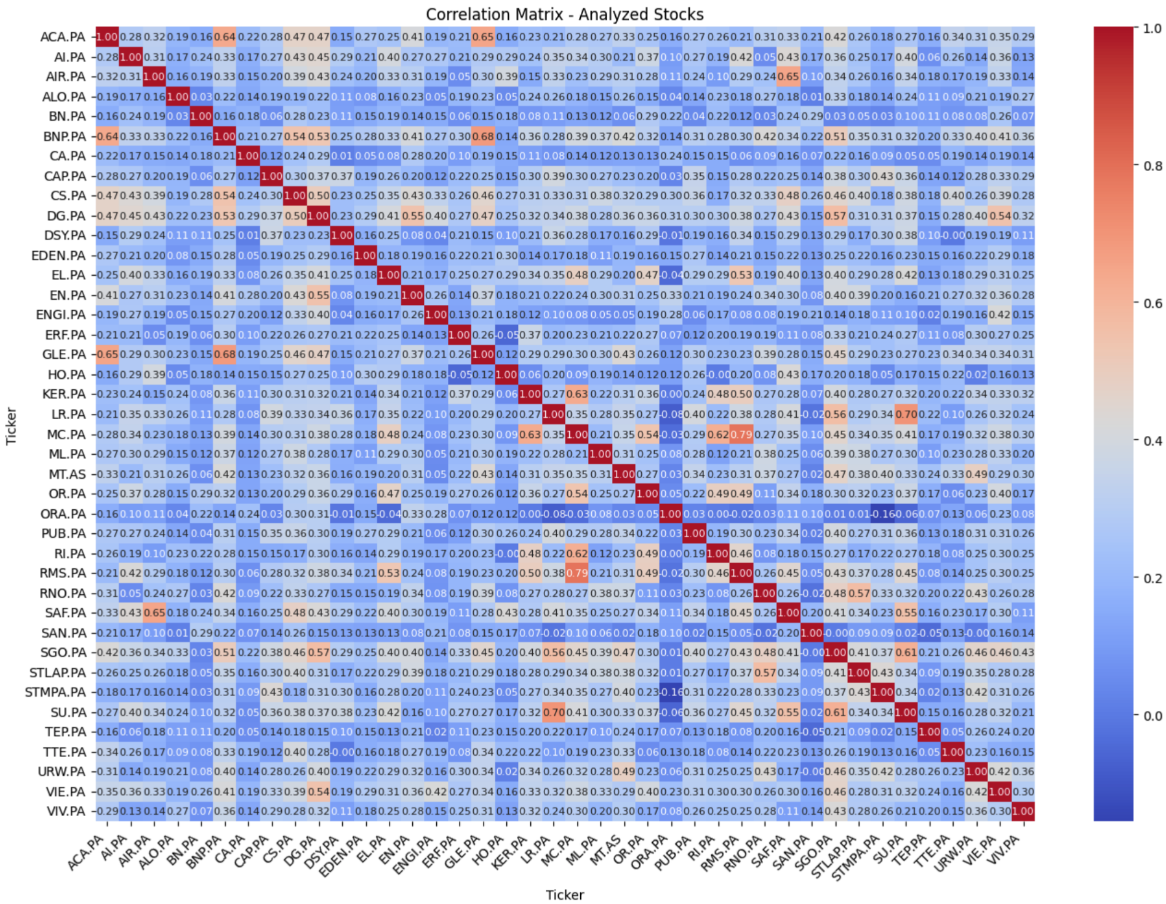


Figure 7: Correlation of 40 stocks of CAC40

Metric	Value
Stock 1	SAF.PA
Stock 2	AIR.PA
Test Statistic	-0.9589477436954627
p-value	0.9103605918033804
Critical Value 1%	-3.9009194099958884
Critical Value 5%	-3.3386270944334977
Critical Value 10%	-3.046182970225127

Table 3: Cointegration Test Results for SAF.PA and AIR.PA

The cointegration test between SAF.PA (SAFRAN) and AIR.PA (AIRBUS) yields the following results:

- **p-value:** The p-value is 0.9104, which is much higher than the typical significance levels of 1%, 5%, and 10%. This high p-value indicates a lack of evidence to reject the null hypothesis of no cointegration.

- **Critical Values:** The test statistic is -0.9589 which is greater than the critical values at 1%, 5%, and 10% levels (-3.9009, -3.3386, and -3.0462 respectively). Since the test statistic does not fall below any of these critical values, we fail to reject the null hypothesis of no cointegration.

Conclusion: Based on the test statistic and the high p-value, there is no evidence to suggest that SAF.PA and AIR.PA are cointegrated. Therefore, these two stocks do not exhibit a long-term equilibrium relationship according to the cointegration test performed. But These two stocks show high correlation pattern. We decide to try to back test our strategy anyway.

