

A: programmingteam, 2 + 4
B: flightplans, 7 + 17
C: entirelyunsortedsequences, 3 + 5
D: primalrepresentation, 12 + 19
E: disastrousdoubling, 11 + 19
F: eatingout, 14 + 19
G: babelfish, 14 + 20
H: brickwall, 8 + 14
I: wheretolive, 11 + 19
J: coupons, 1 + 0
K: pipes, 1 + 1
L: whiteboard, 2 + 3
M: rainfall, 1 + 3

Going through problems in descending order of solve counts:

G: babelfish, 14 + 20

Implement map

F: eatingout, 14 + 19

The menu has m items, and Alice, Bob, and Clara will order a , b , and c items respectively. Is it possible for them to pick some items such that no item is picked by everyone?

Check the 'bad' conditions: if any $\geq m$, or if $a + b + c > 2m$
Think of the people not choose something

D: primalrepresentation, 12 + 19

Do prime factorization for things up to 2^{31} , take care of negatives too (by adding a -1)

How many primes are there up to 2^{16} : 7000 of them, somehow this passed.
Apparently even brute force check passed.

(need to mentally check: are these jerks asking me to code pollard rho....), this is a problem where you can wait, and let the scoreboard tell you whether you need pollard rho

I: wheretolive, 11 + 19

100 instances of minimize the point with total squared distance to everyone, and then round that point to an integer point

“If the best location is not exactly on a grid point, choose the grid point closest to the best location”

The dimensions double: this is 2 1-D problems.

You can solve each dimension separately, it is just minimizing a quadratic (turns out to be average of the points)

E: disastrousdoubling, 11 + 19

Simulate the process of

For $i = 1 \dots n$

$x[0] = 1$

$x[i + 1] = x[i] * 2 - b[i]$

Output whether $x[]$ ever gets negative, and $x[n] \% \text{something}$

KEY: $b[i]$ can have 60 bits, and n has 20 bits.

Difficulty of this problem is to fight the precision issue. This is scary because the max one can store in a 64-bit signed int is $2^{63} - 1$.

Key observation is once you have more than 2^{61} bacteria, you will just not run out...

H: brickwall, 8 + 14

Given bricks of length 1, 2, 3, of up to 300 each

Check if one can arrange a subset of them (with a specified total) so that no prefix sum is in a forbidden subset.

Do dynamic program: state = # of bricks of size 1, 2, 3 that have been used, and just check if the sum is in the ‘bad’ set.

Naive: precompute lookup table for ‘bad’, then fill table, $O(300^3)$ time, $O(300^3)$ memory

??? (was not needed) Can get memory down to $O(300^2)$ via lookup tables

??? ?????????? can probably also push $O(300^3 / 64)$ using bitset.

B: [flightplans](#), 7 + 17

Interesting phenomenon: in afternoon session (<https://naptc21.kattis.com/sessions/ta8g26>), B got solved by way more teams because it was solved early (26 vs. 59 (after 4 wrongs) in morning session, <https://naptc21.kattis.com/sessions/sbceig>). Usually you want to check, ~ 180ish, whether any of the 'hard' problems are actually hard... because if they are easy, you can still solve them as if they are easy.

Do shortest path on a unit weighted graph where vertices' neighbors are listed either as neighbors list, or complement(neighbors).

Simulate BFS:

- * neighbors: remove all neighbors from unvisited vertices.

- * complement(neighbors): unvisited becomes unvisited intersect complement(neighbors), and add unvisited \ complement(neighbors) into the queue.

C: [entirelyunsortedsequences](#), 3 + 5

Given n ($n \leq 5000$) numbers, count the # of permutations where every number has either: a predecessor that's more, or a successor that's less.

(some thoughts by Richard) Assume all distinct:

Think about what happens to the max:

Say n is placed at location i , then locations $i + 1 \dots n$ can be w/e

DP state: $DP[x][k]$: I'm placing $1 \dots x$, considering x , & there are k 'free' slots

Transition:

$[x][k] \rightarrow [x - 1][k - 1]$

$\rightarrow [x - 1][k + w]$ (for all w , as long as you don't run out of free slots)

For each i , count the # of sequences where i is the first sorted elements

Any sorted element is equal to its index (up to some ties)

$ans[i] = ans[i - 1] * (\# \text{ of possible arrangements after } i)$.

Use multinomial coefficients to count # of arrangements after i (to handle duplicate items)

A: [programmingteam](#), 2 + 4

Given tree on n vertices, pick a connected chunk including root with size k that maximizes (sum of $a[i]$) / (sum of $b[i]$).

$n, k \leq 2500$, $a[i], b[i]$ positive.

1. Turn it into single value via binary search, aka. Search if i can make $\sum_{\{i \text{ picked}\}} a[i] - \lambda * b[i]$ positive.

So you create $c[i] \leftarrow a[i] - \lambda * b[i]$, and check if there is a set of k vertices where $c[i]$ sums to ≥ 0 .

2. Min convolution on a tree ($DP[u][x] = \max_{\{y_1\}} DP[v][y_1] + DP[w][y_2]$ for v & w children of u) takes $O(n^2)$ time (if $y_1 \leq \text{size}[v]$, $y_2 \leq \text{size}[w]$).

Rather messy proof of this is in Lemma 5.5. of https://www.cc.gatech.edu/~rpeng/CS4540_F20/Sep14TreeDP.pdf

More tree convolution DP that looks like $O(N^3)$ but is actually $O(N^2)$

https://atcoder.jp/contests/abc207/tasks/abc207_f

<https://open.kattis.com/problems/kthsubtree>

<https://codeforces.com/problemset/problem/1280/D>

J: [coupons](#), 1 + 0

Can prove:

1st ticket bought goes as long as possible til it expires

Won't buy another ticket until this one expires.

Zac's code: <https://pastebin.com/zKsXVx0k>

Works because the # of distinct stations is so small (≤ 11). With more stations, we would need to use data structures.

(A counterexample was found!)

```
6
2 0
5 5000
2 5001
0 9999
2 10000
5 15000
```

The answer should be 9, (but this solution outputs 11):

Get ticket 1 at time 0 for zones $[0, 2]$ for \$4.

Get ticket 2 at time 5000 for zones $[2, 5]$ for \$5.

Get and use ticket 1 at time 0 to go from zone 0 to zone 2.

Get and use ticket 2 at time 5000 to go from zone 2 to zone 5 and (at time 5001) back to zone 2.

Use ticket 1 at time 9999 to go from zone 2 to zone 0 and (at time = 10000) to zone 2.

Use ticket 2 at time 15000 to go from zone 2 to zone 5.

L: [whiteboard](#), 2 + 3

M: [rainfall](#), 1 + 3

K: [pipes](#), 1 + 1