CSE330(Numerical Methods)

LECTURE 35 – NUMERICAL INTEGRATION

[MULTIPLE INTEGRATION]

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Overview

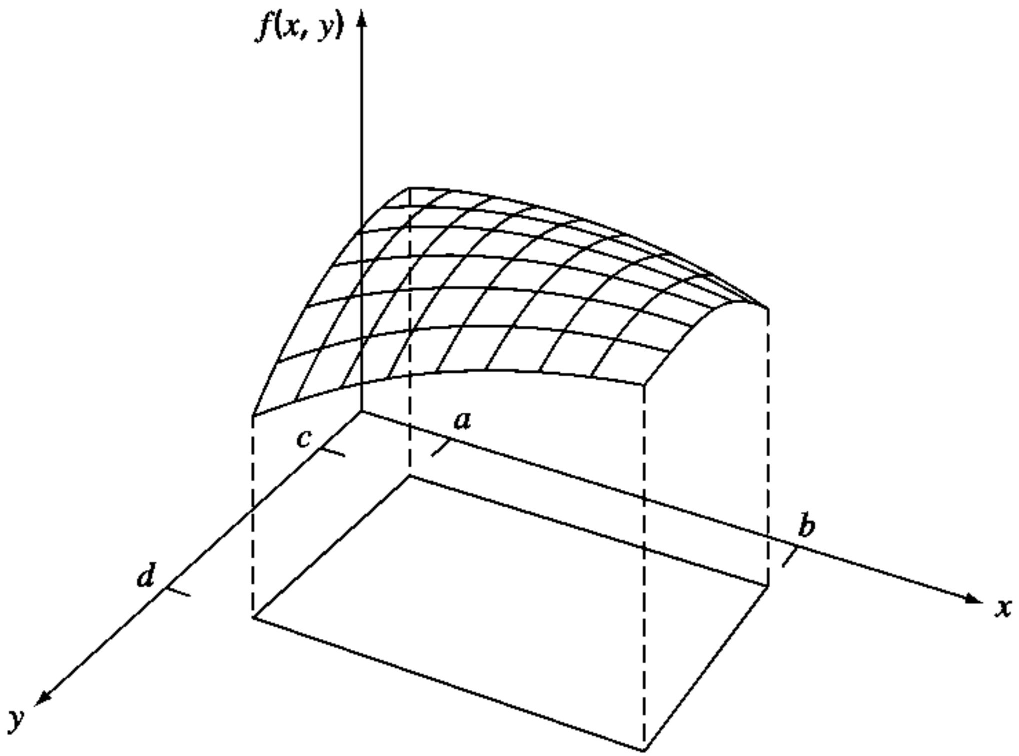
❑Multiple integrals involve double or triple integrals. For example:

$$\bar{f} = \frac{\int_c^d \left(\int_a^b f(x, y) dx \right) dy}{(d - c)(b - a)}$$

The numerator of the function contains a double integral.

Double Integral determines the volume under the curve.

$$\int_c^d \left(\int_a^b f(x, y)\, dx \right) dy = \int_a^b \left(\int_c^d f(x, y) dy \right) dx$$