Strategy Implementation in Python

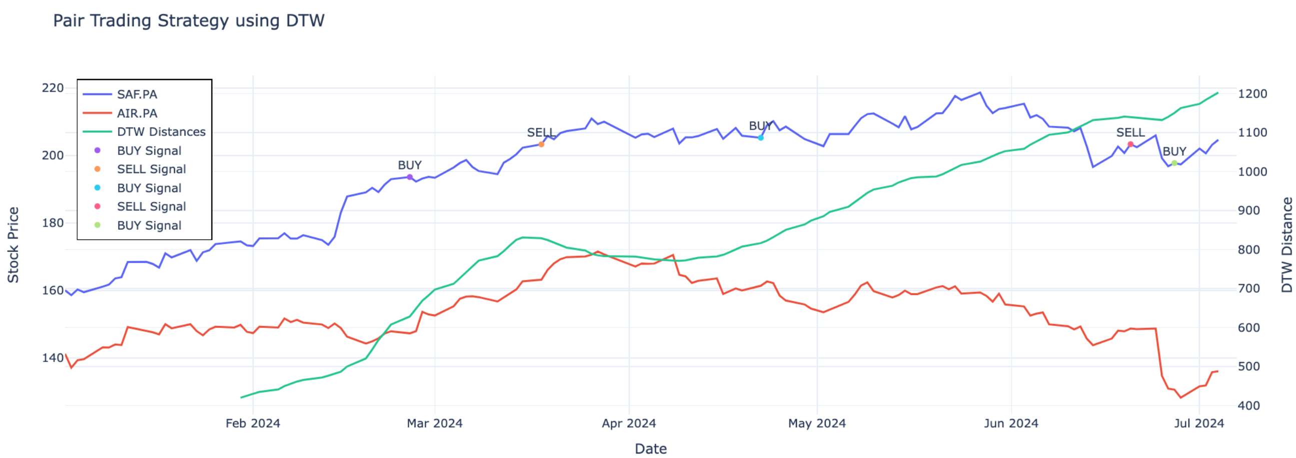


Figure 8: DTW Approach

Action	Stock	Price	Date	Number of Shares
BUY	SAF.PA	193.66	2024-02-26	10
SELL	SAF.PA	203.35	2024-03-18	10
BUY	SAF.PA	205.30	2024-04-22	10
SELL	SAF.PA	203.40	2024-06-20	10
BUY	SAF.PA	197.80	2024-06-27	10

Table 4: Trading Signals

The final return from this strategy is 1.47%

3.5.2 Pair Trading Strategy with Regression

Mathematical Formulation

The following steps describe the pair trading strategy using a dynamic threshold based on the rolling standard deviation of the spread between two stocks. The strategy involves fetching stock price data, calculating the hedge ratio, estimating the value of one stock based on the other, calculating errors and dynamic thresholds, and executing trades based on the error crossing the threshold.

- **Fetching Data:**

  $$P_{i,t} \text{ and } P_{j,t}$$

  where $P_{i,t}$ and $P_{j,t}$ are the prices of stock i and stock j at time t, respectively.

- **Calculating Hedge Ratio:**

  $$P_{j,t} = \alpha + \beta P_{i,t} + \epsilon_t$$

  where α and β are the intercept and slope coefficients obtained from the Ordinary Least Squares (OLS) regression of $P_{j,t}$ on $P_{i,t}$, and ϵ_t is the error term.

- **Estimating the Value:**

  $$\hat{P}_{j,t} = \alpha + \beta P_{i,t}$$

  where $\hat{P}_{j,t}$ is the estimated value of stock j based on the price of stock i.

- **Calculating Error and Rolling Standard Deviation:**

  $$e_t = P_{j,t} - \hat{P}_{j,t}$$

  $$\sigma_{rolling,t} = \sqrt{\frac{1}{n}\sum_{k=0}^{n-1}(e_{t-k} - \bar{e})^2}$$

  where e_t is the error (spread) at time t, $\sigma_{rolling,t}$ is the rolling standard deviation of the error over a window of size n, and $\bar{e}$ is the mean of the error over the window.

- **Trading Signals:**

  $$\begin{cases} e_t > \sigma_{rolling,t} \Rightarrow \text{Enter SHORT position on stock } j \\ e_t < -\sigma_{rolling,t} \Rightarrow \text{Enter LONG position on stock } j \\ \text{Close positions if conditions are reversed} \end{cases}$$

- **Position Management:**
  - **Initial Investment:** $C_0 = \text{initial cash} = 10,000$
  - **Cash and Shares Management:**

    $$C_t = C_{t-1} + (\Delta S_t \times P_{j,t})$$

    $$S_t = S_{t-1} + \Delta S_t$$

    where C_t is the cash at time t, S_t is the number of shares held at time t, and ΔS_t is the change in the number of shares based on the trading signal.

