Sect 8.1

Quan Nguyen

3/

a) $10\ T\ 1$: True : $10-1=9$ and $3 \mid 9$

$1\ T\ 10$: True : $1-10=-9$ and $3 \mid -9$

$(2,2) \in T$: True : $2-2=0$ and $3 \mid 0$

$(8,1) \in T$: False : $8-1=7$; $3 \nmid 7$

b) Let $S = \{n \mid n \in \mathbb{Z}$ and $n\,T\,0\} = \{0, 3, 6, 9, 12\}$

c) Let $A = \{n \mid n \in \mathbb{Z}$ and $n\,T\,1\} = \{1, 4, 7, 10, 13\}$

d) Let $B = \{n \mid n \in \mathbb{Z}$ and $n\,T\,2\} = \{2, 5, 8, 11, 14\}$

e) Let a be the integer that $a\,T\,0$, which means $3 \mid (a-0) = 3 \mid a$.
therefore, $a = 3k$ for some $k \in \mathbb{Z}$

Let b be integer that $b\,T\,1$ It means $3 \mid (b-1)$
Therefore, $b-1 = 3m$ for some $m \in \mathbb{Z}$, so $b = 3m+1$

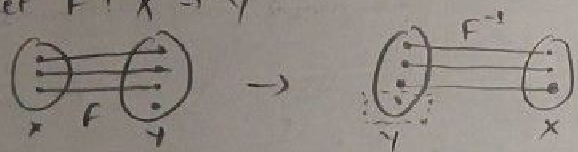
Let c be integer that $c\,T\,2$. It means $3 \mid (c-2)$
Therefore, $c-2 = 3n$ for some $n \in \mathbb{Z}$, so $c = 3n+2$

6/ a) $\{a\}\ J\ \{c\}$: False. Because: $\{a\} \cap \{c\} = \emptyset$

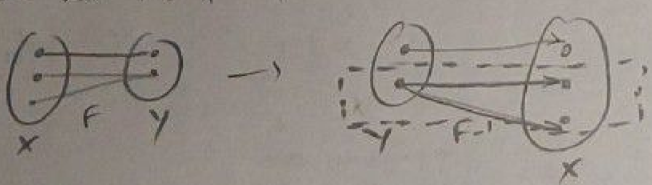
b) $\{a,b\}\ J\ \{b,c\}$: True.

c) $\{a,b\}\ J\ \{a,b,c\}$: True

12/ a) No. Let $F: X \to Y$



b) No. Let $F: X \to Y$



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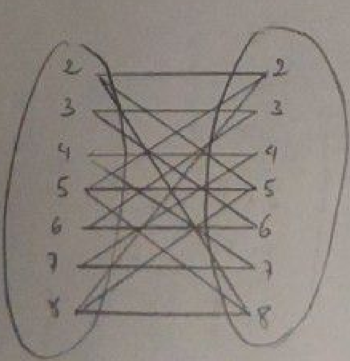

we have: $2\,T\,2$; $2\,T\,5$; $2\,T\,8$
$3\,T\,3$; $3\,T\,6$
$4\,T\,4$; $4\,T\,7$
$5\,T\,2$; $5\,T\,5$; $5\,T\,8$
$6\,T\,3$; $6\,T\,6$
$7\,T\,4$; $7\,T\,7$
$8\,T\,2$; $8\,T\,5$; $8\,T\,8$

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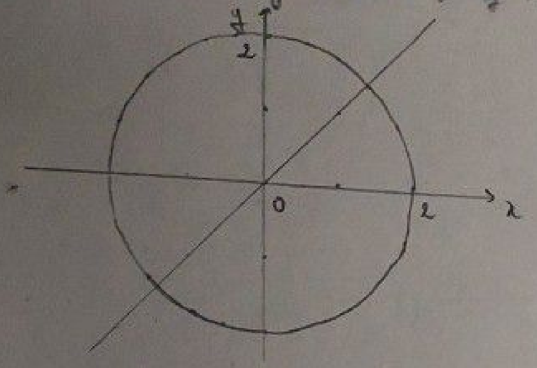
R : all x, y coordinates in circle created from $x^2 + y^2 = 4$

S : all x, y coordinates in line created from $x = y$.

$R \cup S$: all x, y coordinates in circle or line.

$R \cap S$: are x, y coordinates of intersections between circle and line.



