CSE330(Numerical Methods)

LECTURE 11 – ROOTS OF EQUATIONS

[OPEN METHODS: SECANT METHOD]

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Secant Method: Overview

- **Secant method** is an extension to the **Newton-Raphson method.**
- The Newton-Raphson formula from lesson 10 is –

$$x_{i+1} = x_i - \frac{f(xi)}{f'(xi)} \;\ldots\ldots\ldots\ldots\ldots.(1)$$

The value of derivative $[f'(xi)]$ can be determined using **first order backward divided difference formula** from lesson 4 as follows-

$$f'(xi) \cong \frac{f(x_{i\_1}) - f(xi)}{x_{i\_1} \_ x_i} \;\ldots\ldots\ldots\ldots\ldots.(2)$$

Substituting the value from equation (2) into equation (1), we get-

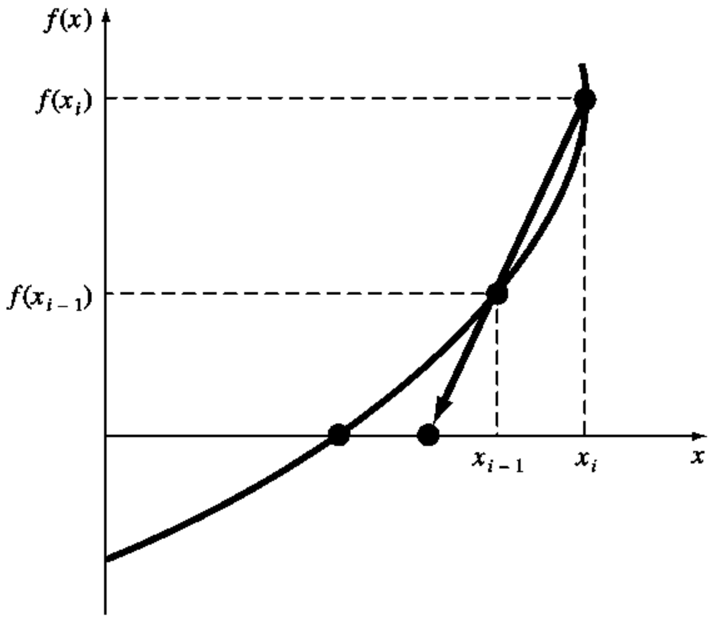
$$x_{i+1} = x_i - \frac{f(x_i)(x_{i\_1} \_ x_i)}{f(x_{i\_1}) - f(xi)} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Secant Method: Overview

- Equation (3) is referred to as the **Secant Formula.** It is also called **linear interpolation method.**

- Notice that the approach requires two initial estimates of x. However, because f(x) is not required to change signs between the estimates, it is not classified as a bracketing method.

- The name **'Secant Formula'** is derived from the fact that at the initial starting point(x_i), we draw a secant through the function. The secant intersects the x-axis at point (x_{i+1}), which is referred to as the next approximation of root. In **Newton-Raphson** formula, this was a tangent line; not a secant line.



- The Secant formula can be derived using the slope information of the secant line. **Try to derive it.**

Secant Method: Algorithm

Step 1:

Take two initial guesses, such as x_i and x_{i-1}

Step 2:

Find out the new approximation of the root (x_{i+1}) with the following formula-

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Step 3: Calculate relative percentage error with the formula –

$$\epsilon_a = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\ \%$$

Step 4:

--> If $\epsilon_a < \epsilon_s$, terminate the calculation. x_{i+1} is the estimated root.

--> If $\epsilon_a > \epsilon_s$, set $x_i = x_{i+1}$ and $x_{i-1} = x_i$; go to step 2.

Secant Method: Example

- **Example 1:**

Use the Secant method to estimate the root of f(x) =e^{-x} - x , employing an initial guess of x_0=1 and x_{-1} = 0.

Solution:

The solution process is same as that for the Newton-Raphson method, except there are two starting points and the secant formula is used for approximating the root. Try to solve the problem!

Q: Although the secant method may be divergent, when it converges it usually does so at a quicker rate than the false-position method. Can you find out the difference between False-Position Method and the Secant method although they both require two starting points?

Modified Secant Method: Overview

- Rather than using two arbitrary values to estimate the derivative, an alternative approach involves a fractional perturbation of the independent variable to estimate f'(x)-

$$f'(x_i) \cong \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i} \quad \text{..............................(4)}$$

where δ = a small perturbation fraction. This approximation can be substituted into Eq. (1) to yield the following iterative equation-

$$x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)} \quad \text{..............................(5)}$$

- **Modified Secant Method** is interesting, because now it requires one single starting point, rather than two in the **Original Secant Method.**

Modified Secant Method: Example

- Use the modified secant method to estimate the root of f(x) = e^{-x}-x. Use a value of 0.01 for δ and start with x_0=1.0. Recall that the true root is 0.56714329. . .
- **Solution:**

Iteration 1:

Here, we will be using **True Percentage Relative Error** (ϵ_t) since we know the true value of the root.

$$x_0 = 1 \qquad f(x_0) = -0.63212$$

$$x_0 + \delta x_0 = 1.01 \qquad f(x_0 + \delta x_0) = -0.64578$$

$$x_1 = 1 - \frac{0.01(-0.63212)}{-0.64578 - (-0.63212)} = 0.537263 \qquad |\varepsilon_t| = 5.3\%$$

Modified Secant Method: Example

Iteration 2:

$$x_0 = 0.537263 \qquad f(x_0) = 0.047083$$

$$x_0 + \delta x_0 = 0.542635 \qquad f(x_0 + \delta x_0) = 0.038579$$

$$x_1 = 0.537263 - \frac{0.005373(0.047083)}{0.038579 - 0.047083} = 0.56701 \qquad |\varepsilon_t| = 0.0236\%$$

Iteration 3:

$$x_0 = 0.56701 \qquad f(x_0) = 0.000209$$

$$x_0 + \delta x_0 = 0.572680 \qquad f(x_0 + \delta x_0) = -0.00867$$

$$x_1 = 0.56701 - \frac{0.00567(0.000209)}{-0.00867 - 0.000209} = 0.567143 \qquad |\varepsilon_t| = 2.365 \times 10^{-5}\%$$

Modified Secant Method: Choosing δ

- The choice of a proper value for δ is not automatic. If δ is too small, the method can be swamped by round-off error caused by subtractive cancellation in the denominator of Eq. (5).

- If it is too big, the technique can become inefficient and even divergent.

- However, if chosen correctly, it provides a nice alternative for cases where evaluating the derivative is difficult and developing two initial guesses is inconvenient.

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Conclusion

- In this lesson, we have
  - learnt **Secant Method**, one of the most widely used root finding method next to Newton-Raphson Method.
  - developed the algorithm for implementing the method.
  - Known about an alternative version of Secant Method, named **'Modified Secant Method'**. The modified version is useful, because it involves single starting point and it does not require calculating complex derivative.

- Please try out the MATLAB code of Secant method and Modified Secant Method.

In the next lesson, we will learn about an extension of Secant method, names as "Inverse Quadratic Interpolation Method (IQIM)"

Thank You