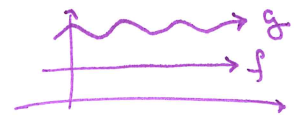
Consider:



Name: Key **CS301 F12**

1. (__ /2 pts) Consider $f, g : \mathbb{N} \to \mathbb{R}^+$ such that $\lim_{n\to\infty} f/g$ does not exist. Circle all that *must* be false: $f = O(g)$, $f = o(g)$, $g = O(f)$, and $g = o(f)$.

2. (__ /2 pts) Consider the proof, from homework, of Theorem 7.11. Provide details that justify $O(t(n)b^{t(n)}) = O(2^{t(n)})$.

$$O(t(n)\cdot b^{t(n)}) = O(2^{\log_2 t(n)} \cdot 2^{t(n)\log_2 b})$$
$$= O(2^{O(t(n)) + O(t(n))})$$
$$= O(2^{O(t(n))}) = 2^{O(t(n))}$$

3. (__ /2 pt) Explain why P $\subseteq$ NP.

Let $A \in P$. Then A has a polytime decider, which can be interpreted as a polytime verifier that doesn't use its certificate. Thus $A \in NP$.

4. (__ /2 pt) One can show CLIQUE $\in$ NP by using the nondeterministic decider on page 296. Prove that this decider runs in polynomial time.

1.) $O(n)$ to scan through node list and nondeterministically select a subset; then check that k nodes were selected.

2.) loop over all pairs of nodes in c: $O(n^2)$
for each pair, check for edge in G: $O(n)$

3.) $O(1)$.

5. (__ /2 pts) Prove that P is closed under concatenation.

Let $A, B \in P$ w/ polytime deciders M_A and M_B, whose complexities are $t_A(n)$ and $t_B(n)$. Consider the decider of $A \circ B$ which tries each possible "splitting point" of an input string and running M_A on left, M_B on right.

Complexity $= O(n \cdot (t_A(n) + t_B(n)))$

number of "splits" polynomial.