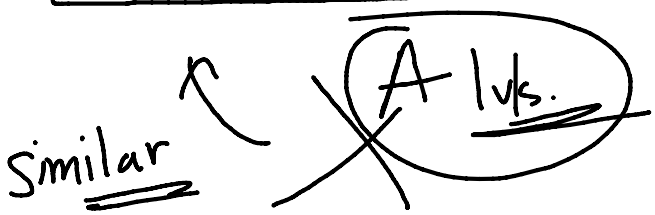
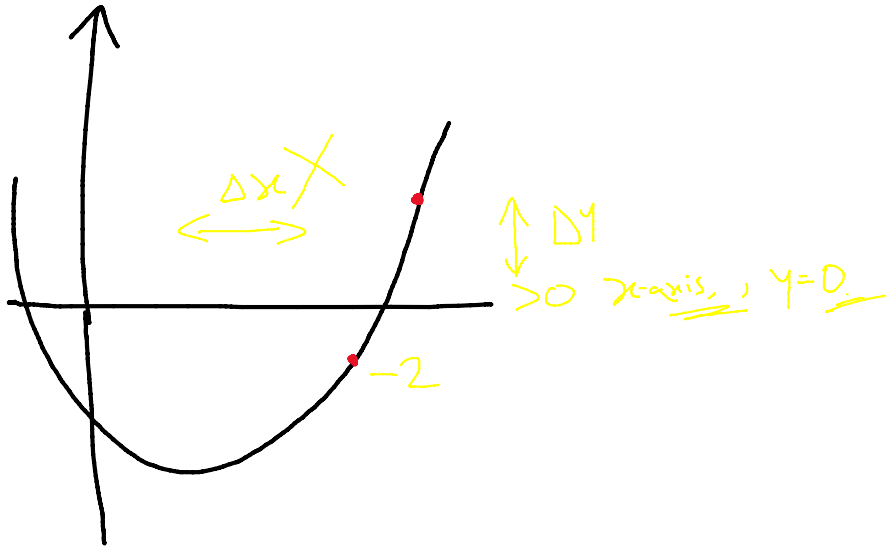
False Position Method.

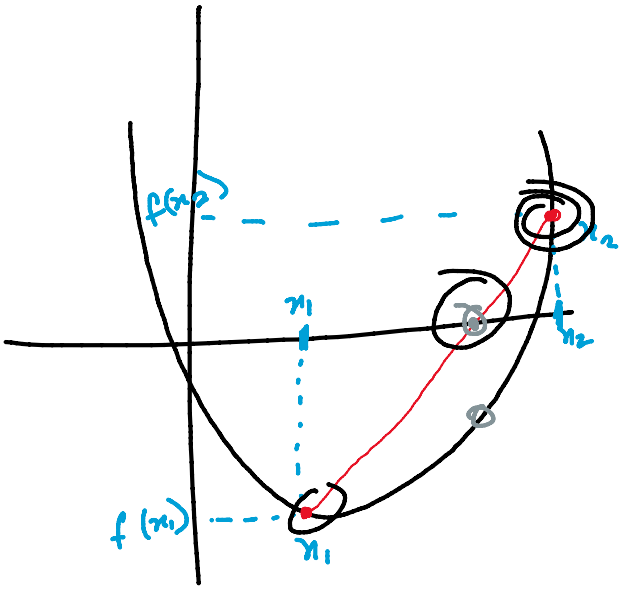
Monday, 13 June, 2022 07:22 PM

$$x_{i+1} = x_i^2 + 2x_i + 3$$

iterative solutions to finding roots numerically







$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{0 - f(x_2)}{x_1 - x_2} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{-f(x_2)}{x_1 - x_2} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

x_i	x_{i+1}

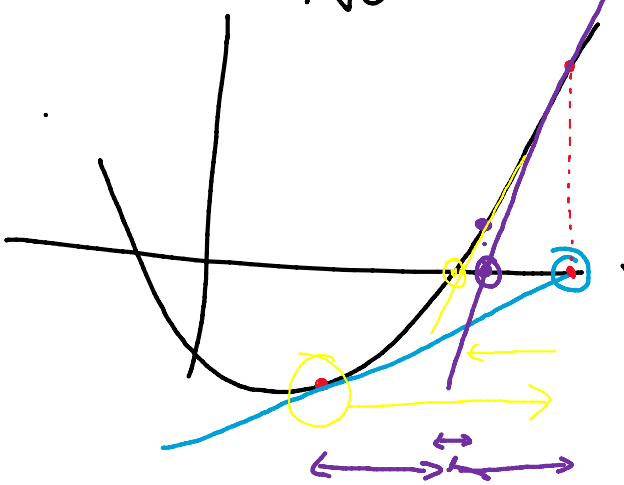
$x_i - x_2$ $\quad x_2 \to x_1$

$$-f(x_2) \times \frac{x_2 - x_1}{f(x_2) - f(x_1)} = x_i - x_2$$

$$\boxed{x_{i+1} = x_2 - f(x_2) \times \frac{x_2 - x_1}{f(x_2) - f(x_1)}}$$

(4)

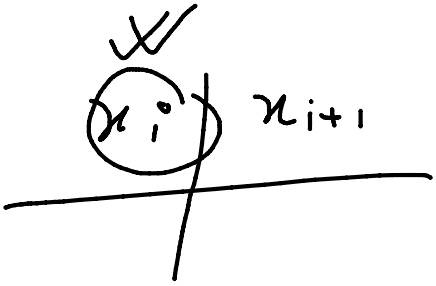
Newton Rhapson



→ Open

↳ starts from any point.

