Linear Regression Assignment 4

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1. Chapter 3, problem 3 (a, c, d, e, f)

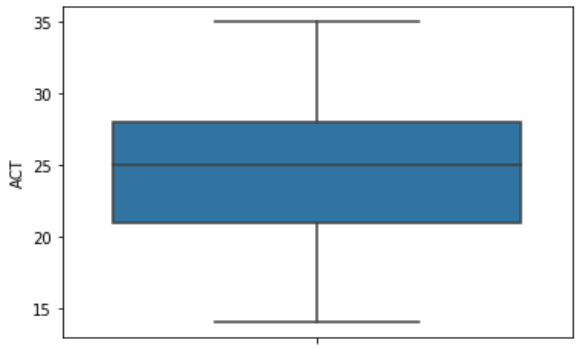
(a) The code is showing below

```
import pandas as pd
import numpy as np
from sklearn import linear_model
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as stats
%matplotlib inline
```

```
# data
data = pd.read_table('CH01PR19.txt', sep = ' ', header = None).dropna(axis = 1)
data.columns = ['GPA', 'ACT']
```

```
# (a) boxplot
sns.boxplot(data['ACT'], orient = 'v')
print("The mean of ACT is around 25 and the dataset don't have much outliers.")
print("The distriubtion of ACT score is almost like normal distribution")
```

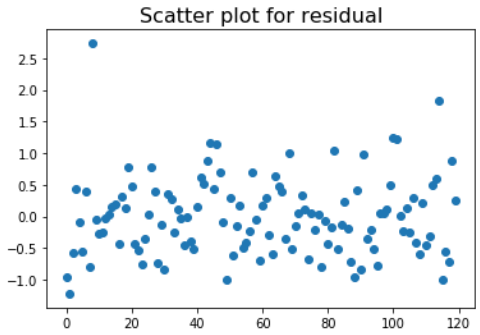
```
The mean of ACT is around 25 and the dataset don't have much outliers.
The distriubtion of ACT score is almost like normal distribution
```



(b)

```
# (b) dot plot of the residuals
LReg33 = LinearReg(data = data)
beta_ols = LReg33.sklearn_ols(x = 'ACT', y = 'GPA')
intercept = beta_ols[0][0]
slope = beta_ols[1][0]
f33 = lambda x: slope*x + intercept
plt.scatter(data.index, (data['ACT'].map(f33) - data['GPA']))
plt.title('Scatter plot for residual', fontsize = 16)
print('The residual have equal variance')
```

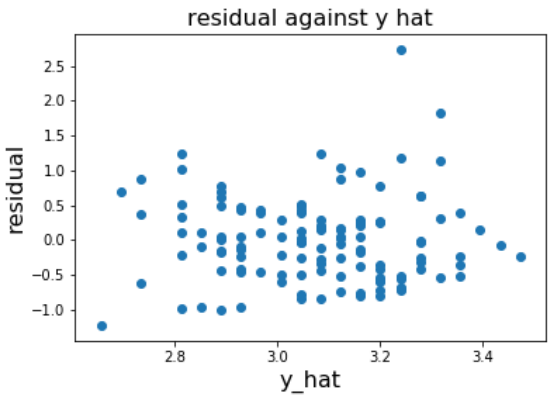
```
The residual have equal variance
```



(c)

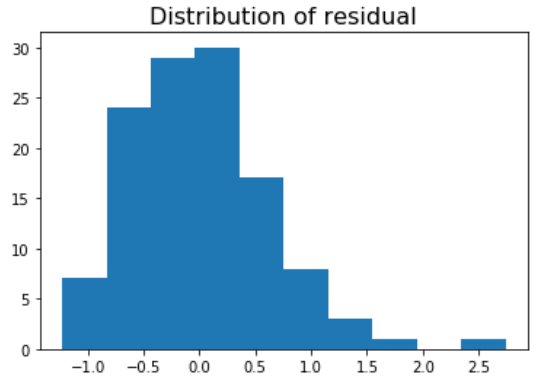
```
# (c)
plt.scatter(data['ACT'].map(f33), (data['ACT'].map(f33) - data['GPA']))
plt.xlabel('y_hat', fontsize = 16)
plt.ylabel('residual', fontsize = 16)
plt.title('residual against y hat', fontsize = 16)
```

```
Text(0.5,1,'residual against y hat')
```



```
plt.hist(data['ACT'].map(f33) - data['GPA'])
plt.title('Distribution of residual', fontsize = 16)
print('the distribution of residual is positively skewed distribution')
```

