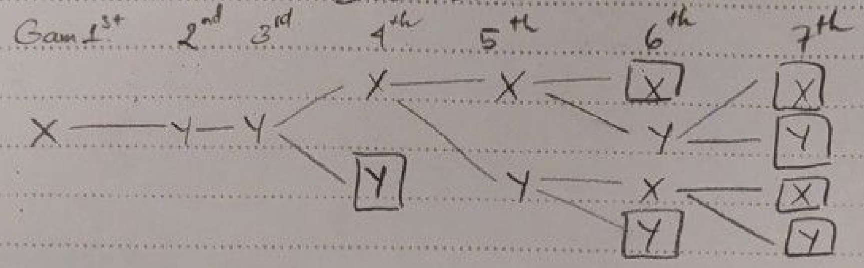
CS 201 Quan Nguyen

Sect 9.2: 5, 12, 18, 30, 39
Sect 9.3: 8, 24, 34, 41

Sect 9.2

5/



} Total 7 ways

12/ a) $\underline{9} \times \underline{16} \times \underline{16} \times \underline{16} \times \underline{11} = 9 \times 16^3 \times 11$

b) $\underline{10} \times \underline{16} \times \underline{16} \times \underline{16} \times \underline{16} \times \underline{13} = 10 \times 16^4 \times 13$

18/ a) $\underline{2}\ \underline{1}\ \underline{3}\ \underline{3}$
4 ways 3 ways 4 ways 4 ways $\Longrightarrow$ total $4^3 \cdot 3$ ways

b) 5 0 3 1
4 ways 1 way 4 ways 3 ways $\Rightarrow$ Total: $4^2 \times 1 \times 3$ ways

c) $\underline{10} \times \underline{9} \times \underline{8} \times \underline{7}$ ways

30/ Assume $n = a \cdot b$

a) Each element $p_i^{k_i}$ has 2 ways to choose: $n = p_i^{k_i} \cdot p_i^0$ or $n = p_i^0 \cdot p_i^{k_i}$
So, with m elements: 2^m ways

b) Given $n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdots p_m^{k_m}$
Since order does not matter $n' = p_m^{k_m} \cdot p_1^{k_1} \cdot p_2^{k_2} \cdots p_{m-1}^{k_{m-1}}$ is same as n
We have m! ways that are same
So we have in total $\frac{2^m}{m!}$ ways

39/ a) use r-permutation: $\frac{9!}{(9-3)!} = \frac{9!}{6!} = 9 \cdot 8 \cdot 7$

b) $\frac{9!}{(9-6)!} = \frac{9!}{3!}$

c) A□□□□□: $\frac{8!}{(8-5)!} = \frac{8!}{3!}$

d) OR□□□□: $\frac{7!}{(7-4)!} = \frac{7!}{3!}$

Sect 9.3

8/ Total symbols: $26 + 10 + 14 = 50$

a) $\left.\begin{array}{l} \text{Case } 3: \ 50^3 \\ \text{Case } 4: \ 50^4 \\ \text{Case } 5: \ 50^5 \end{array}\right\} \Sigma = \underbrace{50^3 + 50^4 + 50^5}_{\text{ways}}$

b) $\left.\begin{array}{l} \text{Case } 3: \ 50 \cdot 49 \cdot 48 \\ \text{Case } 4: \ 50 \cdot 49 \cdot 48 \cdot 47 \\ \text{Case } 5: \ 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \end{array}\right\}$ Total is # of ways

c) $\left.\begin{array}{l} \text{Case } 3: \ 50^3 - (50 \cdot 49 \cdot 48) \\ \text{Case } 4: \ 50^4 - (50 \cdot 49 \cdot 48 \cdot 47) \\ \text{Case } 5: \ 50^5 - (50 \cdot 49 \cdot 48 \cdot 47 \cdot 46) \end{array}\right\} \Sigma = \#$ of ways

d) Let $1 \leq n \leq \#$ symbols

Case 3: $\dfrac{50^3 - (50 \cdot 49 \cdot 48)}{50^3} = 1 - \dfrac{50 \cdot 49 \cdot 48}{50^3}$

Case 4: $1 - \dfrac{50 \cdot 49 \cdot 48 \cdot 47}{50^4}$

Case 5: $1 - \dfrac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{50^5}$

24/ a) $\left.\begin{array}{l} \text{Mult of } 2: \ 500 \\ \text{Mult of } 9: \ 111 \end{array}\right\} \Sigma = 611$

b) $\dfrac{611}{1000}$

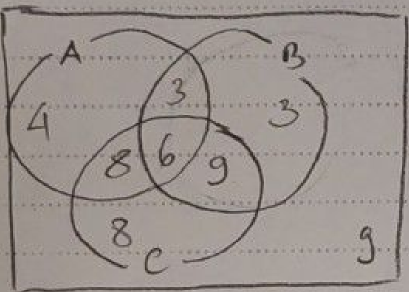
c) $1000 - 611 = 389$

34/ a) $|U| - |(A \cup B \cup C)| = 50 - 41 = 9$

b) $|(A \cap B \cap C)| = |A| + |B| + |C| - |(A \cap B)| - |(A \cap C)| - |(B \cap C)| + |(A \cup B \cup C)|$

$= 21 + 21 + 31 - 9 - 14 - 15 + 41 = 6$

c)



d) 4

41/ Let $A = \{m \leq pqr \mid \gcd(m,p) = 1\}$

$B = \{m \leq pqr \mid \gcd(m,q) = 1\}$

$C = \{m \leq pqr \mid \gcd(m,r) = 1\}$

$|A| = pqr - \dfrac{pqr}{p} = pqr - qr$

$|B| = pqr - \dfrac{pqr}{q} = pqr - pr$

$|C| = pqr - \dfrac{pqr}{r} = pqr - pq$

$|A \cap B| = pqr - \dfrac{pqr}{pq} = pqr - r$

$|A \cap C| = pqr - \dfrac{pqr}{pr} = pqr - q$

$|B \cap C| = pqr - \dfrac{pqr}{qr} = pqr - p$

$|A \cap B \cap C| = pqr - \dfrac{pqr}{pqr} = pqr - 1$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= (pqr - qr) + (pqr - pr) + (pqr - pq) - (pqr - r) - (pqr - q) - (pqr - p) + (pqr - 1)$

$= pqr - qr - pr - pq + r + p + q - 1$

$= (pq - q - p + 1)(r - 1)$

$= (p-1)(q-1)(r-1)$