Sect 11.3

12/

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for k := 1 to n-1:
    max := a[k]
    for i := k+1 to n
        If max < a[i] then max := a[i]
    next i
    a[k] := max
next k
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a)

[From (1): $\theta(n^2)$

$\in$ 1 operation/loop

$\Big\}$ Number of operations is $\theta(n^2)$

b) $\frac{1}{2}n^2 - \frac{1}{2}n$ is $\theta(n^2)$

Note: $k=1$; $i=2$ to n has: $n-2+1 = n-1$ loops

$k=2$; $i=3$ to n has: $n-3+1 = n-2$ loops

$k=n-1$; $i=n$ to n has: $n-n+1 = 1$ loop

So there are $(n-1)-1+1 = n-1$ elements

$$\Sigma = \frac{(n-1)[(n-1)+1]}{2} = \frac{n^2-2n+1+n-1}{2} \text{ is } \theta(n^2) \quad (1)$$

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a) $n-1$ times $\rightarrow$ $\theta(n)$

b) $n-2$ times $\rightarrow$ $\theta(n)$

c) $n-k$ times $\rightarrow$ $\theta(n)$

d) worst case: $\theta(n^2)$

Sect 11.4

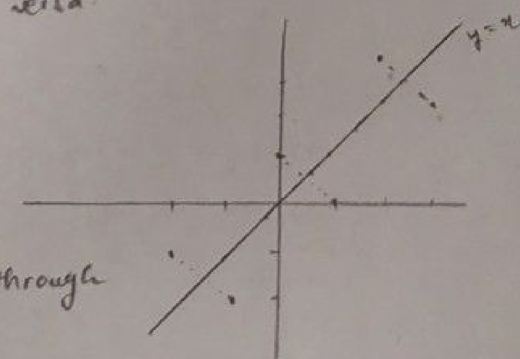
11/

a) Let $f(x) = \log_b x$; $g(x) = b^x$

Let $u: f(v) = \log_b v$ (*) $\quad v = b^u$ (1)

Let $v = g(u) = b^u$ (2)

$\Big\}$ If (u, v) is on $f(x)$, then it is also on $g(x)$ and vice versa



b) With $b > 1$.

I choose (u, v) pairs: $(0, 1)$; $(2; 3)$; $(-2; -1)$.

then (v, u) pairs should be: $(1, 0)$; $(3; 2)$; $(1, -2)$

Describe: for each pair (u, v) and (v, u), the points are symmetric through line $y = x$.

c) Describe: graphs drawn by $y = \log_2 x$ and $y = 2^x$ are symmetric through line $y = x$

38) Let $H_n = \frac{1}{5} + \frac{4}{5^2} + \frac{4^2}{5^3} + \dots + \frac{4^n}{5^{n+1}} = \frac{1}{5}\left(1 + \frac{4}{5} + \frac{4^2}{5^2} + \dots + \frac{4^n}{5^n}\right)$

$= \frac{1}{5} \cdot \frac{(4/5)^{n+1} - 1}{4/5 - 1}$

We have $\left(\frac{4}{5}\right)^{n+1} = \frac{4}{5} \cdot \left(\frac{4}{5}\right)^n \rightarrow \Theta\left(\frac{4^n}{5^n}\right) = \Theta(1)$

41/ Let $A = k_1 n + k_2 n \log_2 n$

_ Prove A is $\Omega(n \log_2 n)$:

We have: $k_1 n + k_2 n \log_2 n = n \log_2 n \left(\frac{k_1}{\log_2 n} + k_2\right)$

Note: $\exists N > 0$ st $\forall x > N$:

we have $k_2 - \frac{1}{2} \leq \frac{k_1}{\log_2 n} + k_2$

So $n \log_2 n \left(k_2 - \frac{1}{2}\right) \leq n \log_2 n \left(\frac{k_1}{\log_2 n} + k_2\right) = A$

Therefore: A is $\Omega(n \log_2 n)$ (1)

_ Prove A is $O(n \log_2 n)$:

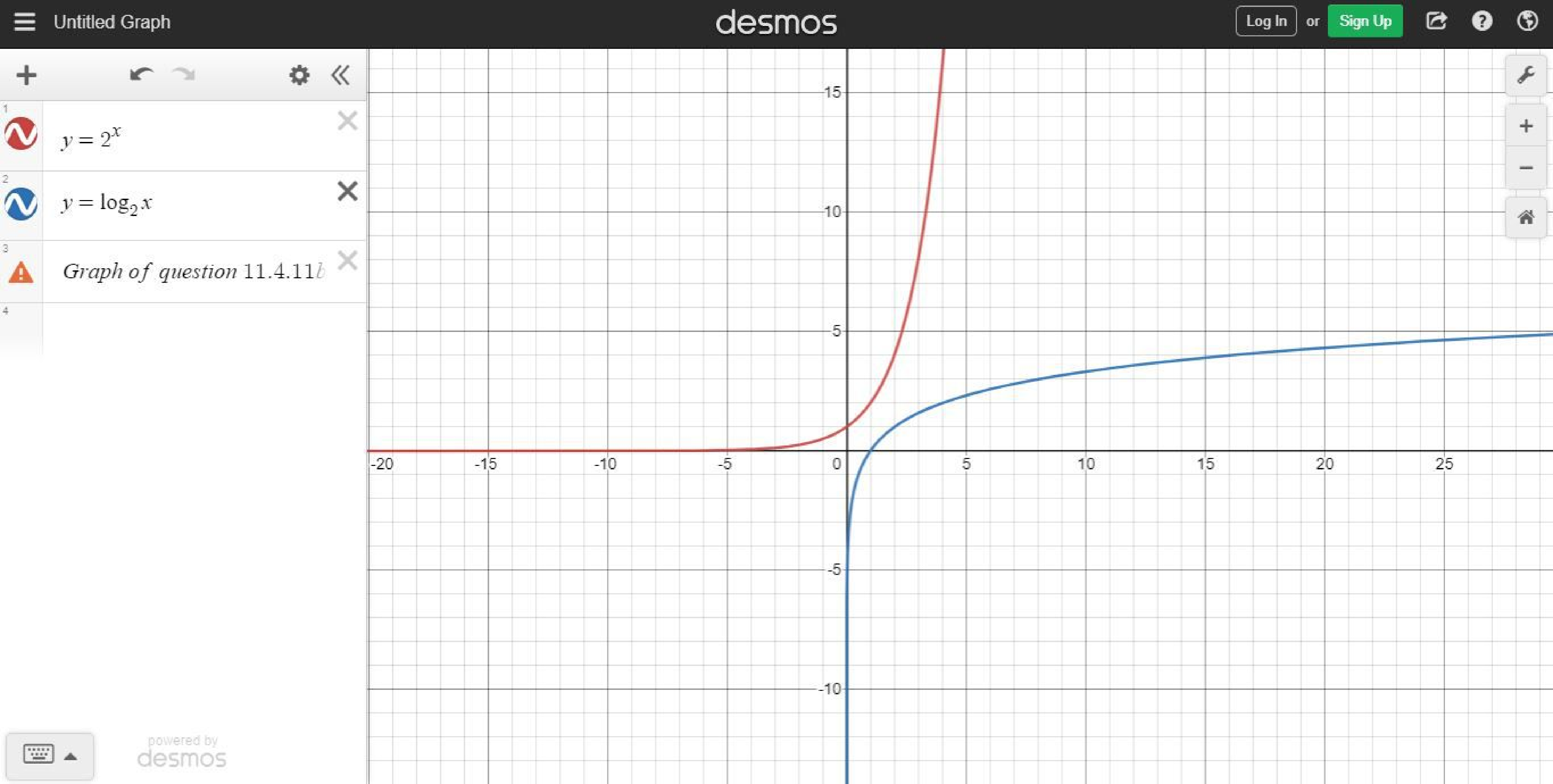
We have $k_1 n \leq k_1 n \cdot \log_2 n$

$\Leftrightarrow k_1 n + k_2 n \log_2 n \leq k_1 n \log_2 n + k_2 n \log_2 n$

$\Leftrightarrow \quad A \leq n \log_2 n (k_1 + k_2)$

Therefore: A is $O(n \log_2 n)$ (2)

From (1), (2): A is $\Theta(n \log_2 n)$



Untitled Graph
desmos
Log In
or
Sign Up
$y = 2^x$
$y = \log_2 x$
Graph of question 11.4.11
15
10
5
-5
-10
-20
-15
-10
-5
0
5
10
15
20
25
powered by
desmos