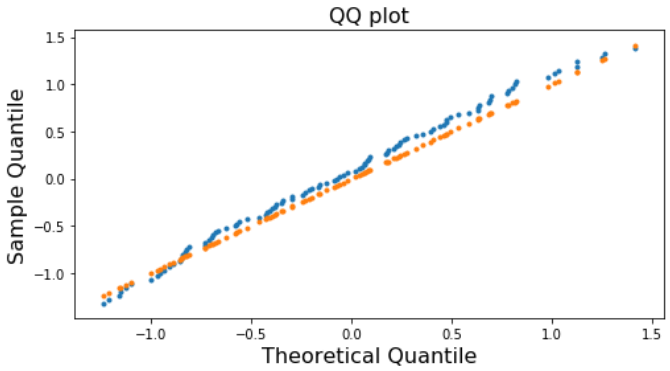
```
the distribution of residual is positively skewed distribution
```



(d)

```
# (d)
def qq_plot(data):
    # calculate Q-Q
    S_mean = np.mean(data)
    S_std = np.std(data)
    S_return = pd.DataFrame(data)
    S_return_norm = (S_return - S_mean)*1.0/S_std
    S_return_norm.columns = ['return_norm']
    S_return_norm_sort = S_return_norm.sort_values(by = 'return_norm')
    S_return_norm_sort.index = range(len(S_return_norm_sort))
    S_return_norm_sort['percentage'] = [(i+1)*1.0/len(S_return_norm_sort) for i in range(len(S_return_norm_sort))]
    S_return_norm_sort['norm'] = S_return_norm_sort['percentage'].map(stats.norm(0,1).ppf)
    x = S_return_norm_sort.iloc[10:-10]['return_norm']
    y = S_return_norm_sort.iloc[10:-10]['norm']
    # plot
    plt.figure(figsize=(8, 4))
    plt.scatter(x, y, marker = ".")
    plt.scatter(x, x, marker = ".")
    plt.xlabel('Theoretical Quantile', fontsize = 16)
    plt.ylabel('Sample Quantile', fontsize = 16)
    plt.title('QQ plot', fontsize = 16)
```

```
qq_plot(data['ACT'].map(f33) - data['GPA'])
```




```
residual = sorted(data['ACT'].map(f33) - data['GPA'])
standard_residual = (residual - np.mean(residual))/np.std(residual)
norm_res = sorted(np.random.normal(0, 1, len(residual)))
print('The correlation coefficient is ', round(np.corrcoef(standard_residual, norm_res)[0][1], 4))
print('Since n = 120, alpha = 0.05, from Table B.6, the critical value is 0.987 >= 0.978 . We can conclude that the distribution of erro terms
```

```
The correlation coefficient is  0.98
Since n = 120, alpha = 0.05, from Table B.6, the critical value is 0.987 >= 0.978 . We can conclude that the distribution of erro terms depa
rts from a normal distribution.
```

(e)

After calculating, n1 = 65, the mean of d1 = 0.438, n2 = 55, the mean of d2 = 0.5065

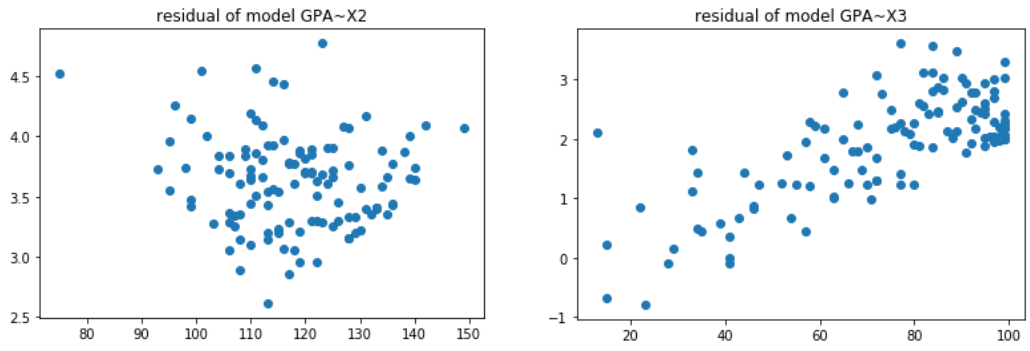
$$s^2 = \frac{\sum(d_{i1} - \overline{d_1})^2 + \sum(d_{i2} - \overline{d_2})^2}{n - 2} = 0.1741$$

$$t^*_{BF} = \frac{\overline{d_1} - \overline{d_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.438 - 0.5065}{0.4173\sqrt{\frac{1}{65} + \frac{1}{65}}} = -0.8967$$

$|t^*_{BF}| = 0.8967 < t(0.995, 118) = 2.618$, thus the error variance is constant and does not vary with the level of X.

