Math 201 Homework week 2

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Section 2.2

Question 4

Statement: "Fix my ceiling or I won't pay my rent"
Rewrite: "If my ceiling is fixed, then I will pay my rent"

Question 6

$$(p \vee q) \vee (\neg p \wedge q) \to q$$

Truth table:

p	q	$(p \vee q)$	$(\neg p \wedge q)$	$(p \vee q) \vee (\neg p \wedge q)$	$(p \vee q) \vee (\neg p \wedge q) \to q$
T	T	T	F	T	T
T	F	T	F	T	F
F	T	T	T	T	T
F	F	F	F	F	T

Question 27

Statement: "The converse and inverse of a conditional statement are logically equivalent to each other"
Truth table:

p	q	$\neg p$	$\neg q$	$q \to p$	$\neg p \to \neg q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

Since both converse and inverse have the same truth table, they are logically equivalent to each other

Question 35

Statement: "Sam will be allowed on Signe's racing boat only if he is an expert sailor"
Let p: "Sam will be allowed on Signe's racing boat"; and q: "he is an expert sailor"
We have $p \to q$: if Same is an expert sailor, then he will be allowed on Signe's racing boat.
Contrapositive of $p \to q$ is $\neg q \to \neg p$: if Sam is not an expert sailor, then he will not be allowed on Signe's racing boat

Question 41

Statement: "Having two 45° angles is a sufficient condition for this triangle to be a right triangle"
if-then statement: If this triangle has two 45°, then it is a right triangle

Question 43

Statement: "Doing homework regularly is a necessary condition for Jim to pass the course"
This statement is $p \to q$ when p: "pass the course"; q: "do homework regularly"
The contrapositive ($\neg q \to \neg p$): If Jim does not do homework regularly, then he won't pass the course

Section 2.3

Question 4

If this figure is a quadrilateral, then the sum of its interior angles is $360\circ$.
The sum of the interior angles of this figure is not $360\circ$.
$\therefore$ this figure is **not** a quadrilateral.

Question 9

$$
\begin{aligned}
& p \wedge q \to \neg r \\
& p \vee \neg q \\
& \neg q \to p \\
\therefore & \neg r
\end{aligned}
$$

Truth table:

	p	q	r	$p \wedge q \to \neg r$	$p \vee \neg q$	$\neg q \to p$	$\neg r$
1	T	T	T	F	T	T	
2	T	T	F	T	T	T	T
3	T	F	T	T	T	T	F
4	T	F	F	T	T	T	T
5	F	T	T	T	F	T	
6	F	T	F	T	F	T	
7	F	F	T	T	T	F	
8	F	F	F	T	T	F	

The truth table shows that the premises and the conclusion are all true in rows 2 and 4, there is a situation (row 3) where the premises are true and the conclusion is false.
This statement is not valid since there is a critical row which its conclusion is false (row 3)

Question 23

Oleg is a math major or Oleg is an economics major.
If Oleg is a math major, then Oleg is required to take Math 362.
$\therefore$ Oleg is an economics major or Oleg is not required to take Math 362.

Let:
- p: is a math major
- q: is an economics major

r: is required to take Math 362
Then we have:

$$p \vee q$$
$$p \to r$$
$$\therefore q \vee \neg r$$

Truth table:

	p	q	r	$p \vee q$	$q \to r$	$q \vee \neg r$
1	T	T	T	T	T	T
2	T	T	F	T	F	
3	T	F	T	T	T	F
4	T	F	F	T	T	T
5	F	T	T	T	T	T
6	F	T	F	T	F	
7	F	F	T	F	T	
8	F	F	F	F	T	

This argument is not valid.

Question 28

If there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite.
The set of all irrational numbers is infinite.
∴ There are as many rational numbers as there are irrational numbers.

Let
p: there are as many rational numbers as there are irrational numbers
q: the set of all irrational numbers is infinite

We then have the argument:

$$p \to q$$
$$q$$
$$\therefore p$$

This argument has converses error.

Question 32

If I get a Christmas bonus, I'll buy a stereo.
If I sell my motorcycle, I'll buy a stereo.
∴ If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo.

Let
p: I get a Christmas bonus
q: I'll buy a stereo
r: I sell my motorcycle

We then have the argument:

$$
\begin{aligned}
&p \to q \\
&r \to q \\
&\therefore (p \lor r) \to q
\end{aligned}
$$

Division into Cases:

$$
\begin{aligned}
&p \to q \\
&r \to q \\
&p \lor r \\
\therefore\, &q
\end{aligned}
$$

Question 37

The treasure is buried under the flagpole.

Section 2.4

Question 8

We have:

$$(p \lor q) \lor \neg(q \land r)$$

Truth table:

	p	q	r	$p \lor q$	$\neg(q \land r)$	$(p \lor q) \lor \neg(q \land r)$
1	T	T	T	T	F	T
2	T	T	F	T	T	T
3	T	F	T	T	T	T
4	T	F	F	T	T	T
5	F	T	T	T	F	T
6	F	T	F	T	T	T
7	F	F	T	F	T	T
8	F	F	F	F	T	T

Now we just need to replace T by 1 and F by 0, then we get the input for the circuit. The result is that whatever the input is, the result will be 1