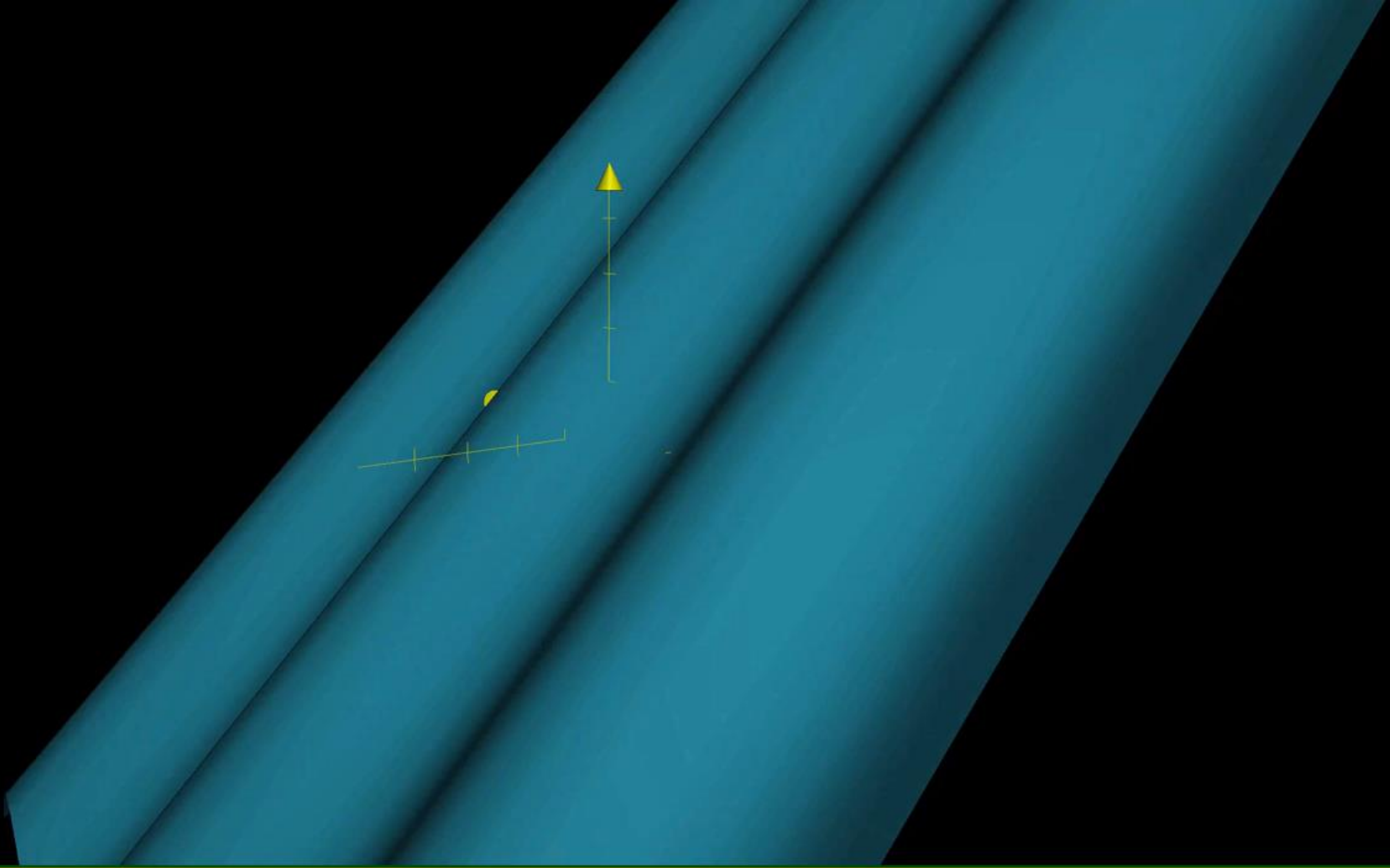


Overview

$$\int_{y_1}^{y_2} \overbrace{\left(\int_{x_1}^{x_2} f(x,y)dx \right)}^{\text{This is a function of } y} dy \qquad \int_{x_1}^{x_2} \overbrace{\left(\int_{y_1}^{y_2} f(x,y)dy \right)}^{\text{This is a function of } x} dx$$

- First, methods like the multiple-segment trapezoidal or Simpson's rule would be applied in the first dimension with each value of the second dimension held constant.
- Then the method would be applied to integrate the second dimension.
- Let's look at a function:

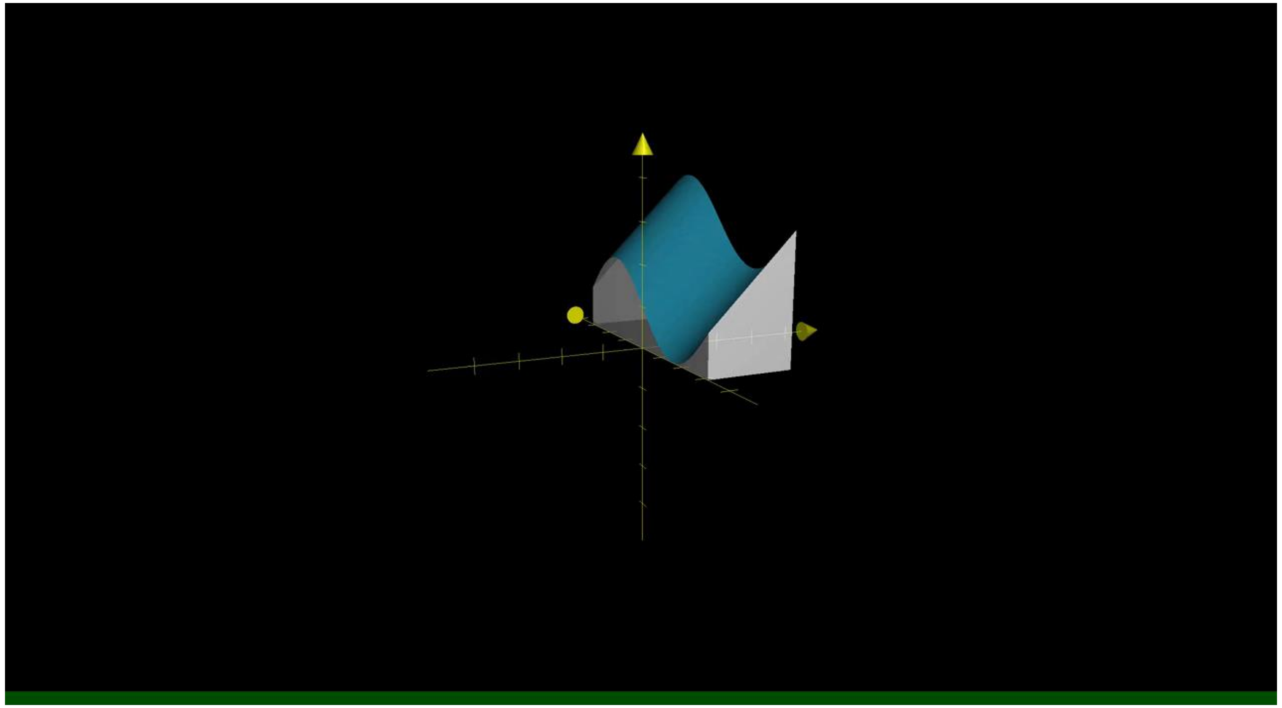
$$f(x, y) = x + \sin(y) + 1$$



Overview

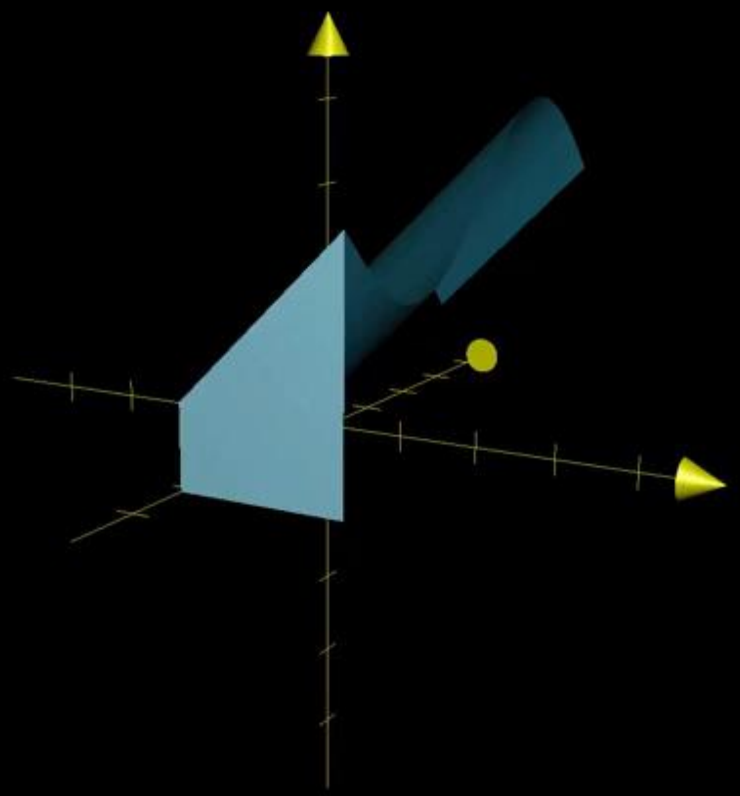
Now consider the rectangle on the xy-plane defined by-

