

GR5203: PROBABILITY
Autumn 2018
Homework 3

Homework is not for submission but for study for final.

Read Sections 3.7-3.9 and 4.1-4.7 of DeGroot and Schervish.

You may find solutions to some problems in various sources, including solution manual for the textbook. I encourage you to solve the problems yourself rather than obtain solutions, homework is your most valuable tool in studying and preparing for the exams. As I mentioned previously, you may work in groups but do write up on your own and mention the people you collaborated with (no points will be taken off for collaboration as long as there is no evidence of copying).

1. Let U_1 and U_2 be independent and uniform on $[0, 1]$. Find the density of $S = U_1 + U_2$.
2. Let X have the density

$$f(x) = \frac{1 + \alpha x}{2}, \quad -1 \leq x \leq 1, \quad -1 \leq \alpha \leq 1.$$

Find $E(X)$ and $\text{Var}(X)$.

3. If U_1, \dots, U_n are independent uniform random variables, find $E(U_{(n)} - U_{(1)})$.
4. A random square has a side length that is uniform $[0, 1]$ random variable. Find the expected area of the square.
5. Find $E[1/(X + 1)]$, where X is a Poisson random variable with mean λ .
6. Let X and Y have the joint pdf

$$f(x, y) = e^{-y}, \quad 0 \leq x \leq y.$$

- (a) Find $\text{Cov}(X, Y)$ and the correlation of X and Y .
 - (b) Find $E(X|Y = y)$ and $E(Y|X = x)$.
 - (c) Find the density functions of the random variables $E(X|Y)$ and $E(Y|X)$.
7. If X_1, X_2, X_3, X_4 are (pairwise) uncorrelated random variables each with mean 0 and variance 1, compute the correlations of X
 - (a) $X_1 + 2X_2$ and $X_2 + X_3 + 1$.
 - (b) $X_1 + X_2 + 3$ and $5X_3 + X_4$.

8. A coin having the probability p of coming up heads is flipped until both heads and tails have appeared. Find
 - (a) the expected number of flips;
 - (b) the probability that the last flip lands heads.
9. An urn contains 30 balls, of which 10 are red and 8 are blue. From this urn, 12 balls are drawn without replacement. Let X be the number of red and Y be the number of blue balls that are drawn. Find $\text{Cov}(X, Y)$.
10. DeGroot & Schervish, #8 on page 167.
11. DeGroot & Schervish, #3 on page 174.
12. DeGroot & Schervish, #4 on page 187.
13. DeGroot & Schervish, #10 on page 240.
14. DeGroot & Schervish, #11 on page 240.
15. DeGroot & Schervish, #5 on page 247.
16. DeGroot & Schervish, #12 on page 255.