- **Portfolio Value:**

  $$V_t = C_t + (S_t \times P_{j,t})$$

  where V_t is the total portfolio value at time t.

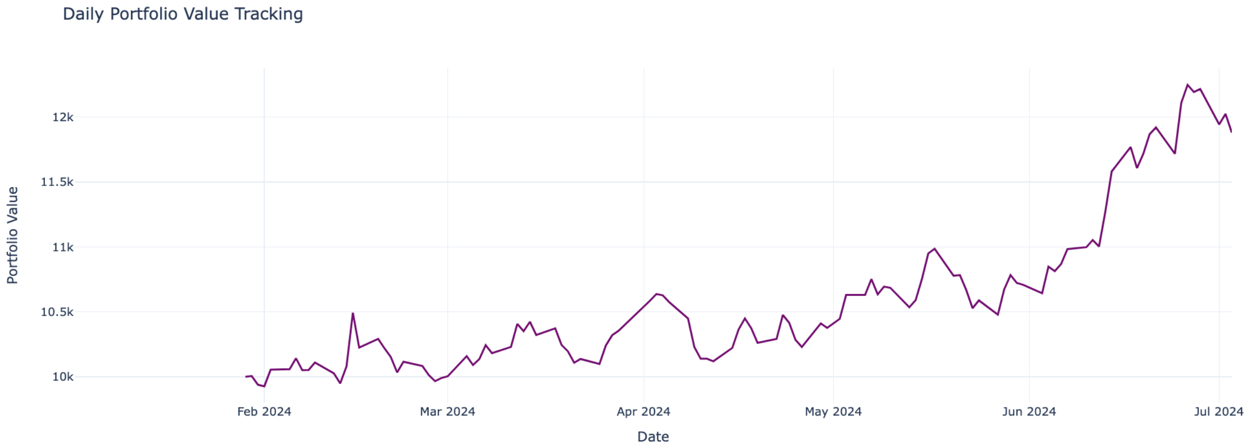
- **Total Return:**

  $$R = \frac{V_{\text{final}} - V_{\text{initial}}}{V_{\text{initial}}} \times 100\%$$

  where V_{final} is the final portfolio value, and V_{initial} is the initial investment amount.

Implementation in python

We choose



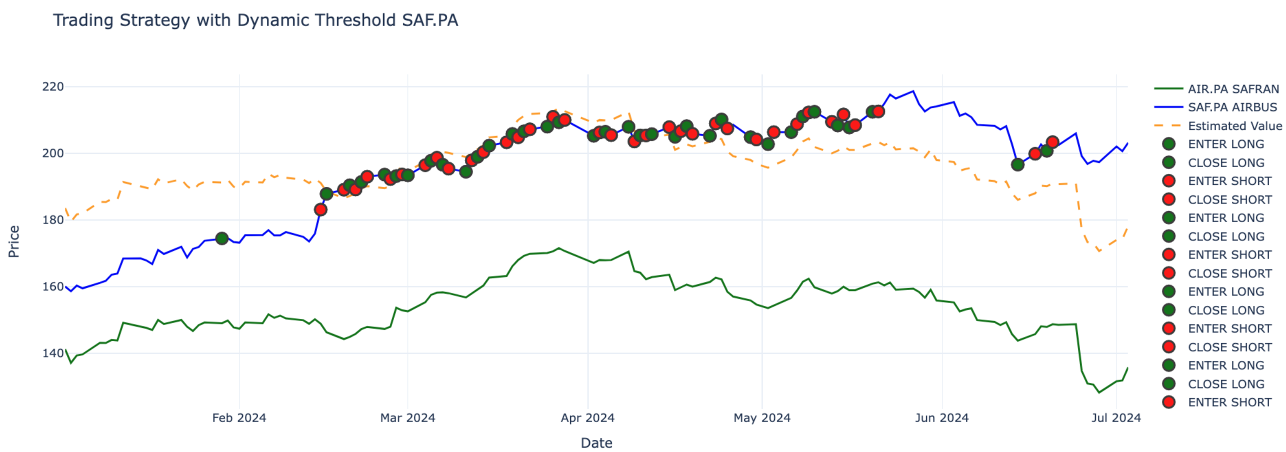


Figure 9: Regression Approach

For the same period, we back test our strategy and it shows a significant increase of our trading strategy using Regression approach with a final return: 18.79%

Limitations of Pair Trading

While pair trading offers several advantages, it also comes with certain limitations that traders should be aware of:

