# CS 201 Homework week 4

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## Section 3.4

### Question 4

If the first two premises of universal transitivity are written as "Any $x$ that makes $P(x)$ true makes $Q(x)$ true"
and "Any $x$ that makes $Q(x)$ true makes $R(x)$true,"
Conclusion can be written as "Any $x$ that makes $P(x)$ true makes $R(x)$ true".

### Question 14

If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
$\therefore$ This program is correct.
Invalid: inverse error
JUSTIFIED:
If compilation of a computer program produces error messages, then the program is not correct.
This program is correct.
$\therefore$ Compilation of a computer program does not produce error messages.

### Question 19

Statement: No good cars are cheap
Rewrite: For all cars, if it is a good car, then it is not cheap.

#### a

Invalid. Inverse error

#### b

Valid

#### c

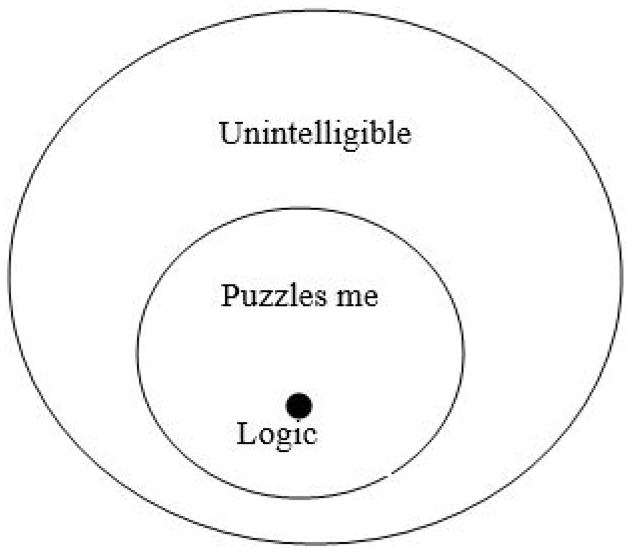
Invalid. Converse error

#### d

Valid

### Question 27

It can be rewritten as:
For all things, if that things is intelligible, then it does not puzzle me.
Logic puzzles me
$\therefore$ Logic is unintelligible.
It is valid (inverse form)




### Question 29

If an object is not above all the black objects, then it is not a square
All the objects that are above all the black objects are to the right of all the triangles
All the objects that are to the right of all the triangles are above all the circles.
$\therefore$ All the squares are above all the circles.

## Section 4.1

### Question 8

Prove this statement: There is a real number $x$ such that $x > 1$ and $2^x > x^{10}$.
Let $x \in \mathbb{Z}$ such that $x > 1, 2^x > x^{10}$. Assume that $x = 100$, then $2^{100} - 100^{10} > 0$, so $2^{100} > 100^{10}$.
Therefore, with $x = 100$, $x > 0$ and $2^x > x^{10}$.

### Question 16

Statement: The average of any two odd integers is odd.
Prove: let $x, y \in \mathbb{Z}$ are two odd integers. Assume that $x = 2k+1$ and $y = 2m+1$ with $k, m \in \mathbb{Z}$.
The average of these two numbers is: $(2k + 2m + 2)/2 = k + m + 1$.
Case 1: Both $k, m$ are odd. Let $k = 2t + 1; m = 2i + 1$. We have $k + m + 1 = 2t + 2i + 3 = 2(t + i + 1) + 1$, which is odd. Therefore, average of two odd integers is odd

Case 2: Both $k, m$ are even. Let $k = 2t; m = 2i$. We have $k + m + 1 = 2t + 2i + 1 = 2(t + i) + 1$, which is odd. Therefore, average of two odd integers is odd
Case 3: One of $k$ or $m$ is odd, the another is even. Let $k = 2t; m = 2i + 1$, we have $k + m + 1 = 2t + 2i + 2 = 2(t + i + 1)$, which is even. Therefore, average of these two is even. Conclusion, the statement is false

## Question 19

Theorem: "The sum of any even integer and any odd integer is odd."

### a

- $\forall x, y \in \mathbb{Z}$, If $x = 2k$ and $y = 2k + 1$ with $k \in \mathbb{Z}$, then $x + y = 4k + 1$ is an odd integer.
- $\forall x, y \in \mathbb{Z}$ such that $x = 2k$ and $y = 2k + 1$ with $k \in \mathbb{Z}$, $x + y = 4k + 1$ is not divisible by 2.
- If there is $x = 2k$ and $y = 2k + 1$ with $k \in \mathbb{Z}$, then $x + y = 4k + 1$ is an odd integer.

### b

- a: any odd integer
- b: integer $r$
- c: $2r + 2s + 1$
- d: $m + n$ is odd

## Question 30

Claim: For all integers $m$,if $m$ is even then $3m + 5$ is odd.
Prove:

- Let $m = 2t$ for some $t \in \mathbb{Z}$
- We have $3m + 5 = 3 \cdot 2t + 5 = 6t + 5 = 2(3t + 2) + 1$, which is odd
- Therefore, the statement is true

## Question 36

Claim: There exists an integer $n$ such that $6n^2 + 27$ is prime.
Prove: We have $6n^2 + 27 = 3(2n^2 + 9)$.
Based on the definition, a number $p$ is prime if and only if

- $p > 1$ ($51 > 1$: true)
- $\forall r, s \in \mathbb{Z}_{>0}$; if $rs = p \to r = p \vee s = p$. However, $\exists$ 3 and $2n^2 + 9$ such that $3 \cdot (2n^2 + 9) = 3(2n^2 + 9)$, but $3 = 3(2n^2 + 9) \vee (2n^2 + 9) = 3(2n^2 + 9)$ is false

Therefore, there does not exist $n$ such that $6n^2 + 27$ is prime

## Section 4.2

### Question 18

Claim: If $r$ and $s$ are any two rational numbers, then $\frac{r+s}{2}$ is rational.

