Name: Key **CS301 Q10**

1. Consider the problem of determining whether a Turing machine ever writes a particular symbol $\gamma \in \Gamma - \Sigma$ on its tape when it is run on input w.

   (a) **(__ /1 pt)** Formulate this problem as a language.

   $C = \{ \langle M, w \rangle \mid M$ is TM and writes $\gamma \in \Gamma - \Sigma$ on its tape when given $w. \}$

   (b) **(__ /2 pts)** How would the proof that the language above is undecidable differ from the proof, on homework, of the undecidibility of

   $B = \{\langle M, w\rangle \mid M$ 2-tape TM that writes $\gamma \neq \sqcup$ on 2nd tape given $w\}$?

   T = "On input x:
   1. Simulate M' on x
   2. If simulation shows that M' accepts, write γ anywhere on tape."

   Where M' is M, except if it uses γ, instead it uses γ'.

2. Let $R_{\text{TM}} = \{\langle M\rangle \mid M$ is a TM and $L(M)$ is regular$\}$. Consider, from homework, the proof that R_{TM} is undecidable, and answer the following.

   (a) **(__ /2 pts)** M_2 is constructed to first accept input x of the form $0^n 1^n$. If x does not have this form, run M on input w and accept if M accepts w.

   (b) **(__ /2 pts)** Consider $C_{\text{TM}} = \{\langle M\rangle \mid M$ is TM and $L(M)$ is CFL$\}$. How can you change this proof to instead show that C_{TM} is undecidable?

   Change $0^n 1^n$ to an expression which describes a language which is not context-free.

   E.g. $0^n 1^n 2^n$, as established in class via the pumping lemma.

3. Let $Z_{\text{CFG}} = \{\langle G\rangle \mid G$ is CFG and $L(G) = \Sigma^*\}$. Consider, from homework, the proof of Theorem 5.13: *Z_{CFG} is undecidable.*

   (a) **(__ /1 pt)** The CFG G is designed to generate all strings that are not accepting computational histories for M on input w.

   (b) **(__ /2 pts)** Instead of constructing CFG G, we instead create PDA D. Since D uses the stack structure, we write every other configuration (C_i) in reverse order.