- **Model Risk:**
  - The strategy relies on statistical models, such as cointegration or correlation, which may not always accurately capture the relationship between asset pairs.
  - Model parameters, such as hedge ratios, can change over time, leading to potential misalignment between the paired assets.
- **Market Risk:**
  - During periods of market stress or extreme volatility, historical relationships between paired assets may break down, leading to unexpected losses.
  - The strategy may be less effective in trending markets where the divergence between asset prices is persistent.
- **Execution Risk:**
  - Timely execution of trades is crucial for capturing the expected price reversion. Delays or slippage in execution can significantly impact returns.
  - High-frequency trading and market impact can further exacerbate execution risks, especially for large trade sizes.
- **Transaction Costs:**
  - Pair trading often involves frequent trading, leading to higher transaction costs that can erode overall returns.
  - Costs associated with bid-ask spreads, commissions, and market impact should be carefully considered and managed.
- **Technology and Data Risk:**
  - Accurate and timely data is essential for model calibration and trade execution. Data errors or delays can lead to incorrect signals and trades.
  - Dependence on technology and trading platforms introduces risks related to system failures, connectivity issues, and cybersecurity threats.

3.6 Single Moving Average

Concept

The Single Moving Average Strategy leverages the empirical finding that stocks crossing above or below their moving averages often indicate potential future price movements. Specifically, when a stock's price crosses above its moving average, it may suggest bullish momentum, whereas a cross below its moving average may indicate bearish momentum. It exists few types of moving average, we gonna look at Simple Moving Average (SMA) & Exponential Moving Average.

Construction

- **Data Collection:**
  - Obtain historical stock price data for the universe of stocks. For this example, focus on a single stock or a set of stocks for backtesting.
- **Calculation of Exponential Moving Average:**
  - Compute the Exponential Moving Average (EMA) over a specific period, using the smoothing factor λ:
  $$\text{EMA}(T,\lambda) = \frac{\sum_{t=1}^{T} \lambda^{t-1} P(t)}{\sum_{t=1}^{T} \lambda^{t-1}} \tag{3}$$
  where λ is the smoothing factor (with $\lambda < 1$) that determines the weight of the most recent observations.
- **Signal Generation:**
  - Generate trading signals based on the relationship between the stock price and the EMA:
    * **Buy Signal:** Establish a long position if the stock price P crosses above the EMA.
    * **Sell Signal:** Establish a short position if the stock price P crosses below the EMA.
- **Portfolio Formation:**
  - For a long-only strategy, open a long position when the buy signal is generated and close it when the next sell signal occurs.
  - For a short-only strategy, open a short position when the sell signal is generated and close it when the next buy signal occurs.
  - For a long-short strategy, simultaneously take long positions on buy signals and short positions on sell signals.
- **Rebalancing:**
  - Rebalance the portfolio according to the trading signals, ensuring positions are adjusted or closed as per the latest buy or sell signals.
  - Ensure that any opened position is closed before opening a new one to avoid overlapping trades.
- **Risk Management:**
  - Monitor the performance of the portfolio and adjust position sizes according to volatility and market conditions.
  - Implement stop-loss and take-profit levels to manage potential losses and lock in gains.

Expected Performance

The Moving Average strategies (SMA and EMA) aim to achieve several key objectives:

- **Trend Identification:** By using moving averages, the strategies are designed to capture and follow prevailing market trends, potentially leading to profitable trades in trending markets.
- **Reduced Noise:** Moving averages help smooth out short-term price fluctuations, making it easier to identify the underlying trend and reduce market noise.
- **Improved Timing:** The strategies aim to provide clear buy and sell signals based on the relationship between the stock price and the moving average, potentially improving trade timing.
- **Consistency:** By focusing on price trends and smoothing out volatility, the strategies seek to deliver more consistent performance compared to strategies that react to every market fluctuation.

Practical Considerations

While Moving Average strategies offer several benefits, they also present certain challenges and considerations:

- **Lag Effect:** Both SMA and EMA can exhibit lag, particularly in rapidly changing markets. The lag may result in delayed signals and missed opportunities.
- **Whipsaw Risk:** In choppy or sideways markets, moving average strategies may generate frequent buy and sell signals, leading to potential whipsaw losses and increased transaction costs.
- **Parameter Sensitivity:** The effectiveness of the strategy is sensitive to the choice of moving average period and smoothing factor (for EMA). Optimizing these parameters is crucial for achieving desired performance.
- **Market Conditions:** Moving average strategies may perform better in trending markets and may underperform in range-bound or highly volatile conditions. Adjustments and additional filters may be needed based on market conditions.
- **Transaction Costs:** Frequent trading signals can incur higher transaction costs, which may impact the overall profitability of the strategy.

