Are we actually restricted by NP-hardness for General computing?
Can we have it for classes of problems?
Iteration/optimization field. Look at Ising Machine Papers.

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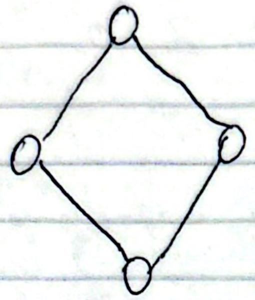
Maximum Cut



$g = \{V, E\}$

$V = V^{(1)} \cup V^{(2)}$

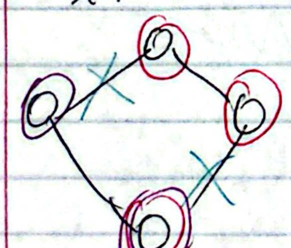
Total # of edges connecting

cut $= \# \{ e \in E : \underbrace{e = (v_1, v_2)}, v_1 \in V^{(1)}, v_2 \in V^{(2)} \}$

is maximized Each edge is a "cut"

Split graph g into two subsets such that the # of edges between the two ~~edges~~ subsets

Ex:



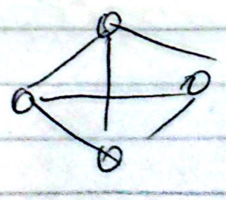
# of cuts = 2

Give each edge a weight (0 or 1) if it is ~~set~~ present then give that edge a weight of 1. Else 0.

~~cut~~ Take complete graph, then remove edges by giving weights of 0.

$cut \leq M = |E|$

Complete graphs



Each node ~~edge~~ goes to all other nodes

Cut is less than # of edges.

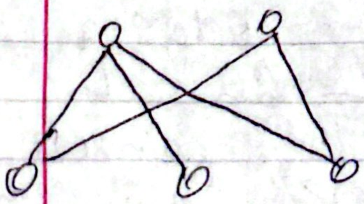
"Excersize:"

Graphs with max-cut $= m$ are <u>bipartite.</u>

Key word V important

bipartite: The graph has a partition of nodes such that each node connects ~~to~~ two nodes in diff partitions.

~~NN~~ Neural Networks are bipartite

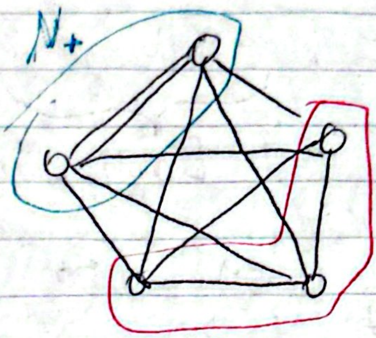


"Excersize":

max-cut $\geq \frac{m}{2}$ Not sure abt $=$ tho.

$m = |E|$

K_5 (complete graph w/ 5 nodes)



Good thing abt complete graphs is that swapping any 2 nodes doesn't change the graph.

$N_+ + N_- = N$

$\text{Cut} = N_+ \cdot N_-$

Every node in N_+ connects to node in N_-. If there are ~~N~~ j nodes in N_+, then # of edges is # of ~~nodes~~ edges ~~in~~ is $N_+ \cdot N_-$ which is the cut.

Now maximize $N_- = N - N_+$ cut.

$$\text{Cut} = N_+(N - N_+) = N_+^2 - NN_+$$

Derivative w/ discrete variable $\rightarrow$ $$\frac{dC}{dN_+} = 2N_+ - N = 0 \quad \boxed{N_+ = \frac{N}{2}}$$

Fails to work for ~~odd~~ odd vals of N.

$$\text{max cut} = \bar{C} = \frac{N^2}{4} = \underbrace{\frac{N}{2}}_{N_+} \cdot \underbrace{\frac{N}{2}}_{N_-}$$

$$C(N_+) = N_+(N - N_+)$$

$$C(N_+ + 1) - C(N_+) = (N_+ + 1)(N - N_+ - 1) - N_+(N - N_+)$$

$$= N - N_+ - 1 - N_+ = \boxed{N - 2N_+ - 1}$$

change when node increases by 1. ΔC

if $2N_+ + 1 \leq N \Rightarrow \Delta C > 0$

$2N_+ + 1 > N \Rightarrow \Delta C < 0$

Maximum N_+: $2N_+ + 1 \leq N \qquad 2N_+ + 1 \leq 2p$

If $N = 2p \Rightarrow \bar{N}_+ = p$

$N = 2p + 1$

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Odd:

$2\bar{N}_+ + 1 \le 2p+1$

$N_+ \le p$

Even:

Even

$2\bar{N}_+ + 1 \le 2p$

$\bar{N}_+ \le \frac{2p-1}{2}$

$N = 2p+1$

$c(N_+=p) = p(2p+1-p)$
$= p^2 + p$
$= p(p+1)$

$c(N_+=p-1) = (p-1)(2p+1-p-1)$
$= p(p+1)$

$M = \frac{N(N-1)}{2}$ N nodes connecting to N-1 and remove overcounts

M = # of total edges

$N = 2p$

$c(\frac{N}{2}) = p(2p-p) = p^2$

$c(N_+=p+1) = (p+1)(2p-p-1)$
$= p^2 - 1$

$\Delta c < 0$

$c(p+1) - c(p) < 0$

$c(p-1) = p^2 - 1$

$c(p) - c(p-1) > 0$

$\Rightarrow \bar{N}_+ = p$

$N = 2p$

$N_+ = p$ $N_- = p$

$N = 2p+1$

$N_+ = p$

$N_+ = p+1$

Switching changes groups, but not cut.

