Probability HW1. 522769

1. (a) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

(b) $A = \{HHH, HHT, HTH, THH\}$

$B = \{HHH, HHT\}$

$C = \{HHT, HTT, THT, TTT\}$.

(c) $A^c = \{HTT, THT, TTH, TTT\}$.

$A \cap B = \{HHH, HHT\}$

$A \cup C = \{HHH, HHT, HTH, THH, HTT, THT, TTT\}$.

2. (a) $P(E) = \dfrac{4}{\binom{52}{5}}$

(b) $P(E) = \dfrac{4 \times 9}{\binom{52}{5}} = \dfrac{36}{\binom{52}{5}}$

(c) $P(E) = \dfrac{13 \times 48}{\binom{52}{5}}$

(d) $P(E) = \dfrac{\left[\binom{13}{5} - 10\right] \times 4}{\binom{52}{5}}$

(e) $P(E) = \dfrac{\binom{4}{3}\binom{48}{1}\binom{44}{1} \times 13}{2 \cdot \binom{52}{5}}$

(f) $P(E) = \dfrac{\binom{4}{2}\binom{4}{2}\binom{44}{1} \cdot \binom{13}{2}}{\binom{52}{5}}$

3. (a) $P(E) = \dfrac{16 \times 47}{48 \times 47} = \dfrac{1}{3}$, $P(F) = \dfrac{32 \times 47}{48 \times 47} = \dfrac{2}{3}$

$P(G) = \dfrac{16 \times 15 + 32 \times 31}{48 \times 47} = \dfrac{77}{141}$

(b) $P(E \cap F) = \dfrac{16 \times 32}{48 \times 47} = \dfrac{32}{141}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \dfrac{1}{3} + \dfrac{2}{3} - \dfrac{32}{141} = \dfrac{109}{141}$

$P(E \cap F \cap G) = 0$

$P(G \mid E \cup F) = \dfrac{16 \times 15 + 32 \times 31}{48 \times 47 \times \frac{109}{141}} = \dfrac{77}{109}$

4. $P(E) = \dfrac{P_4 \times P_{48} \times 49}{P_{52}}$

5. (a) $P(E) = \dfrac{\binom{55}{25} + \binom{55}{25}}{\binom{60}{30}}$.

(b) $P(E) = \dfrac{\binom{55}{26}\binom{5}{4} \times 2}{\binom{60}{30}}$

(c) $P(E) = \dfrac{\binom{55}{26} \times 2}{\binom{60}{30}}$

6.(a) $E = \{\uparrow\downarrow, \downarrow\uparrow\}$

$P(E) = 2p(1-p) = 2p - 2p^2$

(b) $E = \{\uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \downarrow\uparrow\uparrow\}$.

$P(E) = 3 \cdot p \cdot p \cdot (1-p) = 3p^2 - 3p^3$

(c) $F = \{\uparrow \cancel{\downarrow} \cancel{\downarrow}\}$.

$P(F|E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{2p^2 - 2p^3}{3p^2 - 3p^3} = \dfrac{2}{3}$

7. $P(a) = p$

$P(b) = p^2 + \frac{1}{2}p(1-p) \cdot 2 = p^2 + p - p^2 = p = P(a)$.

so, both strategies come out the same probability of correct answer.

8. E = correct F = agree. G = disagree

$P(E|F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{p^2}{p^2 + (1-p)^2} = \dfrac{0.36}{0.36 + 0.16} = \dfrac{9}{13}$

$P(E|G) = \dfrac{P(E \cap G)}{P(G)} = \dfrac{\frac{1}{2}(1-p)p \cdot 2}{(1-p) \cdot p \cdot 2} = \dfrac{1}{2}$

9. E = at least 1 head

E^c = no head

$P(E) = 1 - P(E^c) = 1 - (1-p)^n \geq \frac{1}{2}$

$(1-p)^n \leq \frac{1}{2}$ $\quad n \geq \log_{(1-p)} \frac{1}{2}$

10. E = one drawer is a silver coin

$= \{SS, SG\}$ ~~GS~~

F = the other drawer is a silver coin

$$P(F|E) = \frac{P(F\cap E)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{2}{3}$$

11. (a) E = red into B

F = blue into B

G = green into B

H = red from B

$$p(H) = P(H|E) + P(H|F) + P(H|G)$$

$$= \frac{4}{9}\cdot\frac{3}{10} + \frac{3}{9}\cdot\frac{2}{10} + \frac{2}{9}\cdot\frac{2}{10}$$

$$= \frac{11}{45}$$

(b) F = red from B

E = red from A.

$$P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{4}{9}\times\frac{3}{10}}{\frac{11}{45}} = \frac{6}{11}$$

P21 #7 $\max(P(A\cap B)) = 0.4$ when $A \subset B$

$\min(P(A\cap B)) = 0.1$ when $A \cup B = S$

P25 #3

first dice	second dice.
1	1.2.3
2	1.2.3.4
3	1.2.3.4.5
4	2.3.4.5.6
5	3.4.5.6
6	4.5.6

$$P(E) = \frac{3+4+5+5+4+3}{36} = \frac{2}{3}$$

P32 #7 $\quad P(E) = \dfrac{\binom{20}{12}\cdot 12!}{20^{12}}$

P41 #12 $\quad P(E) = \dfrac{\binom{33}{8}+\binom{33}{10}}{\binom{35}{10}}$

P46 #8 $\quad P(E) = \dfrac{\binom{40}{10,10,10,10}\binom{12}{3,3,3,3}}{\binom{52}{13,13,13,13}}$

P50 #6 $\quad P(E) = \dfrac{\binom{60}{10}\times 3 - \binom{30}{10}\times 3}{\binom{90}{10}}$