

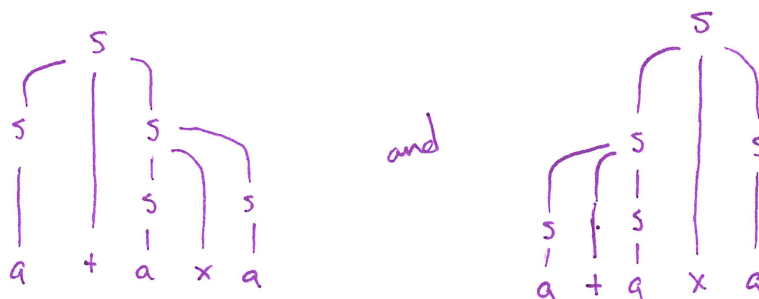
Name: Key

CS301 Q5

1. (/2 pts) Give a CFG that generates the language $\{ww^R \mid w \in \{0,1\}^*\}$.

$$S \rightarrow 0S0 \mid 1S1 \mid \epsilon$$

2. (/2 pts) Show that the CFG $S \rightarrow S + S \mid S \times S \mid a$ is ambiguous by providing two distinct parse trees for $a + a \times a$.



3. (/3 pts) Complete the statement of the CFL pumping lemma below.

If A is a CFL, then there is a number p , the pumping length, where if $s \in A$ has length at least p , then we may write $s = uvxyz$ satisfying the following conditions:

- for each $i \geq 0$, $uv^i xy^i z \in A$,
- $|vy| > 0$, and
- $|vxy| \leq p$.

4. (/3 pts) Consider the language $B = \{ab^q \mid q \text{ is prime}\}$. Use the CFL pumping lemma to prove that B is not context-free.

HINT:

Pump to $i = q + 1$.

By way of contradiction, assume B is context-free. Consider $ab^q \in B$ of length at least p , the pumping length of B . By the CFL pumping lemma, we may write $s = uvxyz$ satisfying the properties in problem (3). Note that vy can not contain the symbol a . Therefore, $vy = b^k$ where $k \geq 1$. Now pump

$$uv^i xy^i z = ab^{q + (i-1)k} \in B$$

and set $i-1 = q$ to get $ab^{q+qk} = ab^{q(1+k)} \in B$. Note that $q(1+k)$ is not prime. Contradiction.