- We know that sum of 2 rational numbers is rational
- Let $r, s \in \mathbb{Q}$ such that $r = \frac{a}{b}, a, b \in \mathbb{Z}$ and $s = \frac{c}{d}, c, d \in \mathbb{Z}$ and $b, d \neq 0$
- $r + s = \frac{ad+bc}{bd}$ with $a, b, c, d \in \mathbb{Z} \implies ad, bc \in \mathbb{Z}$ and $b, d \neq 0 \implies bd \neq 0$
- $\frac{r+s}{2}$, the numerator is the same, but the denominator: $2bd$. Since $bd \neq 0$, so $2bd \neq 0$
- Therefore, $\frac{r+s}{2}$ is also rational

### Question 22

Claim: If $a$ is any odd integer, then $a^2 + a$ is even.

- Let $a \in \mathbb{Z}$ is an odd integer. We have $a = 2k + 1$
- We have $a^2 + a = (2k+1)^2 + (2k+1) = (4k^2 + 4k + 1) + (2k + 1)$
  $= 4k^2 + 6k + 2 = 2(2k^2 + 3k + 1)$, which is even
- Therefore, the statement is true

### Question 36

Fixed: "Proof: Let rational numbers $r = \frac{1}{4}$ and $s = \frac{1}{2}$ be given with 1, 2, 4 are integers and 2, $4 \neq 0$.Then $r + s = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$, which is also a rational number since 3, 4 are also integers and $4 \neq 0$."

## Section 4.3

### Question 13

If $n = 4k + 3$, does 8 divides $n^2 - 1$ ?

$$\begin{aligned} n^2 - 1 &= (4k+3)^2 - 1 \\ &= 16k^2 + 24k + 9 - 1 \\ &= 16k^2 + 24k + 8 \\ &= 8(2k^2 + 3 + 1) \end{aligned}$$

Therefore, 8 divides $n^2 - 1$

### Question 15

For all integers $a, b$, and $c$, if $a|b$ and $a|c$ then $a|(b + c)$.
Proof:
Let $b = a \cdot k$ and $c = a \cdot m$ with $k, m \in \mathbb{Z}$. We have:

$$\begin{aligned} b + c &= a \cdot k + a \cdot m \\ &= a(k + m) \end{aligned}$$

Therefore, $a|a(k + m)$, so $a|(b + c)$

## Question 42

### a

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

### b

$20! = 20 \cdot 19 \cdot \ldots 2 \cdot 1$

### c

The last digit of 0 appears when 2x5, or a number multiplied by 10 and multiples of 10.
for 20!, there are $2 * 5$ creates a 0, then $9! * 10$ creates another 0, $12 * 15$ creates a 0, and finally, $19! * 20$ results in another 0. In total, 20! will have 4 zeros.
Therefore, $(20!)^2$ will have $4 * 2 = 8$ zeros

# Section 4.6

## Question 7

Statement: There is no least positive rational number.
Negation: There exists a least positive rational number.
Let $M$ is the least positive rational number with $M = \frac{a}{b}$ with $a, b \in \mathbb{Z}$. If we have $c = b + 1$, then let $N = \frac{a}{c} = \frac{a}{b+1}$, which means that $N < M$. Therefore, $N$ must be the least positive rational number instead of $M$, which is contradict to the assumption. Thus, there is no least positive rational number

## Question 20

Statement: If a sum of two real numbers is less than 50, then at least one of the numbers is less than 25.
Formal form: $P(x) \rightarrow Q(x)$ with $P(x)$ is the sum of 2 real numbers which is less than 50, and $Q(x)$ is at least one of the numbers is less than 25.
Contrapositive: $\neg Q(x) \vee \neg P(x)$: if both numbers are greater than or equal to 25, then the sum of 2 real numbers which is greater than or equal to 50 Let $m, n \in \mathbb{Z}$. If $m = 25$ and $n = 26$, then their sum is 51 which is greater than 50. Thus, this statement is true

## Question 28

Statement: For all integers $m$ and $n$, if $mn$ is even then $m$ is even or $n$ is even.

### a: contraposition

contrapositive: For all integers $m$ and $n$, if $m$ is odd and $n$ is odd, then $mn$ is odd
Proof:

- Let $m, n \in \mathbb{Z}$. Assume that $m, n$ are odd, so $m = 2a + 1, n = 2b + 1$ with $a, b \in \mathbb{Z}$.
- $mn = (2a + 1)(2b + 1) = 2ab + 2a + 2b + 1 = 2(ab + a + b) + 1$. It is obvious that $2(ab + a + b) + 1$ is an odd number, so $mn$ is odd
- Therefore, the statement is true

**b: contradiction**

Contradiction: There exists integers $m, n$, $mn$ is even and $m, n$ are odd integers.
Proof:

- Let $m, n \in \mathbb{Z}$. Assume that $m, n$ are odd, so $m = 2k + 1$ and $n = 2t + 1$ for some $k, t \in \mathbb{Z}$
- We have: $mn = (2k+1)(2t+1) = 4kt + 2k + 2t + 1 = 2(2kt + k + t) + 1$. From this, however, $2(2kt + k + t) + 1$ is odd, so $mn$ must be odd.
- In conclusion, the contradiction is false, which means that the statement is true.