Math 201 Homework week 3

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Section 3.2

Question 3

a

$\forall$ fish x, x has grill.
Negation: $\exists$ fish x, x does not has grill.

b

$\forall$ computers c, c has a CPU.
$\exists$ computers c, c does not has a CPU.

c

$\exists$ a movie m such that m is over 6 hours long.
$\forall$ movies m such that m are not over 6 hours long

d

$\exists$ a band b such that b has won at least 10 Grammy awards.
$\forall$ bands b such that b have not won at least 10 Grammy awards

Question 7

Statement: "There are no orders from store A for item B."
The statement is existential.
Negation: For all orders, they are all from store A for item B.
Formal: let "orders from store A for item B": p. We have: "$\exists\neg p$". Negation will be "$\forall p$"

Question 21

"$\forall$ integers n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3".
Negation: $\exists$ integers n such that n is divisible by 6 and n is neither divisible by 2 nor divisible by 3

Question 38

"All occurrences of the letter u in Discrete Mathematics are lowercase."
False. Because there exists letters u in Discrete Mathematics are uppercase. For example, letter u from "CPU" in question 3b, section 3.2

Question 44

"Having a large income is not a necessary condition for a person to be happy."
Negation: Having a large income is a condition that everyone must have to be happy.

Section 3.3

Question 11

a

There are some students that have seen Casablanca

b

All the students have seen Star Wars

c

All the students have seen a released movie m

d

There are some movies that all the students have seen

e

There are some students have seen the movies m, and some other students also have seen these movies m

f

If some student have seen all the movies, then some other students have also seen all movies

Question 12

$D = E = \{-2, -1, 0, 1, 2\}$

a

$\forall x$ in D, $\exists y$ in E such that $x + y = 1$. False
Negate: $\exists x$ in D such that $\forall y$ in E, $x + y \neq 1$. True. There exists $x = -2$ and for all $y \in E$, $x + y$ does not equal to 1

b

$\exists x$ in D such that $\forall y$ in E, $x + y = -y$. False
Negate: $\forall x$ in D, $\exists y$ in E such that $x + y \neq -y$. True. If $y = -2$ and with all $x \in D$, the sum of x and y won't equals to $-y$

c

$\forall x$ in D, $\exists y$ in E such that $xy \geq y$. True: $y = 0$ and for all values of x in D, $xy = 0$
Negate: $\exists x$ in D such that $\forall y$ in E, $xy < y$. False

d

$\exists x$ in D such that $\forall y$ in E, $x \leq y$. True, there exists $x = -2$ such that for all $y \in E$, $x \leq y$
Negate: $\forall x$ in D, $\exists y$ in E such that $x > y$. False

Question 24

a

$$\begin{aligned}\neg\{\forall x \in D[\forall y \in E(P(x,y))]\} &\equiv \exists x \in D[\neg(\forall y \in E(P(x,y))] \\ &\equiv \exists x \in D[\exists y \in E(\neg P(x,y))]\end{aligned}$$

b

$$\begin{aligned}\neg\{\exists x \in D[\exists y \in E(P(x,y))]\} &\equiv \forall x \in D[\neg(\exists y \in E(P(x,y))] \\ &\equiv \forall x \in D[\forall y \in E(\neg P(x,y))]\end{aligned}$$

Question 43

Let $(\forall \varepsilon \in \mathbb{R}) \wedge (\epsilon > 0)$: E
Let $(\exists \delta \in \mathbb{R}) \wedge (\delta > 0)$: D
Let $a - \delta < x < a + \delta$ and $x \neq a$: $P(x)$
Let $L - \epsilon < f(x) < L + \epsilon$: $Q(x)$
Definition: E, D such that $\forall x \in \mathbb{R}$, $P(x) \rightarrow Q(x)$.
Negation: $\neg E, \neg D$, $\exists x \in \mathbb{R}$ such that $P(x) \wedge \neg Q(x)$
$\rightarrow (\exists \epsilon \in \mathbb{R})$ OR $(\epsilon \leq 0)$, AND $(\forall \delta \in \mathbb{R})$ OR $(\delta \leq 0)$ such that $(a - \delta < x < a + \delta$ AND $x \neq a)$ and $[(f(x) \geq L + \epsilon)$ OR $(f(x) \leq L - \epsilon)]$

Question 45

Statement: "$\exists! x \in D$ such that $P(x)$".
Rewrite: There exists only one value of $x \in D$ such that $P(x)$. Or: $E = \{x \in D | P(x)\} \wedge |E| = 1$

Section 2.5

Question 21

a

$S = 0; T = 1$

b

$S = 0; T = 1$

c

$S = 0; T = 0$

Question 30

$10111010_2 = -70_{10}$

Question 34

$89_{10} + (-55)_{10} = 01011001_2 + 11001001_2 = 00100010_2$

Question 36

$123_{10} + (-94)_{10} = 29_{10} = 00011101_2$

Question 42

$B53DF816_{16} = 10110101001111011111100000010110_2$