$p \in \mathbb{N}$

Even $N=2p$

Max cut for 2p nodes

$$\bar{C} = \frac{N^2}{4} = \frac{(2p)^2}{4} = p^2$$

$$M = \frac{N(N-1)}{2} = \frac{2p(2p-1)}{2} = \frac{4p^2-2p}{2} = 2p^2-p$$

total # of edges in comp. graph

$$\frac{M}{2} = p^2 - \frac{p}{2} \Rightarrow \boxed{\bar{C} > \frac{M}{2}}$$

diving by two to get

much larger than $\bar{C}$

uneven dist.

Odd

$$\bar{C} = \frac{N-1}{2} \cdot \frac{N+1}{2} = \frac{N^2-1}{2}$$

$$\bar{C} = N_+ \cdot N_-$$

$$M = \frac{N(N-1)}{2} = \frac{(2p+1)(2p+1-1)}{2} = p(2p+1)$$

$$\bar{C} = \frac{(2p+1)^2-1}{2} = \frac{4p^2+4p}{2} = 2p(p+1) = 2p^2+2p$$

$$\frac{M}{2} = p^2 + \frac{p}{2} \qquad \boxed{\bar{C} > \frac{M}{2}}$$

5 ~~6~~ $\frac{\bar{C}}{\frac{M}{2}} > 1$ but $\lim_{N\to\infty} \frac{\bar{C}}{\frac{M}{2}} = 1$

$$\boxed{\frac{M}{2} \le \bar{C} \le M}$$

Finding $\bar{C}$ for arbitrary graph is NP-hard problem.

ΔC — C

How close we are to true value

if $\frac{\Delta C}{C} <$ "9[illegible]%"

then it requires exponential time. If $P \neq NP$

Before that we good.

Unique game conjecture:

If UGC then $P \neq NP$

$\rightarrow \frac{\Delta C}{C} <$ "13%"

APX-hard:

This is what Ising machines try to do.

Hard to approximate.

Partitioning

$$\sigma_m + \sigma_n = 0$$

$$\frac{1}{4}\left[4 - (\sigma_m + \sigma_n)^2\right] = \begin{cases} 1 & \text{if cut} \\ 0 & \text{otherwise} \end{cases}$$

we need to check this

$\sigma = +1 \rightarrow N_+$

$\sigma = -1 \rightarrow N_-$

$$A_{m,n}x(e) = A_{m,n} \underbrace{\frac{1}{2}(1 - \sigma_m \sigma_n)}_{\begin{cases} 1 & \text{if } (m,n) \checkmark \\ 0 & \text{otherwise} \end{cases}}$$

$\sigma_+ + \sigma_- = 0 \Rightarrow$ cut

but we need to count them

$$A = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \quad w_{ij} = \begin{cases} 1 & \text{if edge exists} \\ 0 & \text{else} \end{cases}$$

~~Notes~~ = Excersize:

$A_{m,n} \quad \frac{1}{2}(1-\sigma_m \sigma_n)$

$A_{m,n} \quad \frac{1}{4}[4-(\sigma_m+\sigma_n)^2]$

$A_{mn} \quad \frac{1}{4}[\sigma_m - \sigma_n]^2$

Task 4 Expand squares and simplify

$$C(\underline{\sigma}) = \frac{1}{4}\sum_{m,n} A_{m,n}(1-\sigma_m\sigma_n)$$

checks to see if diff sets or not.

$\frac{1}{2}$ from overcount

$\frac{1}{2}$

$A_{m,n}$: 1 if (m,n), 0 else

$$= \underbrace{\frac{1}{4}\sum_{m,n} A_{m,n}}_{2W \text{ counted all edges twice}} - \underbrace{\frac{1}{4}\sum_{m,n} A_{m,n}\sigma_m\sigma_n}_{\text{just ising hamiltonian}}$$

W = weight of each edge if weights are only (0 or 1)

$$\frac{W}{2} = \frac{2W}{4} = \frac{2M}{4} = \frac{M}{2}$$

$$C(\underline{\sigma}) = \frac{W}{2} - \frac{H(\sigma)}{2}$$

counting edges too much, so divi by 2.

max cut w/ ising hamiltonian

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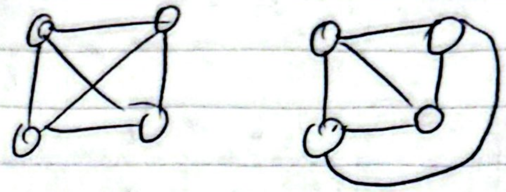
Ising machine deal w/ a specific class of problems. Much like GPUs. We can make a bunch of cores/Parellization. Even tho its NP-Hard, its v easy to parellize.

MC is not ~~NP~~ NP-hard if

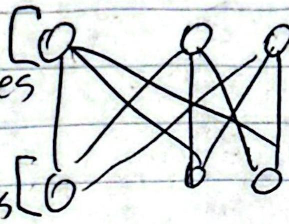
- Planar graph — Drawn on plane w/o edges intersection.



- Bipartite
- Complete
- when g does not contain K_5 as a minor.

Edges need not be straight

3 nodes



3 nodes

$K_{3,3}$ is non planar

Kuratonsky thm

Famous Problem

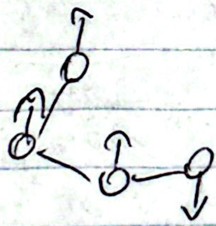
	E	W	G
you	o	o	o

nect

o	o	o	o
sing	1	2	3

r lines?

Heuristics are tough.

$H(\underline{\sigma}) =$ 

$\mathcal{E}(\text{edge}) = A\,\sigma_m \sigma_n$
coupling

You overcount.

$$H(\underline{\sigma}) = \frac{1}{2}\sum_{m,n} A_{m,n}\,\sigma_m \sigma_n$$

mization oblems

[redacted]ard but to be solved.
↓
we only need to solve certain classes of problems?

Good quality solutions & how to ensure that.