Math 201 Homework week 1

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September 5 2021

Section 1.1

Question 4

a

Given any real number r, there is a real number s such that s is greater than r

b

For any real number r, there is a number s such that $s > r$

Question 13

a

Some real number has the property that its product with every real number equals zero

b

There is a real number a such that the product of a with every real number equals zero

c

There is a real number a with the property that for every real number b, the product of a and b equals zero

Section 1.2

Question 4

a

$2 \in \{2\}$: True

b

$\{2, 2, 2, 2\}$ has 4 elements

c

$\{0, \{0\}\}$ has 2 elements

d

$\{0\} \in \{\{0\}, \{1\}\}$: True

e

$0 \in \{\{0\}, \{1\}\}$: False

Question 7

b

$T = \{m \in \mathbb{Z} | m = 1 + (-1)^i, \text{for some integer } i\} = \{0, 2\}$

e

$W = \{t \in \mathbb{Z} | 1 < t < -3\} = \{\}$

f

$X = \{u \in \mathbb{Z} | u \leq 4 \text{ or } u \geq 1\} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Question 8

$A = \{c, d, f, g\}$; $B = \{f, j\}$; $C = \{d, g\}$

a

$B \subseteq A$: False. Because $j \in B$, but $j \notin A$

b

$C \subseteq A$: True. Because for all elements in C, $d, g \in C$, and $d, g \in A$

c

$C \subseteq C$: True. Actually, it should be written like this: $C = C$

d

$C \subset A$: True. Because $d, g \in C$ (d, g are all elements in C), $d, g \in A$, and $c, f \in A$, $c, f \notin C$

Question 10

b

$(5, -5) = (-5, 5)$: False

d

$\left(\frac{-2}{-4}, (-2)^3\right) = \left(\frac{3}{6}, -8\right)$: True

Question 12

$S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$

a

$SXT = \{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)\}$

d

$TXT = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

Section 1.3

Question 4

$G = \{-2,0,2\}$ and $H = \{4,6,8\}$
So, $GXH = \{(-2,4),(-2,6),(-2,8),(0,4),(0,6),(0,8),(2,4),(2,6),(2,8)\}$

a

$2V6$: True
$(-2)V(-6)$: True
$(0,6) \in V$: False
$(2,4) \in V$: False

b

$V = \{(-2,6),(0,4),(0,8),(2,6)\}$

c

Domain of V is G and Co-domain of V is H

d

Quan: "I include the image at the end of this pdf file"

Question 13

a

Domain of F is A. Co-domain of F is B

b

$F(-1) = u$
$F(0) = w$
$F(1) = u$

Question 15

d

Question 20

Starting with the function H:

$$\begin{aligned} H(x) &= (x-2)^2 \\ &= x^2 - 4x + 4 \end{aligned}$$

Function K:

$$\begin{aligned} K(x) &= (x-1)(x-3) + 1 \\ &= x^2 - 4x + 3 + 1 \\ &= x^2 - 4x + 4 \end{aligned}$$

It is obviously that function $H = K$ since both functions are $x^2 - 4x + 4$

Section 2.1

Question 2

If 2 is odd, then all prime numbers are odd.
2 is not odd.
Therefore, it is not the case that all prime numbers are odd.

Question 13

$\neg(p \wedge q) \vee (p \vee q)$
Truth tables:

p	q	$(p \wedge q)$	$\neg(p \wedge q)$	$(p \vee q)$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

Question 24

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$
Truth tables:

p	q	r	$(p \vee q)$	$(p \wedge r)$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	T
F	F	F	F	F	F	F

So, in conclusion, these two are not equivalent since they have different truth tables

Question 30

The dollar is at an all-time high and the stock market is at a record low.
Let a = The dollar is at an all-time high; b = the stock market is at a record low.
Thus, we have $a \wedge b$.

Negation: $\neg(a \wedge b) = \neg a \vee \neg b$.
So we have: "The dollar is **not** at an all-time high **OR** the stock market is **not** at a record low"

Question 45

Statement (a):

- Bob: major in both Math and Computer Science.
- Ann: major in Math, but not major in both Math and Computer Science. So, this means: Ann: major in Math only.

