Name: Key **CS301 F4**

1. **(__ /3 pts)** Complete the statement of the pumping lemma below.

   If A is a regular language, then there is a number p, the pumping length, where if $s \in A$ has length at least p, then we may write $s = xyz$ satisfying the following conditions:

   - for each $i \geq 0$, $xy^iz \in A$,
   - $|y| > 0$, and
   - $|xy| \leq p$.

2. Answer the following questions about the proof of the pumping lemma.

   (a) **(__ /2 pts)** Recall that p is taken to be the number of states in a given DFA recognizing a given regular language. Why is $|xy| \leq p$?

   The string y is chosen to be the characters leading from the first instance of the first repeated state to the second instance, and x the string that preceeds y. Note that, at maximum, p characters are needed to force a repeated state.

   (b) **(__ /2 pts)** How is the substring z chosen?

   The string z is uniquely chosen such that, for any $s \in A$ with length at lest p, we have $s = xyz$ where xy was defined above.

3. **(__ /3 pts)** Let $A = \{a^j b^k \mid j \geq k \geq 0\}$. Write a formal proof that A is not regular using the pumping lemma.

   By way of contradiction, suppose A is regular. We may apply the pumping lemma to $s = a^p b^p \in A$, where p is the pumping length of A. We can write $s = xyz$ such that $y \neq \varepsilon$ and $|xy| \leq p$. This implies $y = a^k$ with $k \geq 1$. Further, the pumping lemma demands that $xz \in A$ (taking y^i with $i = 0$), which implies $a^{p-k} b^p \in A$. ~~However,~~ $p - k \ngeq p$, contradiction.