Name: Key **CS301 Q9**

1. Let $I_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is infinite}\}$. Consider, from homework, the proof that I_{DFA} is decidable, and answer the following about it.

   (a) **(__ /2 pts)** The decider D is constructed to accept all strings of length k or more. What is k chosen to be and *why*? Be concise.

   K = number of states in A.
   If A accepts any string of length k, it will accept arbitrarily long strings via the pumping lemma.

   (b) **(__ /2 pts)** The TM M is constructed so that $L(M) = L(A) \cap L(D)$. Which decidable language is useful when analyzing this intersection?

   $E_{DFA} = \{\langle A \rangle \mid A \text{ is DFA and } L(A) = \emptyset\}$ (See proof on pg. 213)

2. Consider, from class, the proof that A_{TM} is recognizable, but not decidable.

   (a) **(__ /1 pts)** Provide the definition of A_{TM}.

   $A_{TM} = \{\langle M, w \rangle \mid M \text{ is TM and } M \text{ accepts } w\}$.

   (b) **(__ /2 pts)** Why can't we prove A_{TM} is decidable by simulating any $\langle M, w \rangle$ on a *universal Turing machine*?

   If a universal Turing machine simulates M on input w, it will not halt if M does not halt on w. Therefore, if M can't decide w, neither can the universal Turing machine.

3. Consider, from homework, the proof of Theorem 4.22: *A language is decidable if and only if it is Turing-recognizable and co-Turing-recognizable.*

   (a) **(__ /1 pts)** What is the definition of co-Turing-recognizable?

   A language is co-T-recognizable if it is the (set) complement of a T-recognizable language.

   (b) **(__ /2 pts)** Suppose A is decidable. Explain why we can "easily see" (as our textbook says) that $\overline{A}$ is also decidable.

   If A is decidable, it must be decided by some Turing machine M. Just swap the "accept" and "reject" states of M.