Name: Key **CS301 F8**

1. Consider the proof that every multitape Turing machine has an equivalent single-tape Turing machine.

   (a) **(__ /2 pts)** How does the simulation machine first format its tape given the input string $w = \varepsilon$?

   $\# \dot{\sqcup} \# \dot{\sqcup} \# \cdots \# \dot{\sqcup} \#$

   (b) **(__ /2 pts)** What happens if a simulation tape head is instructed to move left and encounters #?

   It instead does not move, just like on a "normal" tape.
   The # to the left indicates the boundary of the tape.

2. Consider the proof that every nondeterministic Turing machine has an equivalent deterministic Turing machine.

   (a) **(__ /2 pts)** Consider N's nondeterministic computation tree. At most how many children can a node in that tree have? Why?

   Remember that δ has co-domain $\mathcal{P}(Q \times \Gamma \times \{L, R\})$, so each tree node — which can follow any possibility — has at most $|Q \times \Gamma \times \{L,R\}|$ children.

   (b) **(__ /2 pts)** Describe what it *means* in step 3. of the proof where it is stated, "If no more symbols remain on tape 3 or if this nondeterministic choice is invalid, abort this branch..."

   no more symbols: The simulation has explored N's tree to the exact address on the address tape.

   choice invalid: The simulation is being asked to explore an impossible computational step in N. No need to go that direction.

3. **(__ /2 pts)** Describe a reasonable encoding $\langle p \rangle$ for a polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$. E.g. if $a_i \in \mathbb{Z}$, then we could have

   $\langle a_n x^n + \cdots + a_1 x + a_0 \rangle = a_0, a_1, a_2, \ldots, a_n$.