Name: Key **CS301 Q4**

1. (__ **/3 pts)** Complete the statement of the pumping lemma below.

   If A is a regular language, then there is a number p, the pumping length, where if $s \in A$ has length at least p, then we may write $s = xyz$ satisfying the following conditions:

   - for each $i \geq 0$, $xy^iz \in A$,
   - $|y| > 0$, and
   - $|xy| \leq p$.

2. Answer the following questions about the proof of the pumping lemma.

   (a) (__ **/2 pts)** Why is p taken to be the number of states of a given finite automaton recognizing a given regular language?

   Any string of length p or greater is guaranteed to pass through some state of the machine more than once by the pigeon-hole principle.

   (b) (__ **/2 pts)** How is the substring y chosen?

   Let q' be the first repeated state in the machine. The string y is the string of characters that lead from q' back to q'.

3. (__ **/3 pts)** Let $A = \{ww^{\mathcal{R}} \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$, where $w^{\mathcal{R}}$ is the reverse string of w. Write a formal proof that A is not regular using the pumping lemma.

   By way of contradiction, suppose A is regular. We may apply the pumping lemma to $s = a^p bba^p \in A$, where p is the pumping length of A. Therefore, $s = xyz$ where $y \neq \varepsilon$ and $|xy| \leq p$. This implies $y = a^k$ with $k \geq 1$. Note that xy^2z has no bs in the left-half of the string, so therefore cannot be written as ww^R. Contradiction.