Question 12

We have:

$$(p \lor q) \lor \neg(q \land r)$$

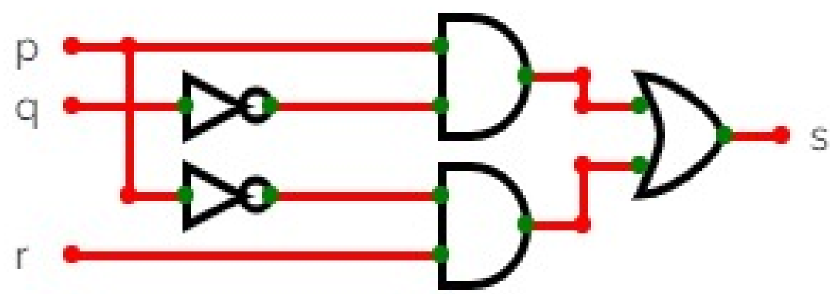
Truth table:

	p	q	r	$p \vee q$	$\neg(q \wedge r)$	$(p \vee q) \vee \neg(q \wedge r)$
1	T	T	T	T	F	T
2	T	T	F	T	T	T
3	T	F	T	T	T	T
4	T	F	F	T	T	T
5	F	T	T	T	F	T
6	F	T	F	T	T	T
7	F	F	T	F	T	T
8	F	F	F	F	T	T

Question 17

$$(p \wedge \neg q) \vee (\neg p \wedge r)$$

Draw diagram



Question 25

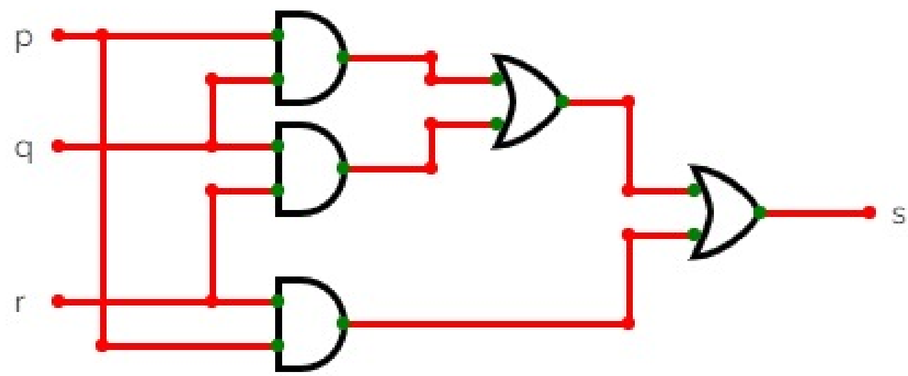
Let p, q, and r are control panels in three different locations. Then we have:

$$(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$$

This system means that it only in the on position if and only if it satisfies one of these requirement:

- p and q are in the on position
- p and r are in the on position
- q and r are in the on position

Diagram:



Question 32

$$\begin{aligned}&(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)\\ =&[p \wedge (q \wedge r)] \vee [p \wedge (\neg q \wedge r)] \vee [p \wedge (\neg q \wedge \neg r)]\\ =&p \wedge [(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)]\\ =&p \wedge [\neg(q \wedge \neg r)]\end{aligned}$$

From truth table, we can proof that $(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$ is equivalent to $\neg(q \wedge \neg r)$.

p	q	r	$q \wedge r$	$\neg q \wedge r$	$\neg q \wedge \neg r$	$(q \wedge r) \vee (\neg q \wedge r) \vee (\neg q \wedge \neg r)$	$\neg(q \wedge \neg r)$
T	T	T	T	F	F	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	F	T	T
T	F	F	F	F	T	T	T
F	T	T	T	F	F	T	T
F	T	F	F	F	F	F	F
F	F	T	F	T	F	T	T
F	F	F	F	F	T	T	T

Section 3.1

Question 6

"If m is a factor of n^2 then m is a factor of n", with $m, n \in \mathbb{Z}$

a

Because $n^2 = 100$, and $m = 25$ is the factor of n^2 since 100 is divisible by 25. However, with $n = 10$, $m = 25$, m is not a factor of n since 10 is not divisible by 25. Therefore, $R(m, n)$ is false

b

$R(m, n)$ is false when $m = 8$ and $n = 4$.

c

$R(m, n)$ is true when $m = 5$ and $n = 10$ because the remaining when $10 \div 5$ $(n \div m)$ is 0, and when $n^2 \div m$ result in 0 as a remaining

d

$R(m, n)$ is true when $m = 2$ and $n = 4$.

Question 12

$\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

That statement is false when $x = 2$, $y = 2$ since $\sqrt{2+2} = 2$ while $\sqrt{2} + \sqrt{2} > 2$

Question 18

a

$\exists s \in D, E(s) \wedge M(s)$

b

$\forall s \in D, C(s) \rightarrow E(s)$

c

$\forall s \in D, \neg C(s) \wedge E(s)$

d

$\exists s \in D, C(s) \wedge M(s)$

e

$\exists s \in D, (C(s) \wedge E(s))) \vee (C(s) \wedge \neg E(s)))$

Question 24

$\exists$ hater x such that x is mad.
$\exists x$ such that x is a hatter and x is mad
$\exists$ question x such that x is easy.
$\exists x$ such that x is a question and x is easy

Question 29

a

Rewrite: There is a rectangle that is also a square. True because a rectangle with width = 1 and height = 1 is also a square.

b

Rewrite: There are some rectangles that are not squares. True since rectangle with width = 2 and height = 3 is a rectangle but is not a square.

c

Rewrite: All squares are rectangles. True