(f)

```
# (f)
CH03PR03 = pd.read_excel('CH03PR03.xlsx', header = None)
CH03PR03.columns = ['GPA', 'ACT', 'X2', 'X3']
CH03 = LinearReg(CH03PR03)
# X2
intercept1 = CH03.sklearn_ols(x = 'X2', y = 'GPA')[0][0]
slope1 = CH03.sklearn_ols(x = 'X2', y = 'GPA')[1][0]
f1 = lambda x: x*slope + intercept
# X3
intercept2 = CH03.sklearn_ols(x = 'X3', y = 'GPA')[0][0]
slope2 = CH03.sklearn_ols(x = 'X3', y = 'GPA')[1][0]
f2 = lambda x: x*slope + intercept
# residual
res1 = f1(CH03PR03['X2']) - CH03PR03['GPA']
res2 = f2(CH03PR03['X3']) - CH03PR03['GPA']
# plot
plt.scatter(CH03PR03['X2'], res1)
plt.title('residual of model GPA~X2')
plt.show()
plt.scatter(CH03PR03['X3'], res2)
plt.title('residual of model GPA~X3')
```



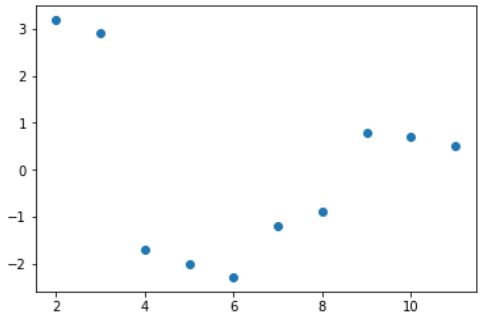
The residual of linear model GPA~X2 has equal variance. The residual of linear model GPA~X3 are increasing with X3. Thus, we can add intelligence test score (X2) to the original model.

2. Chapter 3 problem 9

```
CH03PR09 = pd.read_table('CH03PR09.txt', header = None, sep = ' ').replace(np.nan, 0)
CH03PR09['x'] = CH03PR09[2] + CH03PR09[3]
CH03PR09['residual'] = CH03PR09[5] + CH03PR09[6]
CH03PR09 = CH03PR09[['x', 'residual']]
```

```
plt.scatter(CH03PR09['x'], CH03PR09['residual'])
```

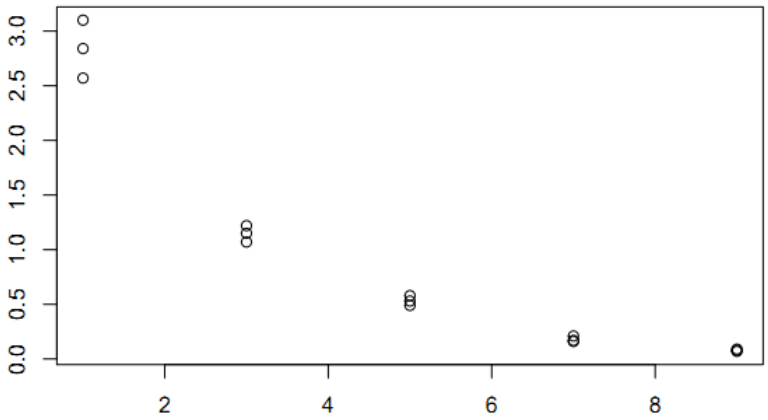
```
<matplotlib.collections.PathCollection at 0xdd2b9b0>
```



The variance of residual is big when x is small and small when x is big. We might use sqrt transformation to x to alleviate this problem.

3. Chapter 3 problem 16

(a)



Based on the plot, Y and e^{-x} seems has correlated relationship, so we might apply log transformation to y.

(b)

$$W_i = \begin{cases} K_1\left(Y_i^{\lambda} - 1\right), & \lambda \neq 0 \\ K_2(log_e Y_i), & \lambda = 0 \end{cases}$$

Where:

$$K_2 = \left(\prod_{i=1}^{n} Y_i\right)^{1/n}$$

$$K_1 = \frac{1}{\lambda {K_2}^{\lambda-1}}$$

```
def w(lambda_, predictor):
    Y = Ch3Pr15['Y']
    n = length(Y)
    K2 = (prod(Y))**(1/n)
    K1 = 1/(lambda_*K2^(lambda_ - 1))
    return(ifelse(lambda_==0, K2*log(predictor), K1*Predictor**lambda_-1))
lam = [-0.2, -0.1, 0, 0.1, 0.2]
sse = []
for i in range(1, 6):
    YY = []
    for j in range(1, 16):
        YY = [YY, w(lam[i], Y[j])]
    L = lm(YY~X)
    SSE = [SSE, sum(L['residual'])**2]
print('The best lambda is ', lam[sorted(SSE)[1]])
```

