

Name: Key

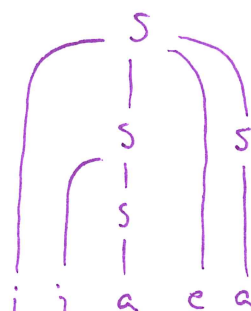
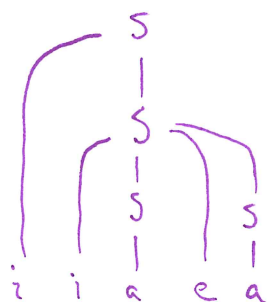
CS301 F5

1. (    /2 pts) Give a CFG that generates the language of all  $w \in \{0,1\}^*$  with odd length.

$$A \rightarrow BAB \mid 0 \mid 1$$

$$B \rightarrow 0 \mid 1$$

2. (    /2 pts) Show that the CFG  $S \rightarrow iS \mid iSeS \mid a$  is ambiguous by providing two distinct parse trees for  $iaaea$ .



3. (    /3 pts) Complete the statement of the CFL pumping lemma below.

If  $A$  is a CFL, then there is a number  $p$ , the pumping length, where if  $s \in A$  has length at least  $p$ , then we may write  $s = uvxyz$  satisfying the following conditions:

- for each  $i \geq 0$ ,  $uv^i xy^i z$ ,
- $|vy| > 0$ , and
- $|vxy| \leq p$ .

4. (    /3 pts) Consider the language  $B = \{b^l a \mid l = 2^j, j \geq 0\}$ . Use the CFL pumping lemma to prove that  $B$  is not context-free.

By way of contradiction, suppose  $B$  is a CFL. Let  $s = b^l a \in B$  have length at least  $2p$ , the <sup>double</sup> pumping length of  $B$ . We may write  $s = uvxyz$  satisfying (3) above. Note that  $vy$  is non-empty and contains only bs. Write  $vy = b^{k \cdot 2^j}$ , where  $k \geq 1$ . By pumping,

$$uv^i xy^i z = b^{l+k(i-1)} a \in B \quad \text{for all } i \geq 0.$$

Note that  $k < 2$  so that  $l < l+k < 2l$ . However,  $l = 2^j$  and  $2l = 2^{j+1}$ , which implies  $l+k$  is not a power of 2. i.e.  $b^{l+k} a \notin B$ , contradiction.