Name: KEY **CS301 Q12**

1. **(__ /2 pts)** Consider $f, g : \mathbb{N} \to \mathbb{R}^+$ such that $\lim_{n\to\infty} f(n)/g(n) = 1$. Circle all that are true: $f = O(g)$, $f \neq o(g)$, $g = O(f)$, and $g \neq o(f)$.

2. Consider the proof, from homework, of Theorem 7.11: Every $t(n)$ time nondeterministic TM has an equivalent $2^{O(t(n))}$ time deterministic TM.

   (a) **(__ /1 pts)** Let N be such a nondeterministic machine. In N's computational tree, $t(n)$ is the maximum length path from root to leaf.

   (b) **(__ /1 pts)** If every node in N's computational tree has at most b children, the tree has at most $O(b^{t(n)})$ nodes.

3. **(__ /2 pt)** Provide definitions for the classes P and NP using TIME.

$$P = \bigcup_{k>0} TIME(n^k)$$

$$NP = \bigcup_{k>0} NTIME(n^k)$$

4. **(__ /2 pts)** Let SUBSUM $= \{\langle S, t\rangle \mid \exists R \subseteq S, \sum_{r\in R} r = t\}$. Prove that SUBSUM $\in$ NP by constructing a polynomial time verifier.

On input $\langle\langle S, t\rangle, c\rangle$:
1.) Test if c is set of numbers summing to t.
   → Scan c to check format : $O(n)$
   → Sum c bit-wise : $O(n^3)$
2.) Test if $c \subseteq S$ → Scan back-and-forth : $O(n^2)$
3.) If both pass, accept; otherwise, reject.

5. **(__ /2 pts)** Prove that NP is closed under the star operation.

Let $A \in NP$. Consider the following NTM decider for A^*. On input w:
1.) Divide $w = x_1 x_2 \cdots x_k$ nondeter.
   → scan through w and place •'s at division points. : $O(n)$
2.) Guess all possible certificates for each x_i to check $x_i \in A$.
   → $|c| \leq O(t(n))$ for A's verifier : $O(n \cdot t(n))$ to guess all.
3.) If all $x_i \in A$, accept; otherwise, reject.
   → Run verifier : $O(n \cdot t(n))$.

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