Backtesting and Optimization

- **Model Overfitting:**
  - *Overfitting to Historical Data*: Moving Average strategies may appear to perform well on historical data used for backtesting. However, strong historical performance does not guarantee future results. Overfitting can occur when the strategy is too closely tailored to past market conditions and may not perform effectively in future or different market environments.
  - *Parameter Sensitivity*: The effectiveness of Moving Average strategies can be highly sensitive to parameters such as the moving average period (for SMA) or the smoothing factor (for EMA). Minor adjustments to these parameters can significantly impact performance. It is crucial to optimize these parameters carefully to avoid suboptimal results.
- **Backtesting:**
  - Assess the strategy using historical price data to evaluate performance metrics such as cumulative returns, maximum drawdown, and the Sharpe ratio.
  - Analyze the strategy's performance across different market conditions (e.g., trending vs. range-bound markets) to ensure it is robust and adaptable.

- **Optimization:**
  - Employ techniques such as grid search or cross-validation to find optimal parameters for the moving average periods and smoothing factors.
  - Refine the parameters used in generating trading signals and portfolio rebalancing to enhance the strategy's performance and robustness.

Our Implementation

The implementation of the Moving Average Strategies, specifically the 10-day Simple Moving Average (SMA) and the 12-day Exponential Moving Average (EMA) for AAPL stock, presents several challenges and limitations, as outlined below:

1. **Data Quality Issues:**
   - *Data Lag*: Moving averages are based on historical price data and may not reflect real-time market conditions. This lag can impact the timing of buy and sell signals, affecting the overall performance of the strategy.
2. **Strategy Parameters:**
   - *Parameter Selection*: The choice of 10 days for SMA and 12 days for EMA may not be optimal for all market conditions. The performance of these parameters can vary, and fine-tuning may be required to match different market environments.
   - *Sensitivity to Changes*: The strategies' effectiveness is sensitive to the chosen periods for moving averages. Small changes in these parameters can lead to significant variations in trading signals and overall performance.
3. **Stock-Specific Factors:**
   - *Company-Specific Events*: Significant events such as earnings announcements or management changes can cause abrupt shifts in AAPL's stock price, impacting the stability of moving average signals.
   - *Volatility Patterns*: AAPL's stock may experience unique volatility patterns, which can influence the performance of the moving average strategies based on the specific dynamics of the stock.

SMA Final Return: 23.74%
EMA Final Return: 47.79%

Figure 10: Moving Average

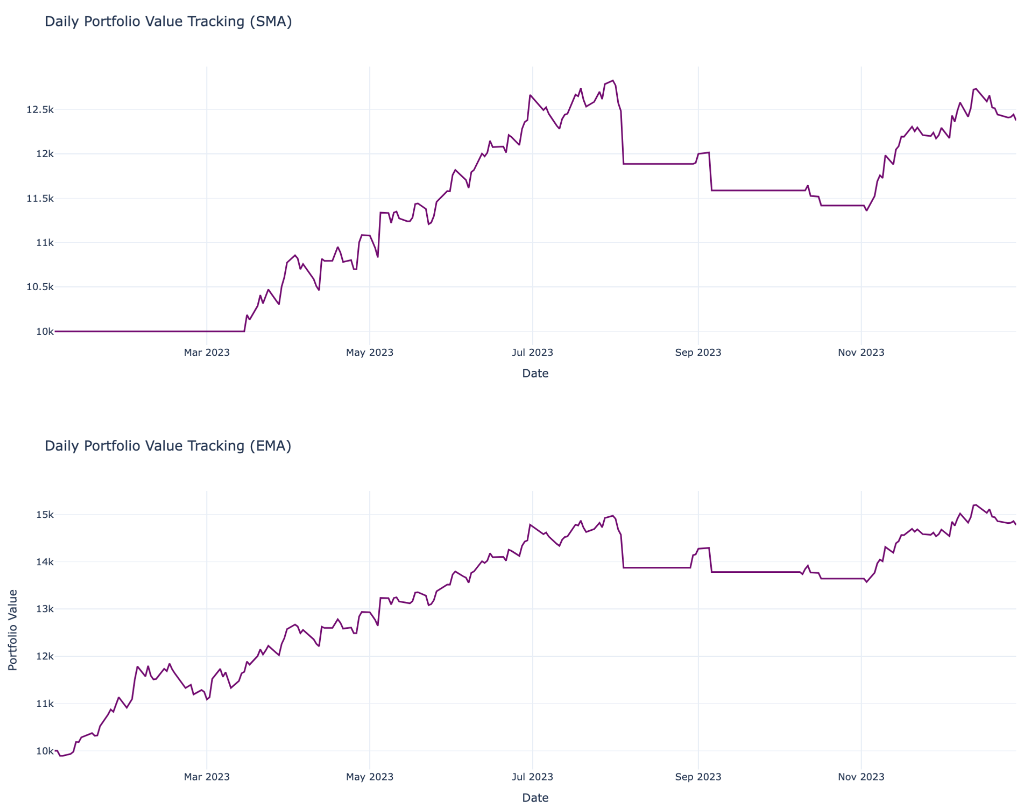


Figure 11: Moving Average Perfomance

3.7 Two Moving Average Strategy

Concept

The Two Moving Average Strategy involves using two different moving averages to generate buy and sell signals based on their crossover points. Specifically, a shorter-term moving average crossing above a longer-term moving average indicates bullish momentum, while a cross below suggests bearish momentum. This strategy helps in identifying trends and making trading decisions based on these crossover points. We will look at Simple Moving Averages (SMA) and Exponential Moving Averages (EMA).