$$0 < x < 2 \qquad -\pi < y < \pi$$

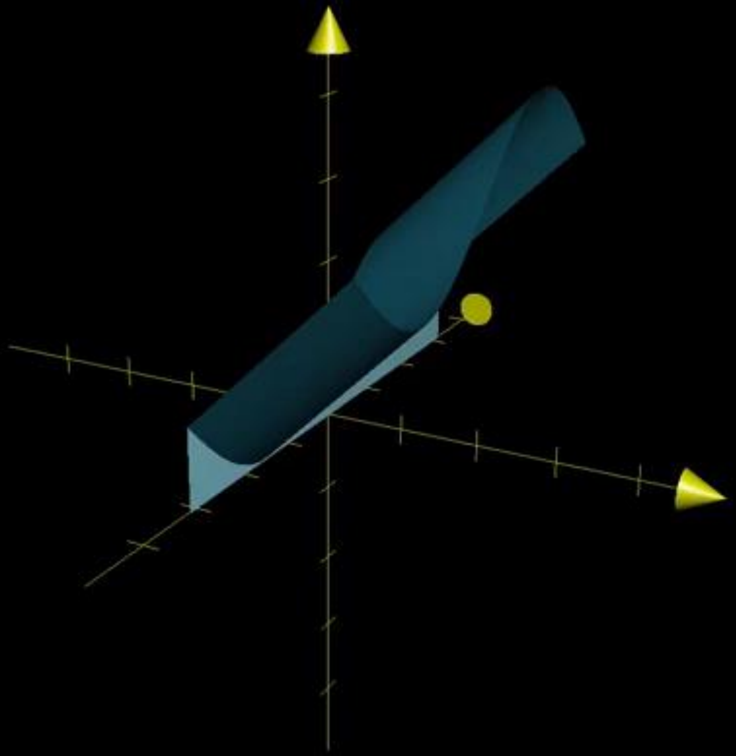


$$\int_{y_1}^{y_2}\left(\int_{x_1}^{x_2} f(x,y)dx\right)dy$$

$$\overbrace{\int_{-\pi}^{\pi}\left(\underbrace{\int_{0}^{2} f(x,y)dx}_{\text{Area of a slice}}\right)dy}^{\text{Volume under } f(x,y)}$$



$$\int_{x_1}^{x_2}\left(\int_{y_1}^{y_2} f(x,y)dy\right)dx$$



Multiple Integration: Example

- Suppose that the temperature of a rectangular heated plate is described by the following function:

$$T(x, y) = 2xy + 2x - x^2 - 2y^2 + 72$$

If the plate is 8-m long (x dimension) and 6-m wide (y dimension), compute the average temperature.

Solution:

Let’s use two-segments along both axis.

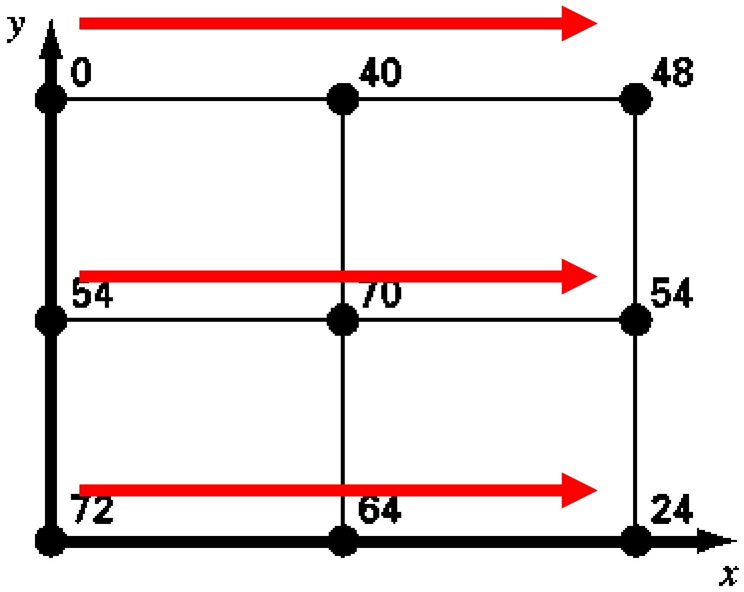
- Along the x-axis, first segment is from x=0 to x=4

  second segment is from x=4 to x=8

- Similarly, along the y-axis, first segment is from y=0 to y=3

  second segment is from y=3 to y=6

Multiple Integration: Example



$$(8-0)\frac{0+2(40)+48}{4} \rightarrow 256$$

$$(8-0)\frac{54+2(70)+54}{4} \rightarrow 496$$

$$(8-0)\frac{72+2(64)+24}{4} \rightarrow 448$$

$$(6-0)\frac{256+2(496)+448}{4} = 2688$$

Multiple Integration: Example

Can we extend our idea of single integration and double integration towards **triple integration**?

Yes.

Conclusion

- In this lesson, we have
  - leant how to extend the idea of single integration to double and triple integration.
  - analyzed a double integration graphically.
  - solved a double integral problem numerically.

- Please try out the MATLAB code for double integration.

NB: Part of the video animation is due to the courtesy of khan academy.
[https://www.khanacademy.org/]

In the next lesson we will learn about numerical method of integration of an equation or a function.

Thank You