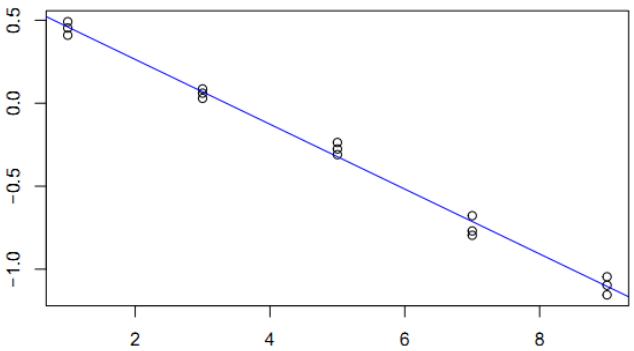
```
The best lambda is  0
```

(c)

Run the regression, we could get:

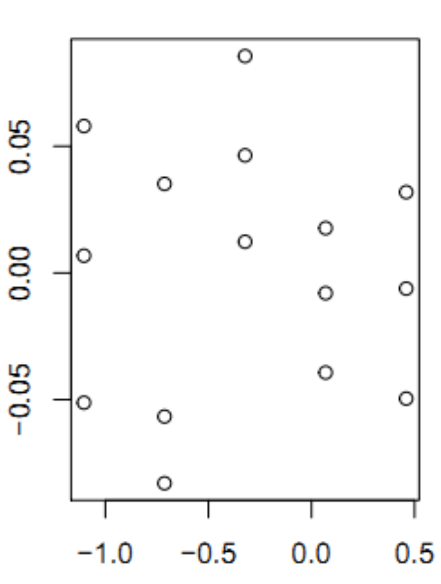
$$y = -0.1954x + 0.6549$$

(d)



Based on the plot, the regression result seems good.

(e)



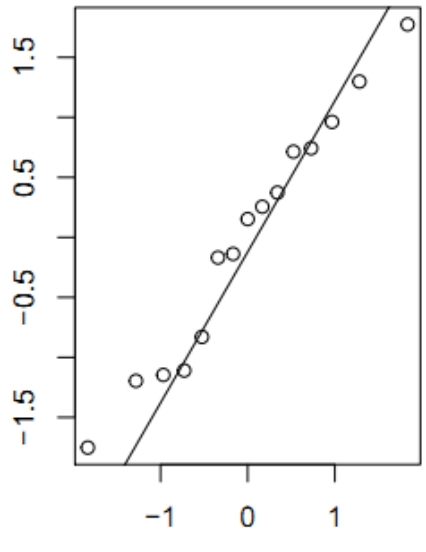


Figure 1: Residuals against fitted values **Figure 2: Q-Q plot**

Based on the plot, we could see the residuals are normally distributed.

(f)

$$\mathrm{Y} = 4.519 \times 10^{-0.195\mathrm{X}}$$

4. Chapter 3 problem 23

Set $X_1, X_2, \ldots, X_n$ as different levels of X. On each level, there are $\mathrm{n_j}$ observations. Therefore, $n = \sum_{i=1}^{c} n_j$. The observed value of the response variable for the ith jth level of X by Y_{ij}.

Full model:

$$Y_{ij} = \mu_j + \varepsilon_{ij}$$

Where:

μ_j are parameters $j = 1, \ldots, c$

ε_{ij} are independent $N(0, \sigma^2)$

$$\mathrm{SSPE} = \sum_j \sum_i (Y_{ij} - \overline{Y_j})^2$$

$$\mathrm{d_F} = \mathrm{n} - \mathrm{c}$$

Reduced model:

$$Y_{ij} = \beta_1 X_j + \varepsilon_{ij}$$

$$\mathrm{SSE} = \sum_j \sum_i (Y_{ij} - \widehat{\beta_1} X_j)^2$$

$$\mathrm{d_{f_R}} = \mathrm{n} - 2$$

Test Statistics:

$$F^* = \frac{SSE - SSPE}{c-2} / \frac{SSPE}{n-c} \sim F(c-2, n-c)$$

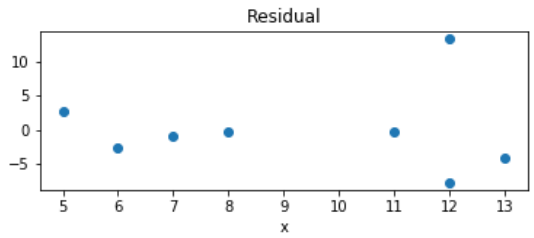
When n = 20, c = 10, d_F = 10, $d_{f_R} = 18$. $F^* \sim F(8, 10)$

