CS 201 Homework week 9

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Section 6.4

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a

- Let P : this sentence is true, and $Q : 1 + 1 = 3$
- We have $P \to Q$
- If P is false, then the statement is still true because $False \to False$ has the truth value. Thus, that sentence is true and $1 + 1 = 3$
- If P is true, then Q must also be true, so $1 + 1 = 3$

Also, from another perspective, it is not clear in the definition of what is meant by the occurrence of "this sentence":

- If "this sentence" is true, then we have 1+1=3
- It is can be understand that:

  if "If this sentence is true, then 1+1=3." is true, then 1+1=3. In this case, it is obvious that the whole part "If this sentence is true, then 1+1=3." is a truth statement (since $False \to False$), so it is true that 1+1=3
- And again, if "If "If this sentence is true, then 1+1=3." is true, then 1+1=3." is true, then 1+1=3. and so on which create a recursion

b

We cannot deduce anything from the status of "This sentence is true". Because what ever the status of "This sentence is true", 1+1=3 is provable

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No
Suppose there is a computer program like that

- Case 1: If the computer program mentions itself in output, but the computer program (as an output) does not mention itself. Contradict
- Case 2: If the computer program mentions does not mention itself in output, then its output does not mention itself, which is contradict since the computer program mentions all the computer programs (including itself)

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We have truth: The word heterological refers to an adjective that does not describe itself

- Case 1: if "heterological" is heterological, then "heterological" does not describe itself. Thus, the word "heterological" is not heterological. Contradict
- Case 2: if "heterological" is not heterological, then "heterological" describes itself, which means that "heterological" is heterological. Contradict

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Let n be "the smallest integer not describable in fewer than 12 English words." Note that, even n is described in just 11 words (from the definition: "the smallest integer not describable in fewer than 12 English words") which is contradict

Section 7.2

8

a

- H is not one-to-one. Because $H(b) = H(c)$ since both have y as result. However, $b \neq c$
- H is not onto because $x, z \in Y$, but $\nexists e \in X$ such that $H(e) = x$ or $H(e) = z$ knowing that e is an arbitrary element in X

b

- Yes. Since $K(a) = K(a)(= y)$, $K(b) = K(b)(= w)$, $K(c) = K(c)(= x)$, and $a = a, b = b, c = c$
- Not onto. Because $z \in Y$, but there are no elements from X such that $K(element) = z$

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a

Not one-to-one. Counter-example:
$N("0011") = N("01")$(both equal to 0), but "0011" is different from "01"

b

Yes, onto
Proof:

- Let x be number of 1's in S. Let y be number of 0's in S. Therefore, $x, y \in \mathbb{Z}$. Thus, $x - y$ must be in $\mathbb{Z}$, also

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a

Yes, one-to-one.
Proof:

- Let $s_1, s_2 \in S$
- We have $C(s_1) = C(s_2)$. By definition of C, $as_1 = as_2$. But this can be true only if $s_1 = s_2$

b

No. It is not onto
We have $b \in S$, but $\forall s \in S, C(s) = as \neq b$, so $\forall s \in S, b \neq C(s)$

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- Let $f(x) = x$ and $g(x) = -x$ which satisfy that they are both onto in $\mathbb{R}$. Therefore, $(f+g)$ has codomain of $\mathbb{R}$
- By definition: $(f+g)(x) = f(x) + g(x) = x - x = 0$. However, we have $1.5 \in \mathbb{R}$, but there are no a in domain $\mathbb{R}$ such that $(f+g)(a) = 1.5$ since $f(a) + g(a) = a - a = 0$

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a

Case 1: $F^{-1}(F(A)) \subseteq A$

- Let $x \in F^{-1}(F(A))$. We need to prove that $x \in A$, which means that $F(x) \in F(A)$, so there exists $a \in A$ such that $F(x) = F(a)$
- Since F is one to one, it means that $x = a$. Thus, $x \in A$

Case 2: $A \subseteq F^{-1}(F(A))$

- Let $a \in A$, then $F(a) \in F(A)$
- By definition, $a \in F^{-1}(F(A))$
- Thus, $A \subseteq F^{-1}(F(A))$

b

Case 1: Prove $F(A_1 \cap A_2) \subseteq F(A_1) \cap F(A_2)$

- Let $y \in F(A_1 \cap A_2)$, since it is one-to-one, the exists unique $a \in A_1 \cap A_2$ such that $F(a) = y$
- By definition of intersection, $a \in A_1$ and $a \in A_2$
- That implies that $y \in F(A_1) \cap F(A_2)$

Case 2: Prove $F(A_1) \cap F(A_2) \subseteq F(A_1 \cap A_2)$

- Let $y \in F(A_1) \cap F(A_2)$, which means that $y \in F(A_1)$ and $y \in F(A_2)$
- Since $F : X \to Y$ is one-to-one, there exists unique a_1 such that $F(a_1) = y$ and unique a_2 such that $F(a_2) = y$

- From those two, we note that there exists $a \in A_1 \cap A_2$ such that $F(a) = y$
- Thus, $F(A_1) \cap F(A_2) \subseteq F(A_1 \cap A_2)$

Therefore, from 2 cases, $F(A_1 \cap A_2) = F(A_1) \cap F(A_2)$

Section 7.3

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- $(K \circ H)(0) = K(H(0)) = K(0) = 0$
- $(K \circ H)(1) = K(H(1)) = K(6) = 2$
- $(K \circ H)(2) = K(H(2)) = K(12) = 0$
- $(K \circ H)(3) = K(H(3)) = K(18) = 2$

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$$H(x) = \frac{x+1}{x-1}; H^{-1}(x) = \frac{x+1}{x-1}$$

- $(H \circ H^{-1})(x) = H(H^{-1}(x)) = H\left(\frac{x+1}{x-1}\right)$

$$\begin{aligned} H\left(\frac{x+1}{x-1}\right) &= \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1} \\ &= \frac{\left(\frac{2x}{x-1}\right)}{\left(\frac{2}{x-1}\right)} \\ &= \frac{2x}{2} = x = I_x \end{aligned}$$

- $(H^{-1} \circ H)(x) = H^{-1}(H(x)) = H^{-1}\left(\frac{x+1}{x-1}\right)$

$$\begin{aligned} H^{-1}\left(\frac{x+1}{x-1}\right) &= \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1} \\ &= \frac{\left(\frac{2x}{x-1}\right)}{\left(\frac{2}{x-1}\right)} \\ &= \frac{2x}{2} = x = I_x \end{aligned}$$

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- Suppose $f : X \to Y$ is a one-to-one and onto function with the inverse function $f^{-1} : Y \to X$. Let y is arbitrary element in Y
- By definition, we have: $(f \circ f^{-1})(y) = f(f^{-1}(y))$
- Note that the inverse function satisfies this:
  $f^{-1}(b) = a$ and $\Longleftrightarrow f(a) = b$ for $a \in X$ and $b \in Y$

- Let $y' = f(f^{-1}(y))$
- Then we have:

$$f(y') = f(y)$$

- Since f is one-to-one, $y' = y$. Thus:

$$y = f(f^{-1}(y))$$
$$y = (f \circ f^{-1})(y)$$

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Statement: If $f : X \to Y$ and $g : Y \to Z$ are functions and $g \circ f$ is onto, must f be onto? Prove or give a counterexample.
False