→ tangents of the curves.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$m = \frac{0 - f(x_1)}{x_2 - x_1}$$

$$m = \frac{-f(x_1)}{x_2 - x_1}$$

$f(x)$

$$f'(x_1) = \frac{-f(x_1)}{x_2 - x_1} \qquad m = f'(x)$$

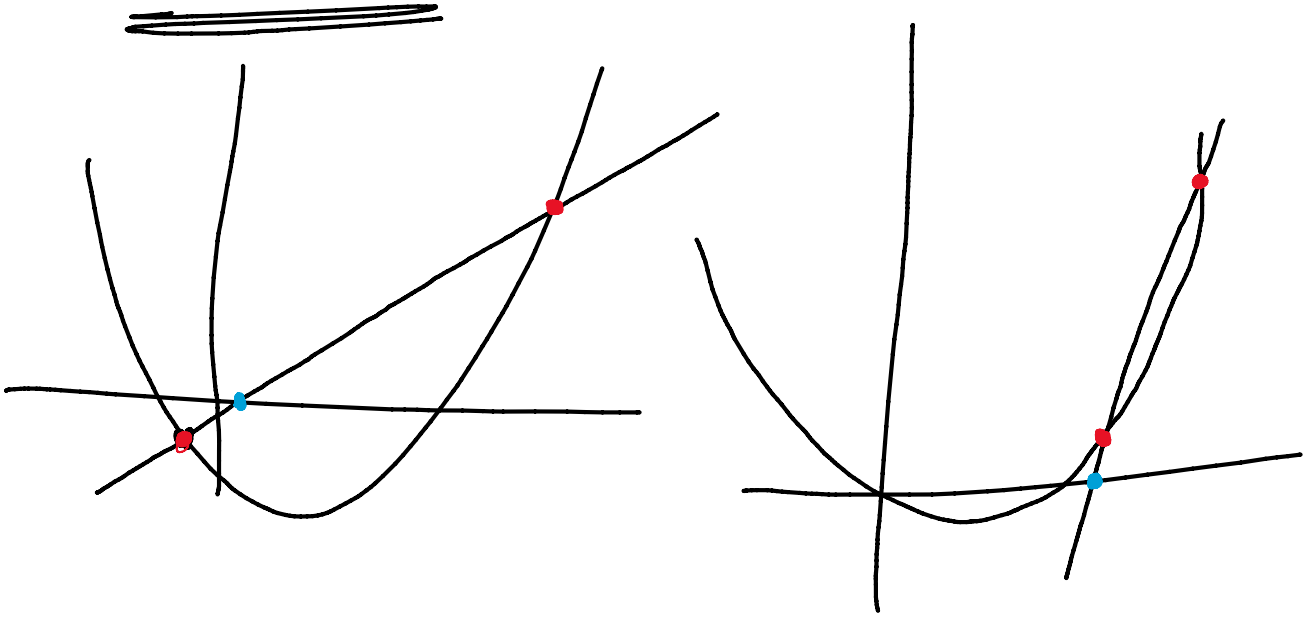
$$\Rightarrow x_2 - x_1 = \frac{-f(x_1)}{f'(x_1)}$$

$$\Rightarrow \boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

$\frac{df(x)}{dx}$

0.0017
0.002 } exactly same (5)

Secant



$$\boxed{x_{i+1} = x_2 - f(x_2) \times \frac{x_2 - x_1}{f(x_2) - f(x_1)}}$$

Gaussian Elimination.

$$a_1 x_1^3 + b_1 x_1^2 + c_1 x_1 = d_1$$
$$a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 = d_2$$
$$a_3 x_3^3 + b_3 x_3^2 + c_3 x_3 = d_3$$

$$\left[\begin{array}{ccc|c} x^3 & x^2 & x & \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & x & y & a \\ 0 & 1 & z & b \\ 0 & 0 & 1 & c \end{array}\right]$$

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \left[\begin{array}{ccc|c} 3 & 1 & -1 & 10 \\ 0 & 1 & 6 & 7 \\ 1 & 0 & -6 & 60 \end{array}\right]$$

$\frac{1}{3} r_1 \Rightarrow r_1$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 3.33 \\ 0 & 1 & 6 & 7 \\ & & & \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 6 & 7 \\ 1 & 0 & -6 & -60 \end{array}\right]$$

$r_3 - r_1 \Rightarrow r_3$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 3.33 \\ 0 & 1 & 6 & 7 \\ 0 & -1/3 & -19/3 & 56.66 \end{array}\right]$$

$r_3 + r_2 \times \frac{1}{3} \Rightarrow r_3$

$$\left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 7.33 \\ 0 & 1 & 6 & 7 \\ 0 & 0 & -13/3 & 58.999 \end{array}\right]$$

$r_3 \times -\frac{3}{13} \Rightarrow r_3$

$+\frac{3}{13} \times \frac{13}{5} = 1$

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}\left[\begin{array}{ccc|c} 1 & 1/3 & -1/3 & 7.33 \\ 0 & 1 & 6 & 7 \\ 0 & 0 & 1 & -13.61 \end{array}\right]$$

$$\begin{matrix} 1 \\ 0\,1 \\ 0\,0\,1 \end{matrix}$$

$z = -13.61$

$y + 6z = 7$

$\Rightarrow y = 7 - z(-13.61)$

$y = y_m$

$\Rightarrow x + \frac{y}{3} - \frac{z}{3} = 7.33$

$\Rightarrow x + \frac{y_m}{3} - \frac{z_m}{3} = 7.33$

$$\Rightarrow x + \frac{y_m}{3} - \frac{z_m}{3} = 7.33$$

$$\Rightarrow x = x_m \text{ V (Ans).}$$