5. Chapter 3 problem 24

(a)

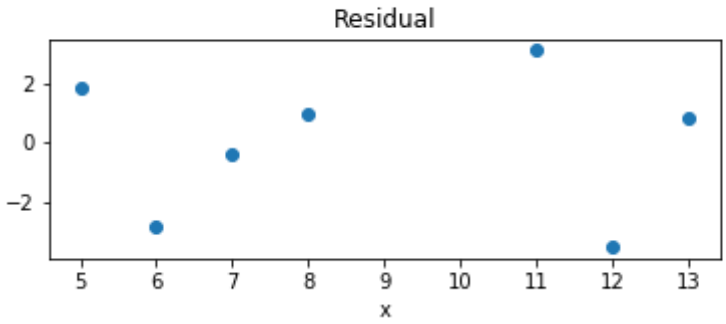
```
# (a)
x = [5, 8, 11, 7, 13, 12, 12, 6]
y = [63, 67, 74, 64, 75, 69, 90, 60]
data324 = pd.DataFrame({'x': x, 'y': y})
Lm34 = LinearReg(data324)
coefs = Lm34.sklearn_ols(x = 'x', y = 'y')
intercept = coefs[0][0]
slope = coefs[1][0]
fun324 = lambda x: slope*x + intercept
plt.figure(figsize = (6, 2))
plt.scatter(data324['x'], data324['y'] - data324['x'].map(fun324))
plt.title('Residual')
plt.xlabel('x')
print('y = {0}x + {1}'.format(round(slope,3), round(intercept, 3)))
```

```
y = 2.333x + 48.667
```



(b)

```
y = 1.621x + 53.068
```



The slope and intercept has changed a lot which means (12, 90) might be a outlier.

(c)

$$Y \pm t(1 - \frac{\alpha}{2}; n-2)\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum(X_i - \bar{X})^2}}$$

Where:

$$\widehat{Y_h} = \widehat{\beta_0} + \widehat{\beta_1}X = 49.8 + 2.16 \times 12 = 75.52$$

$$t(0.995; 5) = 4.0321$$

$$\hat{\sigma} = 2.5860$$

$$\sqrt{1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum(X_i - \bar{X})^2}} = 1.1695$$

Thus,

$$Y_h \in [63.5252, 87.9148]$$

Y_h is not in this interval which means it is an outlier.

6. Chapter 3 problem 31

Notice that in this question, we use the class coded in homework1 and qq_plot function coded in this homework.

```
# define our linear regression
class LinearReg:
    def __init__(self, data):
        '''data: type: pandas dataframe'''
        self.data = data
        self.length = len(data)
    def ols(self, x, y):
        '''
        x: column name
        y: column name
        '''
        X = np.matrix(np.vstack([np.ones(self.length), self.data[x].values]).T)
        y = np.matrix(self.data[y].values).T
        beta = np.linalg.inv(X.T*X)*X.T*y
        return beta
    def sklearn_ols(self, x, y):
        X = np.matrix(self.data[x].values).T
        y = np.matrix(self.data[y].values).T
        # Create linear regression object
        OLS = linear_model.LinearRegression()
        # Train the model using the training sets
        OLS = OLS.fit(X, y)
        y_hat = OLS.predict(X)
        res = y - y_hat
        return {'intercept': OLS.intercept_[0], 'coef': OLS.coef_[0][0]}, res
```

```
    def sklearn_ols(self, x, y):
        X = np.matrix(self.data[x].values).T
        y = np.matrix(self.data[y].values).T
        # Create linear regression object
        OLS = linear_model.LinearRegression()
        # Train the model using the training sets
        OLS = OLS.fit(X, y)
        y_hat = OLS.predict(X)
        res = y - y_hat
        return {'intercept': OLS.intercept_[0], 'coef': OLS.coef_[0][0]}, res
    def visual(self, x, y, step = 0.01):
        para = self.ols(x, y)
        X = self.data[x]
        Y = self.data[y]
        min_x, max_x = min(X), max(X)
        # x is also matrix
        func = lambda x: x*para
        x_sim = np.arange(min_x, max_x, step)
        xm = np.vstack([np.ones(len(x_sim)), x_sim]).T
        y_sim = func(xm)
        self.data.plot.scatter(x,y)
        plt.plot(x_sim, y_sim)
        plt.title('The Relationship Between {0} and {1}'.format(x, y))
```

