CSE330(Numerical Methods)

LECTURE 34 – NUMERICAL INTEGRATION

[INTEGRATION WITH UNEQUAL SEGMENTS]

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Overview

- Experimentally derived data are often of unequal distribution.
- For these cases, one method is to apply the trapezoidal rule to each segment and sum the results:

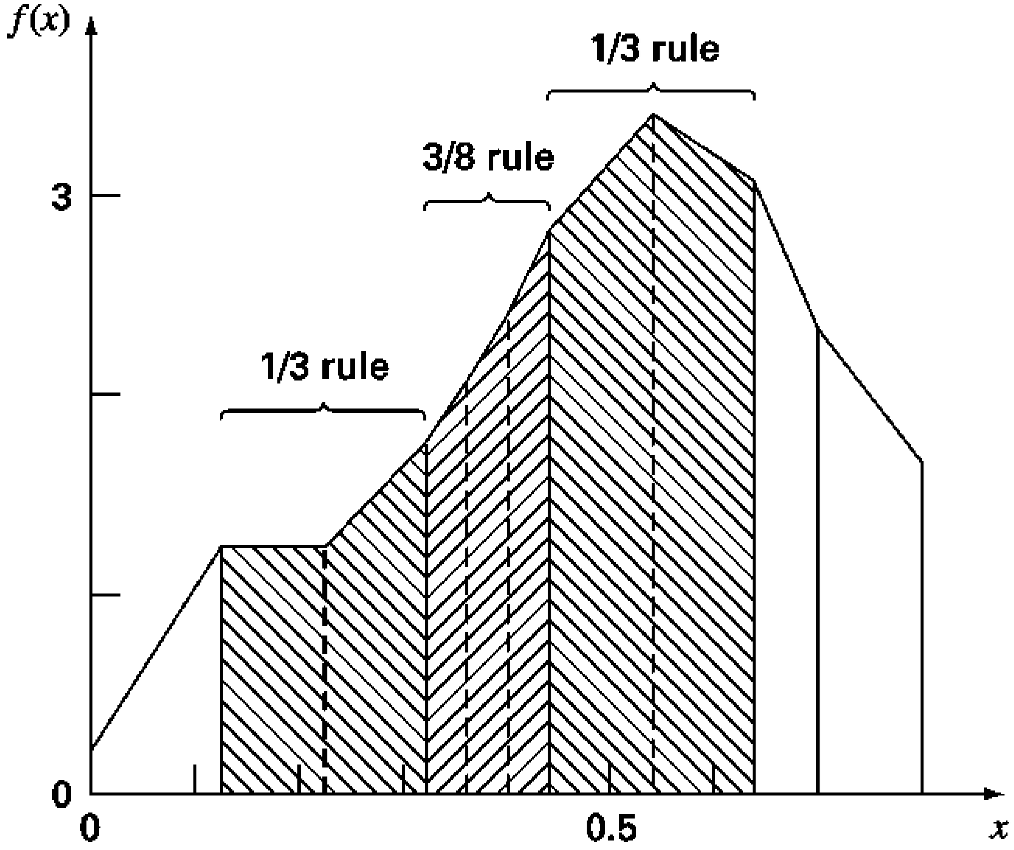
$$I = h_1 \frac{f(x_0) + f(x_1)}{2} + h_2 \frac{f(x_1) + f(x_2)}{2} + \cdots + h_n \frac{f(x_{n-1}) + f(x_n)}{2}$$

where h_i = the width of segment i.

- Note that this was the same approach used for the multiple-application trapezoidal rule. The only difference is that the h's in the former are constant.

Overview

- In the evenly segmented data points among an **unequal distribution**, Simpson's rule could be used.



Unequal Data Distribution: Example

- **Example:** Find the integral result of the following table. The correct answer is 1.640533.

x	f(x)	x	f(x)
0.0	0.200000	0.44	2.842985
0.12	1.309729	0.54	3.507297
0.22	1.305241	0.64	3.181929
0.32	1.743393	0.70	2.363000
0.36	2.074903	0.80	0.232000
0.40	2.456000		

- **Solution:**

$$I = h_1 \frac{f(x_0) + f(x_1)}{2} + h_2 \frac{f(x_1) + f(x_2)}{2} + \cdots + h_n \frac{f(x_{n-1}) + f(x_n)}{2}$$

$$I = 0.12 \frac{1.309729 + 0.2}{2} + 0.10 \frac{1.305241 + 1.309729}{2} + \cdots + 0.10 \frac{0.232 + 2.363}{2}$$

Unequal Data Distribution

x	f(x)	x	f(x)
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0.12	1.309729	0.54	3.507297
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0.32	1.743393	0.70	2.363000
0.36	2.074903	0.80	0.232000
0.40	2.456000		

$$I = 0.090584 + 0.130749 + \cdots + 0.12975 = 1.594801$$

which represents an absolute percent relative error of $\varepsilon_t = 2.8\%$

But as we said we could combine Simpson's rules and Trapezoidal rule in the same problem.

The first segment is evaluated with the trapezoidal rule:

$$I_1 = 0.12 \frac{1.309729 + 0.2}{2} = 0.09058376$$

Because the next two segments from x = 0.12 to 0.32 are of equal length, their integral can be computed with Simpson's 1/3 rule:

$$I_2 = 0.2 \frac{1.743393 + 4(1.305241) + 1.309729}{6} = 0.2758029$$

Unequal Data Distribution

x	f(x)	x	f(x)
0.0	0.200000	0.44	2.842985
0.12	1.309729	0.54	3.507297
0.22	1.305241	0.64	3.181929
0.32	1.743393	0.70	2.363000
0.36	2.074903	0.80	0.232000
0.40	2.456000		

The next three segments are also equal and, as such, may be evaluated with the 3/8 rule to give

$$\boldsymbol{I_3 = 0.2726863.}$$

Similarly, the 1/3 rule can be applied to the two segments from x=0.44 to 0.64 to yield-

$$\boldsymbol{I_4 = 0.6684701.}$$

Finally, the last two segments, which are of unequal length, can be evaluated with the trapezoidal rule to give values of-

$$\boldsymbol{I_5 = 0.1663479}$$
$$\boldsymbol{I_6 = 0.1297500}$$

respectively.

Unequal Data Distribution: Example

Summing all the separate integration results-

$$I = 1.603641$$

Which has an error of $\boxed{\varepsilon_t = 2.2\%}$

Conclusion

- In this lesson we have
  - learnt how to perform integration on unevenly distributed data.
  - learnt how the Simpson's rules and trapezoidal rule, **combined**, can perform better on unevenly distributed data.
  - solved an integration approach using both approaches.

- Please try out the MATLAB code for Integration with uneven data distribution.

In the next lesson, we will learn multiple integration (double/triple integration) solving technique.

Thank You