Construction

- **Data Collection:**
  - Obtain historical stock price data for the universe of stocks. For this example, focus on a single stock or a set of stocks for backtesting.
- **Calculation of Moving Averages:**
  - Compute the shorter-term moving average (e.g., 10-day SMA or EMA) and the longer-term moving average (e.g., 50-day SMA or EMA) over specific periods.
- **Signal Generation:**
  - Generate trading signals based on the relationship between the shorter-term and longer-term moving averages:
    - **Buy Signal:** Establish a long position if the shorter-term moving average crosses above the longer-term moving average.
    - **Sell Signal:** Establish a short position if the shorter-term moving average crosses below the longer-term moving average.
- **Portfolio Formation:**
  - For a long-only strategy, open a long position when the buy signal is generated and close it when the next sell signal occurs.
  - For a short-only strategy, open a short position when the sell signal is generated and close it when the next buy signal occurs.
  - For a long-short strategy, simultaneously take long positions on buy signals and short positions on sell signals.
- **Rebalancing:**
  - Rebalance the portfolio according to the trading signals, ensuring positions are adjusted or closed as per the latest buy or sell signals.
  - Ensure that any opened position is closed before opening a new one to avoid overlapping trades.
- **Risk Management:**
  - Monitor the performance of the portfolio and adjust position sizes according to volatility and market conditions.
  - Implement stop-loss and take-profit levels to manage potential losses and lock in gains.

Expected Performance

The Two Moving Average Strategy aims to achieve several key objectives:

- **Trend Identification:** By using two moving averages, the strategy is designed to capture and follow prevailing market trends more effectively, potentially leading to profitable trades in trending markets.
- **Reduced Noise:** Moving averages help smooth out short-term price fluctuations, making it easier to identify the underlying trend and reduce market noise.
- **Improved Timing:** The strategy aims to provide clear buy and sell signals based on the relationship between the two moving averages, potentially improving trade timing.
- **Consistency:** By focusing on price trends and smoothing out volatility, the strategy seeks to deliver more consistent performance compared to strategies that react to every market fluctuation.

Practical Considerations

While the Two Moving Average Strategy offers several benefits, it also presents certain challenges and considerations:

- **Lag Effect:** Both SMA and EMA can exhibit lag, particularly in rapidly changing markets. The lag may result in delayed signals and missed opportunities.
- **Whipsaw Risk:** In choppy or sideways markets, the strategy may generate frequent buy and sell signals, leading to potential whipsaw losses and increased transaction costs.
- **Parameter Sensitivity:** The effectiveness of the strategy is sensitive to the choice of moving average periods. Optimizing these parameters is crucial for achieving desired performance.

Backtesting and Optimization

- **Model Overfitting:**
  - *Overfitting to Historical Data*: The strategy may appear to perform well on historical data used for backtesting. However, strong historical performance does not guarantee future results. Overfitting can occur when the strategy is too closely tailored to past market conditions and may not perform effectively in future or different market environments.
  - *Parameter Sensitivity*: The effectiveness of the strategy can be highly sensitive to parameters such as the moving average periods. Minor adjustments to these parameters can significantly impact performance. It is crucial to optimize these parameters carefully to avoid suboptimal results.
- **Backtesting:**
  - Assess the strategy using historical price data to evaluate performance metrics such as cumulative returns, maximum drawdown, and the Sharpe ratio.
  - Analyze the strategy's performance across different market conditions (e.g., trending vs. range-bound markets) to ensure it is robust and adaptable.
- **Optimization:**
  - Employ techniques such as grid search or cross-validation to find optimal parameters for the moving average periods.
  - Refine the parameters used in generating trading signals and portfolio rebalancing to enhance the strategy's performance and robustness.

Our Implementation

The implementation of the Two Moving Average Strategy, specifically the 10-day Simple Moving Average (SMA) and the 50-day Simple Moving Average (SMA) for AAPL stock, presents several challenges and limitations, as outlined below:

1. **Data Quality Issues:**
    - *Data Lag*: Moving averages are based on historical price data and may not reflect real-time market conditions. This lag can impact the timing of buy and sell signals, affecting the overall performance of the strategy.
2. **Strategy Parameters:**
    - *Parameter Selection*: The choice of 10 days for the short-term SMA and 50 days for the long-term SMA may not be optimal for all market conditions. The performance of these parameters can vary, and fine-tuning may be required to match different market environments.
    - *Sensitivity to Changes*: The strategy's effectiveness is sensitive to the chosen periods for moving averages. Small changes in these parameters can lead to significant variations in trading signals and overall performance.
3. **Stock-Specific Factors:**
    - *Company-Specific Events*: Significant events such as earnings announcements or management changes can cause abrupt shifts in AAPL's stock price, impacting the stability of moving average signals.
    - *Volatility Patterns*: AAPL's stock may experience unique volatility patterns, which can influence the performance of the moving average strategy based on the specific dynamics of the stock.