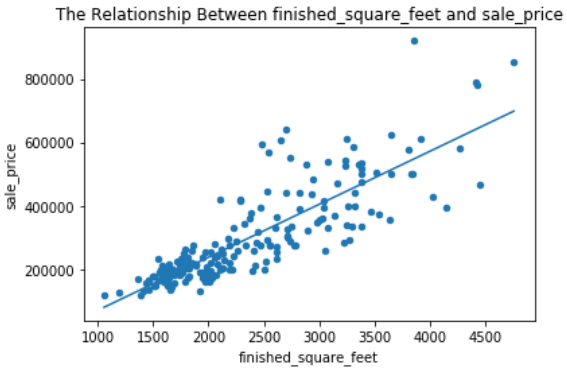
Clean the data and run the regression

```
APPENC07 = pd.read_excel('APPENC07.xlsx', header = None)
APPENC07.columns = ['id', 'sale_price', 'finished_square_feet', 'number_of_bedrooms', 'number_of_bathrooms',\
                    'air_condition', 'garage_size', 'pool', 'year_built', 'quality', 'style', 'lot_size', \
                    'adjacent_to_highway']
# random sample
APPENC07_sample = APPENC07.sample(200)
lm331 = LinearReg(APPENC07_sample)
coefs331 = lm331.sklearn_ols(x = 'finished_square_feet', y = 'sale_price')
coefs331[0]
```

```
{'intercept': -96783.12450020027, 'coef': 167.27152795772716}
```

Plot the regression line

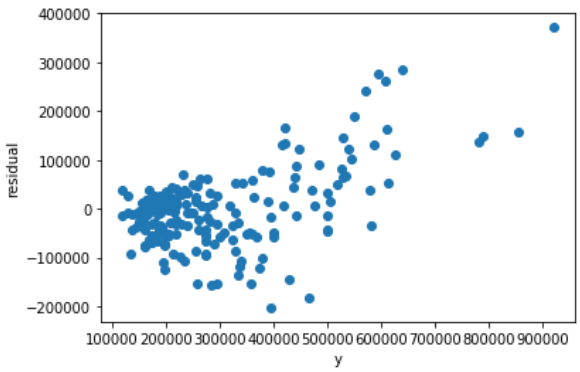
```
lm331.visual(x = 'finished_square_feet', y = 'sale_price')
```



Plot the residual

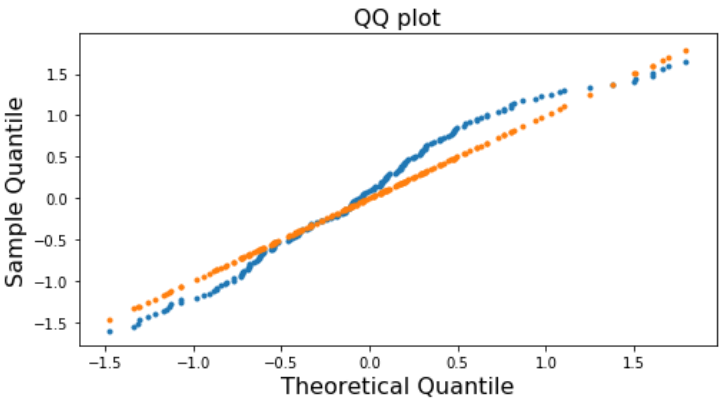
```
plt.scatter(APPENCO7_sample['sale_price'], list(coefs331[1]))
plt.xlabel('y')
plt.ylabel('residual')
```

```
Text(0,0.5,'residual')
```



QQ plot

```
qq_plot(coefs331[1])
```

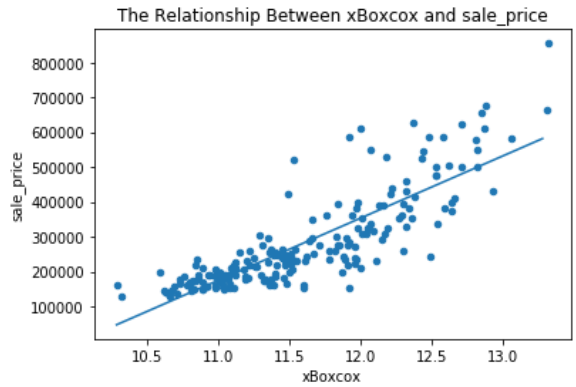


Box Cox transformation

```
from scipy import stats
APPENCO7_sample['xBoxcox'] = stats.boxcox(APPENCO7_sample['finished_square_feet'], 0.1)
lm331new = LinearReg(APPENCO7_sample)
coefs331new = lm331new.sklearn_ols(x = 'xBoxcox', y = 'sale_price')
coefs331new[0]
```

```
{'intercept': -1783825.5398023499, 'coef': 178079.94028817271}
```