Statement (b):

- Not the case: Bob and Ann both choose double major.
- Bob: major in both Math and Computer Science
- Ann: major in Math

Therefore, statements (a) and (b) are equivalent since the result is that:

- Only Bob is a Math and CS major
- Ann is a Math major
- Both Bob and Ann are not Math and CS majors

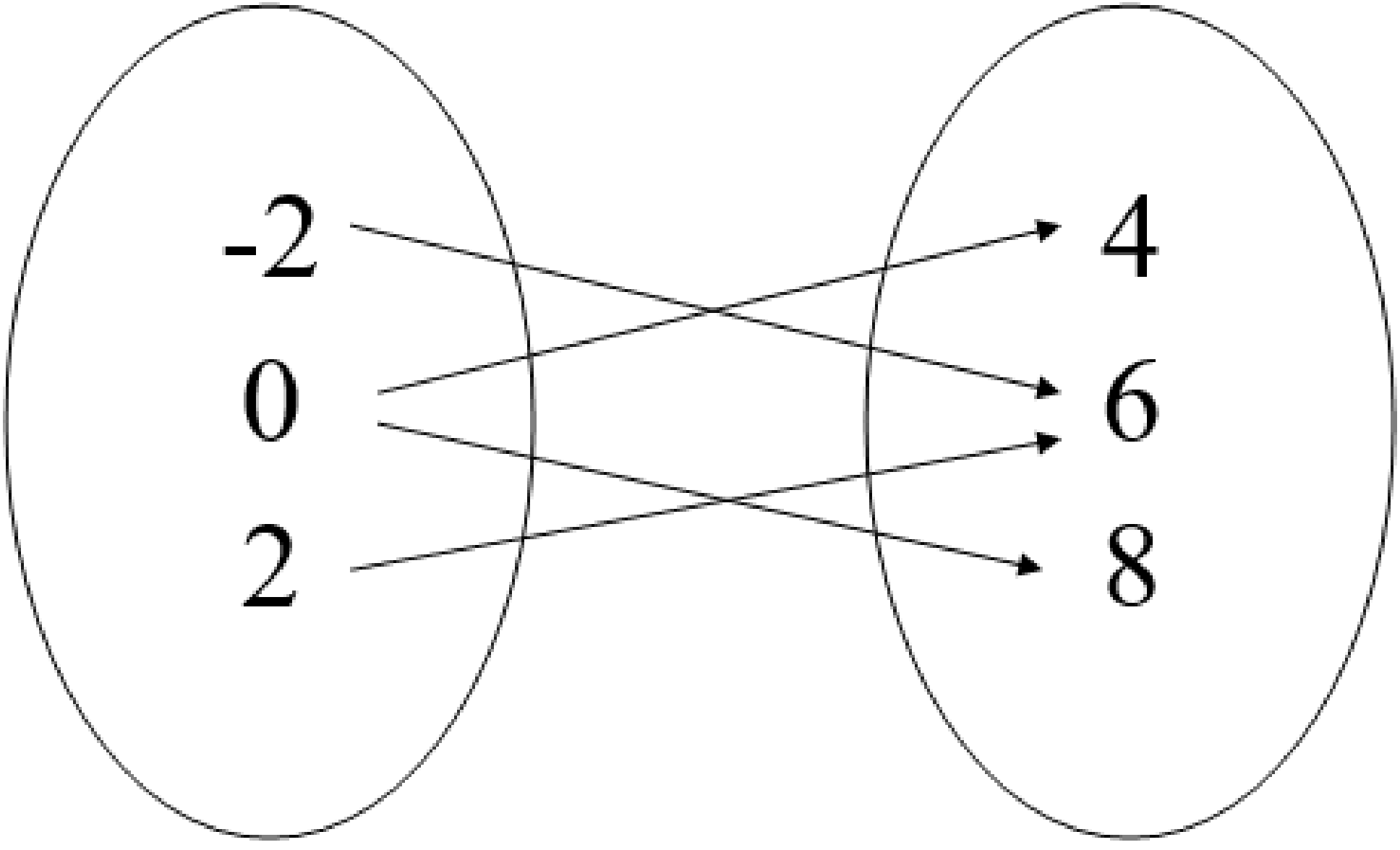
Question 52

$$\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$$

Solve:

$$\begin{aligned}
\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) &\equiv (\neg p \wedge q) \vee (\neg p \wedge \neg q) \text{ (De Morgan's laws)} \\
&\equiv \neg p \wedge (q \vee \neg q) \text{ (Distributive laws)} \\
&\equiv \neg p \wedge t \text{ (Negation laws)} \\
&\equiv \neg p \text{ (Identity laws)}
\end{aligned}$$

Therefore, it is obviously that $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$.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