Figure 12: Two Moving Average

Figure 13: EMA SMA Performance tracking

SMA Final Return: 17.22%
EMA Final Return: 38.91%

3.8 Three Moving Averages

Concept

The Three Moving Average Strategy leverages the relationship between three different moving averages to generate trading signals. Specifically, it utilizes a short-term, a medium-term, and a long-term moving average. When the short-term moving average crosses above the medium-term and the medium-term is above the long-term moving average, it indicates a bullish trend. Conversely, when the short-term moving average crosses below the medium-term and the medium-term is below the long-term moving average, it indicates a bearish trend. This strategy helps in capturing longer-term trends while avoiding short-term market noise.

Construction

- **Data Collection:**
  - Obtain historical stock price data for the universe of stocks. For this example, focus on a single stock or a set of stocks for backtesting.
- **Calculation of Moving Averages:**
  - Compute the short-term, medium-term, and long-term moving averages over specific periods:

$$\mathrm{MA}_{\text{short}}(T) = \frac{1}{T_{\text{short}}} \sum_{t=1}^{T_{\text{short}}} P(t) \tag{4}$$

$$\mathrm{MA}_{\text{medium}}(T) = \frac{1}{T_{\text{medium}}} \sum_{t=1}^{T_{\text{medium}}} P(t) \tag{5}$$

$$\mathrm{MA}_{\text{long}}(T) = \frac{1}{T_{\text{long}}} \sum_{t=1}^{T_{\text{long}}} P(t) \tag{6}$$

    where $T_{\text{short}}, T_{\text{medium}}, T_{\text{long}}$ are the periods for the short-term, medium-term, and long-term moving averages, respectively.
- **Signal Generation:**
  - Generate trading signals based on the relationship between the moving averages:
    * **Buy Signal:** Establish a long position if the short-term moving average is above the medium-term moving average and the medium-term moving average is above the long-term moving average.
    * **Sell Signal:** Establish a short position if the short-term moving average is below the medium-term moving average and the medium-term moving average is below the long-term moving average.
- **Portfolio Formation:**
  - For a long-only strategy, open a long position when the buy signal is generated and close it when the next sell signal occurs.
  - For a short-only strategy, open a short position when the sell signal is generated and close it when the next buy signal occurs.
  - For a long-short strategy, simultaneously take long positions on buy signals and short positions on sell signals.
- **Rebalancing:**
  - Rebalance the portfolio according to the trading signals, ensuring positions are adjusted or closed as per the latest buy or sell signals.
  - Ensure that any opened position is closed before opening a new one to avoid overlapping trades.

- **Risk Management:**
  - Monitor the performance of the portfolio and adjust position sizes according to volatility and market conditions.
  - Implement stop-loss and take-profit levels to manage potential losses and lock in gains.

Expected Performance

The Three Moving Average Strategy aims to achieve several key objectives:

- **Trend Identification:** By using three moving averages, the strategy is designed to capture and follow longer-term market trends, potentially leading to profitable trades in trending markets.
- **Reduced Noise:** The use of three moving averages helps smooth out short-term price fluctuations, making it easier to identify the underlying trend and reduce market noise.
- **Improved Timing:** The strategy aims to provide clear buy and sell signals based on the relationship between the moving averages, potentially improving trade timing.
- **Consistency:** By focusing on longer-term trends and smoothing out volatility, the strategy seeks to deliver more consistent performance compared to strategies that react to every market fluctuation.

Practical Considerations

While the Three Moving Average Strategy offers several benefits, it also presents certain challenges and considerations:

- **Lag Effect:** Moving averages can exhibit lag, particularly in rapidly changing markets. The lag may result in delayed signals and missed opportunities.
- **Whipsaw Risk:** In choppy or sideways markets, the strategy may generate frequent buy and sell signals, leading to potential whipsaw losses and increased transaction costs.
- **Parameter Sensitivity:** The effectiveness of the strategy is sensitive to the choice of moving average periods. Optimizing these parameters is crucial for achieving desired performance.
- **Market Conditions:** The strategy may perform better in trending markets and may underperform in range-bound or highly volatile conditions. Adjustments and additional filters may be needed based on market conditions.
- **Transaction Costs:** Frequent trading signals can incur higher transaction costs, which may impact the overall profitability of the strategy.