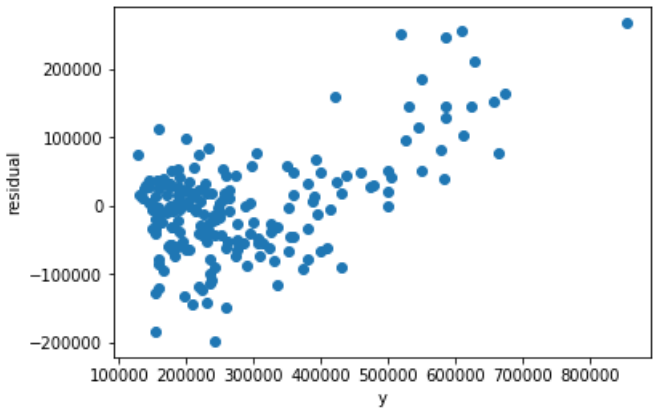
```
lm331new.visual(x = 'xBoxcox', y = 'sale_price', step = 0.1)
```



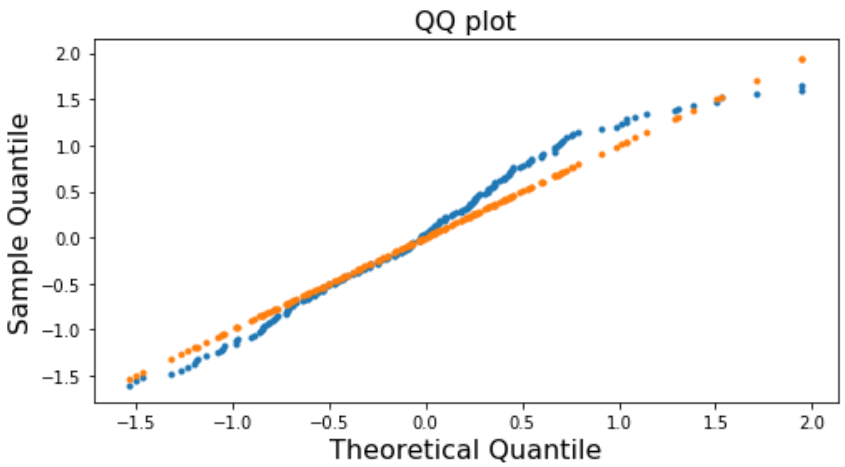
New residual plot

```
plt.scatter(APPENC07_sample['sale_price'], list(coefs331new[1]))
plt.xlabel('y')
plt.ylabel('residual')
```

```
Text(0,0.5,'residual')
```




```
qq_plot(coefs331new[1])
```




```
final331_coef = coefs331new[0]['intercept']
final331_slope = coefs331new[0]['coef']
final_fun331 = lambda x: final331_slope*x + final331_coef
# X = 1100
print('X = 1100: ', final_fun331(stats.boxcox(1100, 0.1)))
# X = 4900
print('X = 4900: ', final_fun331(stats.boxcox(4900, 0.1)))
```

```
X = 1100:  22564.14583428623
X = 4900:  600563.0655822346
```

Based on the analysis and plot, we could see that after box-cox transformation, the residual almost have equal variance and follow the normal distribution. The strength of the model is that it satisfies all the assumptions, but the weakness is that it not perfectly satisfies all and the parameters are not robust enough.

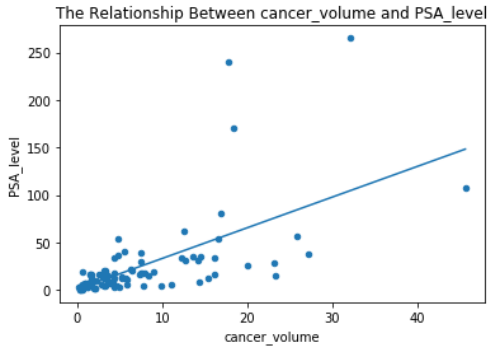
7. Chapter 3 problem 32

The techniques we use are almost the same as techniques we used in problem 31. The difference between those two problems is we remove the outliers in the dataset in this question.

```
APPENC05 = pd.read_excel('APPENC05.xlsx', header = None)
APPENC05.columns = ['id', 'PSA_level', 'cancer_volume', 'weight', \
                    'age', 'benign_prostatic', 'seminal_vesicle_invasion', \
                    'capsular_penetration', 'gleason_score']
lm332 = LinearReg(APPENC05)
coefs332 = lm332.sklearn_ols(x = 'cancer_volume', y = 'PSA_level')
coefs332[0]
```

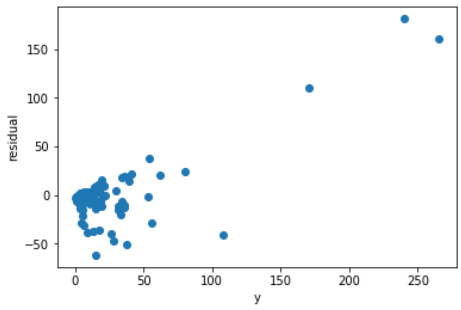
```
{'intercept': 1.1248504112185174, 'coef': 3.2299341615576593}
```

```
lm332.visual(x = 'cancer_volume', y = 'PSA_level')
```

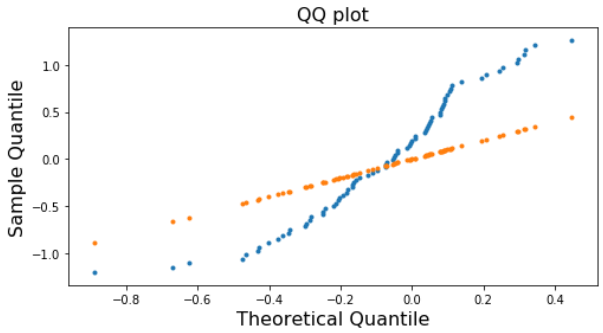



```
plt.scatter(APPENC05['PSA_level'], list(coefs332[1]))
plt.xlabel('y')
plt.ylabel('residual')
```

```
Text(0,0.5,'residual')
```



```
qq_plot(coefs332[1])
```

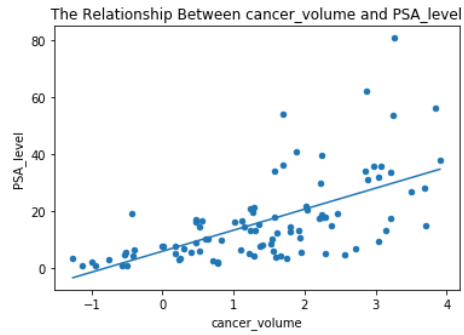



```
APPENC05new = APPENC05[['cancer_volume', 'PSA_level']][APPENC05['PSA_level'] < 100]
APPENC05new['cancer_volume'] = stats.boxcox(APPENC05new['cancer_volume'], 0.1)
```

```
lm332 = LinearReg(APPENC05new)
coefs332 = lm332.sklearn_ols(x = 'cancer_volume', y = 'PSA_level')
coefs332[0]
```

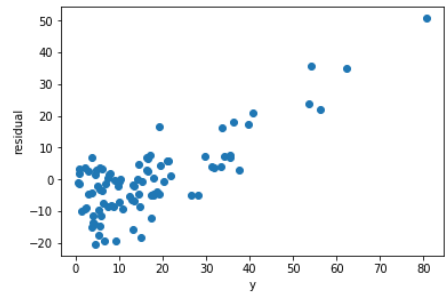
```
{'intercept': 5.849799111724051, 'coef': 7.368909235407426}
```

```
lm332.visual(x = 'cancer_volume', y = 'PSA_level')
```

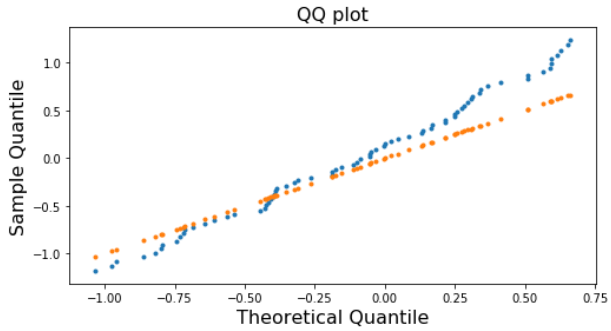



```
plt.scatter(APPENC05new['PSA_level'], list(coefs332[1]))
plt.xlabel('y')
plt.ylabel('residual')
```

```
Text(0,0.5,'residual')
```



```
qq_plot(coefs332[1])
```




```
final332_coef = coefs332[0]['intercept']
final332_slope = coefs332[0]['coef']
final_fun332 = lambda x: final332_slope*x + final332_coef
# X = 20
print('X = 20: ', final_fun332(stats.boxcox(20, 0.1)))
```

```
X = 20:  31.588135131635312
```

Based on the analysis and plot, we could see that after removing outliers and box-cox transformation, the residuals almost have equal variance and follow the normal distribution. The strength of the model is that it satisfies all the assumptions and the parameters are robust, but the weakness is that the residuals seem not follow normal distribution when x above 2.