Backtesting and Optimization

- **Model Overfitting:**
  - *Overfitting to Historical Data*: The Three Moving Average Strategy may appear to perform well on historical data used for backtesting. However, strong historical performance does not guarantee future results. Overfitting can occur when the strategy is too closely tailored to past market conditions and may not perform effectively in future or different market environments.
  - *Parameter Sensitivity*: The effectiveness of the strategy can be highly sensitive to parameters such as the moving average periods. Minor adjustments to these parameters can significantly impact performance. It is crucial to optimize these parameters carefully to avoid suboptimal results.
- **Backtesting:**
  - Assess the strategy using historical price data to evaluate performance metrics such as cumulative returns, maximum drawdown, and the Sharpe ratio.

- Analyze the strategy's performance across different market conditions (e.g., trending vs. range-bound markets) to ensure it is robust and adaptable.

- **Optimization:**
  - Employ techniques such as grid search or cross-validation to find optimal parameters for the moving average periods.
  - Refine the parameters used in generating trading signals and portfolio rebalancing to enhance the strategy's performance and robustness.

Our Implementation

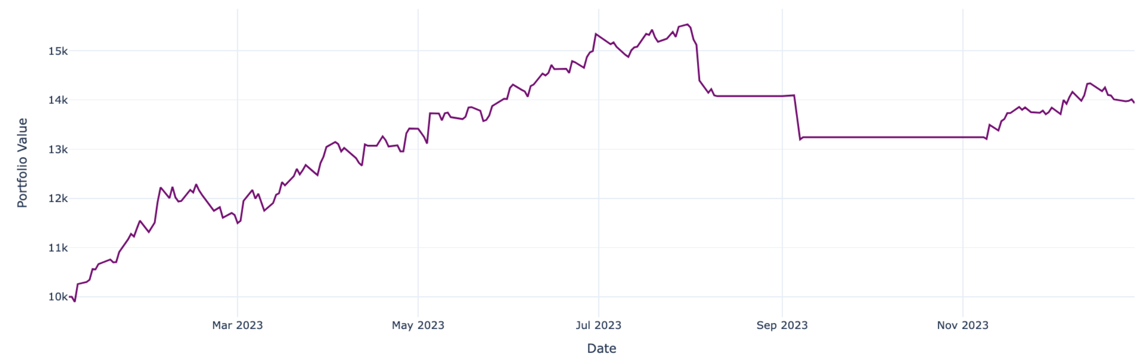
The implementation of the Three Moving Average Strategy, specifically the 10-day short-term, 50-day medium-term, and 200-day long-term moving averages for AAPL stock, presents several challenges and limitations, as outlined below:

Strategy Parameters:

- *Parameter Selection*: The choice of 10, 50, and 200 days for the moving averages may not be optimal for all market conditions. The performance of these parameters can vary, and fine-tuning may be required to match different market environments.

- *Sensitivity to Changes*: The strategies' effectiveness is sensitive to the chosen periods for moving averages. Small changes in these parameters can lead to significant variations in trading signals and overall performance.

Figure 14: Three Moving Average Strategy



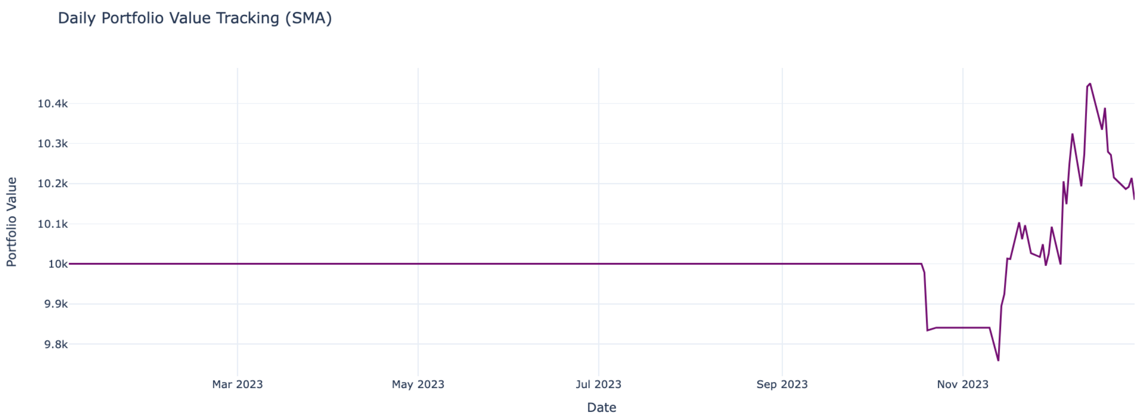


Figure 15: performance of portfolio

SMA Final Return: 1.60%
EMA Final Return: 39.37%